# Game Rules GBG

## Content

2048	1
Connect Four	2
Hex	2
Nim	
Nim3P	
Othello (Reversi)	
Rubik's Cube	¬
Sim	
Tic Tac Toe	5

#### 2048

1-player game.

The game starts with game score 0 and two tiles with value '2' or '4' placed randomly on the otherwise empty 4x4 game board. Whenever the player makes an action, the environment adds a random tile '2' (90%) or '4' (10%) in one of the cells being empty after the player's action.

The player has at most these four actions: LEFT, UP, RIGHT, DOWN. An action is only **legal** if it moves at least one tile. If the player chooses LEFT, all tiles are moved to the left until they are adjacent to another tile. If now in a row two adjacent tiles have the same number 'X', the leftmost pair of those is merged to a new tile '2X'. In Figure 1, if a LEFT action is triggered, the 1st row gets the two leftmost '8's merged into one '16'. In the 2<sup>nd</sup> row the two '2's are merged into one '4'.

Whenever two tiles are merged, the merged value is added to the **game score**.

8	8	8	2
2	2	4	8
4		2	4

Figure 1: Intermediate position in the game 2048. The possible legal actions are LEFT, RIGHT and DOWN. The action UP is not legal since it would not move a single tile

The game **ends** if no legal move is possible (all cells filled and no merge possible).

The first goal of the game is to reach the '2048' tile. But beyond this, the broader goal is to reach even higher tiles and the highest possible game score.

## **Connect Four**

2-player game.

Each player in turn places a piece of his/her color on the (6 rows x 7 columns)-board. The pieces fall in each column down to the lowest empty cell. Yellow (X) starts.

The player who gets four-in-a-row (horizontal, vertical or diagonal) wins. If no player reaches this and no moves are left, it is a tie.

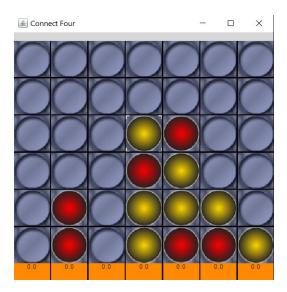


Figure 2: Yellow (X) wins in Connect Four.

#### Hex

2-player game.

Scalable game with scalable parameter N = side length of board.

The game board is a diamond with side length N of hexagonal cells. Two adjacent rims of the diamond are colored black, the other two white. Each player in turn places a piece of his/her color on a free cell.

Black (X) starts.

The player who forms a connected line of his/her color between the adjacent rims of his/her color **wins** the game. Hex **cannot** end in a tie.

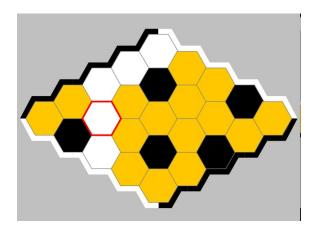


Figure 3: White wins in 5x5 Hex

#### Nim

2-player game.

Scalable game with three scalable parameters: (a) number of heaps N, (b) size of each heap S (number of items), (c) maximum number of items to take in one move ("Max Minus") M.

The game starts with N heaps of size S. Each player in turn takes away 1,2,...,M items from **one** heap. Black (X) starts.

The player who takes away the last item wins the game.

Nim **cannot** end in a tie.



Figure 4 Starting position for Nim: N=3 heaps of size S=5, Max Minus = 3.

## Nim3P

3-player game.

Same game layout (see Figure 4) and same move rule as for Nim. Scalable parameters: (a) number of heaps N, (b) size of each heap S (number of items), (c) whether the extra rule is active or not. The maximum number of items to take from each heap ("Max Minus") is in Nim3P always M=S.

Now three players P0, P1, P2 move in turn and take items away. P0 starts.

**Win rule**: The player **who follows** the player taking the last item is the player who wins. He/she gets a reward 1, both others get reward 0.

**Extra rule (optional)**: If scalable parameter "Extra Rule" is set to true, then the player who follows the winning player gets an extra reward of 0.2. This helps to break ties for otherwise 'undecided' states.

Nim3P cannot end in a tie.

# Othello (Reversi)

2-player game.

This game is played on a 8x8 board. The start position has two white pieces and two black pieces in the center. Each player in turn places a piece of his/her color on the 8x8 board. Placement can be only made in those empty cells that enclose – with a help of another piece of the player – one or more opponent pieces (see Figure 4: the green rectangles mark the legal moves). All enclosed opponent pieces have their color reversed to the player's color. Black (X) starts.

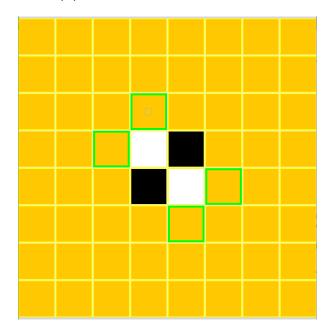


Figure 5 Starting position for Othello. Black starts and the green rectangles mark his/her legal moves.

If a player cannot make a legal move, he has to pass and the opponent moves again. If both players cannot move, the game is over. The player with the most pieces on the board **wins**. If both players have an equal number of pieces, it is a tie.

# Rubik's Cube

1-player game.

(in preparation)

#### Sim

2- or 3-player game.

Scalable game with these scalable parameters: (a) K = number of nodes and (b) P = number of players (2 or 3).

The game board is a complete graph with K nodes. Each player in turn colors a free edge of the graph with his/her color.

Black (X) starts.

The player who completes a monochromatic triangle of his/her color **loses**. In the 2-player variant, the opponent is the winner, in the 3-player variant the two opponents continue to play until one of them also loses. If two or more players do not complete a triangle when the graph is completely colored, it is a **tie**. In the 3-player variant there are two kinds of a tie: (a) a tie between all players or (b) one player loses and it is only a tie between the remaining two opponents.

It is known from Ramsey theory that for  $K \ge K_0$  with  $K_0 = 6$  (2 player) and  $K_0 = 17$  (3 player) the game **cannot** end in an all-player tie. But for smaller  $K < K_0$  the game is likely to end in a tie.

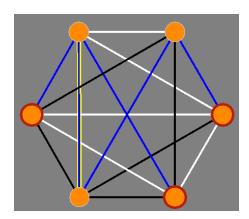


Figure 6: White loses in 3-player Sim with K=6 nodes. It is a tie between Black and Blue.

Within the 3-player variant we allow additionally two *Coalition* variants:

• Coalition = None: all 3 players play for their own, or

• Coalition = 1-2: players 1 and 2 form a coalition against the third player 0 who plays alone.

# Tic Tac Toe

2-player game.

Each player in turn places a piece of his/her color on the 3x3 board. Black (X) starts.

The player who gets three-in-a-row (horizontal, vertical or diagonal) **wins**. If no player reaches this, it is a tie.

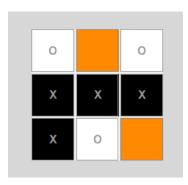


Figure 7: Black (X) wins in Tic Tac Toe.