一、解：(1)、

$$\omega=\begin{cases}\dfrac{1}{EI}\left(ax^4+Lx^3+cx^2\right),0\le x\le L\\[2mm]\dfrac{1}{EI}\left(dx^3+cx^2+ex\right),\ L\le x\le 2L\end{cases}$$

$$\theta=\omega'=\begin{cases}\dfrac{1}{EI}\left(4ax^3+3Lx^2+2cx\right),0\le x\le L\\[2mm]\dfrac{1}{EI}\left(3dx^2+2cx+e\right),\ L\le x\le 2L\end{cases}$$

$$M(x)=-EI\omega''=\begin{cases}-\left(12ax^2+6Lx+2c\right),0\le x\le L\\-\left(6Lx+2c\right),\ L\le x\le 2L\end{cases}$$

(2)、$F_S(x)=\dfrac{\partial M(x)}{\partial x}=\begin{cases}-\left(24ax+6b\right),0\le x\le L\\-\left(6dL+2c\right),\ L\le x\le 2L\end{cases}$

(3)、C 处存在集中外力偶，则对于

在 AC 段上：$M_1(L)=-\left(12aL^2+6bL+2c\right)$

在 BC 段上：$M_2(L)=-\left(6dL+2c\right)$

因此 $M_1(L)\neq M_2(L)$，即

$12aL^2+6bL+2c\neq 0$，$2aL+b+d\neq 0$

(4)、A 处存在外集中力，则 AC 段：$F_s(D)=-6b\neq 0$, 即 $b\neq 0$

(5)、挠度：$\omega_1(L)=\omega_2(L)$，即 $aL^4+bL^3+cL^2=dL^3+cL^2+eL$，

化简为 $aL^3+(b-d)L^2-e=0$

转角：$\theta_1(L)=\theta_2(L)$，即：$4aL^3+3bL^2+2cL=3dL^2+2d+e$

即：$4aL^3+3(b-d)L^2-e=0$

二、解：(1)、以 0° 方向为 x 轴正方向，90° 方向为 y 轴正方向

$$\sigma_{45}=\frac{\sigma_0+\sigma_{90}}{2}+\frac{\sigma_0-\sigma_{90}}{2}\cos 90-\tau_{xy}\sin 90$$

$$110 = \frac{180+100}{2} + \frac{180-100}{2} \times 0 - \tau_{xy}，\text{得}\ \tau_{xy} = 30MPa$$

$$\left.\begin{matrix}\sigma_{max} \\ \sigma_{min}\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{180+100}{2} \pm \sqrt{\left(\frac{180-100}{2}\right)^2 + 20^2}$$

$$= \begin{cases}190MPa \\ 90MPa\end{cases}$$

故 $\sigma_1 = 190MPa$，$\sigma_2 = 90MPa$，$\sigma_3 = 0$

（2）、最大切应力为：$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 95MPa$

$$\varepsilon_0 = \frac{1}{E}\left(\sigma_0 - \upsilon\sigma_{90}\right) = \frac{1}{200 \times 10^9} \times \left(-180 - 0.3 \times 100\right) \times 10^6 = 7.5 \times 10^{-4}$$

$$\varepsilon_{90} = \frac{1}{E}\left(\sigma_{90} - \upsilon\sigma_0\right) = -\frac{1}{200 \times 10^9} \times \left(100 - 0.3 \times 180\right) \times 10^6 = 2.3 \times 10^{-4}$$

$$\sigma_{135} = \frac{\sigma_0 + \sigma_{90}}{2} + \frac{\sigma_0 - \sigma_{90}}{2}\cos 270 - \tau_{xy}\sin 270 = 140 + 30 = 170MPa$$

$$\varepsilon_{45} = \frac{1}{E}\left(\sigma_{45} - \upsilon\sigma_{135}\right) = -\frac{1}{200 \times 10^9} \times \left(100 - 0.3 \times 170\right) \times 10^6 = 2.95 \times 10^{-4}$$

（4）、体积改变能密度：

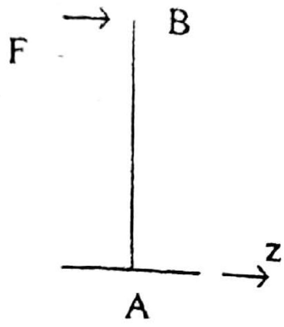
$$V_V = \frac{1-2\nu}{6E}\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2 = \frac{1 - 2 \times 0.3}{6 \times 200 \times 10^9} \times \left(190 + 90 + 0\right)^2 \times 10^{12} = 2.61 \times 10^4\ J/m^3$$

形状改变能密度：

$$V_d = \frac{1+\nu}{6E}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_1 - \sigma_3\right)^2 + \left(\sigma_2 - \sigma_3\right)^2\right]$$

$$= \frac{1+0.3}{6 \times 200 \times 10^9} \times \left[\left(190-90\right)^2 + \left(190-0\right)^2 + \left(90-0\right)^2\right] \times 10^{12} = 5.87 \times 10^4\ J/m^2$$

三、解：（1）、



$$F_y = F\sin\alpha = \frac{1}{2}F$$

$$F_z = F\cos\alpha = \frac{\sqrt{3}}{2}F$$

$$M_y = F_z a = \frac{\sqrt{3}}{2}Fa$$

$$M_z = F_z a = \frac{1}{2}Fa$$

（2）、$I_y = \frac{\pi d^4}{64} - \frac{2b \cdot b^3}{12} = \frac{23b^4}{6}$

$$I_z = \frac{\pi d^4}{64} - \frac{b \cdot (2b)^3}{12} = \frac{10b^4}{3}$$

中性轴方程为：$-\frac{M_y}{I_y}z + \frac{M_z}{I_z}y = 0$

代入数值，$-\frac{\frac{\sqrt{3}}{2}Fa}{\frac{23b^4}{6}}z + \frac{\frac{1}{2}Fa}{\frac{10b^4}{3}}y = 0$

设θ为中性轴与 y 轴的夹角，则 $\tan\theta = \frac{z}{y} = \frac{23}{20\sqrt{3}}$，故$\theta = \arctan\frac{23}{20\sqrt{3}}$

（3）、$\sigma_{max} = \frac{M_y}{I_y}\cdot\frac{d}{2} + \frac{M_z}{I_z}\cdot\frac{d}{2} = \frac{\frac{\sqrt{3}}{2}Fa}{\frac{23}{6}b^4}\cdot\frac{d}{2} + \frac{\frac{1}{2}Fa}{\frac{10}{3}b^4}\cdot\frac{d}{2} = \frac{Fa}{b^3\pi^{\frac{1}{4}}}\left(\frac{6\sqrt{3}}{23} + \frac{3}{10}\right)$

(4)、$\sigma_{II}=\frac{M_y}{I_y}\cdot\frac{b}{2}+\frac{M_z}{I_z}\cdot b=\frac{\frac{\sqrt{3}}{2}Fa}{\frac{23}{6}b^4}\cdot\frac{b}{2}+\frac{\frac{1}{2}Fa}{\frac{10}{3}b^4}\cdot b=\frac{30\sqrt{3}+69}{460}\frac{Fa}{b^3}$

(5)、

$$\sigma_{max}=\frac{M_y}{I_y}\cdot\frac{d}{2}+\frac{M_z}{I_z}\cdot\frac{d}{2}=\frac{Fa\cos\alpha}{\frac{23}{6}b^3\pi^{\frac{1}{4}}}\cdot\frac{1}{2}+\frac{Fa\sin\alpha}{\frac{10}{3}b^3\pi^{\frac{1}{4}}}\cdot\frac{1}{2}=\frac{Fa}{b^3\pi^{\frac{1}{4}}}\left(\frac{12}{23}\cos\alpha+\frac{3}{20}\sin\alpha\right)$$

当$\sigma_z=\sigma_y$时，最大弯曲正应力取得极小值，

故$\frac{Fa\cos\alpha}{\frac{23}{6}b^3\pi^{\frac{1}{4}}}\cdot\frac{1}{2}=\frac{Fa\sin\alpha}{\frac{10}{3}b^3\pi^{\frac{1}{4}}}\cdot\frac{1}{2}$，$\tan\alpha=\frac{20}{23}$，解得 $\alpha=\arctan\frac{20}{23}$

四、解：(1)、$M(x)=-Fx$

$$V_{\varepsilon b}=\int_0^L\frac{M^2(x)}{2EI}dx=\int_0^L\frac{(Fx)^2}{2EI}dx=\frac{F^2L^3}{4Eb^4}$$

(2)、截面为矩形，故$\alpha=\frac{6}{5}$

$$V_{\varepsilon S}=\int_0^L\alpha\frac{F^2(x)}{2GA}dx=\alpha\frac{F^2L}{2GA}=\frac{3F^2L}{8Eb^2}$$

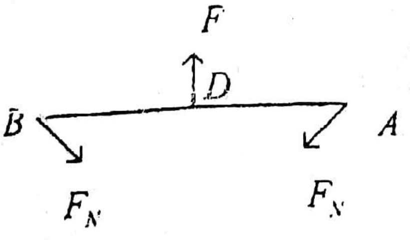
(3)、$\frac{V\varepsilon b}{V\varepsilon S}=\frac{\frac{F^2L^3}{4Eb^4}}{\frac{3F^2L}{8Eb^2}}=\frac{3}{2}\left(\frac{b}{L}\right)^2$

当$\frac{b}{L}=\frac{1}{10}$时，$\frac{V\varepsilon b}{V\varepsilon S}=\frac{3}{200}$

故$V\varepsilon S$所占百分比为$\frac{\frac{3}{200}}{\frac{3}{200}+1}=1.48\%$

五、请参照孙驯服的课本第二册例 3-17

解：正三角形的内力为超静定，利用对称性，截开 A、B 两点，只存在对称的力。



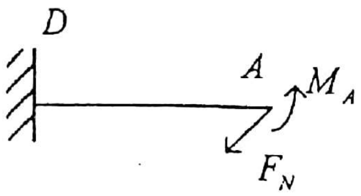
即只可能有轴力和弯矩，由于 AB 段结构和荷载都关于 CD 轴对称，故轴向截面

AB 上的轴力和弯矩相等，其轴力可由静力平衡方程求得，即 $\sum F_y = 0$，

$2F_N\sin60° - F = 0$，$F_N = \dfrac{F}{\sqrt{3}}$

故仅有 M_A 未知，由于 A 截面的转角为 0,（反对称的位移为 0）

将结构化简为



$$M(x) = -\frac{F}{2}x + M_A$$

$$故\ \theta = \int_0^a \frac{M_A - \frac{F}{2}x}{EI}dx = 0$$

解得 $M_A = \dfrac{Fa}{4}$

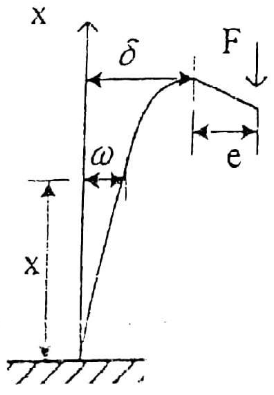
所以 $M_D = M(a) = -\dfrac{Fa}{4}$

故 D 处的内力为 $M_D = -\dfrac{Fa}{4}$

$$F_S = \frac{F}{2}$$

$$F_N = -\frac{F}{\sqrt{3}}\cos60° = -\frac{\sqrt{3}}{6}F \text{（压）}$$

六、这道题严重被所谓的网络答案误导，而且在 15 年考研又考出类似的题，在刘鸿文的材料中绝对找不到，某些商家还申称指定用书只有刘鸿文。其实是孙训方下册，4-6 的课后题，4 章为《压杆稳定问题的进一步研究》，很多同学看到这里误以为超纲而没有仔细研读，当年我考试也是被所谓的答案害惨了。



解：$M(x) = -F(e + \delta - \omega)$

$$EI\omega'' = -M(x) = F(e+\delta-\omega)$$

设 $\frac{F_{cr}}{EI} = k^2$，则 $\omega'' + k^2\omega = k^2(e+\delta)$

$\omega = A\sin kx + B\cos kx + e + \delta = \delta$

$\omega' = Ak\cos kx - Bk\sin kx$

当 $x = 0$ 时，$\omega = 0$ 即 $B + e + \delta = 0$，$B = -e - \delta$

当 $\omega' = 0$，即 $Ak = 0$

当 $x = L$ 时，$\omega = \delta$，即 $-(\delta + e)\cos kL + e + \delta = \delta$，

$$-(\delta + e) = \frac{e}{\cos kL}$$

$$\delta = \frac{1-\cos kL}{\cos kL}e$$

故 B 截面的最大应力 $\sigma = \frac{F}{A} + \frac{M}{W}$

$M = F(e+\delta) = \frac{Fe}{\cos kL}$，故 $\sigma = \frac{F}{A} + \frac{Fe}{\cos kL W}$

临界力表达式 $\sigma_{st} = \frac{F}{A} + \frac{Fe}{W\cos\left(\sqrt{\frac{F}{EI}}L\right)} \le [\sigma]$