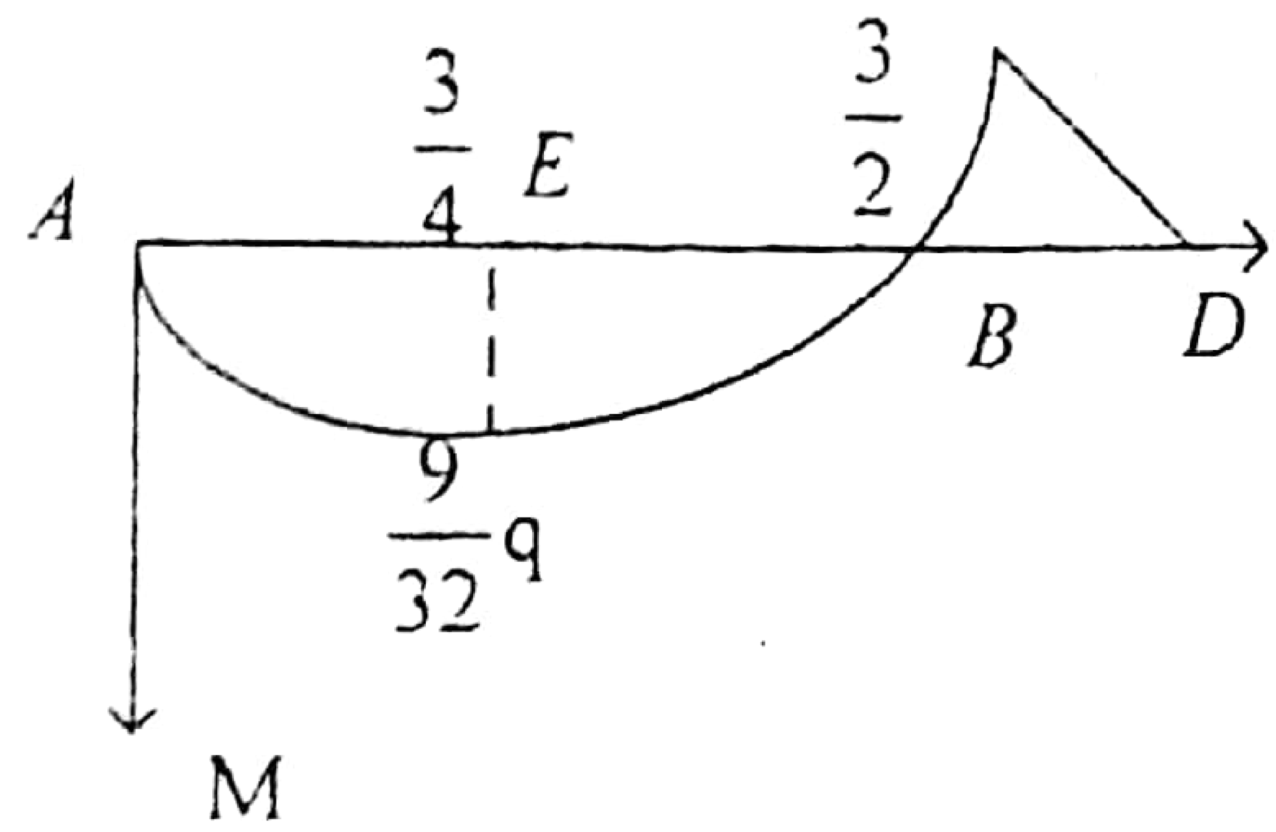
一九九八年答案解析

一、

二、解：（1）、



$$\sum M_A = 0 \qquad -\frac{1}{2}q(L_1+L_2)^2 + F_B L_1 = 0$$

即 $-\frac{1}{2}q9 + F_B 2 = 0$，$F_B = \frac{9}{4}q(\uparrow)$

$$\sum F_y = 0 \qquad -3q + \frac{9}{4}q + F_A = 0，F_A = \frac{3}{4}q(\uparrow)$$

（2）、以过 C 点的竖轴为 y 轴

$$I_Z = \left[\frac{180\times30^3}{12} + (48-15)^2\times180\times30 + \frac{20\times160^3}{12} + (142-80)^2\times160\times20\right]\times10^{-12} = 2.14\times10^{-4}$$

危险截面可能是 B 或距 A $\frac{3}{4}m$ 处的 E 截面

在 B 截面上，$M = \frac{1}{2}q$

$$\sigma_t = \frac{M_B y_1}{I_Z} = \frac{\frac{1}{2}q\cdot48\times10^{-3}}{2.14\times10^{-4}} \le [\sigma_t]，即 q \le 178.33\,kN/m$$

$$\sigma_c = \frac{M_B y_2}{I_Z} = \frac{\frac{1}{2}q\cdot142\times10^{-3}}{2.14\times10^{-4}} \le [\sigma_c]，即 q \le 120.56\,kN/m$$

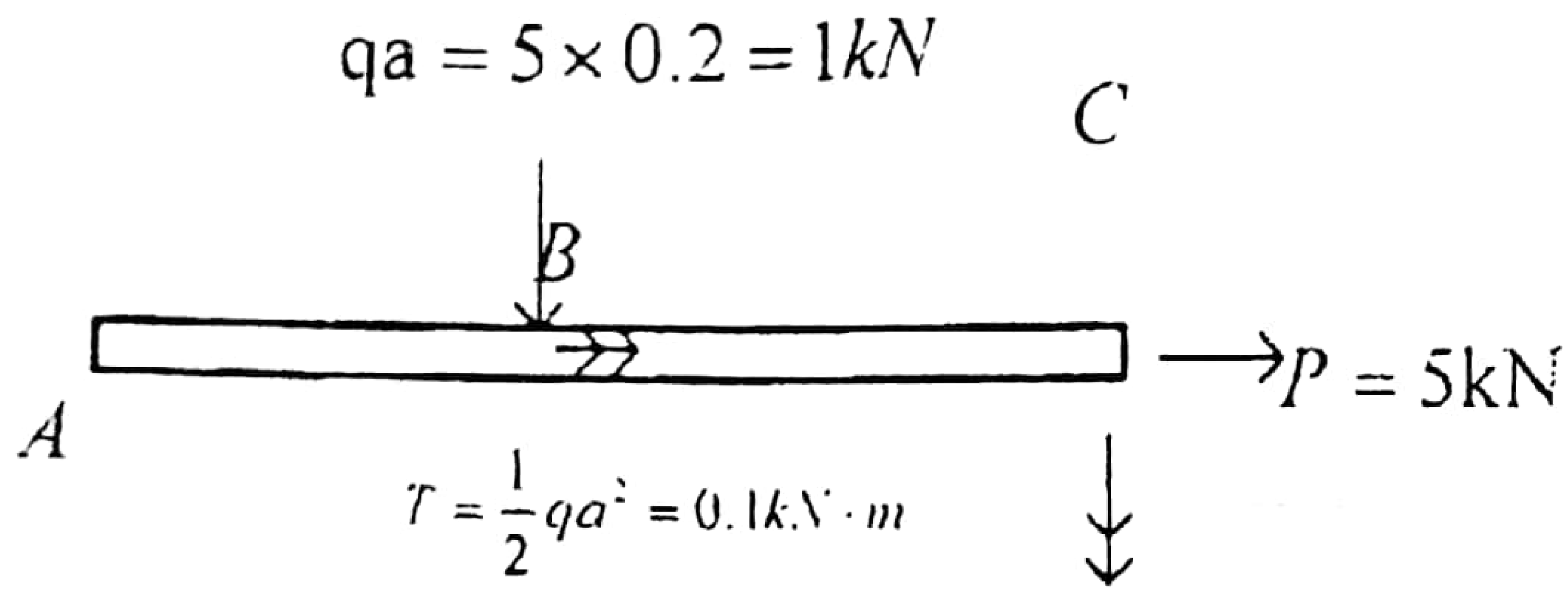
在 E 界面上

$$\sigma_t = \frac{M_E y_2}{I_Z} = \frac{\frac{9}{32}q\cdot142\times10^{-3}}{2.14\times10^{-4}} \le [\sigma_t]，即 q \le 107.17\,kN/m$$

$$\sigma_c = \frac{M_E y_1}{I_Z} = \frac{\frac{9}{32}q\cdot48\times10^{-3}}{2.14\times10^{-4}} \le [\sigma_c]，即 q \le 634.07\,kN/m$$

综上，$q \le 107.17kN/m$

三、解：作 ABC 的受力简图：



$$M_y = P \cdot 0.1 = 0.5\,kN/m$$

易知，A 截面为危险截面

$M_{Ay} = 0.5\text{kN}\cdot\text{m}$， $F_N = 5\text{kN}$， $M_{Az} = 0.2\text{kN}\cdot\text{m}$

故$\sigma = \dfrac{\sqrt{M_{Az}^2 + M_{Ay}^2}}{W} + \dfrac{F_N}{A} = \dfrac{32\sqrt{500^2 + 200^2}}{\pi 0.04^3} + \dfrac{5000}{\frac{1}{4}\pi 0.04^2} = 89.69MPa$

$$\tau = \frac{T}{W_t} = \frac{100\times 16}{\pi\times 0.04^3} = 7.96MPa$$

则$\sigma_{r3} = \sqrt{\sigma^2 + 4\tau^2} = 91.09MPa$

四、a、b 点是单向拉应力状态

$$\begin{cases} \sigma_a = \dfrac{F}{A} + \dfrac{M}{W} = \dfrac{P}{A} + \dfrac{M}{W} = \dfrac{4P}{\pi d^2} + \dfrac{32Pe}{\pi d^3} \\ \sigma_a = \varepsilon_a E \end{cases}$$

$$\begin{cases} \sigma_b = \dfrac{F}{A} - \dfrac{M}{W} = \dfrac{P}{A} - \dfrac{M}{W} = \dfrac{4P}{\pi d^2} - \dfrac{32Pe}{\pi d^3} \\ \sigma_b = \varepsilon_b E \end{cases}$$

代入数值，$\begin{cases} 520\times 10^{-6}\times 200\times 10^9 = \dfrac{4P}{\pi 0.1^2} + \dfrac{32Pe}{\pi 0.1^3} \\ -9.5\times 10^{-6}\times 200\times 10^9 = \dfrac{4P}{\pi 0.1^2} - \dfrac{32Pe}{\pi 0.1^3} \end{cases}$

解得：$P = 400.95\text{kN}$， $e = 25.93\text{mm}$

C 处：$\tau = \dfrac{T}{W_t} = -\dfrac{16m}{\pi d^3}$

$$\sigma = \frac{P}{A} = \frac{4P}{\pi d^2} = 0.51MPa$$

$$\sigma_{-45^\circ}=\frac{\sigma+0}{2}+\frac{\sigma-0}{2}\cos-90^\circ-\tau\sin-90^\circ=\frac{\sigma}{2}+\tau$$

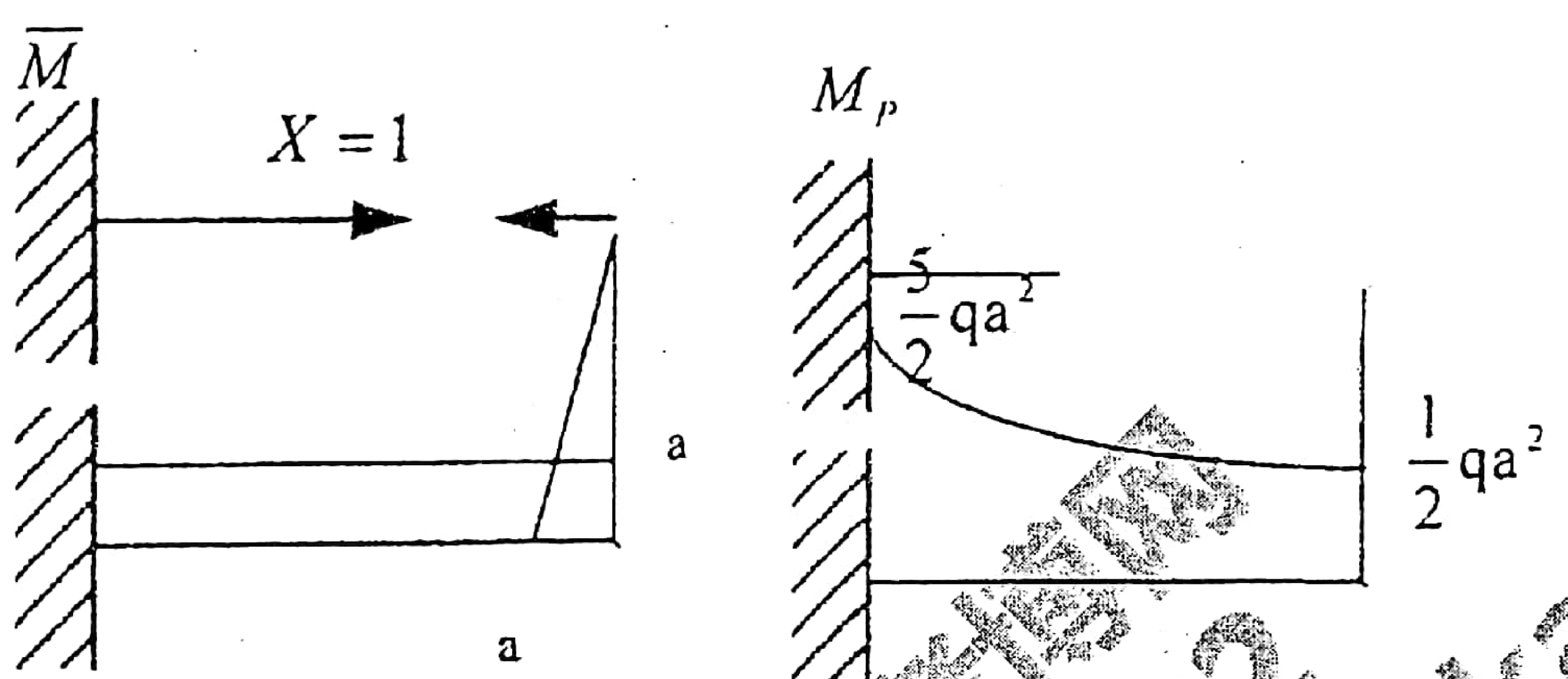
$$\sigma_{-+45^\circ}=\frac{\sigma+0}{2}+\frac{\sigma-0}{2}\cos 90^\circ-\tau\sin 90^\circ=\frac{\sigma}{2}-\tau$$

所以，$\varepsilon_{-45^\circ}=\dfrac{1}{E}\left(\sigma_{-45^\circ}-\upsilon\sigma_{45^\circ}\right)=\dfrac{\sigma}{2}(1-\upsilon)+\tau(1+\upsilon)$

$$\tau=-\frac{\sigma}{2}\frac{(1-\upsilon)}{1+\upsilon}+E\varepsilon_{-45^\circ}\frac{1}{1+\upsilon}=30.77MPa$$

进一步得到 $m=\dfrac{\tau\pi d^3}{16}=6.04\mathrm{kN}\cdot\mathrm{m}$

五、结构为一次超静定，断开多余约束 CD 杆，代之以一对未知反力 X。用单位力法做，令 X=1，则在单位力下，



$$\overline{F}_{NCD}=1,\ \delta_{11}=\frac{1}{EI}\left(\frac{1}{2}a\cdot a\cdot\frac{2}{3}a+2a\cdot a\cdot a\right)+\frac{2a}{EA}=\frac{7a^3}{EI}+\frac{2a}{EA}$$

在实际力作用下：

BA 段：$M(x)=-qa^2-\dfrac{1}{2}qx^2 \qquad 0\le x\le 2a$

故 $\Delta_{1P}=\displaystyle\int_0^{2a}\frac{\overline{M}(x)M(x)}{EI}dx=\int_0^{2a}\frac{\left(-qa^2-\frac{1}{2}qx^2\right)a}{EI}dx=-\frac{10qa^4}{EI}$

故 CD 中轴力 $X=-\dfrac{\Delta_{1P}}{\delta_{11}}=\dfrac{\dfrac{10qa^4}{EI}}{\dfrac{7a^3}{EI}+\dfrac{2a}{EA}}=\dfrac{10qa^3A}{21a^2A+6I}$