二〇〇二年答案解析

一、解：$\sum M_A=0$，$-\frac{1}{2}qL^2-\frac{1}{2}qL^2+\frac{qL^2}{2}+F_C L=0$ $F_C=\frac{qL}{2}(\uparrow)$

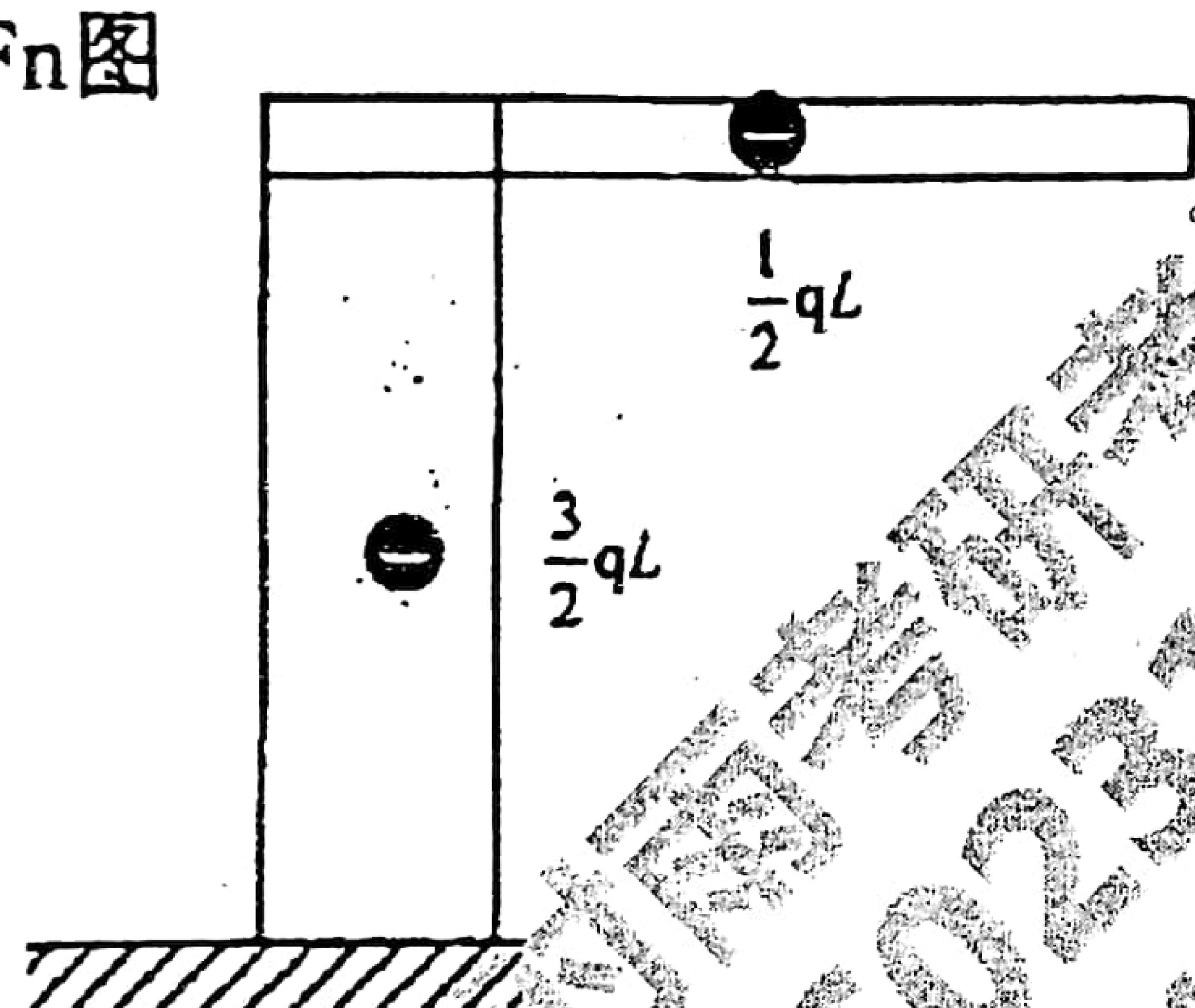
$\sum F_y=0$ $F_{Ay}=\frac{3}{2}qL(\uparrow)$

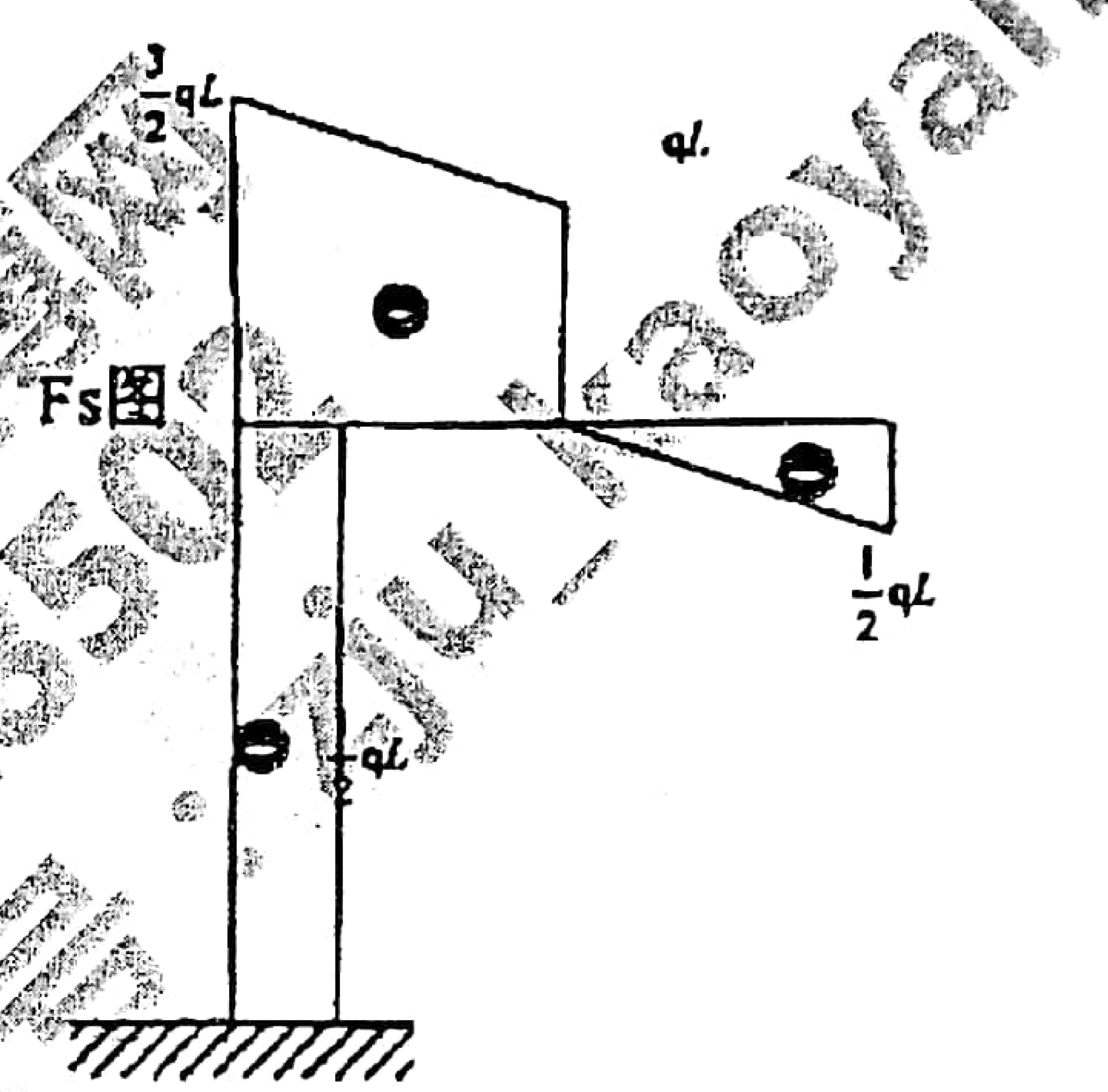
$\sum F_x=0$ $F_{Ax}=\frac{1}{2}qL(\rightarrow)$

AB 段 $M(x)=\frac{qLx}{2}-\frac{qx^2}{2}$

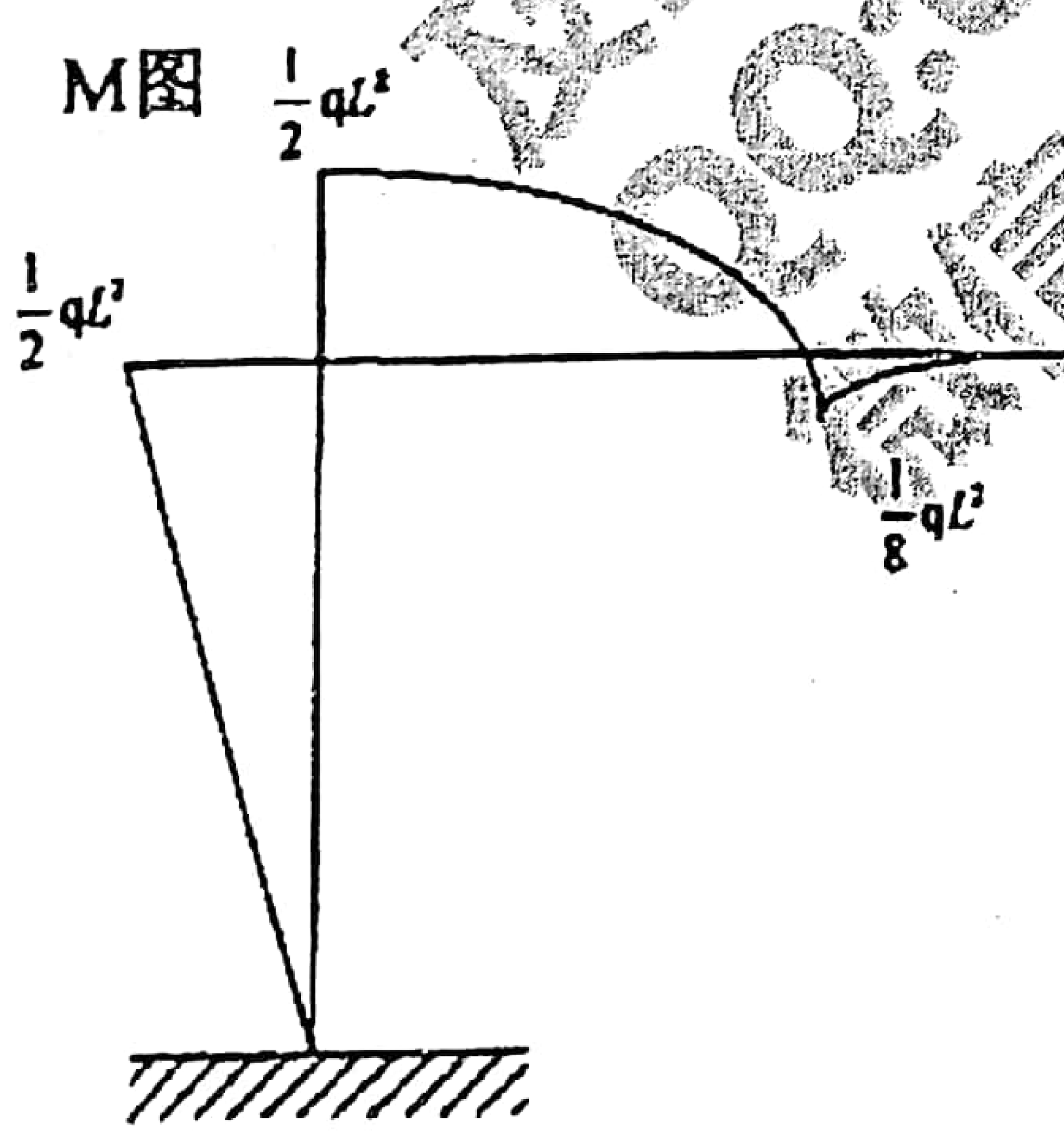
BC 段 $M(x)=\frac{qLx}{2}-\frac{qx^2}{2}-qL\left(x-\frac{L}{2}\right)=-\frac{1}{2}qx^2+\frac{1}{2}qL^2-\frac{qLx}{2}$

Fn图





M图



二、解：（1）、a：

$$\left.\begin{array}{l}\sigma_{max}\\ \sigma_{min}\end{array}\right\}=\frac{3\sigma+\sigma}{2}\pm\sqrt{\left(\frac{3\sigma-\sigma}{2}\right)^2+(\sqrt{3}\sigma)^2}=\begin{cases}4\sigma\\ 0\end{cases}$$

$\sigma_1=4\sigma$，$\sigma_2=\sigma_3=0$

（2）、 b：

$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{-\sqrt{3}\sigma+\sqrt{3}\sigma}{2} \pm \sqrt{\left(\frac{-\sqrt{3}\sigma-\sqrt{3}\sigma}{2}\right)^2+\sigma^2} = \begin{cases}2\sigma\\ -2\sigma\end{cases}$$

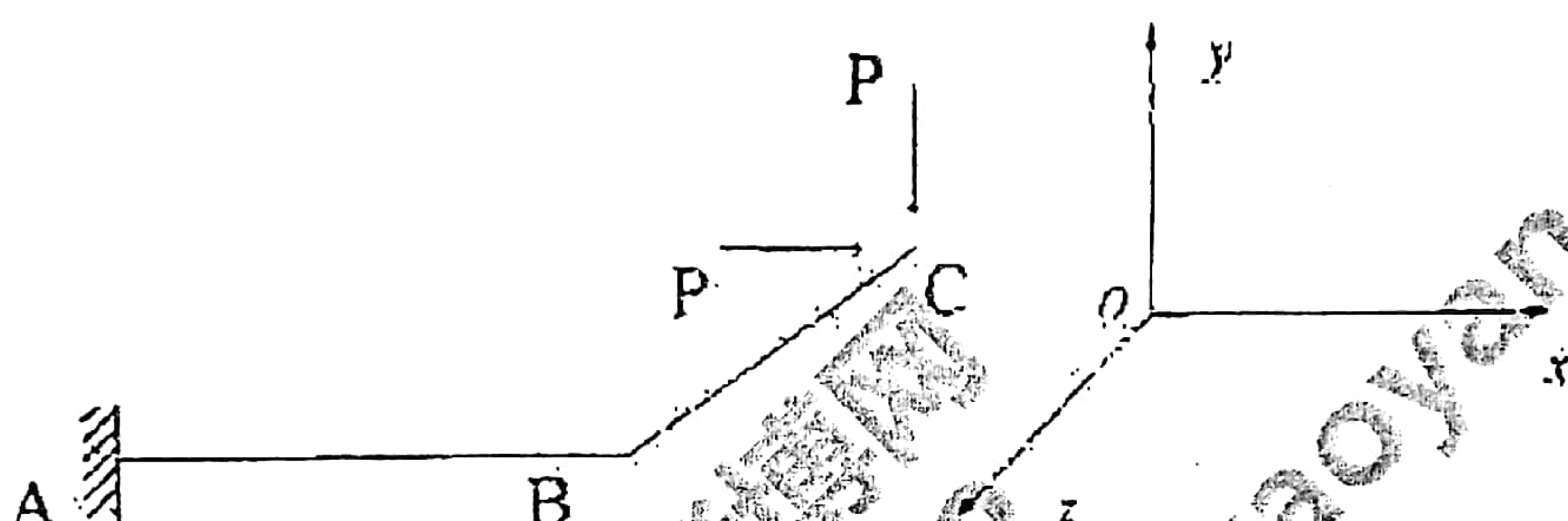
$\sigma_1 = 2\sigma$，$\sigma_2 = 0$，$\sigma_3 = -2\sigma$

（2）、a： $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = 2\sigma$

b： $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = 2\sigma$

（3）、a 单元体处于单轴应力状态

b 单元体处于纯剪切应力状态



三、

（1）、BC 杆 B 端： $M_{B,x} = PL$　　$M_{B,y} = PL$

$$\sigma = \frac{\sqrt{M_{B,x}^2 + M_{B,y}^2}}{W} = \frac{32\sqrt{2}PL}{\pi d^3}$$

$$\sigma_{r3} = \sigma = \frac{32\sqrt{2}PL}{\pi d^3}$$

（2）、AB 杆的 B 截面 $M_{B,y} = PL$

$$T = PL$$

$$\sigma = \frac{M_{B,y}}{W} = \frac{32PL}{\pi d^3} \qquad \tau = \frac{T}{W_t} = \frac{16PL}{\pi d^3}$$

$$\sigma_{r3}' = \sqrt{\sigma^2 + 4\tau^2} = \frac{32\sqrt{2}PL}{\pi d^3}$$

（3）、AB 杆的 A 截面

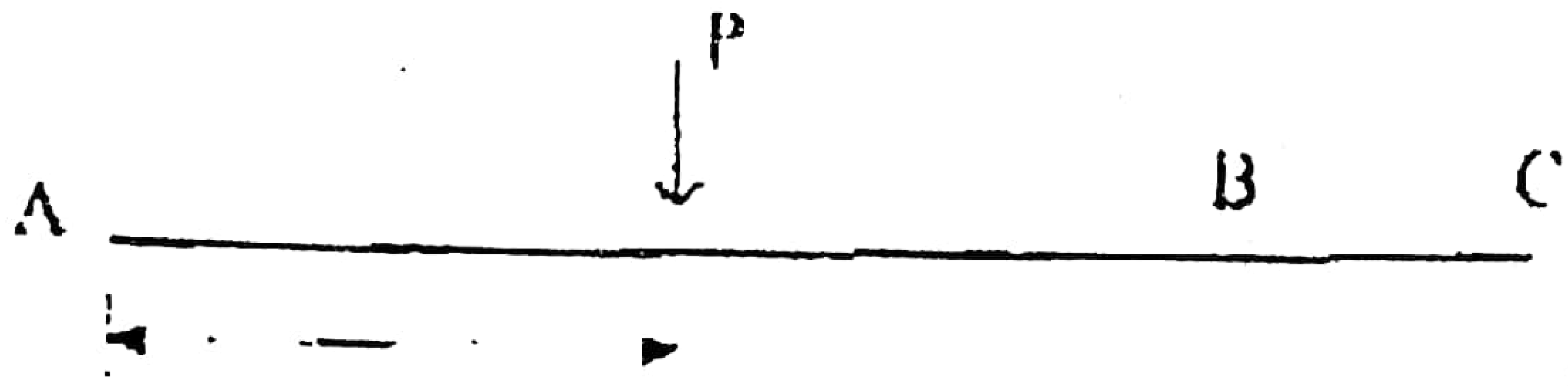
$M_{Ay} = PL$　　　$M_{Az} = PL$

$$\sigma = \frac{\sqrt{M_{Az}^2 + M_{Ay}^2}}{W} = \frac{32\sqrt{2}PL}{\pi d^3}$$

$T = PL \qquad \tau = \frac{T}{W_t} = \frac{16PL}{\pi d^3}$

$$\sigma_{r3}'' = \sqrt{\sigma^2 + 4\tau^2} = \frac{32\sqrt{3}PL}{\pi d^3}$$

四、解：当 P 在 A、B 之间运动时

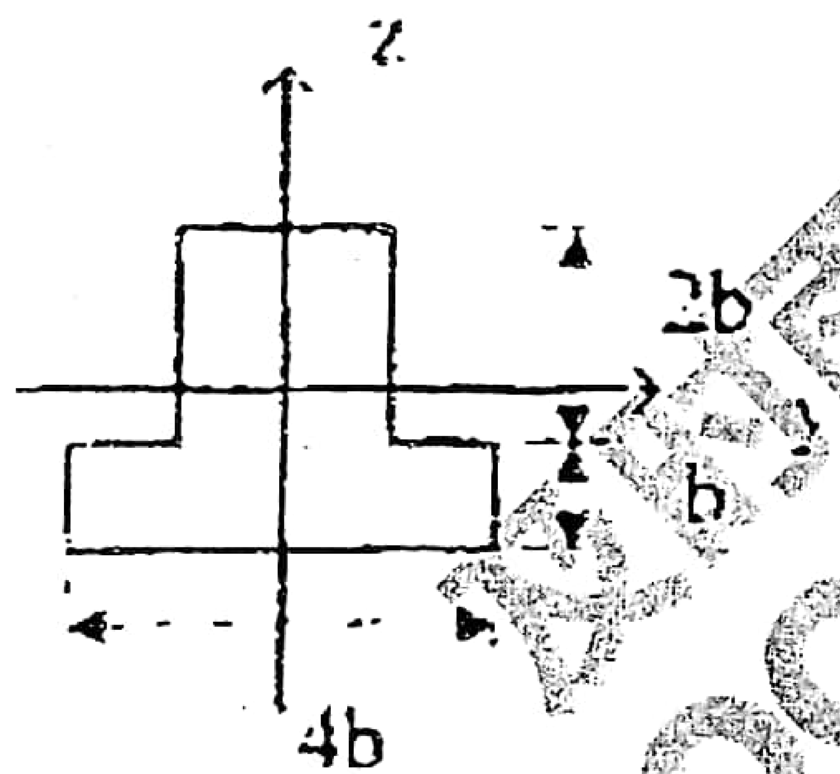


$$\sum M_A = 0, \quad F_B = \frac{Px}{6L}, \quad F_A = \frac{P(6L - x)}{6L}$$

故 $M(x) = \frac{P(6L - x)}{6L}x \qquad 0 \le x \le L$

$$\frac{\partial M(x)}{\partial x} = 0, \quad x = 3L$$

$$M_{max} = M(3L) = \frac{P(3L)^2}{6L} = \frac{3PL}{2}$$



$$\bar{y} = \frac{2b \times 2b \times 2b + b \times 4b \times 0.5b}{2b \times 2b + b \times 4b} = \frac{5}{4}b$$

$$I_{z0} = \frac{2b(2b)^3}{12} + \left(\frac{3}{4}b\right)^2 \cdot 4b^2 + \frac{4bb^3}{12} + \left(\frac{3}{4}b\right)^2 4b^2 = \frac{37b^4}{6}$$

$$\sigma_{max\,t} = \frac{M_{max}\frac{5}{4}b}{I_z} = \frac{\frac{3}{2}PL \times \frac{5}{4}b}{\frac{37}{6}b^4} = \frac{15}{8} \times \frac{6PL}{37b^3} = \frac{45PL}{148b^3} \le [\sigma_t] = 2[\sigma_c]$$

$$\sigma_{max\,c} = \frac{M_{max}\frac{7}{4}b}{I_z} = \frac{\frac{3}{2}PL \times \frac{7}{4}b}{\frac{37}{6}b^4} = \frac{63PL}{148b^3} \le [\sigma_c]$$

所以，$P \le \frac{148b^3[\sigma_c]}{63PL}$

当 P 在 BC 运动时，C 处弯矩最大，$M_{B,\max}=PL$

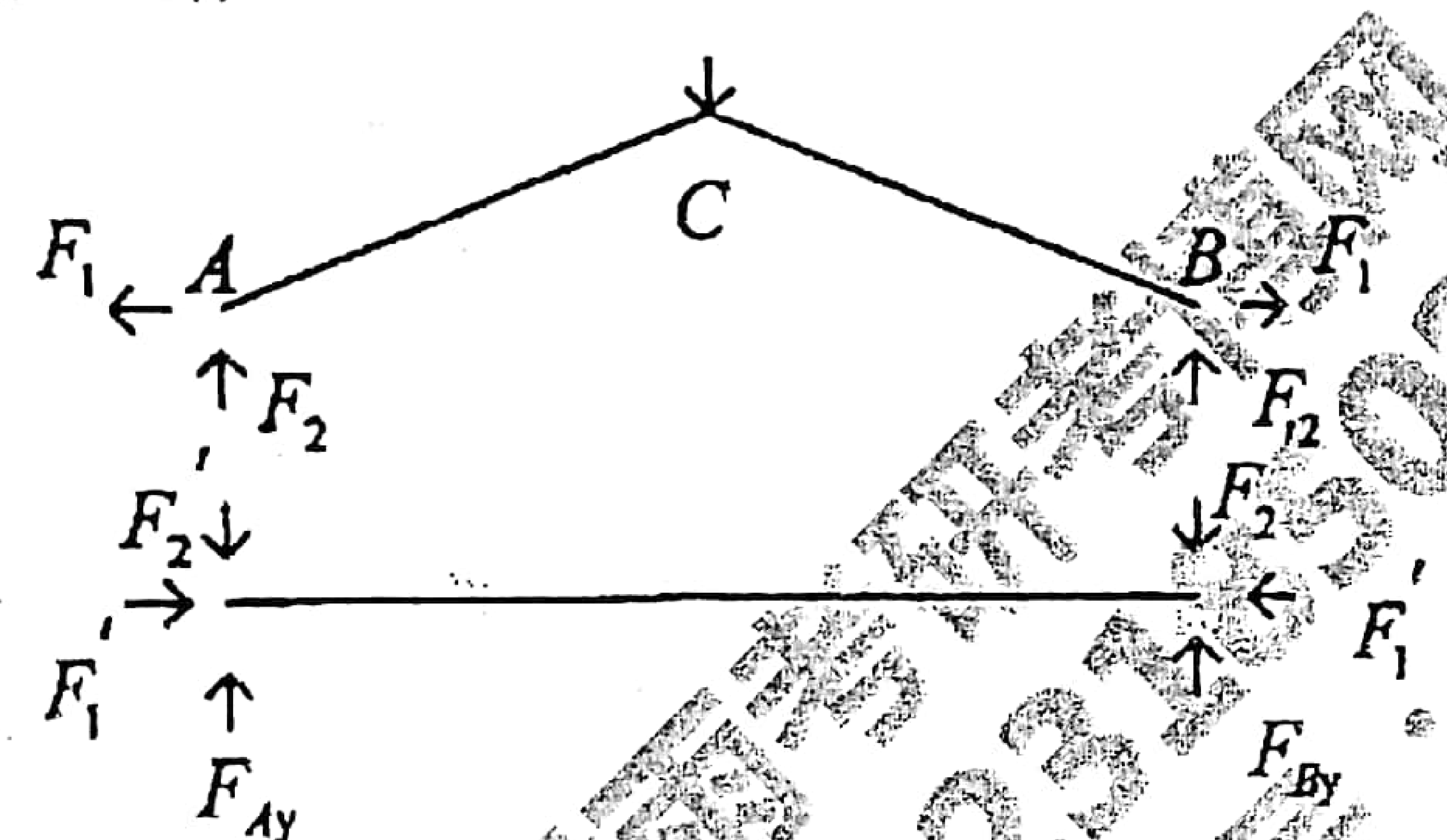
$$\sigma_{\max,c}{}'=\frac{PL\times\frac{5}{4}b}{\frac{37}{6}b^4}=\frac{15PL}{74b^3}\le[\sigma_c]$$

$$\sigma_{\max,t}{}'=\frac{PL\times\frac{7}{4}b}{\frac{37}{6}b^4}=\frac{35PL}{74b^3}\le 2[\sigma_c]$$

所以 $P\le\frac{148b^3[\sigma_c]}{35PL}$

综上，$P\le\frac{148b^3[\sigma_c]}{63PL}$

五、解：（1）、



所以 $F_2=F_2'=\frac{P}{2}$

$F_{Ay}=F_{By}=\frac{P}{2}$

有因为 $\Delta_{AB}=\Delta_{AB}{}'\ \Delta_{AB}{}'=0$

再求 Δ_{AB}，AB 杆弯矩 $M(\mathrm{x})=F_1\cos\theta 45^\circ\mathrm{x}+F_2\sin45^\circ\mathrm{x}\qquad 0\le\mathrm{x}\le L$

$$故\Delta_{AB}=2\int_0^L\frac{\left(F_1\frac{\sqrt2}{2}\mathrm{x}+F_2\frac{\sqrt2}{2}\mathrm{x}\right)\frac{\sqrt2}{2}\mathrm{x}}{EI}\mathrm{dx}=\frac{F_1L^3}{3EI}+\frac{PL^3}{6EI}=0$$

则有 $F_1=-\frac{P}{2}(\rightarrow)$

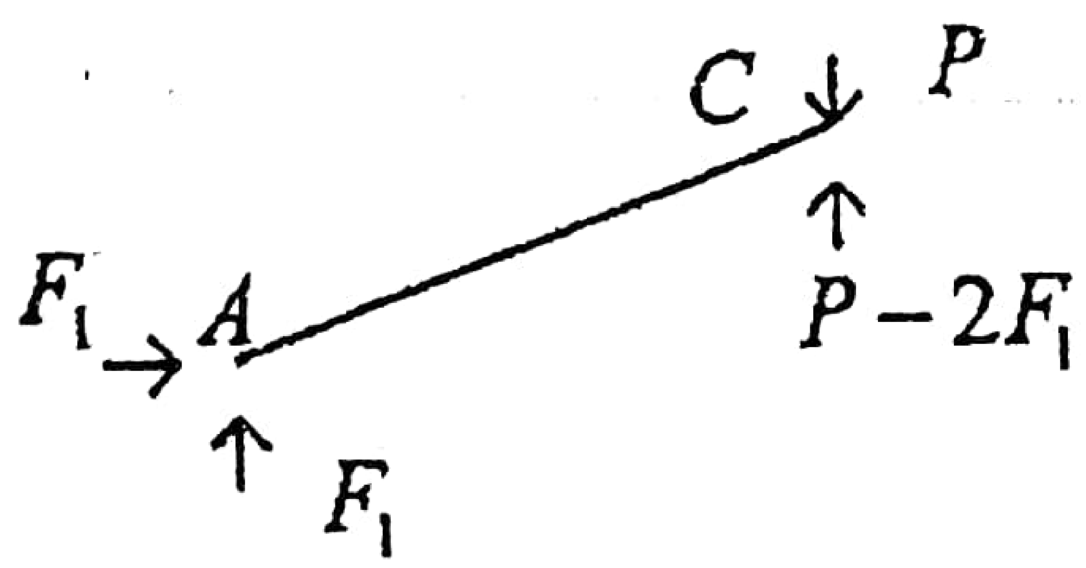
（2）、ΔC 和 BC

$M(\mathrm{x})=0$

$F_{NAC}=-\frac{\sqrt2}{2}P$（压应力）

所以$\Delta_P = \dfrac{\overline{F}_{NAC} F_{NAC} L}{EA} \times 2 = \dfrac{PL}{EA}$

（3）、$\delta = \dfrac{PL}{2EA} < \omega_C$，故C与水平支座接触



易知$M(\mathrm{x}) = 0$，故$F_x = F_y = F_1$，则水平支座$F = P - 2F_1$

$$F_N = -\sqrt{2} F_1$$

$$V_\varepsilon = \frac{2F_1^2 L}{2EA} \times 2 = \frac{2F_1^2 L}{EA} \qquad \delta = \frac{PL}{2EA}$$

故$\dfrac{\partial V_\varepsilon}{\partial P} = \dfrac{PL}{2EA}$

$$V_\varepsilon = \frac{P^2 L}{4EA} = \frac{2F_1^2 L}{EA}$$

因此，$F_1 = \dfrac{1}{2\sqrt{2}} P(\uparrow)$