# 二〇一二年答案解析

1、 解：（1）、设 P 点距 A 点 x 处，则

$$\sum M_A = 0,\quad F_B \cdot \mathrm{a} - P\mathrm{x} = 0,\quad F_B \cdot = \frac{P\mathrm{x}}{\mathrm{a}}$$

$$\sum F_y = 0,\quad F_A = \frac{\mathrm{a}-\mathrm{x}}{\mathrm{a}} P$$

欲使钢梁仍水平，则 $\Delta_A = \Delta_B$

$$\Delta_A = \frac{F_A L}{EA_1} = \frac{\frac{\mathrm{a}-\mathrm{x}}{\mathrm{a}} PL}{E\frac{1}{4}\pi d^2}$$

$$\Delta_B = \frac{\frac{\mathrm{x}}{\mathrm{a}} P0.5L}{E\frac{1}{4}\pi(2d)^2}$$

联立上式，解得 $\mathrm{x} = \frac{4}{5}\mathrm{a}$

即 P 位于距 A $\frac{4}{5}$a 处

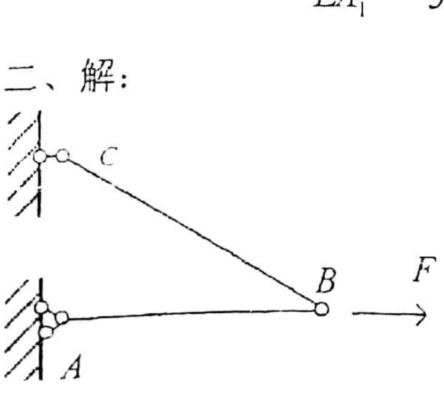
（2）、$F_B \cdot = \frac{4P}{5}, F_A \cdot = \frac{P}{5}$

（3）、$V_{\varepsilon 1} = \frac{F_{N1}{}^2 L}{2EA_1} = \frac{\left(\frac{1}{5}P\right)^2 L}{2E\frac{1}{4}\pi \mathrm{d}^2} = \frac{2P^2 L}{25E\pi \mathrm{d}^2}$

$$V_{\varepsilon 1} = \frac{F_{N2}{}^2 0.5L}{0.5\times 2EA_2} = \frac{\left(\frac{4}{5}P\right)^2 0.5L}{E\frac{1}{4}\pi(2\mathrm{d})^2} = \frac{8P^2 L}{25E\pi \mathrm{d}^2}$$

（4）、$\Delta_A = \Delta_B = \frac{F_{NA} L}{EA_1} = \frac{4PL}{5E\pi \mathrm{d}^2}$

二、解：



$F_x = F\sin\theta$

$F_y = F\cos\theta$

$\sum F_{Bx} = 0$， $F_x - F_{NAB} - F_{NBC}\cos 45° = 0$

$\sum F_{By} = 0$， $F_y - F_{NBC} - F_{NBC}\sin 45° = 0$

得 $F_{NBC} = \sqrt{2}F_y$， $F_{NAB} = F_x - F_y$

则 $\Delta_{Bx} = \dfrac{F_{NBA}\cdot a}{EA}\dfrac{\partial F_{NAB}}{\partial F_x} = \dfrac{(F_x - F_y)a}{EA_2}$

$$\Delta_{By} = \frac{F_{NBA}\cdot a}{EA_2}\frac{\partial F_{NAB}}{\partial F_y} + \frac{F_{NBC}}{EA_1}\frac{\partial F_{NAC}}{\partial F_y} = -\frac{(F_x - F_y)a}{EA_2} + \frac{\sqrt{2}\cdot\sqrt{2}F_y\cdot\sqrt{2}a}{\sqrt{2}EA_2} = \frac{(3F_y - F_x)a}{EA_2}$$

由题知， $\dfrac{\Delta_{Bx}}{\Delta_{By}} = \tan\theta = \dfrac{F_x - F_y}{3F_y - F_x} = \dfrac{\sin\theta - \cos\theta}{3\cos\theta - \sin\theta}$

化简， $3\sin\theta\cos\theta - \sin^2\theta = \cos\theta\sin\theta - \cos^2\theta$

$\sin^2\theta - 2\cos\theta\sin\theta - \cos^2\theta = 0$

$\tan^2\theta - 2\tan\theta - 1 = 0$

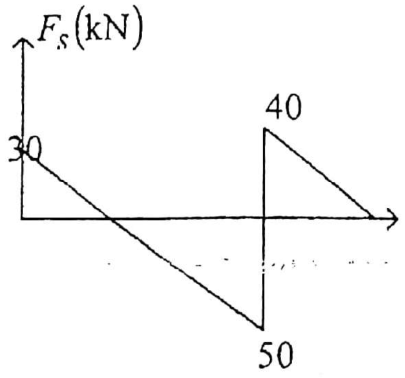
$\tan\theta = \sqrt{2} + 1$， 所以 $\theta = 67.5°$

三、解：(1)、 $M_B = \dfrac{1}{2}ql^2_{BC} = \dfrac{1}{2}\times 40\times 1^2 = 20\text{kN}\cdot\text{m}$

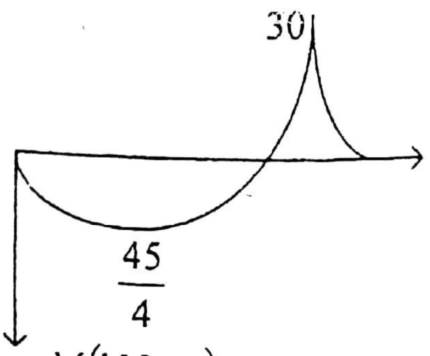
$\sum M_A = 0$， $F_B L_{AB} - \dfrac{1}{2}qL^2_{AC} = 0$， $F_B = \dfrac{1}{4}\times 40\times 9 = 90\text{kN}$

$\sum F_y = 0$， $F_A + F_B - qL_{AC} = 0$， $F_A = 40\times 3 - 90 = 30\text{kN}$

故 B 处剪力 $F_{S,\max} = 90 - 40 = 50\text{kN}$， $M_B = 20\text{kN}\cdot\text{m}$




(2)、剪力图




弯矩图 $M(\mathrm{kN\cdot m})$

$$M(x)=30x-\frac{1}{2}\times 40x^2=30x-20x^2，\quad M\left(\frac{3}{4}\right)=\frac{45}{4}\mathrm{kN\cdot m}$$

$$\sigma_{max}=\frac{M_{max}}{I}y_{max}=\frac{20}{2.59\times 10^{-5}}\times 0.142=110.0MPa$$

$$S_z=20\times 142\times 71\times 10^{-9}=2.02\times 10^{-6}\mathrm{m}^3$$

$$\tau_{max}=\frac{F_{max}S_z}{bI}=\frac{50\times 10^3\times 2.02\times 10^{-5}}{0.02\times 2.59\times 10^{-5}}=19.5MPa$$

四、解：此题源自孙训方下册 5-1 题

第五章为《应变分析，电阻应变计法基础》，请不要忽略

通用方法请参考课后答案，此处用简便方法。

（1）、以轴向设为 x 方向，纵向设为 y 方向

$$\sigma_x=\frac{F}{A}=\frac{F}{bh}，\quad \sigma_y=0$$

$$\varepsilon_x=\frac{\sigma_x-\upsilon\sigma_y}{E}=\frac{F}{Ebh}$$

$$\varepsilon_y=-\upsilon\varepsilon_x=-\frac{F}{Ebh}$$

（2）、$\varepsilon_{30°}=\dfrac{\varepsilon_x+\varepsilon_y}{2}+\dfrac{\varepsilon_x-\varepsilon_y}{2}\cos 60°-\dfrac{\gamma_{xy}}{2}\sin 60°=\dfrac{(3-\upsilon)F}{4Ebh}$

$$L_{BC}=\frac{h}{\sin 30°}=2h$$

故 $\Delta L_{BC}=L_{BC}\cdot\varepsilon_{30°}=\dfrac{(3-\upsilon)F}{2Eb}$

（3）、$-\dfrac{\gamma_{30°}}{2}=\dfrac{\varepsilon_x-\varepsilon_y}{2}sin 60°-\dfrac{\gamma_{xy}}{2}\cos 60°$

故 $\gamma_{30°}=\dfrac{\sqrt{3}}{2}\cdot\dfrac{F(1+\upsilon)}{4Ebh}\times 2\times(-1)=-\dfrac{\sqrt{3}F(1+\upsilon)}{2Ebh}$ （变大）

即∠ABC 变大了 $\dfrac{\sqrt{3}F(1+\upsilon)}{2Ebh}$

五、解：（1）、$F_z = P\cos\alpha \quad I_y = \dfrac{bh^3}{12}$

$F_y = P\sin\alpha \quad I_z = \dfrac{hb^3}{12}$

在 $0 \le x \le L$ 段，$M_z(x) = \dfrac{Px\sin\alpha}{2}$ ，$M_y(x) = \dfrac{Px\cos\alpha}{2}$

中性轴方程：$\dfrac{M_z}{I_z}y - \dfrac{M_y}{I_y}z = 0$，即 $\dfrac{\sin\alpha}{b^2}y - \dfrac{\cos\alpha}{h^2}z = 0$

设 $\theta$ 为 y 轴与 z 轴夹角，则 $\tan\theta = \dfrac{y}{z} = \cot\alpha \cdot \dfrac{b^2}{h^2}$

（2）、最大正应力：

$$\sigma_{max} = \frac{M_{z\max}}{I_z}\cdot\frac{b}{2} + \frac{M_{y\max}}{I_y}\cdot\frac{h}{2} = \frac{\frac{P}{2}\sin\alpha}{\frac{hb^3}{12}}\cdot\frac{b}{2} + \frac{\frac{P}{2}\cos\alpha}{\frac{bh^3}{12}}\cdot\frac{h}{2}$$

$$= \left(\frac{\sin\alpha}{b} + \frac{\cos\alpha}{h}\right)\frac{3P}{bh}$$

（3）、梁变形为非对称弯曲

（4）、利用叠加原理

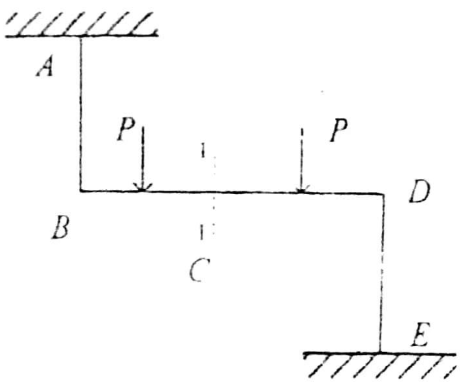
$$\omega_y = \frac{P\cos\alpha(2L)^3}{48EI_y} = \frac{2PL^3\cos\alpha}{Ebh^3}，\quad \omega_z = \frac{2PL^3\sin\alpha}{Ehb^3}$$

故 $\omega = \dfrac{2PL^3}{Ebh}\sqrt{\dfrac{\cos^2\alpha}{h^4} + \dfrac{\sin^2\alpha}{b^4}}$

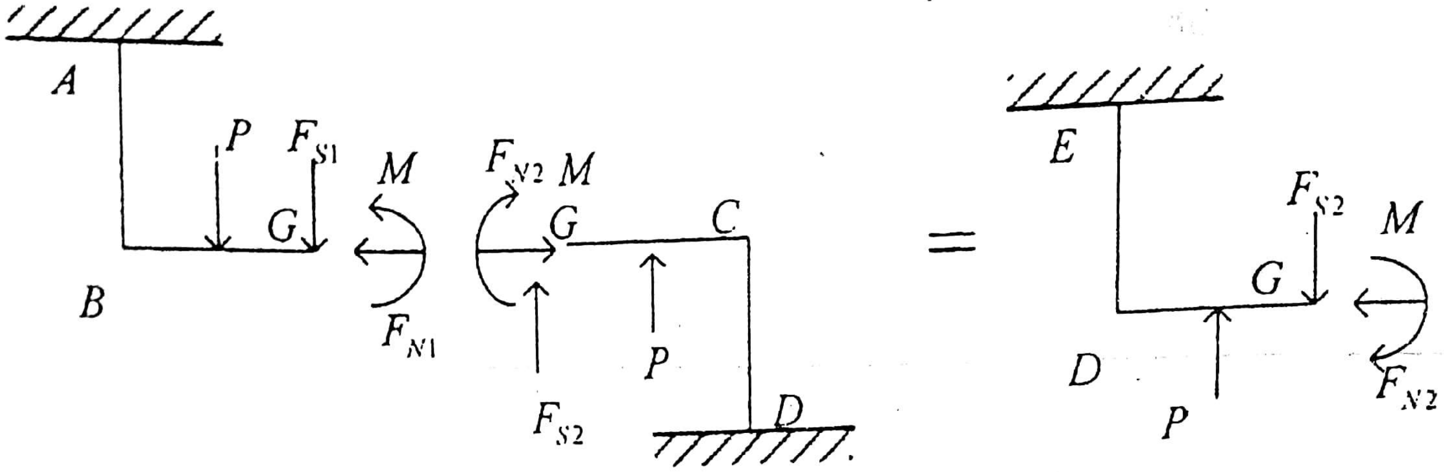
六、解：标准答案请参照刘鸿文教材的 14-16 题

这里说简单方法。

证明：由题可知，结构反对称，利用对称性来做，断开 C 点，存在三个力



将 CDE 旋转 180° 来观察




由于对称性，$F_{N1}+F_{N2}=0$，$F_{N1}=F_{N2}$，故 $F_{N1}=F_{N2}=0$

$$M_1-M_2=0，M_1=M_2，故 M_1=M_2\neq 0$$

$$P-P+F_{S1}+F_{S2}=0，F_{S1}=F_{S2}，故 F_{S1}=F_{S2}=0$$

由此可知，轴力、剪力都为0