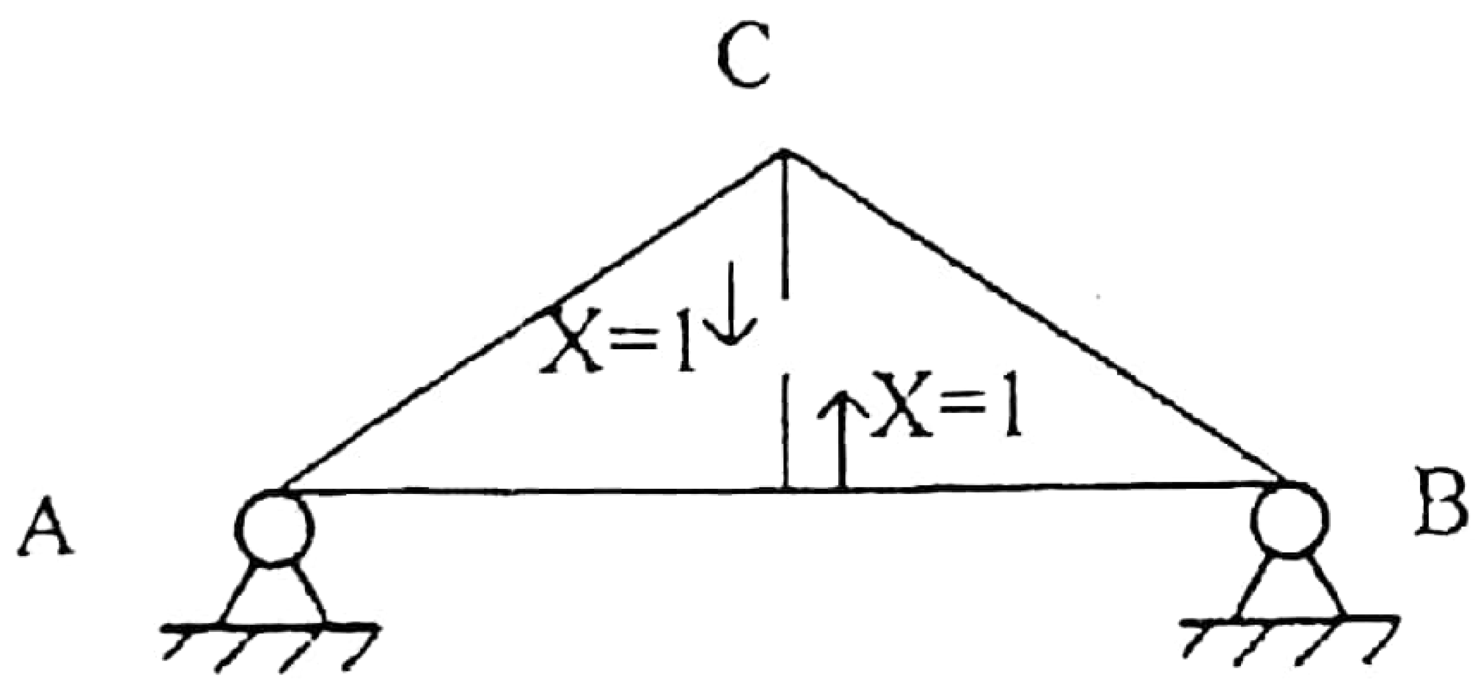
二〇〇〇年答案解析

一、解：用单位力法做

断开 CD 杆，代之以未知反力 X=1，弯矩图如图所示

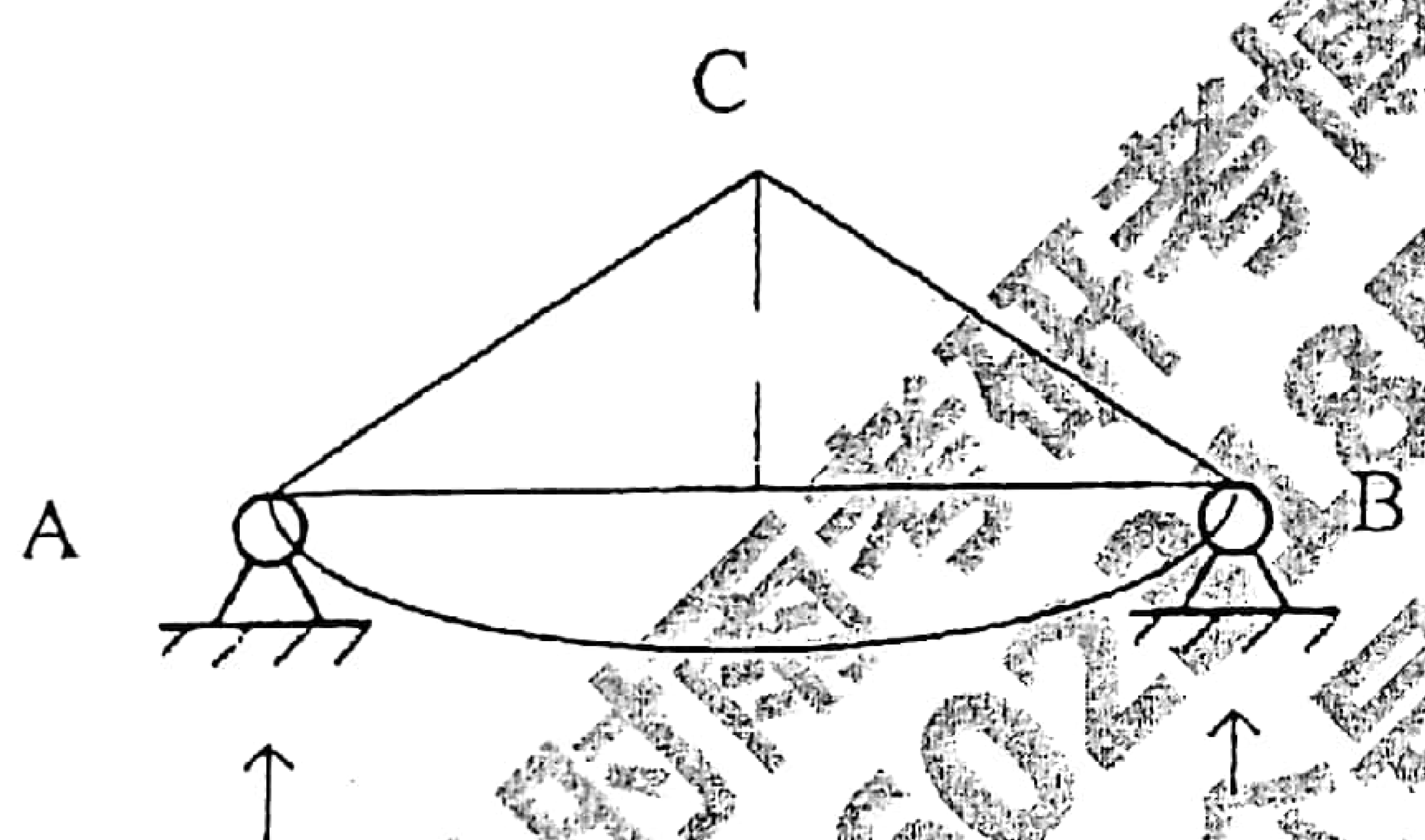


则 $\overline{F}_{NCD}=1$　　$\overline{F}_{NAC}=\overline{F}_{NCB}=-2$

AD 段：$\overline{M}(x)=\dfrac{x}{2}$　　$0\le x\le\sqrt{3}a$

$$\delta_{11}=\frac{1\times1\times a+2\times2\times2a}{EA}+\frac{2}{EI}\int_0^{\sqrt{3}a}\frac{x^2}{4}dx=\frac{9a}{EI}+\frac{\sqrt{3}a^3}{2EI}$$

实际弯矩图如图，



则 $M(x)=\dfrac{qx^2}{2}$　　$0\le x\le\sqrt{3}a$

$$\Delta_{1P}=-\frac{2}{EI}\int_0^{\sqrt{3}a}\frac{1}{2}qx^2\cdot\frac{1}{2}xdx=\frac{-9qa^4}{8EI}$$

$$X=-\frac{\Delta_{1P}}{\delta_{11}}=\frac{\dfrac{9qa^4}{8EI}}{\dfrac{9a}{EI}+\dfrac{\sqrt{3}a^3}{2EI}}=\frac{9Aa^2}{18I+\sqrt{3}Aa^2}qa$$

二、解：1、某塑形材料构件内，存在三处平面应力状态

a：$\sigma_x=\sigma$　　$\sigma_y=\sigma$　　$\tau_{xy}=\sigma$

$$\left.\begin{array}{l}\sigma_{max}\\ \sigma_{min}\end{array}\right\}=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\left(\tau_{xy}\right)^2}=\begin{cases}2\sigma\\0\end{cases}$$

所以$\sigma_1 = 2\sigma$， $\sigma_2 = \sigma_3 = 0$

单元体处于单向应力状态

b: $\sigma_x = -\sigma \quad \sigma_y = \sigma \quad \tau_{xy} = \sigma$

$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \begin{cases}\sqrt{2}\sigma \\ -\sqrt{2}\sigma\end{cases}$$

所以$\sigma_1 = \sqrt{2}\sigma$， $\sigma_2 = 0$， $\sigma_3 = -\sqrt{2}\sigma$

单元体处于二向应力状态 × 纯剪切应力状态

c: $\sigma_x = \frac{3}{2}\sigma \quad \sigma_y = \frac{1}{2}\sigma \quad \tau_{xy} = \frac{\sqrt{3}}{2}\sigma$

$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \begin{cases}2\sigma \\ 0\end{cases}$$

所以$\sigma_1 = 2\sigma$， $\sigma_2 = \sigma_3 = 0$

单元体处于单向应力状态

2、 $\sigma_{r3,1} = \sigma_1 - \sigma_3 = 2\sigma$

$$\sigma_{r3,2} = \sigma_1 - \sigma_3 = 2\sqrt{2}\sigma$$

$$\sigma_{r3,3} = \sigma_1 - \sigma_3 = 2\sigma$$

所以 b 点更容易屈服

备注：请详见刘鸿文第七章关于应力状态的定义

三、解

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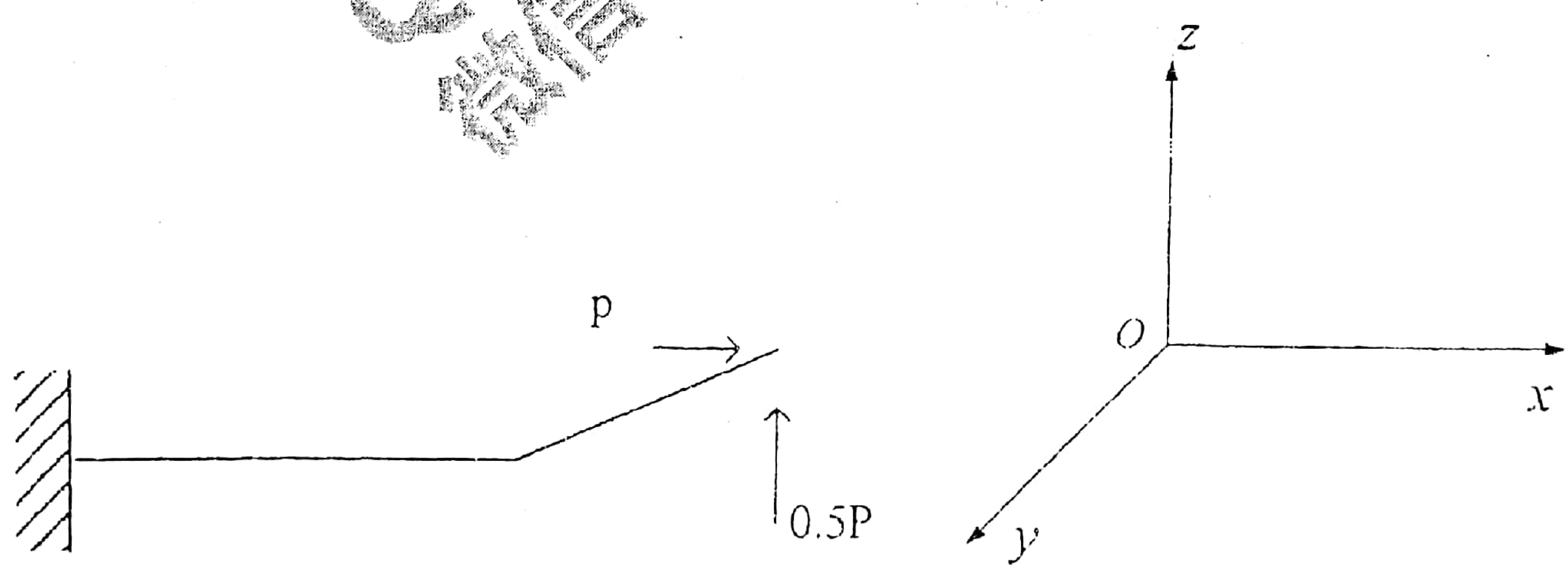
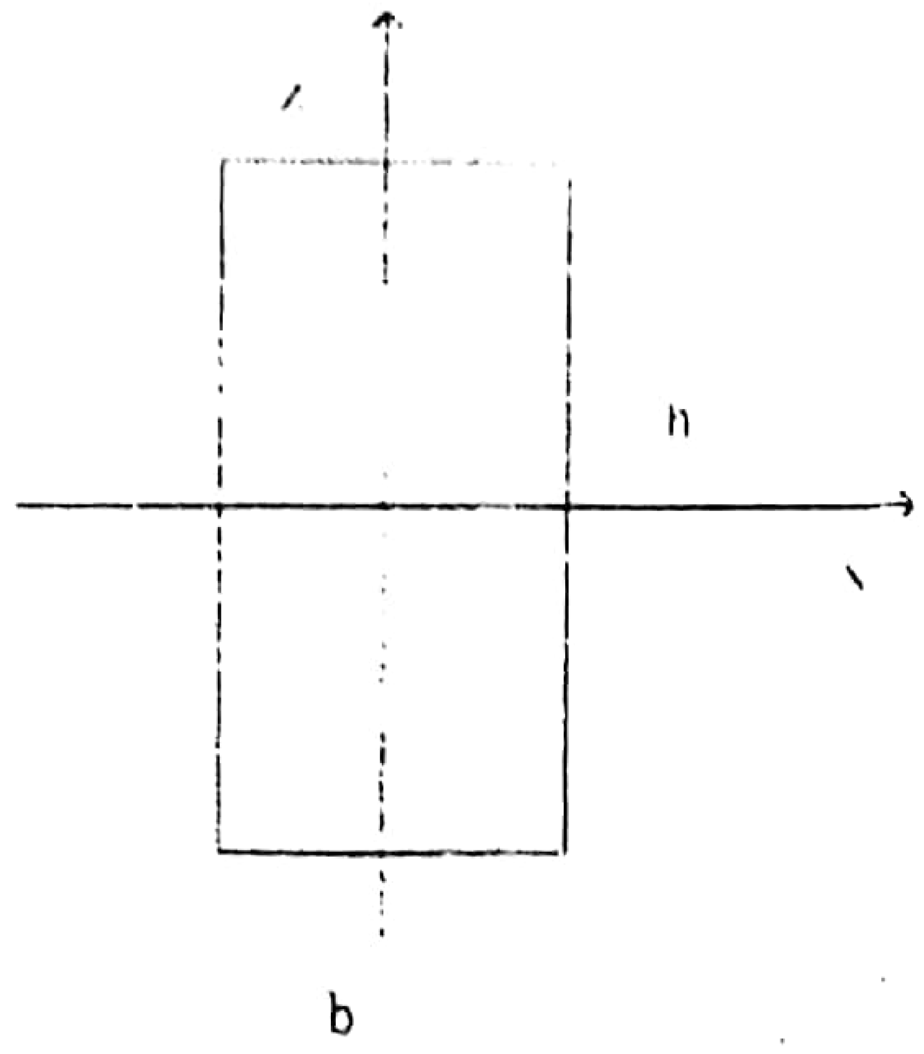




BC 杆：$M_x = \frac{P}{2} y \qquad 0 \le y \le 1$

$$M_z = Py \qquad 0 \le y \le 1$$

B 截面是危险截面，上石和下左是角点是危险点

$$M_x = \frac{P}{2} \cdot 1 = 80\mathrm{N \cdot m}$$

$$M_z = P \cdot 1 = 160\mathrm{N \cdot m}$$

$$\sigma_{max} = \frac{M_x}{W_x} + \frac{M_z}{W_z} = \frac{80}{\frac{1}{6} \times 0.02 \times (0.04)^2} + \frac{160}{\frac{1}{6} \times 0.04 \times (0.02)^2} = 75MPa$$

$$\sigma_{r4} = \sigma_{max} = 75MPa$$

AB 杆：$M_y = \frac{P}{2} x \qquad 0 \le x \le 2$

$$M_z = P \cdot 1 = 160N \cdot \mathrm{m} \qquad 0 \le x \le 2$$

$$T = \frac{P}{2} \cdot 1 = 80N \cdot \mathrm{m} \qquad 0 \le x \le 2$$

故 A 截面是危险截面，$M_y = \frac{P}{2} \cdot 2 = 160\mathrm{N \cdot m}$

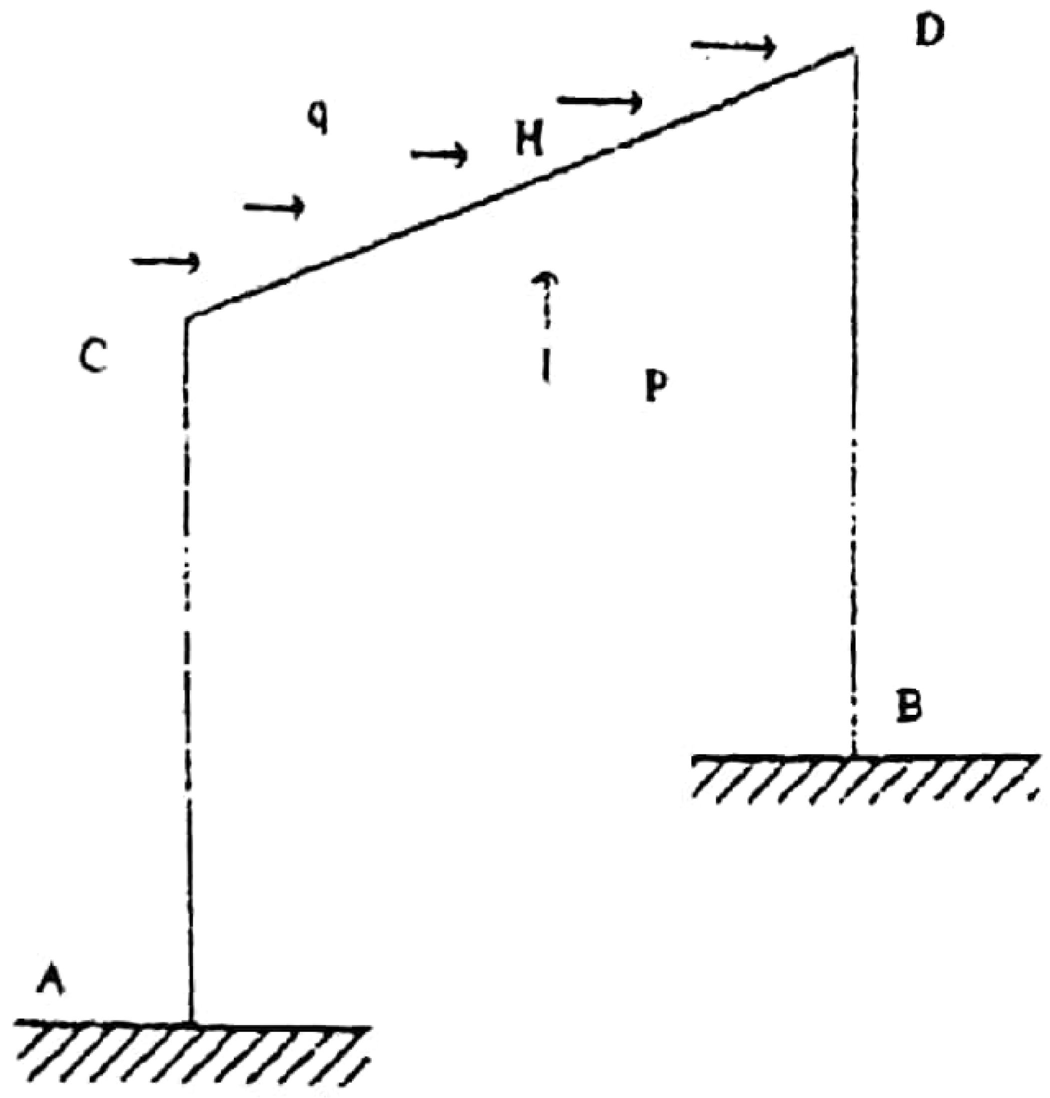
$$\sigma_{max} = \frac{\sqrt{{M_y}^2 + {M_z}^2}}{W} + \frac{P}{A} = (85.36 + 0.27)MPa = 85.63\mathrm{MPa}$$

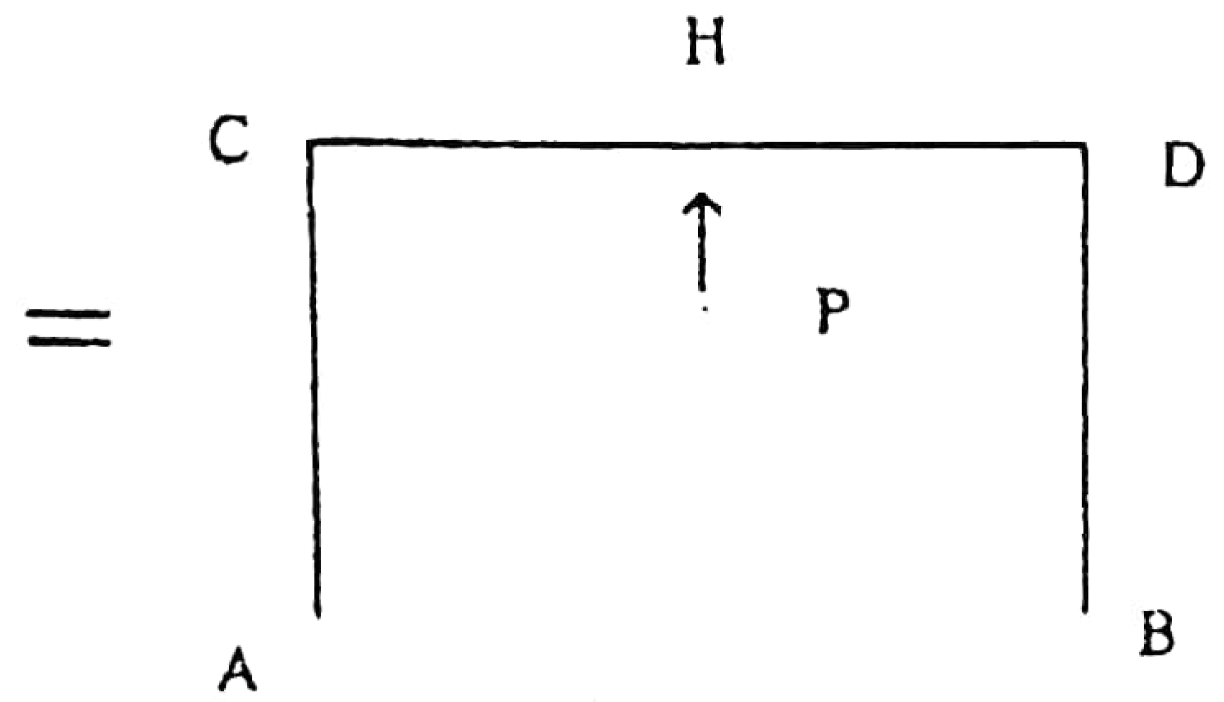
$$\tau_{max} = \frac{T}{W_t} = \frac{16 \times 80}{\pi d^3} = 15.09MPa$$

$$\sigma_{r4} = \sqrt{\sigma^2 + 3\tau^2} = 89.53MPa$$

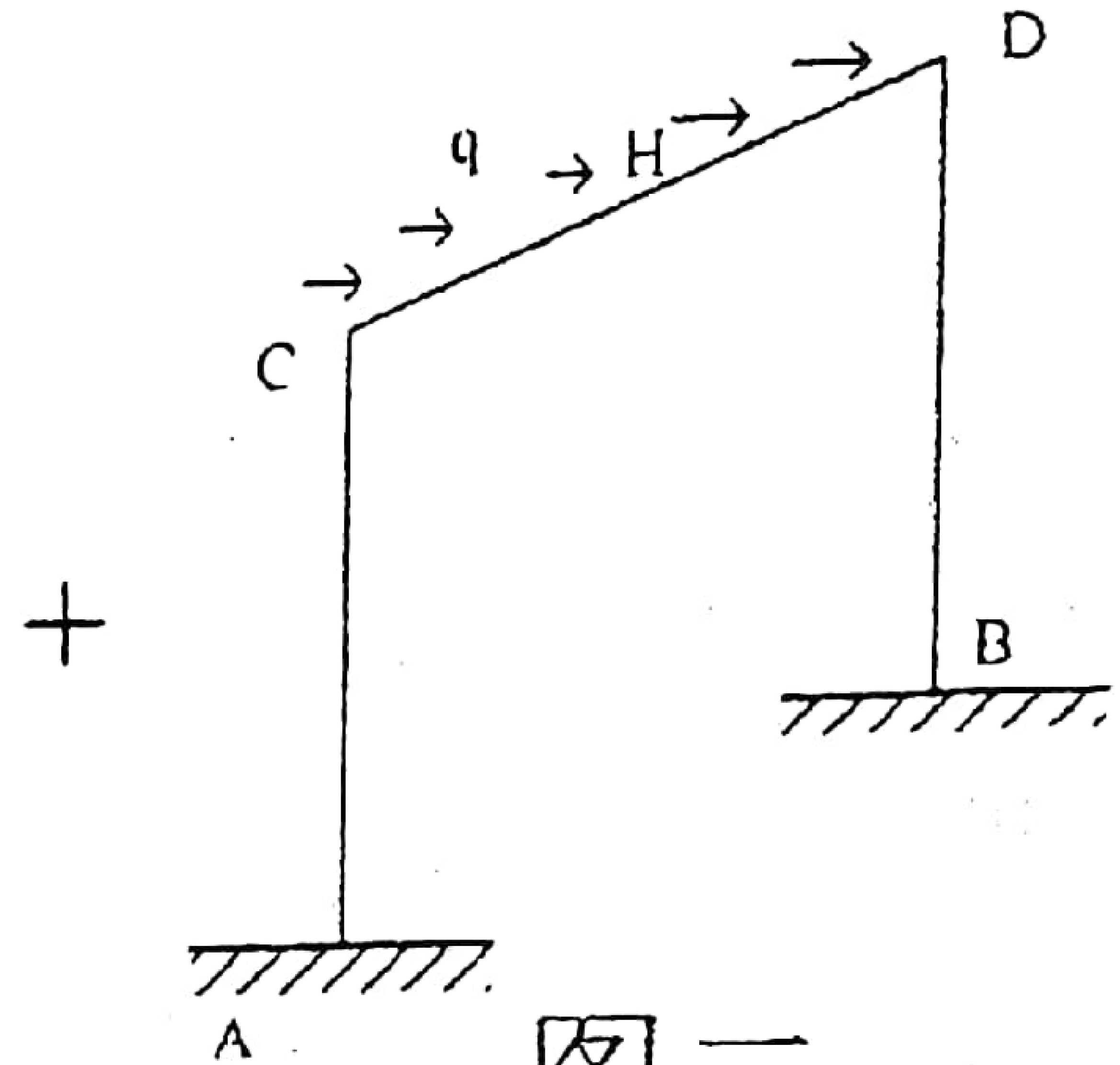
危险点位置位于如图所示 A 点

四、备注：这道题挺难的。正反地称结构，属于计算量大的题型，需要经行大量的对称题型练习才可以在考场上游刃有余，同时这也是浙大材力的一大特色，





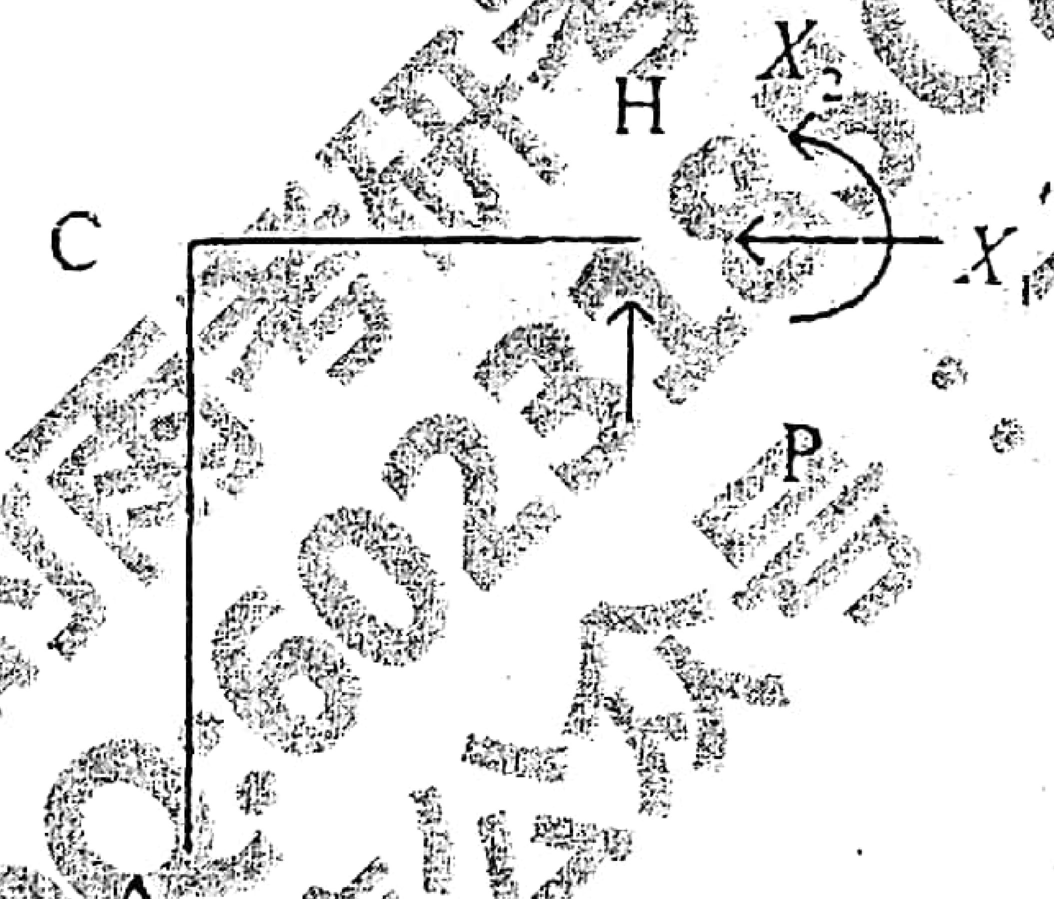
图一



图二

图一中的结构：

正对称，故 H 端截面上只可能存在正对称的力，即轴力和弯矩



分析结构左半部分，分别设轴力和弯矩为 X_1', X_2'

受力分析，$F_{Ay}' = \frac{P}{2}(\downarrow), F_{Ax}' = \frac{3P}{10}(\leftarrow)$

则 HC 段：$M_Z(x) = \frac{P}{2}x + X_2'$ $\quad 0 \le x \le a$

CA 段：$M_Z(x) = \frac{P}{2}a + X_2' + X_1' \cdot x$ $\quad 0 \le x \le a$

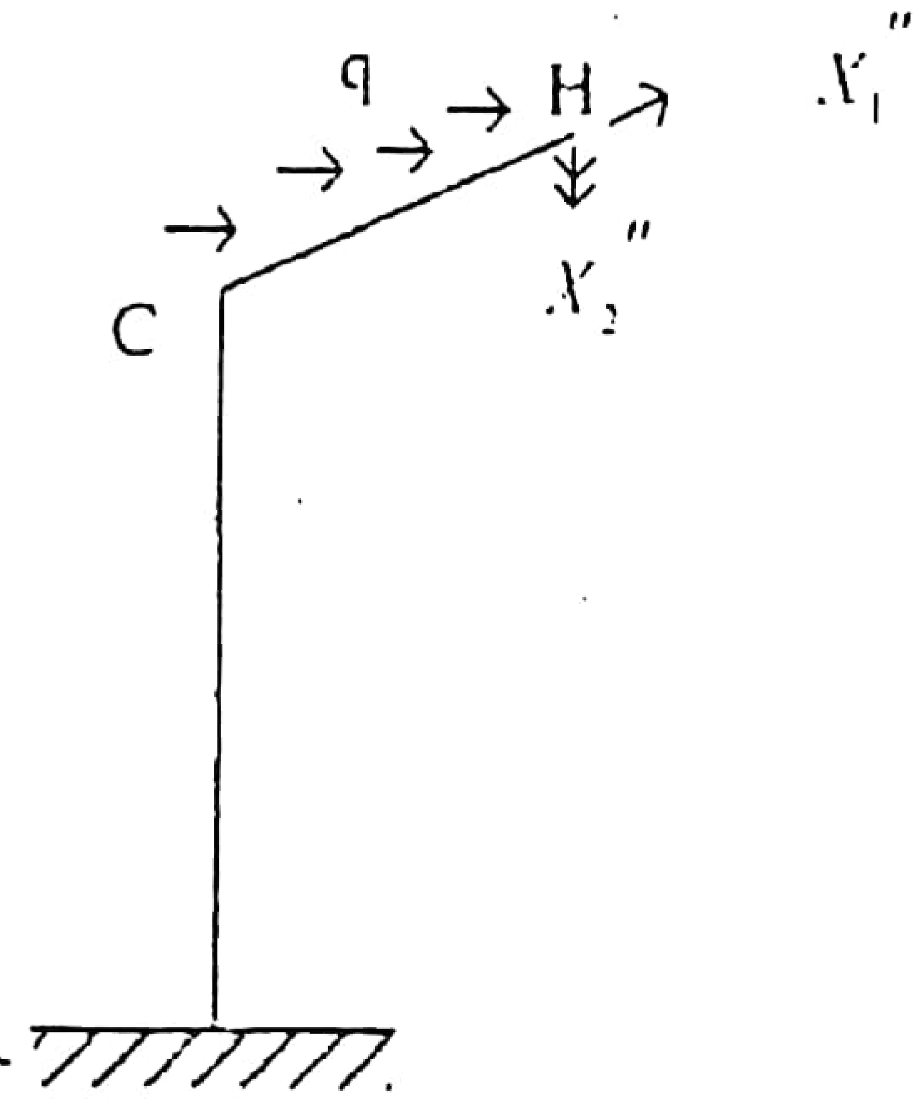
$$\Delta_{Hz} = \int_0^a \frac{\frac{P}{2}ax + X_2'x + X_1'x^2}{EI}dx = \frac{Pa^3}{4EI} + \frac{X_2'a^2}{2EI} + \frac{X_1'a^3}{3EI} = 0 \text{ ①}$$

$$\theta_z = \int_0^a \frac{\frac{P}{2}x + X_2'}{EI}dx + \int_0^a \frac{\frac{P}{2}a + X_2' + X_1'x}{EI}dx = 0 \text{ ②}$$

由①②得，$\begin{cases} X_1{}' = -\dfrac{3}{10}P \\ X_2{}' = -\dfrac{3}{10}Pa \end{cases}$

图二中的结构：

同图一，属于正对称，故 H 处截面只存在轴力和弯矩图，分别设为 $X_1{}'', X_2{}''$



只分析左半部分

则 CH 段：$M_y(x) = \dfrac{P}{2}x + X_2{}''$　　$0 \le x \le a$

CA 段：$T = \dfrac{1}{2}qa^2 + X_2{}''$

$M_Z(x) = qax$　　$0 \le x \le a$

$M_x(x) = X_1{}''x$　　$0 \le x \le a$

故 $\theta_H{}'' = \int_0^a \dfrac{\frac{1}{2}qa^2 + X_2{}''}{EI}dx + \int_0^a \dfrac{\frac{1}{2}qa^2 + X_2{}''}{GI_p}dx$

$= \dfrac{qa^3}{6EI} + \dfrac{X_2{}''a}{EI} + \dfrac{5qa^3}{8EI} + \dfrac{5X_2{}''a}{4EI} = 0$

$X_2{}'' = -\dfrac{19}{54}qa^2$

$\Delta H_Z = \int_0^a \dfrac{X_1{}''x^2}{EI}dx = 0$，则 $X_1{}'' = 0$

所以 $F_{Ax}{}'' = -qa(\leftarrow)$，$T_A{}'' = \dfrac{1}{2}qa^2 - \dfrac{19}{54}Pa$，$M_A{}'' = qa^2$

由图一图二结构叠加。所以

$F_{Ay} = F_{Ay}{}' = \dfrac{P}{2}(\downarrow)$

$F_{Ax} = F_{Ax}{}' + F_{Ax}{}'' = \dfrac{3P}{10} + qa$

$$M_{Az} = M_{Az}' + M_{Az}'' = \frac{Pa}{10} + \mathrm{qa}^2$$

$$T = \frac{1}{2}\mathrm{qa}^2 - \frac{19}{54}P\mathrm{a}$$

（2）、对于图一，H 只有竖直位移

$$\Delta_{Hy} = \int_0^a \frac{\left(\frac{P}{2}\mathrm{x} + X_2'\right)x}{EI}dx + \int_0^a \frac{\left(\frac{P}{2}\mathrm{a} + X_2' + X_1' x\right)x}{EI}dx = \frac{Pa^3}{6EI}$$

对于图二，只有水平位移

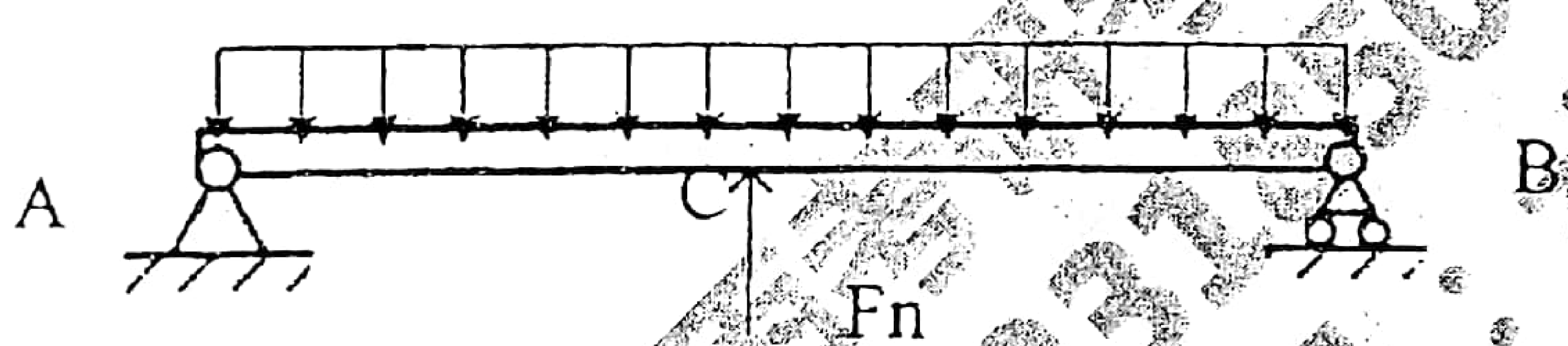
在 H 处虚设一水平力 $\overline{F}_H = 1$，则 HC 段 $\overline{M}(\mathrm{x}) = \mathrm{x}$

CA 段：$\overline{T} = a$，$M = \mathrm{x}$　　　$0 \le \mathrm{x} \le \mathrm{a}$

$$则\ \Delta H_x = \int_0^a \frac{\frac{1}{2}\mathrm{qx}^3 + X_2''\mathrm{x}}{\mathrm{EI}}\mathrm{dx} + \int_0^a \frac{\frac{1}{2}\mathrm{qa}^3 + X_2''\mathrm{a}}{\mathrm{GI_p}}\mathrm{dx} + \int_0^a \frac{\mathrm{qax}^2}{\mathrm{EI}}\mathrm{dx} = \frac{101qa^4}{216EI}$$

图一图二叠加，所以水平位移为 $\frac{101qa^4}{216EI}(\rightarrow)$，竖直位移为 $\frac{Pa^3}{6EI}(\uparrow)$

五、解：



$$\omega_{C1} = \frac{5\mathrm{q}(2\mathrm{a})^4}{384EI} - \frac{F_N \mathrm{a}^3}{48EI} = \frac{5\mathrm{qa}^4}{48EI} - \frac{F_N \mathrm{a}^3}{6EI}$$

$$\Delta L_{CD} = \frac{F_N \mathrm{a}}{EA}$$

$\omega_C = \Delta L_{CD}$，即 $\frac{5\mathrm{qa}^4}{48EI} - \frac{F_N \mathrm{a}^3}{6EI} = \frac{F_N \mathrm{a}}{EA}$

得 $F_N = 0.0265\mathrm{qa}$

则 $F_{Ay} = F_{By} = \frac{\mathrm{q}(2\mathrm{a}) - F_N}{2} = 0.98675\mathrm{qa}$

对于 CD 杆而言，$\lambda_p = \pi\sqrt{\frac{E}{[\sigma]}} = 99.35$，$\lambda = \frac{\mu L}{\frac{\mathrm{d}}{4}} = 80 < \lambda_p$

$\frac{F_N}{An_{st}} \leq [\sigma]$，即 $\frac{265qa}{\frac{\pi}{4}d^2 \times 3} \leq \frac{Ea}{1000}$，解得：$q \leq 2.44 \times 10^{-5} Ea$

对于 AB 杆而言，$M = 0.98675qax - \frac{1}{2}qx^2$

$$令 \frac{\partial M}{\partial x} = 0，得 x = 98675a$$

$$M_{max} = 0.49qa^2$$

又因为 $\sigma_{max} = \frac{M_{max}}{W} = \frac{32 \times 0.49qa^2}{\pi d^3} \leq [\sigma]n_{st}$

所以 $q \leq 2.66 \times 10^{-9} Ea^2$

综上，$[q] = \min\{2.66 \times 10^{-9} Ea^2, 2.44 \times 10^{-5} Ea\}$