ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Electronics Laboratories
Advanced Engineering Course on

CRYPTOGRAPHIC ENGINEERING

Lausanne, Switzerland
September 8–12, 2008

- **Side-Channel Attacks on Cryptographic Tokens**
- **Countermeasures for Preventing Side-Channel Attacks**

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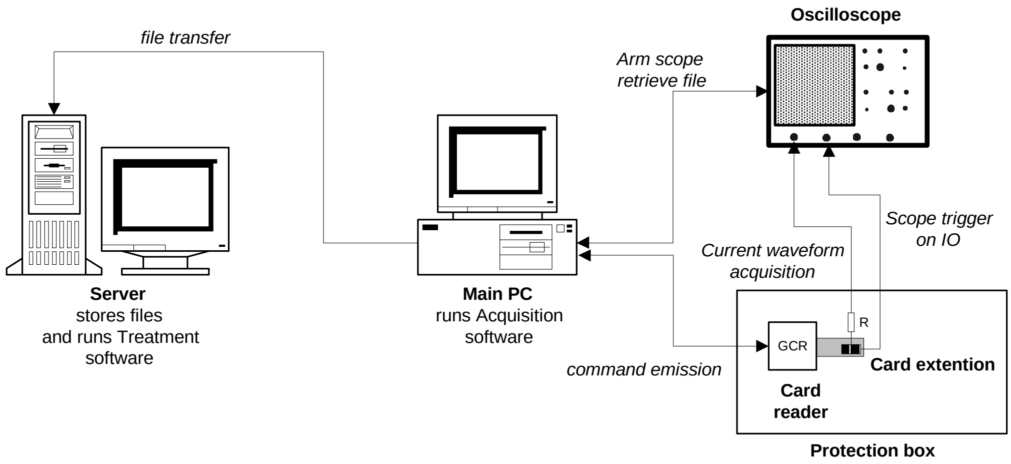
Summary

- **Introduction to Power Analysis**
  - Experimental equipment
  - Information leakage through the power
- **Example: reverse engineering of an algorithm**
  - The algorithm structure
  - Electrical signatures
- **Single Power Analysis (SPA)**
  - Attack against DES key schedule
  - Attack against RSA
- **Conclusion**
  - Countermeasures

Experimental equipment



Devices for monitoring the current consumption of a chip

Information leakage

- **The power consumption of a chip depends on**
  - the manipulated data
  - the executed instruction
- **Leakage models**
  - Hamming Weight of the data, address, Op code
    - HW(0) = 0
    - HW(1) = HW(2) = HW(4) = HW(2^n) = 1
    - HW(3) = HW(5) = HW(6) = HW(9) = 2
    - …
    - HW(255) = HW(0xFF) = 8
  - Transitions weight (flipping bits on a bus state) :
    - HW ($state_i \oplus state_{i-1}$)
  - Other models, chips & technologies …

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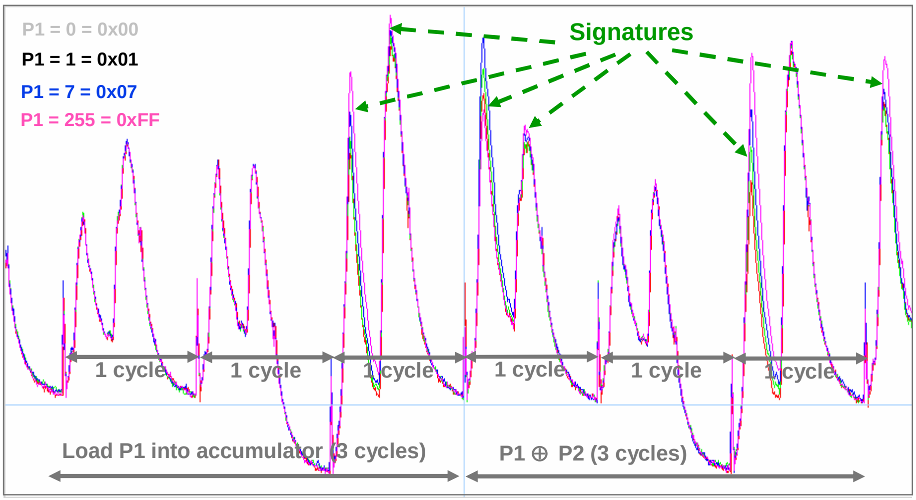
Information leakage

Load P1 and XOR with P2 = 0 (P1 ⊕ P2 with P1 = 0, 1, 7, 255)



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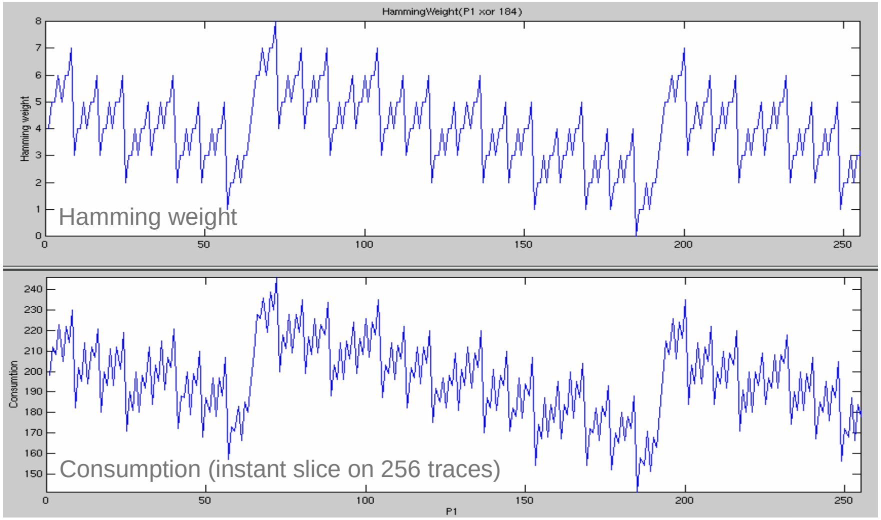
Information leakage

HW (P1 ⊕ 184), for P1 = 0, 255



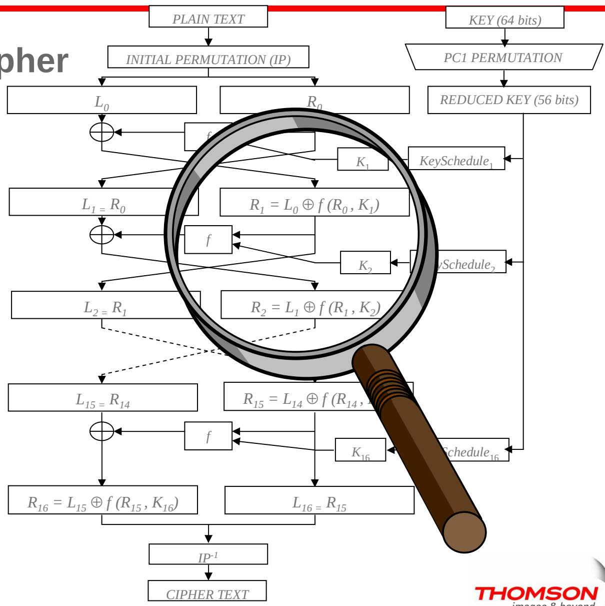
Power trace of an algorithm

- **Typical SK block cipher**
  - key schedule
  - Feistel scheme
  - 16 rounds

Data

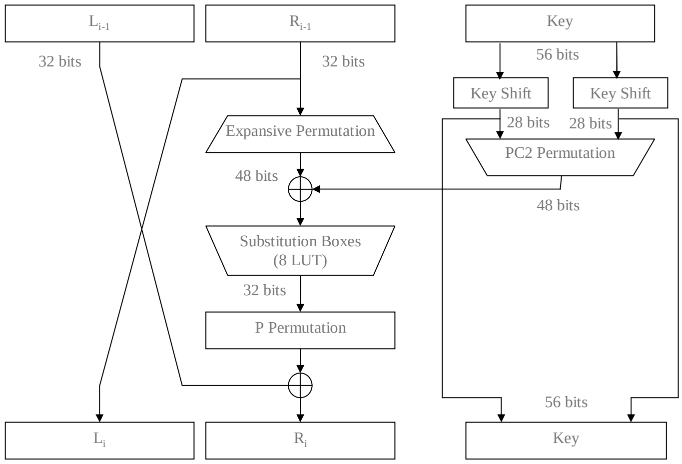
Encryption

Standard



Power trace of an algorithm

- Content of a DES round (with key schedule)



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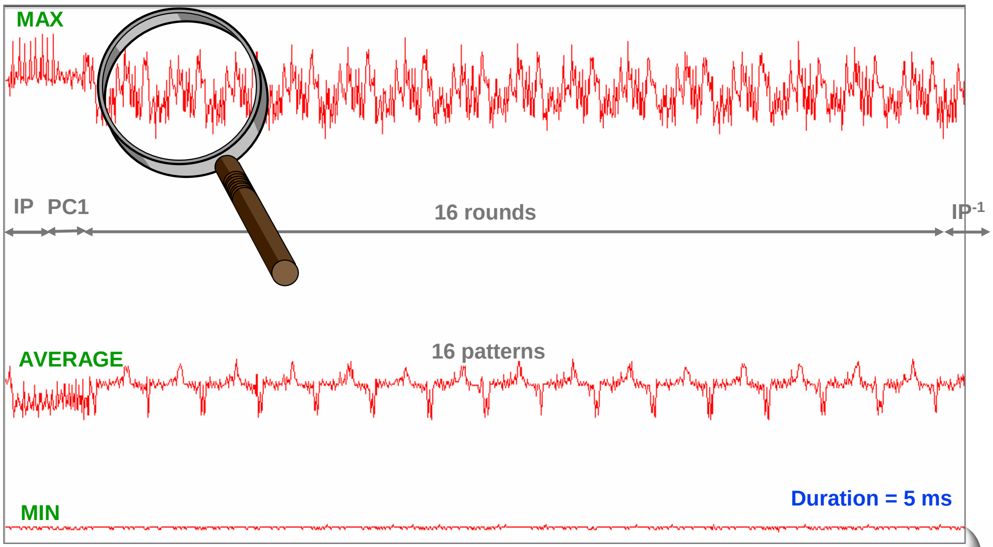
Power trace of an algorithm: DES



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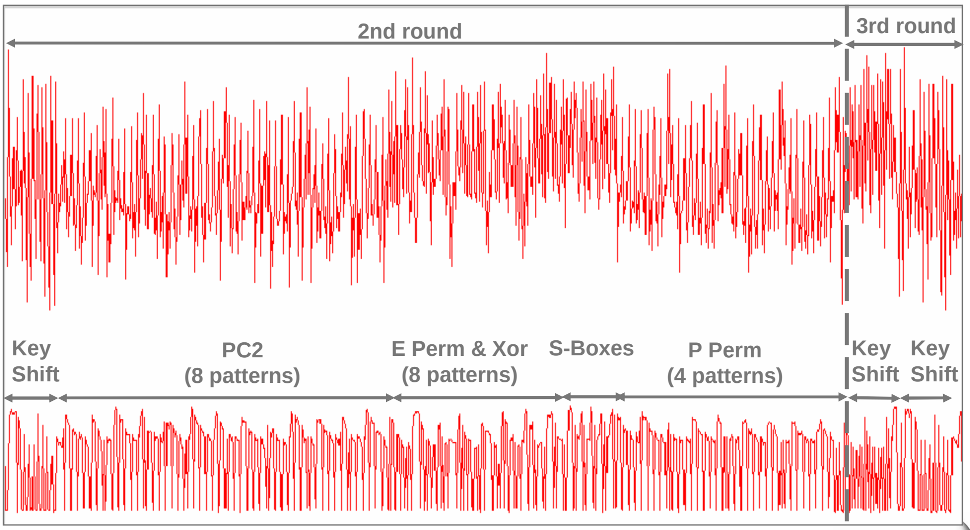
Power trace of an algorithm: DES



SPA attack

- **Simple (Single) Power Analysis context**
  - Find out a secret or private key
  - Known algorithm
  - Unknown implementation (*background culture recommended*)
- **Conditions**
  - 1 card available
  - Reverse engineering phase required (*signature location*)
  - Key inference on a single curve (*with relevant height of view)*
  - Possibly known plain or ciphertext
- **2 target examples:**
  - DES key schedule
  - RSA private exponentiation

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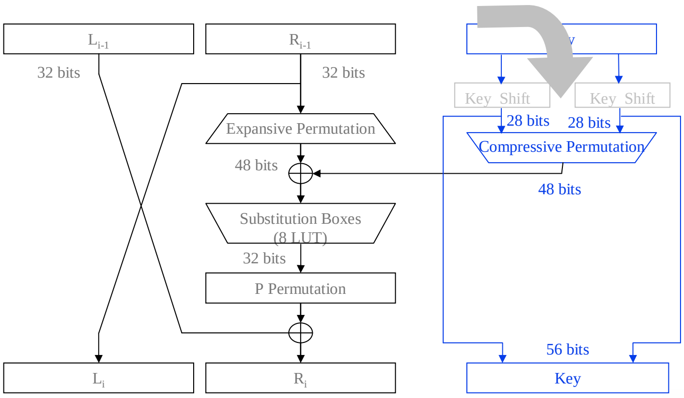
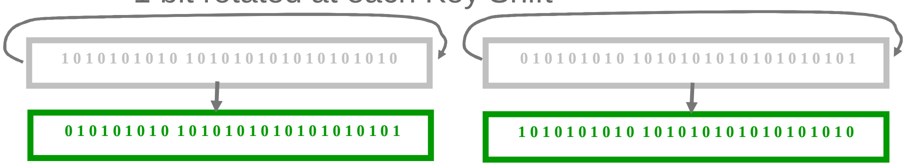
SPA attack on DES

- Goal of the attack: find the DES secret key (56 bits)
- Knowledge on the implementation (assumed hereafter)
- Target of the attack: **key schedule**



SPA attack on DES: Key shift

- **The Key Shift description**
  - Each 28 bits half is shifted separately
  - Shift to the left for DES (to the right for DES^{-1})
  - 1 bit rotated at each Key Shift



  - Number of rotations depends on the round

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
#Shift L (DES)	1	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1
#Shift R (DES-1)	0	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1

SPA attack on DES: Key shift

- **The Key Shift implementation (56 bits stored in 7 bytes)**

byte \ bit	7	6	5	4	3	2	1	0
des_key+0	57	49	41	33	25	17	09	01
des_key+1	58	50	42	34	26	18	10	02
des_key+2	59	51	43	35	27	19	11	03
des_key+3	60	52	44	36	**63**	55	47	39
des_key+4	31	23	15	07	62	54	46	38
des_key+5	30	22	14	06	61	53	45	37
des_key+6	29	21	13	05	28	20	12	04

– Set the carry with bit n°3 of des_key+3: *Carry* = bit 63
– Left rotate des_key+6 (input carry to the right): *Carry* = bit 29
– Left rotate des_key+5 (input carry to the right): *Carry* = bit 30
– … down to des_key+0: *Carry* = bit 57

byte \ bit	7	6	5	4	3	2	1	0
des_key+0	49	41	33	25	17	09	01	58
des_key+1	50	42	34	26	18	10	02	59
des_key+2	51	43	35	27	19	11	03	60
des_key+3	52	44	36	63	55	47	39	31
des_key+4	23	15	07	62	54	46	38	30
des_key+5	22	14	06	61	53	45	37	29
des_key+6	21	13	05	28	20	12	04	63

SPA attack on DES: Key shift

- **The Key Shift implementation (continued)**
  – Clear bit n°4 in des_key+3 (forced to 0)

Carry = bit 57

byte \ bit	7	6	5	4	3	2	1	0
des_key+0	49	41	33	25	17	09	01	58
des_key+1	50	42	34	26	18	10	02	59
des_key+2	51	43	35	27	19	11	03	60
des_key+3	52	44	36	**'0'**	55	47	39	31
des_key+4	23	15	07	62	54	46	38	30
des_key+5	22	14	06	61	53	45	37	29
des_key+6	21	13	05	28	20	12	04	63

  – If Carry is set (= 1) set bit n°4 in des_key+3 (for ced to 1)

byte \ bit	7	6	5	4	3	2	1	0
des_key+0	49	41	33	25	17	09	01	58
des_key+1	50	42	34	26	18	10	02	59
des_key+2	51	43	35	27	19	11	03	60
des_key+3	52	44	36	**'1'**	55	47	39	31
des_key+4	23	15	07	62	54	46	38	30
des_key+5	22	14	06	61	53	45	37	29
des_key+6	21	13	05	28	20	12	04	63

SPA attack on DES: Key shift

- After 16 rounds, 28 key bits have gone through the carry…
- … and have been tested each time!

- If a successful test (with related bit set) is electrically different from an unsuccessful test...
- … then it is possible to read the 28 bit values!

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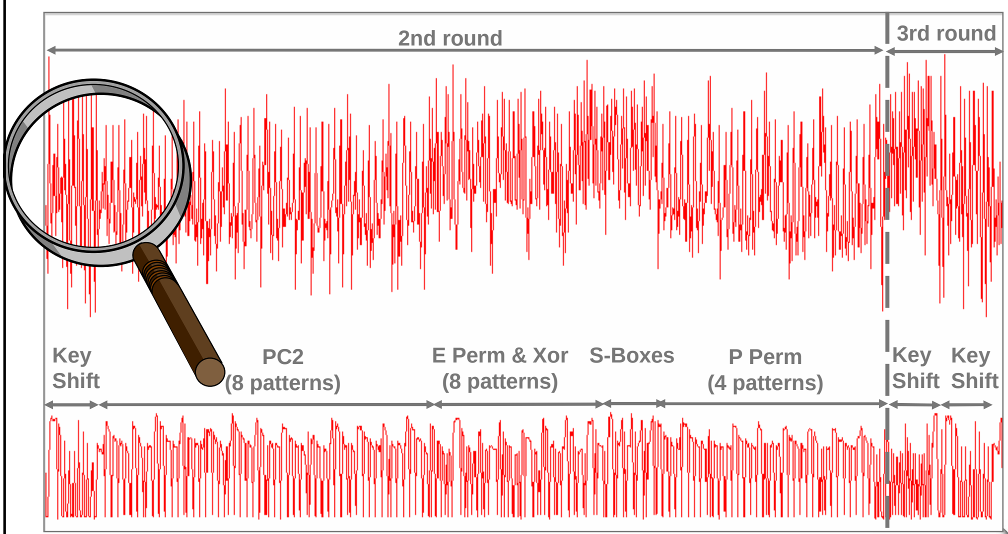
SPA attack on DES: Key shift



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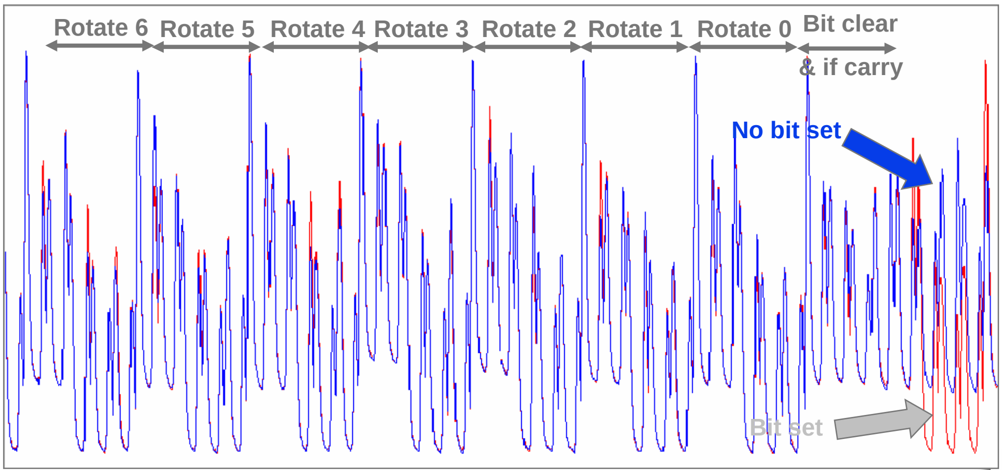
SPA attack on DES: Key shift

Consumption: single "Key Shift" and conditional "bit set"



SPA attack on DES: Conclusion

- 1 or 2 key bits can be read per round
- 28 remaining bits can be retrieved by brute force...
- … or 27 can be found by doing the same on DES^{-1}

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
#Shift L (DES)	1	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1
#Shift R (DES-1)	0	1	2	2	2	2	2	2	1	2	2	2	2	2	2	1

- **BEWARE OF NAIVE PROGRAMMING!**

SPA attack on RSA

- **SPA against RSA private exponentiation**

$$s = \mu(m)^d \bmod N$$

  - N large modulus, say 1024 bits ($N = pq$, with p & q large primes)
  - m message and µ is a padding function (e.g., PSS)
  - s signature
  - d private exponent such that : $ed \equiv 1 \bmod (p-1)(q-1)$, with e public exponent
- **The attacker aims at retrieving *d***

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SPA attack on RSA

- **Implementation points (assumed known hereafter)**
  - N, $\mu(m)$, s and d are 128-byte buffers
  - basic “square and multiply” algorithm
  - exponent bits scanned from MSB to LSB (left to right)

```
k = bitsize(d)
s = 1
For i = k-1 down to 0
  s = s*s mod N            (SQUARE)
  If (d[i]=1)  then
     s = s*m mod N         (MULTIPLY)
  End if
End for
```

Example :	$s = m^9 = m^{1001b}$
init (MSB 1)	$s = m$
round 2 (bit 0)	$s = m^2$
round 1 (bit 0)	$s = (m^2)^2 = m^4$
round 0 (bit 1)	$s = (m^4)^2 * m = m^9$

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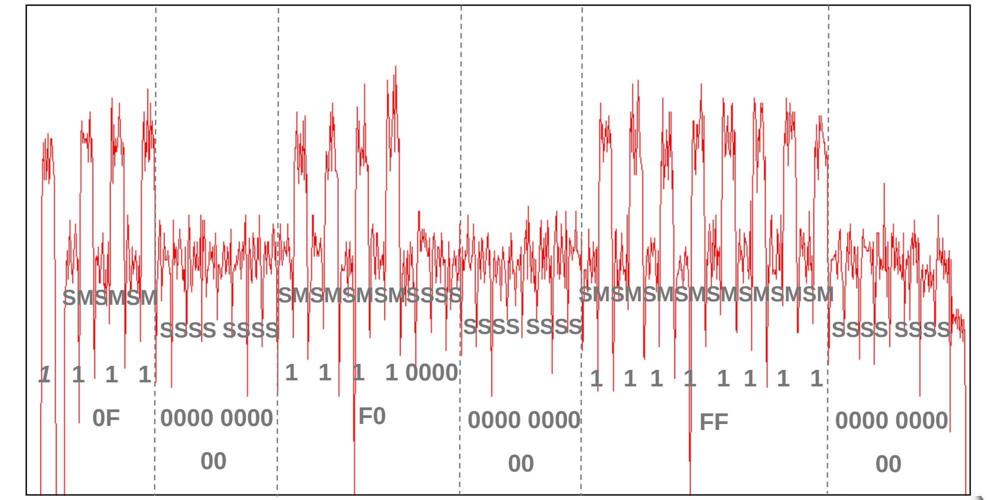
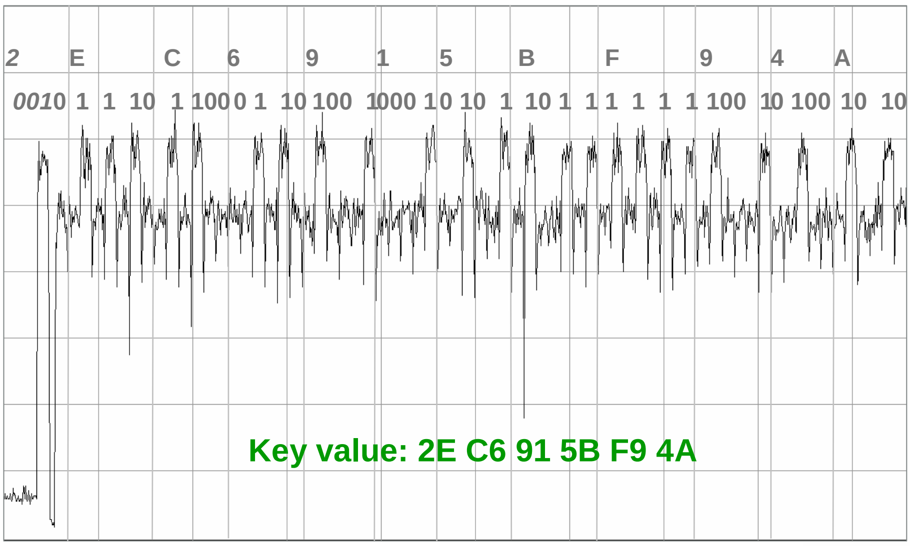
SPA attack on RSA

Test key value : 0F 00 F0 00 FF 00



SPA attack on RSA



Conclusion

- SPA uses **implementation related patterns**
- **SPA strategy**
  - algorithm knowledge
  - reverse engineering phase (*signature location*)
  - representation tuning (*height of view*, *zoom, visualisation*)
  - then play with implementation assumptions...
- **SPA is always specific due to**
  - the algorithm implementation
  - the application constraints
  - the chip's technology (*electrical properties*)
  - possible counter-measures...

Conclusion: Countermeasures

- **Counter-measure: anything that foils the attack !**
- **Trivial countermeasure**
  - prohibit code branches conditioned by the secret bits
- **Advanced counter-measures**
  - algorithm specification refinement
    - code structure
    - data whitening (a.k.a. blinding)
  - implementation design based on the chip's resources
    - play with instructions set
    - hardware electrical behaviour (current scrambler, desynchronisation, cryptoprocessor…)

Part II:
Timing Analysis

Summary

- What are timing attacks?
- Attack on a pin code verification
  - Non constant time execution
  - Randomised execution
- Attack on an RSA computation
- Is there a future for timing attacks?

What are timing attacks?

- The term “Timing Attack” was first introduced at CRYPTO'96 in Paul Kocher's paper
- Few other theoretical approaches without practical experiments up to the end of `97
- GEMPLUS put theory into practice in early `98
- Timing attacks belong to the large family of "side channel" attacks

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What are timing attacks?

- **Principle of Timing Attacks:**
  - Secret data are processed in the card
  - Processing time
    - depends on the value of the secret data
    - leaks information about the secret data
    - can be measured (or at least their differences)
- **Practical attack conditions**
  - Possibility to monitor the processing of the secret data
  - Have a way to record processing duration
  - Have basic computational & statistical tool
  - Have some knowledge of the implementation

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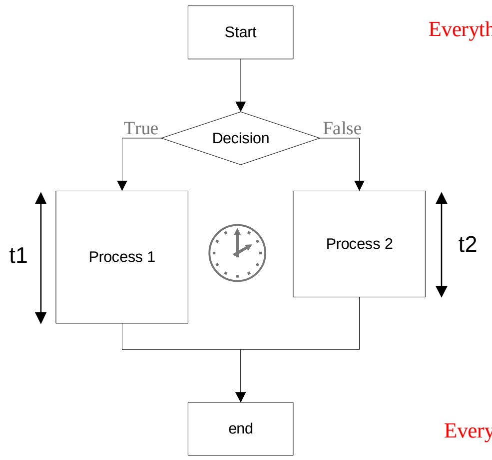
What are timing attacks?



Everything performed unconditionally before the test

A test based on secret data is performed that leads to a boolean decision

Depending on the boolean condition, the process may be long (t1) or short (t2)

Everything performed unconditionally after the test

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PIN code verification

- **Secret data are stored in the smart card**
  - Example: a PIN code, 8 bytes long
- **Like passwords on a PC, authentication is based on this secret**
  - A dedicated function exists in the smart card software : The 'VerifySecret' command which:
    - Receives the challenge (proposed value for the PIN code)
    - Compares the challenge with the stored PIN
    - Grants access rights if the comparison is successful

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PIN code verification
Level 1

Pseudo-code for the "VerifySecret" command

- IN
  - *P* = PIN code value stored in the card
  - *C* = Challenge (proposed value for the PIN)
- OUT
  - 'KO' or 'OK'
- VERIFY SECRET
  - For b = 0 to 7
    - *If* C*[*b*]* != P*[*b*] then return 'KO'*
  - Return 'OK'

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PIN code verification
Level 1

- **Attack implementation**
  - Propose the n possible values of $C[0]$ (256 values)
  - Measure $\tau[n]$ the corresponding command duration
  - Compute the maximum command duration τ , $\tau[n_0]$
    - $\tau[n_0] = \max(\tau[n]),\ n \in \{0, \ldots, 255\}$
    - n_0 is the solution $P[0]$ for the first byte of the PIN code
  - $C[0]$ being known, iterate successively for all $C[i]$
- **Complexity**
  - Number of comparisons: 8 * 256 = 2048 (instead of 256^8)

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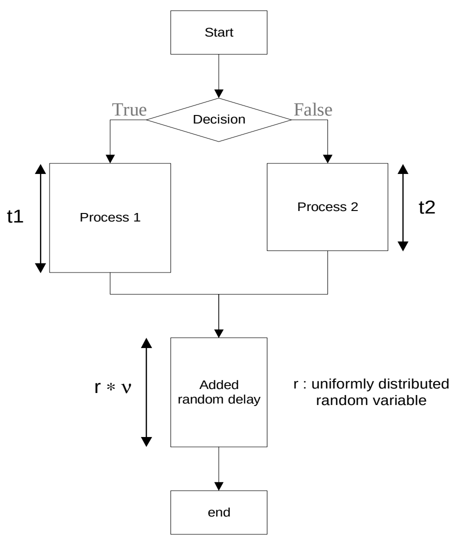
PIN code verification
Level 2

Possible countermeasure

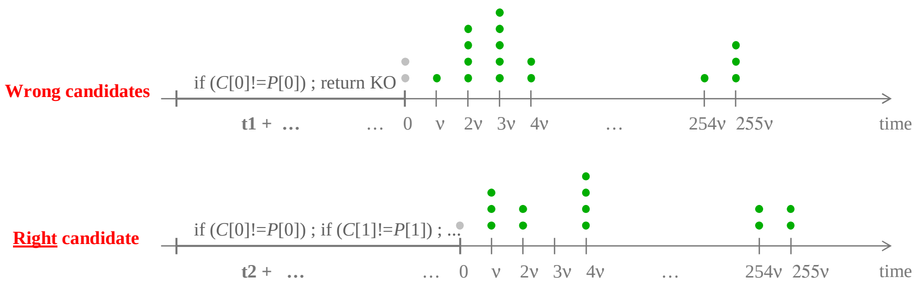
- **To defeat this attack one may think to add a random delay during the execution:**
  - Generate a random delay τ_a uniformly distributed
    - $\tau_a \in \{0, \nu, 2\nu, 3\nu, \ldots, r\nu\}$ with $0 \leq r \leq 255$
    - ν is an elementary time unit
  - Wait τ_a whatever the command status 'KO' or 'OK'
  - Follow the same implementation as the previous one

PIN code verification
Level 2



PIN code verification
Level 2

- **Attack idea**
  - It is possible to know what would be the duration for processing a challenge as if there were no delay



PIN code verification
Level 2

- **Random delay elimination**
  - For each *n* (*n* is the candidate *C*[0] for the first PIN code byte)
    - Acquire a series of *N* command execution durations $\tau_i[n]$
    - The minimum duration corresponds to a $\tau_a = 0$ random delay (with high probability, if *N* is chosen large enough)
  - Consider the corresponding $\tau_{min}[n]$ run time value
- **Attack implementation**
  - Get rid of the random delay for each candidate ($\tau_{min}[n]$)
  - Apply the previous attack scheme
- **Complexity**
  - Number of comparisons: 2048 * *N* (still feasible)

PIN code verification

Level n

- **More complicated counter-measure may be thought of...**
  - Add a binomial (rather than uniform) random delay
  - …
- **…but they also may be defeated by more clever attacks!!**

PIN code verification: Conclusion

- **A typical example of unsecure smart card software**
  - Can happen in any routine processing secret data
    - Secret values comparison
    - Memory scanning and loading
    - Checksum computation
- **Counter-measures evaluation**
  - Add a delay is definitely not the good alternative
  - An inspection of the assembly code for correct implementation may be a warranty

TIME-CONSTANT CODE (for sensitive data)
IS THE SOLUTION

Attack on RSA: Introduction

- **First known practical attacks**
  - During the rump session of CRYPTO'97 by Lenoir
  - In the “Université Catholique de Louvain” (UCL), for the research project Cascade (multi-application smart card)
    - *A practical implementation of the timing attack* (J.F. Dhem, J.L. Willems, F. Koeune & J.J. Quisquater)
- **RSA is not an exception, all cryptosystems may be threatened**
  - Basic mathematical operations
  - Modular exponentiation
  - Cryptographic algorithms

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Attack on RSA: Principle

- **All the requisites**
  - A minimum of knowledge on the RSA algorithm
  - Knowledge and variability of the message are needed
  - Time measurements must be accurate to within few clock cycles
- **Targeted RSA algorithm**
  - A standard RSA exponentiation ($s = m^d \bmod N$)
    - Montgomery method for the modular multiplication on large numbers shows computation time variations
    - The classic square & multiply exponentiation routine allows these variations to be exploited

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Attack on RSA: Square-and-Multiply

- **Straightforward implementation for $s = m^d \bmod N$**
  - Input: m, (d, N)
    - m = message (k bits)
    - (d, N) = RSA private key (k bits)
  - Output: $s = m^d \bmod N$
    - s = signature (k bits)
  - Square & Multiply
    - $s = 1$
    - for $i = k$-1 down to 0
      - $s = s^2 \bmod n$
      - *If* (d[*i*] = 1) *then* s = s*m *mod* N
    - return s

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Attack on RSA: Montgomery multiplication

- **Montgomery modular multiplication (⊗) is dedicated to modular exponentiation**
  - It enhances its efficiency
  - The result of each multiplication lies in [0, 2*N[
    - A subtraction may be needed to fully reduce mod N
- **Multiply step for bit $d[i]$**
  - if ($d[i]$= 1) then $s = s \otimes m \bmod N$
    - Step 1: modular multiplication by m
    - Step 2: optional subtraction by N

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Attack on RSA: Description

- **Working hypothesis**
  - Bits $d[k-1]$ to $d[k-i+1]$ are already known
    - Knowing the message, the intermediate value of s after the square at iteration $k-i$ is computed
    - Whether the subtraction in $s \otimes m \bmod N$ is required may be stated

Attack on RSA: Description

- **The attack is based on an oracle**
  - Sign with same (d, N) for many random messages
  - Make the assumption that $d[k-i] = 1$
  - Construct 2 sets of messages depending on the fact that the subtraction happens or not during the multiplication
    - A = {m : $s \otimes m \bmod N$ implies a subtraction}
    - B = {m : $s \otimes m \bmod N$ implies no subtraction}

<u>The time for the subtraction will be discriminatory</u>

Attack on RSA: Description

- **Case ($d[k-i] = 0$)**
  - Global times for sets A and B are not statistically distinguishable (the split is based on a multiplication which does not occur)
- **Case ($d[k-i] = 1$)**
  - Global times for sets A and B show a statistical difference related to the optional subtraction (the multiplication does occur)

Attack on RSA: Description

- **Time measurements validate or invalidate the oracle**
  - Compute the mean of the global duration for each subset
    - <A>: mean global duration for messages in A
    - <B>: mean global duration for messages in B
  - The oracle criterion is the following
    - <A> - <B> >> 0 ⇒ oracle was right ($d[k-i] = 1$)
    - <A> - <B> ≈ 0 ⇒ oracle was wrong ($d[k-i] = 0$)

Attack on RSA: Conclusion

- **Results** **(on a Pentium 200)**
  - For 128 bits, recovers 2bits / s with 10.000 messages
  - For 512 bits, recovers 1bit / 20s with 100 k messages
- **Conclusion**
  - Time-constant code is a solution
  - Data blinding (randomization) may also be possible

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Is there a future for timing attacks?

- **Associated with other side-channels, it becomes far more efficient**
  - Global measurements are replaced by local ones
- **Timing attacks are still an important threat**
  - Against existing devices applied to secret management
  - Not only a smart cards issue
  - Designers have to think about it
  - Software has still to circumvent hardware flaws
- **Solutions do exist !**

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Part III:
Differential Power Analysis

Summary

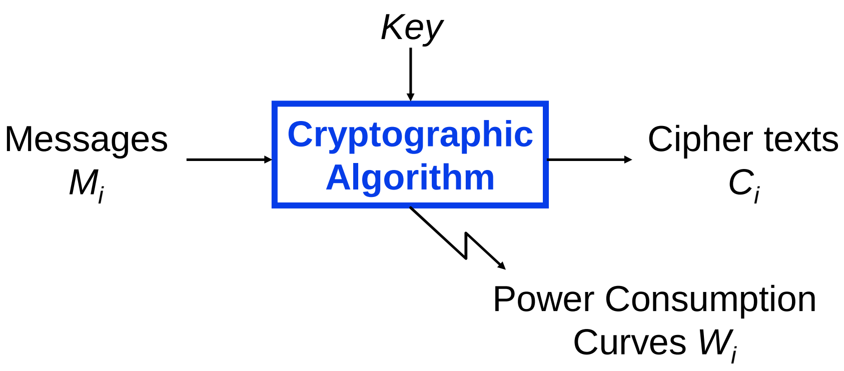
- **DPA Statistical Principle**
  - Acquisition procedure
  - Selection & prediction
  - Differential operator and curves
  - Reverse engineering using the DPA indicator
- **Attacking the DES with DPA**
  - Classical target
  - Hypothesis testing (Guesses management)
- **Generalisation of DPA**
  - Other targets
  - Other algorithms (RSA, AES…)
- **Conclusion: anti-DPA counter-measures**

DPA statistical principle

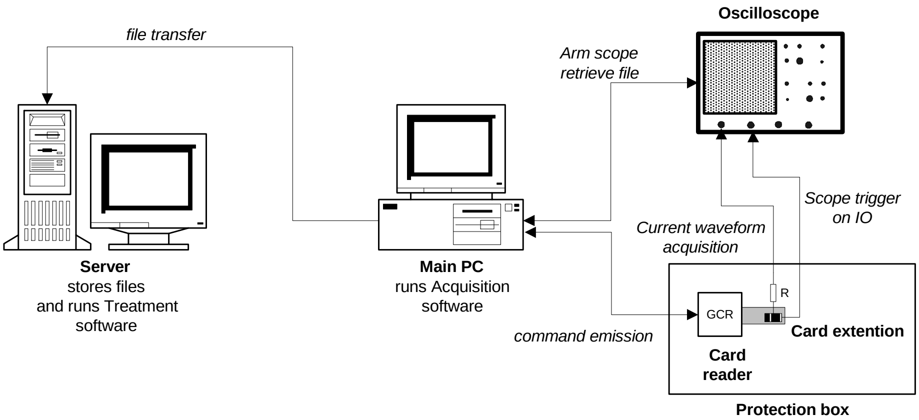
- **Published on the web by Paul KOCHER (1998)**
- **Powerful & generic Power Attack**
  - statistical & signal processing
  - known random messages
  - targeting a known algorithm
  - running on a single smartcard
- **Big noise in the cryptographic community**
- **Big fear in the smartcard industry !**

Acquisition procedure

Play the algorithm N times
(100 < N < 100000)



Acquisition procedure



Monitoring equipment for iterated acquisitions

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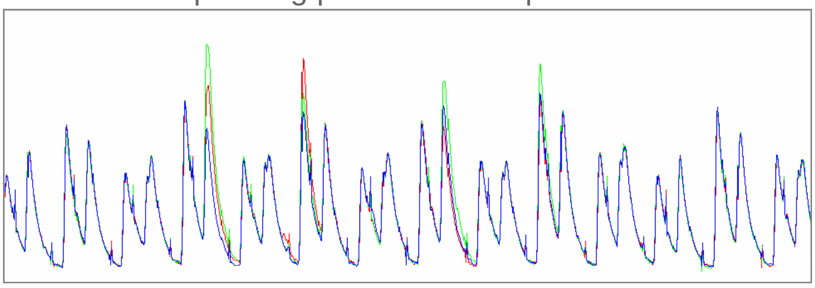
Acquisition procedure

- **After data collection, what is available?**
  - *N* plain or cipher random texts

00	B688EE57BB63E03E
01	185D04D77509F36F
02	C031A0392DC881E6 …

  - *N* corresponding power consumption waveforms



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Selection & prediction

- Assume the message is processed by a known deterministic function *f* (transfer, permutation...)
- Knowing the message, one can recompute off line its image through *f*

$$M_i \longrightarrow \boxed{f} \longrightarrow \quad M'_i = f\,[M_i]$$

- Now **select** a single bit among *M'* bits (in M' buffer)
- One can **predict** the true story of its variations

i	Message	bit		
0	B688EE57BB63E03E	1		
1	185D04D77509F36F	0		
2	C031A0392DC881E6	1	....	*for i = 0, N-1*

DPA operator & curve

- Partition the messages and related curves into two packs, according to the selection bit value...

$$M_i \longrightarrow \boxed{f} \longrightarrow \begin{cases} \boxed{bit\,(M_i') = 0} \\ \boxed{bit\,(M_i') = 1} \end{cases}$$

- … and assign **-1 to pack 0** and +1 to pack 1

0	B688EE57BB63E03E	1	+1	
1	185D04D77509F36F	0	-1	
2	C031A0392DC881E6	1	+1	… *for i = 0, N-1*

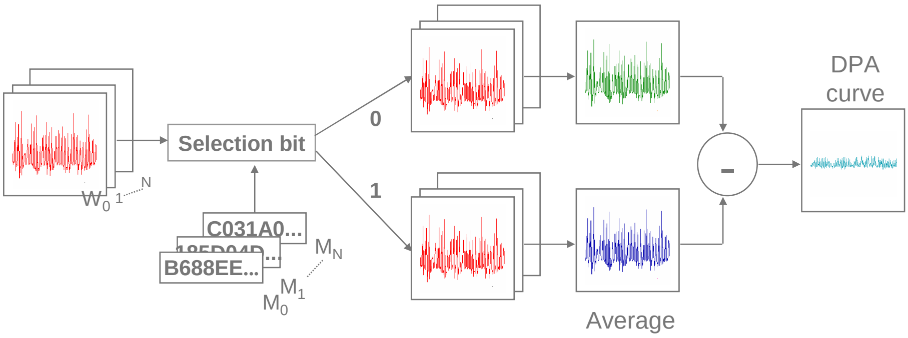
- Sum the signed consumption curves and normalise
- <=> Difference of averages

($N_0 + N_1 = N$)

$$\boxed{DPA = \frac{\sum W_1}{N_1} - \frac{\sum W_0}{N_0}}$$

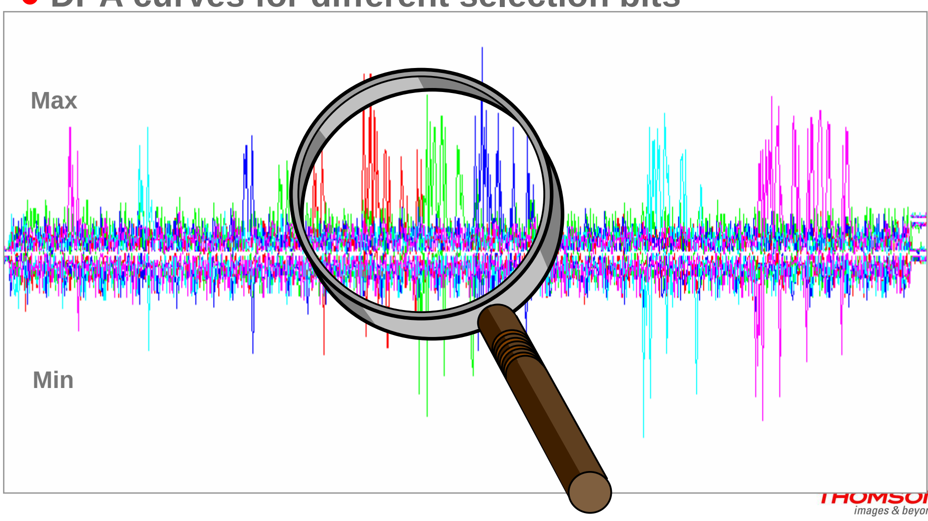
DPA operator & curve

- DPA curve construction



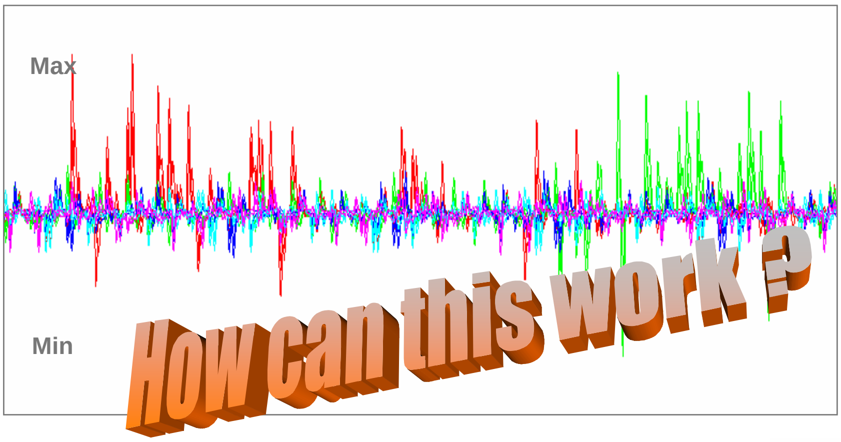
DPA operator & curve

- DPA curves for different selection bits



DPA operator & curve

- Peaks are rising when selection bits are handled



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DPA operator & curve

- Spikes explanation : Hamming Weight of the bit's byte

Selection bit

0	*0 1 0 0 1 0 1 1*	*1 1 0 0 1 1 0 1*
1	*0 1 1 0 1 0 1 0*	*1 0 1 1 0 1 0 0*
2	*1 1 0 0 1 0 0 0*	*0 1 0 1 1 1 1 0*
...	...	...

Average = E [HW_0] = 0 + 3.5 *Average = E [HW_1] = 1 + 3.5*

Δ = E [HW_1] - E [HW_0] = 1

- Contrast (peak height) proportional to $N^{1/2}$ (evaluation criterion)
- If prediction was wrong: selection bit would be random

E [HW0] = E [HW1] = 4 => Δ = 0

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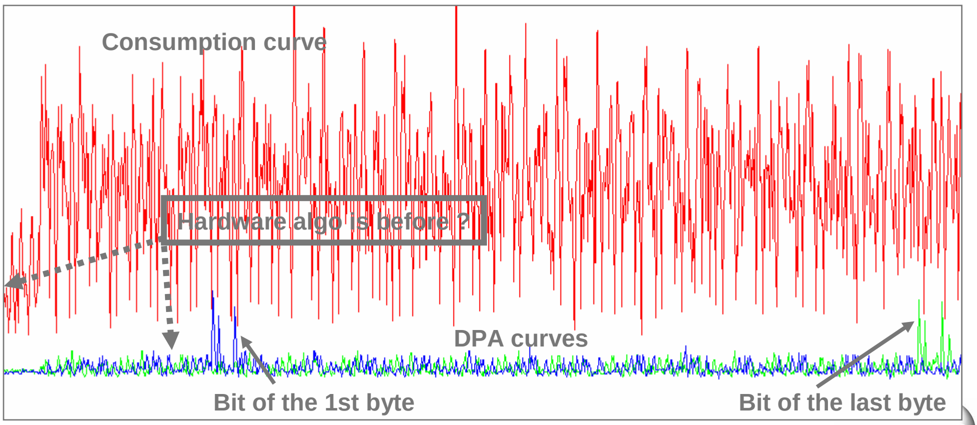
Reverse engineering using DPA

- **Use DPA to locate when predictible things occur**
  - DPA and power curves superposition
  - Example : hardware algo & ciphertext transfer to RAM



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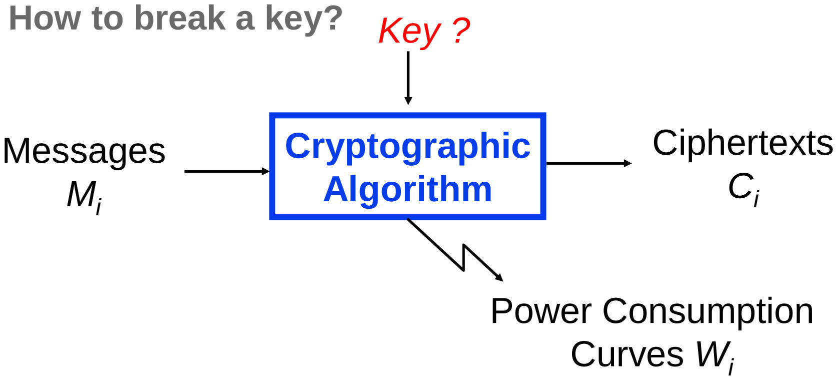
Attacking DES with DPA

- **DPA works thanks to the perfect prediction of the selection bit**
- **How to break a key?**



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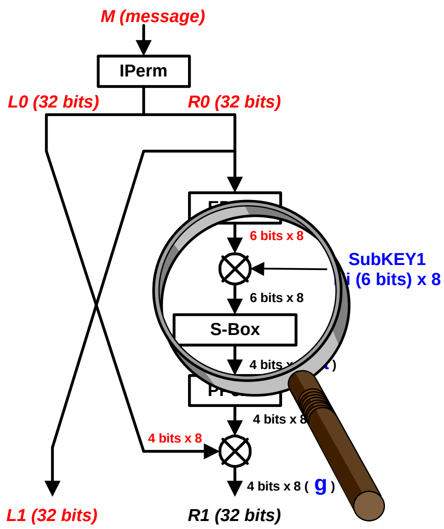
Attacking DES with DPA

- Try different keys a valid them with DPA
- Isn't it like cryptographic exhaustive search ?
- Not exactly …
- … because the research space is drastically reduced!

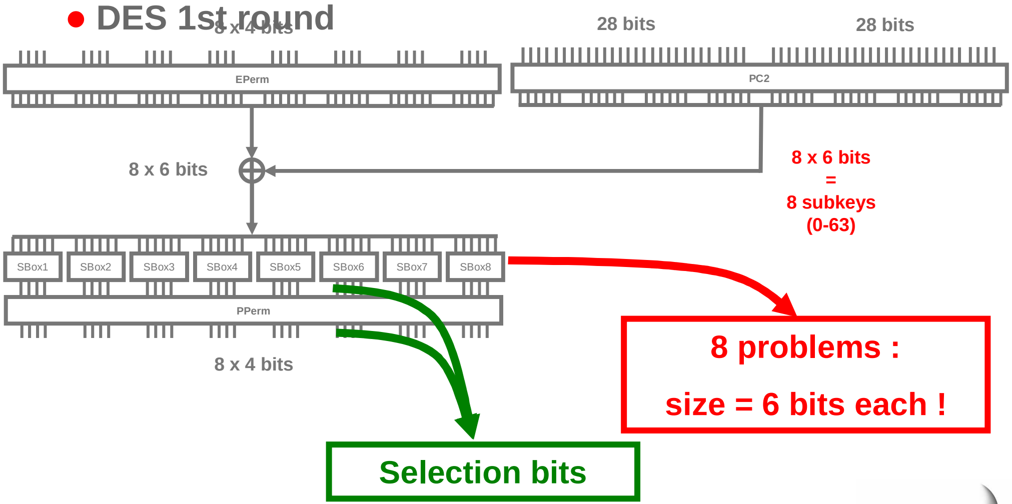
Classical target

- DES 1st round



Classical target

- DES 1st round



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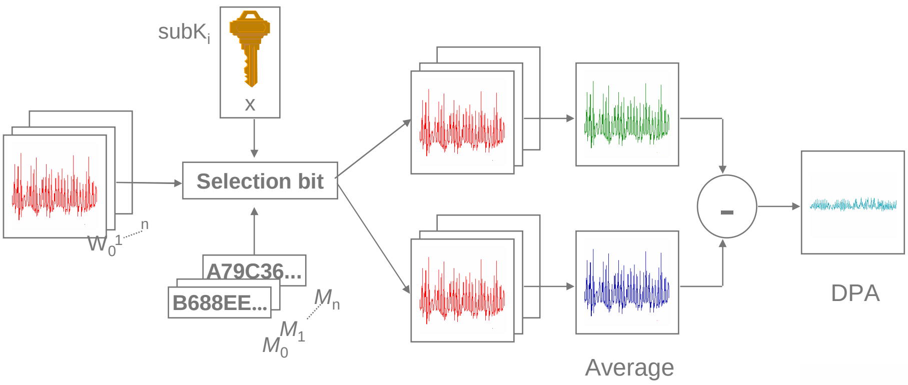
Hypothesis testing (guess)

- **GUESS** = value (0 to 63) of subK$_i$ (i = 1 to 8)
- Try 64 guesses for each subkey: test them with 64 DPA
- 48 key bits disclosed with 8 x 64 DPA (<< 2^{48} !)



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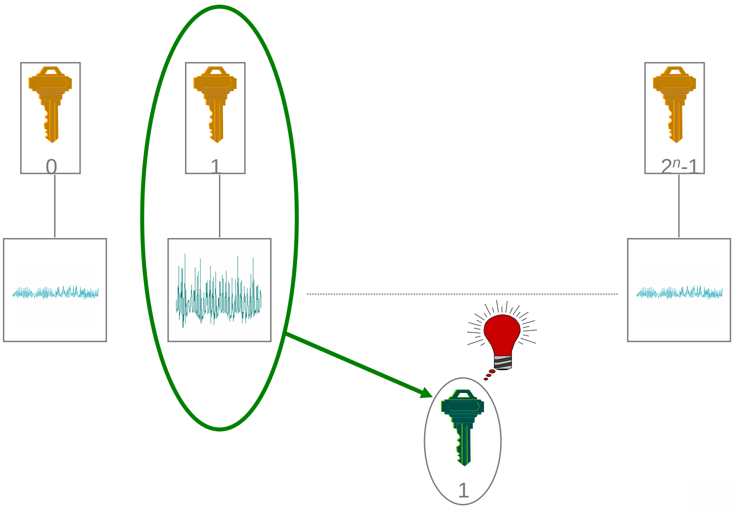
Hypothesis testing (guess)

- The right guess provides the highest spikes !



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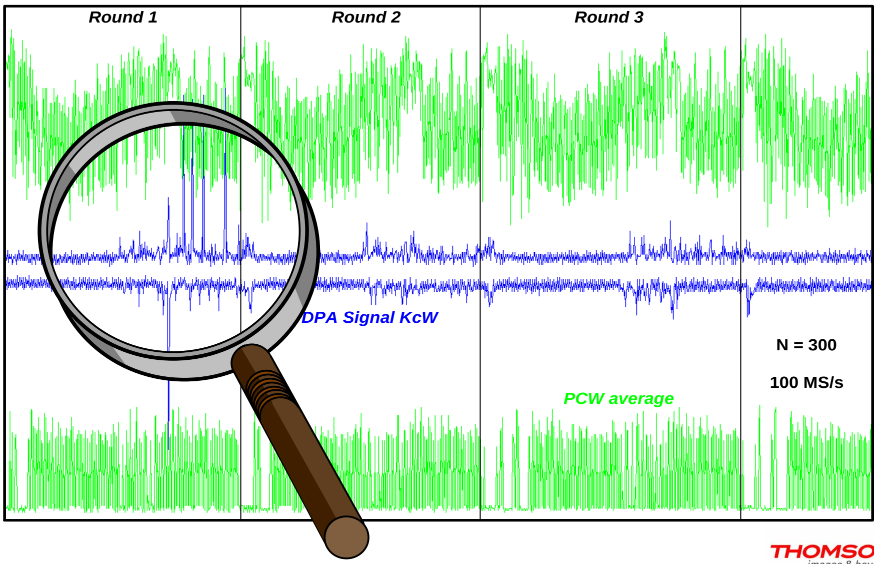
DES DPA signal type a



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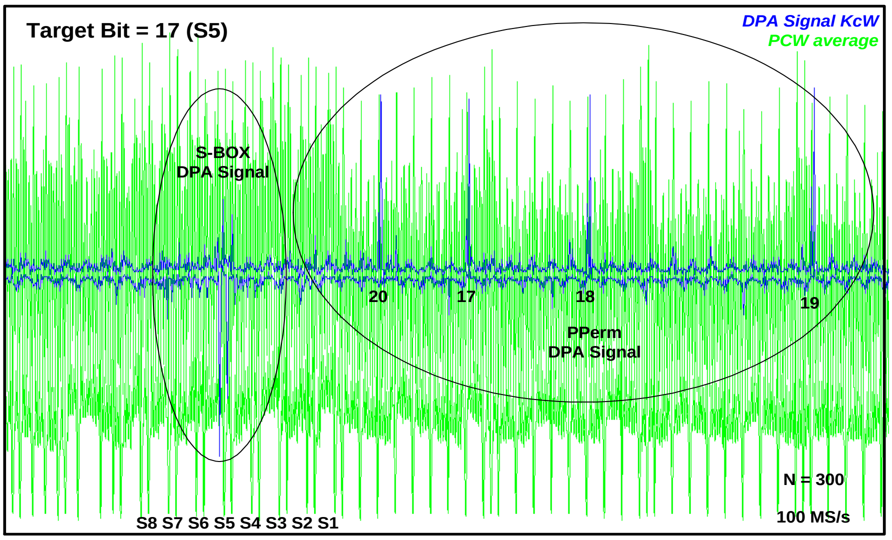
DES DPA signal type a



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Hypothesis testing (right guess)

- Why does the right guess provide the highest spikes!

Right guess

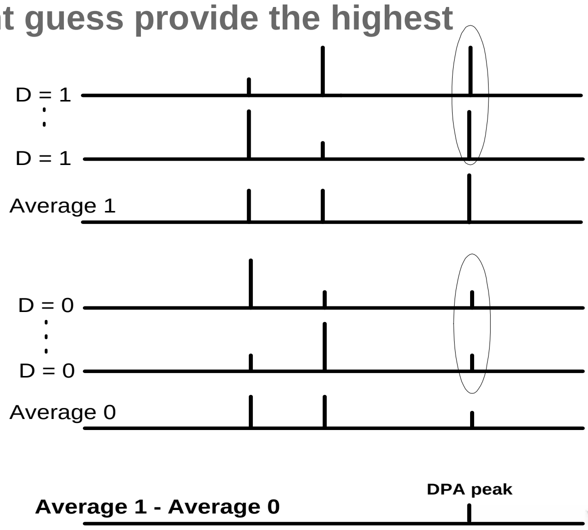
⇓

Exact prediction of the selection bit

0	B688EE57BB63E03E	1	1
1	185D04D77509F36F	0	0
2	C031A0392DC881E6	1	1
...			

Real

Predicted



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Hypothesis testing (wrong guess)

- Why do the wrong guesses provide no spike ?

Wrong guess

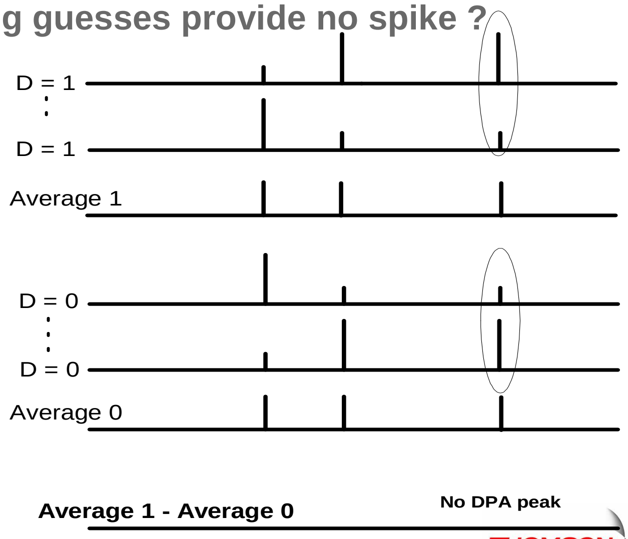
⇓

Wrong prediction of the selection bit

0	B688EE57BB63E03E	1	0
1	185D04D77509F36F	0	1
2	C031A0392DC881E6	1	1
…			

Real

Predicted



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Hypothesis testing

- Reality is not so easy because of
  - low contrast between the guesses
  - wrong guesses leading to higher spikes (wrong model)
- Decision consolidation : compare different equivalent selection bits (4 by SBox)
  - they do not agree necessarily !
- In the best case: 48 subkey bits are broken!
  - Inverse key schedule to recover the “plain” key bits
- 8 significant bits remain to be found
  - by exhaustive search (256 combinations)
  - or by DPA on 2^{nd} round

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Side-Channel Attacks on Cryptographic Tokens

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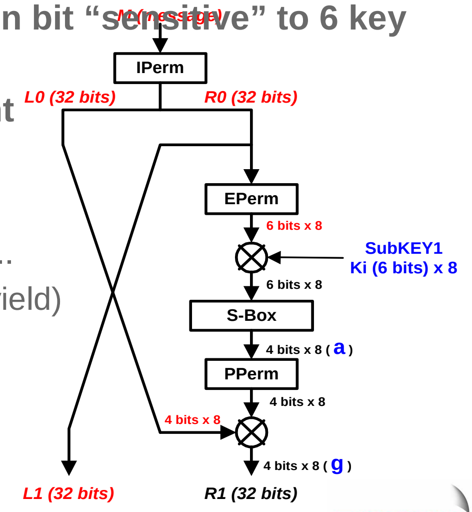
Cryptographic Engineering, Sept. 8-12, 2008, EPFL

Illustrations are courtesy of Gemplus (now Gemalto)

Part I:
Simple Power Analysis

Generalisation of DPA: Other targets

- SBoxes output : 1 selection bit “sensitive” to 6 key bits!
- PPerm output is equivalent
  - More calculation
- SBoxes intput ?
  - 1 selection bit “sensitive”...
  - … to 1 key bit only ! (low yield)



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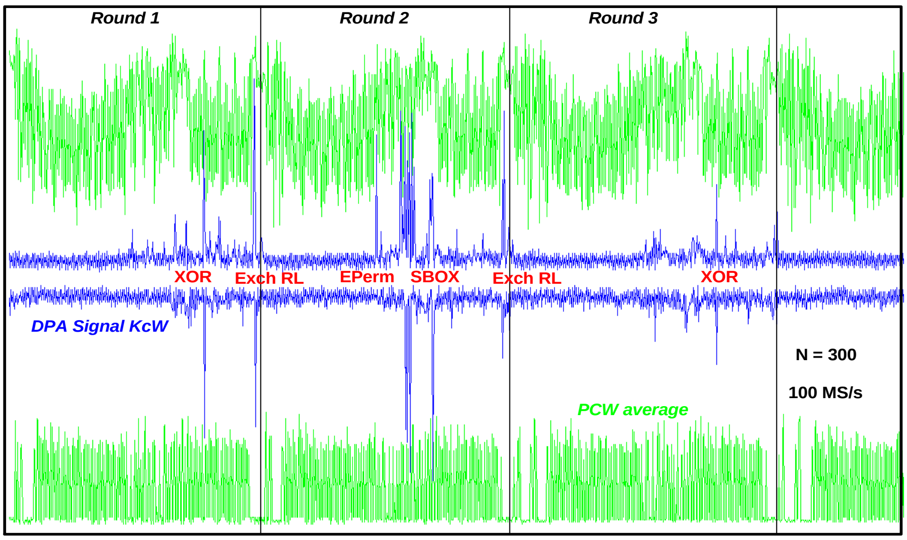
Other targets: DPA type g



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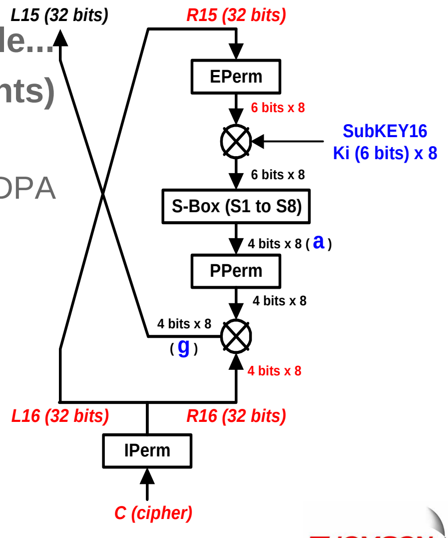
Generalisation of DPA: Other targets

- **If only cipher text is available… (3DES, application constraints)**
- **… do last round DPA !**
  - 16th round is symmetric for DPA
  - key schedule inversion is more complicated



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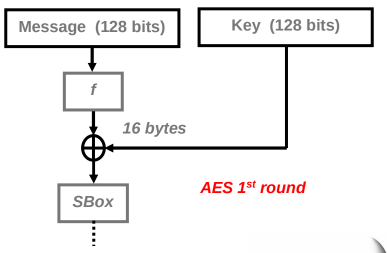
Generalisation to other algorithms

- **DPA on RSA**
  - The key is not entirely handled from the beginning, but progressively introduced
    - Prediction is to be done by time slices: next bit inference requires the previous bit to be broken
- **DPA on AES (Advanced Encryption Standard)**
  - Easier than on DES
  - But larger : 16 x 8 bits subkeys
  - => 16 x 256 guesses



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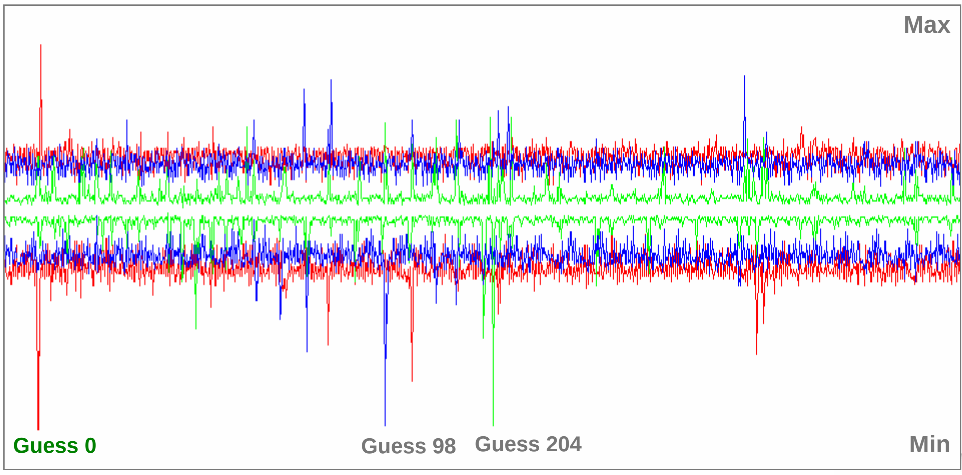
Generalisation to other algorithms

- DPA on AES : 1st round and 1st byte (right guess = 0)



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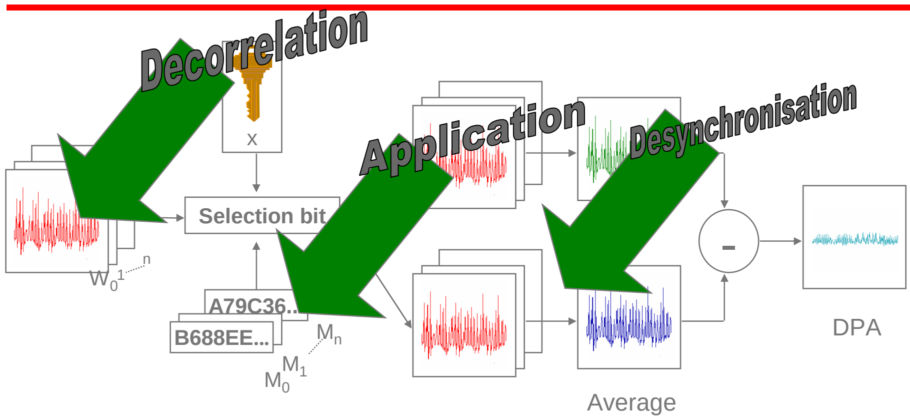
Conclusion: Countermeasures



- DPA is powerful, generic (to many algorithms) and robust (to model errors)…
- … but there are countermeasures!

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DPA countermeasures

- **Application countermeasures: make message free randomization impossible !**
  - Fix some message bytes
  - Constrain the variable bytes (ex: transaction counter)
- **Decorrelate power curves from data**
  - by hardware : current scramblers (additive noise)
  - by software : data whitening
- **Desynchronise the *N* executions**
  - software random delays
  - software random orders (ex: SBoxes in random order)
  - hardware wait states (dummy cycles randomly added by the CPU)
  - hardware unstable internal clock (phase shift)

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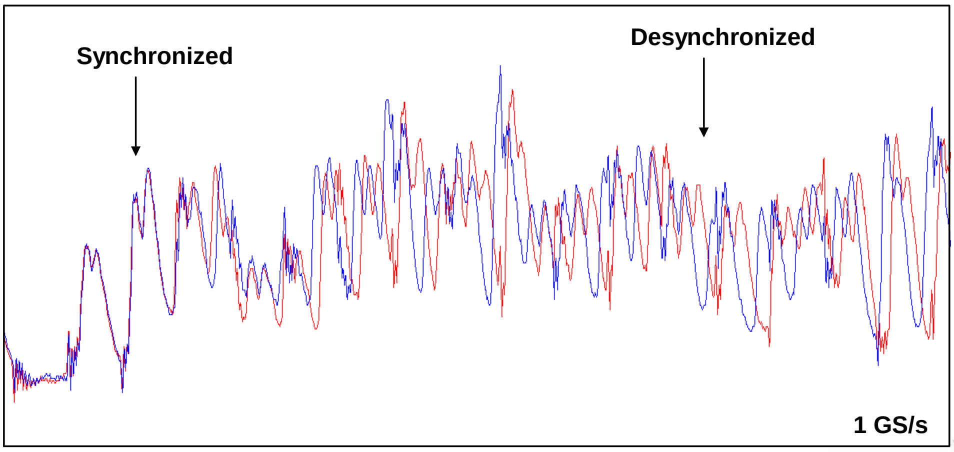
DPA countermeasures

- **Internal clock effects (phase shift)**



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Preventing Side-Channel Attacks

Application to RSA

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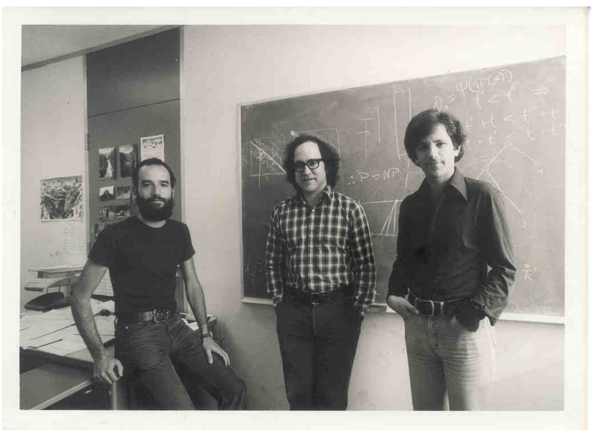
RSA Cryptosystem

- Invented by Ronald Rivest, Adi Shamir and Leonard Adleman in 1977



- Useful for [public-key] encryption and digital signature

RSA Primitive (1/3)

- $(\mathbb{Z}/N\mathbb{Z})^* = \{x \in [0, N[\mid \gcd(x, N) = 1\}$
  - [multiplicative] group
  - Euler totient function $\phi(N) := \#(\mathbb{Z}/N\mathbb{Z})^*$

Example

$(\mathbb{Z}/10\mathbb{Z})^* = \{1, 3, 7, 9\}$ and $\phi(10) = 4$

- Modular exponentiation:

$$(\mathbb{Z}/N\mathbb{Z})^* \times \mathbb{Z} \to (\mathbb{Z}/N\mathbb{Z})^*, \quad (x, e) \mapsto y = x^e \bmod N$$

  - permutation if $\gcd(e, \phi(N)) = 1$

Example

In $(\mathbb{Z}/10\mathbb{Z})^*$, $\{1, 3, 7, 9\} \mapsto \{1, 7, 3, 9\}$ for $e = 3$, and
$\{1, 3, 7, 9\} \mapsto \{1, 9, 9, 1\}$ for $e = 2$ $(\gcd(2, 4) \neq 1)$

RSA Primitive (2/3)

- RSA primitive = modular exponentiation

$$(\mathbb{Z}/N\mathbb{Z})^* \times \mathbb{Z} \to (\mathbb{Z}/N\mathbb{Z})^*, \quad (x, e) \mapsto y = x^e \bmod N$$

  - one-way, trapdoor function for an RSA modulus $N = pq$ where p, q are 512-bit primes

Definition (RSA Problem)

Given an RSA modulus N, $y \in (\mathbb{Z}/N\mathbb{Z})^*$ and an integer $e > 1$ with $\gcd(e, \phi(N)) = 1$, compute $x = y^{1/e} \bmod N$

Solution

$d = e^{-1} \bmod \phi(N)$ with $\phi(N) = (p-1)(q-1)$

$\implies x = y^d \bmod N$

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RSA Primitive (3/3)

- Key generation

  Input keylength k and e

  Output $N = pq$ such that $|N|_2 = k$ and $\gcd(e, \phi(N)) = 1$

  $d = e^{-1} \bmod \phi(N)$

  $pk = \{e, N\}$ and $sk = \{d\}$
- [Plain] RSA encryption

  Input message m and public key pk

  Output ciphertext $c = m^e \bmod N$
- [Plain] RSA decryption

  Input ciphertext c and private key sk

  Output message $m = c^d \bmod N$

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RSA Encryption in Practice

- Plain RSA encryption is insecure
  - encryption should be probabilistic
  - plain RSA is homomorphic

    $\Rightarrow$ e.g., "garbage-man-in-the-middle" attack
- RSA-OAEP
  - Optimal Asymmetric Encryption Padding

$$c = \mu_{\mathrm{OAEP}}(m, r)^e \bmod N \text{ for a random } r$$

  - proposed by Mihir Bellare and Phillip Rogaway in 1994
  - included in PKCS #1
  - highest security level (IND-CCA2) in the ROM

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RSA Signature in Practice (1/4)

- Plain RSA signature is universally forgeable
  $\Rightarrow$ messages should be "appropriately" padded
- RSA signature (with appendix)
  - setup: $N = pq$ with p, q prime
    (e, d) satisfying $e\, d \equiv 1 \pmod{\phi(N)}$
  - public parameters: $\{e, N\}$
  - private parameters: $\{d, N\}$

Signature on message m

$S = \dot{m}^d \bmod N$ where $\dot{m} = \mu(m)$

Verification

$S^e \stackrel{?}{\equiv} \mu(m) \pmod{N}$

RSA Signature in Practice (2/4)

- Deterministic paddings
  - RSA-FDH [Bellare and Rogaway, 1993]
    - Full Domain Hash

$$\mu(m) = H(m) \quad \text{with } H : \{0,1\}^* \to \mathbb{Z}/N\mathbb{Z}$$

    - highest security level (EUF-CMA) in the ROM
  - PKCS #1 v1.5 [RSA Labs]
- Probabilistic paddings
  - RSA-PSS [Bellare and Rogaway, 1996]
    - Probabilistic Signature Scheme

$$\mu(m) = \mu_{\mathrm{PSS}}(m, r) \text{ for a random } r$$

    - highest security level (EUF-CMA) in the ROM
    - tight security proof and can be with message recovery
  - PKCS #1 v2.1 [RSA Labs]

RSA Signature in Practice (3/4)

FDH padding (example)

Signature $S = \mu_{\mathrm{FDH}}(m)^d \bmod N$ with $\mu_{\mathrm{FDH}} : \{0,1\}^* \to \mathbb{Z}/N\mathbb{Z}$,

$$m \mapsto \mu_{\mathrm{FDH}}(m) = \underbrace{\overbrace{h(1\|m)}^{160\text{ bits}} \;\|\; \overbrace{h(2\|m)}^{160\text{ bits}} \;\|\; \ldots}_{1024\text{ bits}} \pmod N$$

where h is a cryptographic hash function (e.g., SHA-1)

RSA Signature in Practice (4/4)

PSS padding

Signature $S = \mu_{\mathrm{PSS}}(m, r)^d \bmod N$ where

$$\mu_{\mathrm{PSS}}(m, r) = 0\|w\|r^*\|g_2(w)$$

with $w = h(m, r)$ and $r^* = g_1(w) \oplus r$

- Signing
  1. pick a random r
  2. compute $w = h(m, r)$ and $r^* = g_1(w) \oplus r$
  3. return $S = [0\|w\|r^*\|g_2(w)]^d \bmod N$
- Verification
  1. compute $S^e \bmod N = b\|\bar{w}\|\rho\|\omega$
  2. compute $\bar{r} = g_1(\bar{w}) \oplus \rho$
  3. check whether (i) $b = 0$, (ii) $h(m, \bar{r}) = \bar{w}$, and (iii) $g_2(\bar{w}) = \omega$

Square-and-Multiply Algorithm (1/3)

- Square-and-multiply algorithm

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

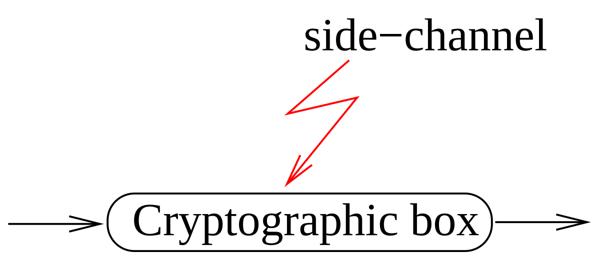
1. $R_0 \leftarrow 1$
2. For $i = k - 1$ downto 0 do
  - $R_0 \leftarrow {R_0}^2 \pmod N$
  - If $(d_i = 1)$ then $R_0 \leftarrow R_0\,\dot{m} \pmod N$

3. Return R_0

  – left-to-right exponentiation
  – $1 + 1$ temporary variables (R_0 and $\dot{m}$)
  – ...subject to SPA-type attacks

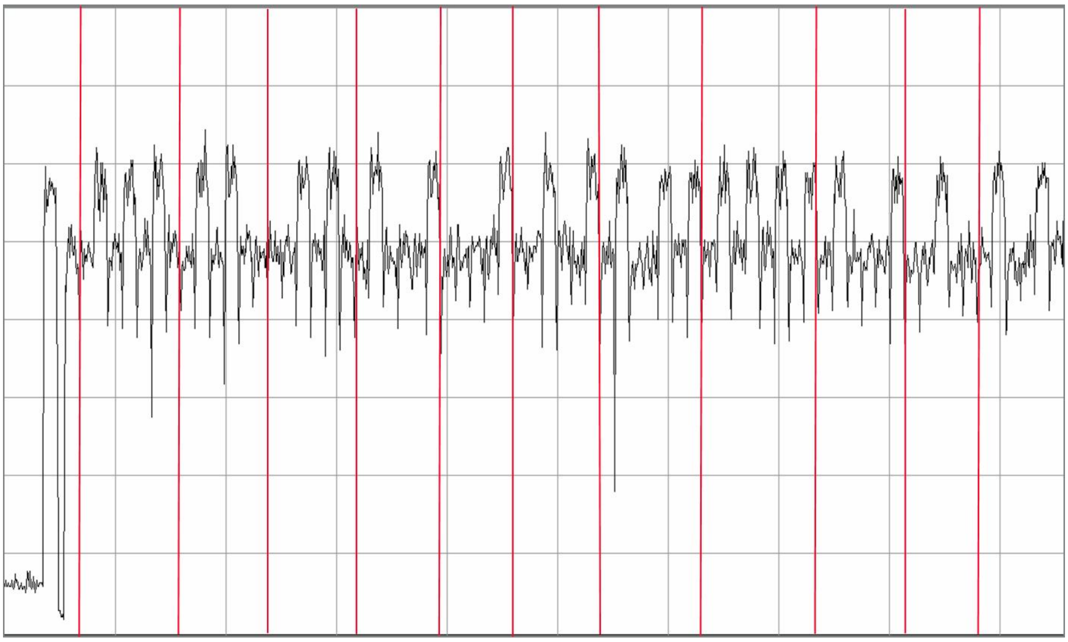
Square-and-Multiply Algorithm (2/3)

- Simple side-channel analysis



  – side-channel = timing, power consumption, ...

Square-and-Multiply Algorithm (3/3)



Key: $d = \texttt{2E C6 91 5B FE 4A}$

Square-and-Multiply-Always Algorithm

- Square-and-multiply-*always* algorithm

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1;\ R_1 \leftarrow 1$
2. For $i = k-1$ downto 0 do
  - $R_0 \leftarrow {R_0}^2 \pmod N$
  - $b \leftarrow 1 - d_i;\ R_b \leftarrow R_b\, \dot{m} \pmod N$

3. Return R_0

- when $b = 1$ (i.e., $d_i = 0$), there is a dummy multiplication
- the power trace now appears as a regular succession of squares and multiplies
- $2 + 1$ temporary variables (R_0, R_1 and $\dot{m}$)
- ...subject to safe-error attacks

Safe-Error Attacks

- Timely induce a fault into the ALU during the multiply operation at iteration i
- Check the output
  - if the result is incorrect (invalid signature or error notification) then the error was effective
    $\Rightarrow d_i = 1$
  - if the result is correct then the multiplication was dummy [safe error]
    $\Rightarrow d_i = 0$
- Re-iterate the attack for another value of i

Lesson

Protection against certain implementation attacks (e.g., SPA) may introduce new vulnerabilities

Montgomery Powering Ladder

- Montgomery exponentiation algorithm

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$
2. For $i = k-1$ downto 0 do
- $b \leftarrow 1 - d_i$; $R_b \leftarrow R_0\, R_1 \pmod N$
- $R_{d_i} \leftarrow {R_{d_i}}^2 \pmod N$

3. Return R_0

  - behaves regularly without dummy operations
  - only 2 temporary variables (R_0, R_1)

Summary

- Comparison

Algorithm	Temp. var.	# mult.
Square-and-multiply	$1+1$	$k+k/2$
Square-and-multiply-*always*	$2+1$	$k+k$
Montgomery ladder	2	$k+k$

- Side-channel atomicity
  - converts any crypto-algorithm into a protected algorithm with (virtually) no penalty

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Chapter 14, CRC Press, 1997

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IEEE Trans. Computers, **49**(9):967–970, 2000

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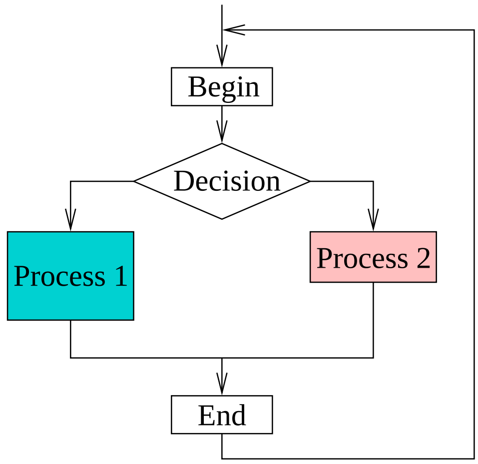
Speeding the Pollard and elliptic curve methods of factorization
Math. Comp. **48**(177):243–264, 1987

General Principle

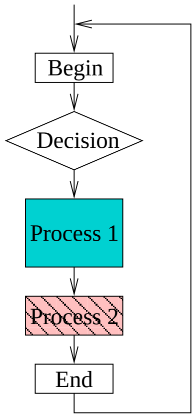
- Example of a crypto-algorithm



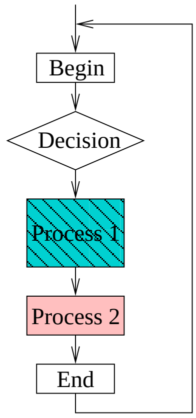
- Side-channel information
  - timing, power consumption, etc. . .

General Principle (1/3)

- Straightforward solution



. . . for executing Process 1



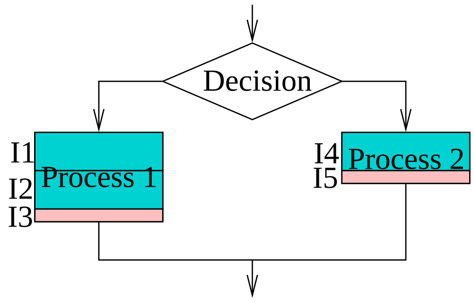
. . . for executing Process 2

provided that a fake execution is indistinguishable from a true execution!

General Principle (2/3)

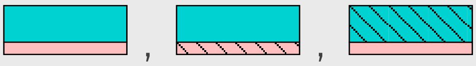
Side-channel atomicity

Refinement of the straightforward solution so that the running time is not (too much) penalized



Process 1 $= \mathrm{I1}\|\mathrm{I2}\|\mathrm{I3}$ and Process 2 $= \mathrm{I4}\|\mathrm{I5}$

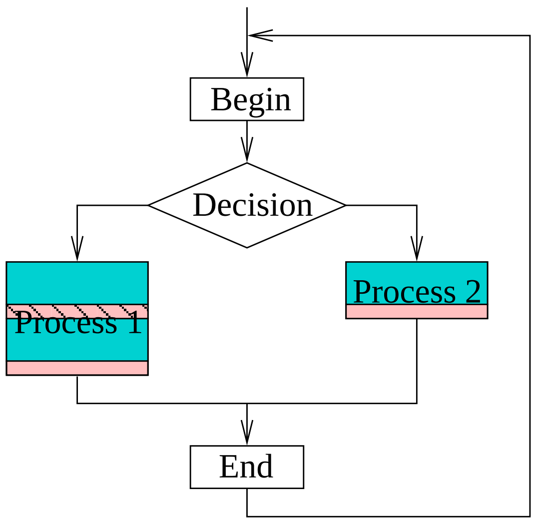
Common atomic block



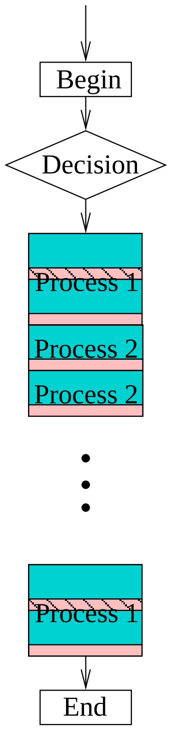
: $\mathrm{I1}\|\mathrm{I3}^{(\mathrm{fake})} \sim \mathrm{I2}\|\mathrm{I3} \sim \mathrm{I4}\|\mathrm{I5}$

General Principle (3/3)

- The whole crypto-algorithm



with chained blocks →

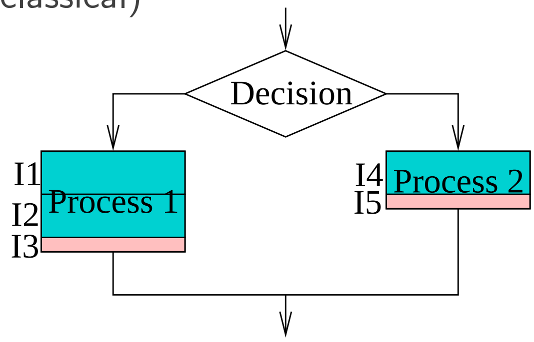


Atomic Square-and-Multiply (1/3)

- Application of the 'General Principle'
  - square-and-multiply algorithm (classical)

---

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow k-1$
2. While $(i \geqslant 0)$ do
   - $R_0 \leftarrow {R_0}^2 \pmod N$
   - If $(d_i = 1)$ then
     $R_0 \leftarrow R_0\, R_1 \pmod N$
   - $i \leftarrow i - 1$
3. Return R_0



Assumptions

- $[R_0 \leftarrow R_0\, R_0 \pmod N)] \sim [R_0 \leftarrow R_0\, R_1 \pmod N)]$
- $[i \leftarrow i - 1] \sim [i \leftarrow i - 0]$

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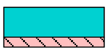
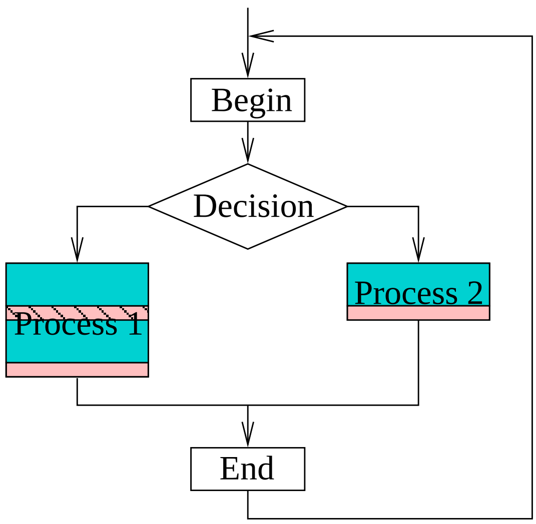
Atomic Square-and-Multiply (2/3)

---

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow k-1$
2. While $(i \geqslant 0)$ do
   Case
   $(d_i = 1)$: /* Process 1 */
   
   - $R_0 \leftarrow R_0\, R_0 \pmod N$
   - $i \leftarrow i - 0$
   
   - $R_0 \leftarrow R_0\, R_1 \pmod N$
   - $i \leftarrow i - 1$

   $(d_i = 0)$: /* Process 2 */
   
   - $R_0 \leftarrow R_0\, R_0 \pmod N$
   - $i \leftarrow i - 1$
3. Return R_0



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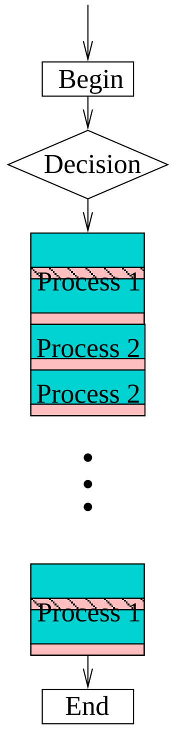
Atomic Square-and-Multiply (3/3)

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow k - 1$
2. While $(i \geqslant 0)$ do
   Case
   $(d_i = 1)$: /* Process 1 */
   - $R_0 \leftarrow R_0\, R_0 \pmod{N}$
   - $i \leftarrow i - 0$
   - $R_0 \leftarrow R_0\, R_1 \pmod{N}$
   - $i \leftarrow i - 1$

   $(d_i = 0)$: /* Process 2 */
   - $R_0 \leftarrow R_0\, R_0 \pmod{N}$
   - $i \leftarrow i - 1$
3. Return R_0



Chaining the blocks

Chaining the Blocks (1/4)

Methodology

1. Each process is divided into common atomic blocks
2. Each block inside a process receives a number r (in chronological order)
3. A bit s is used to keep track whether there remain blocks to be executed in the current process

The trick:

$$r \leftarrow (\neg s) \cdot (r + 1) + s \cdot f(\text{input values})$$

- $s = 0 \implies r \leftarrow r + 1$
- $s = 1 \implies r \leftarrow f(\text{input values})$

Chaining the Blocks (2/4)

- Square-and-multiply algorithm

		r	s
$(d_i = 1)$	$R_0 \leftarrow R_0\, R_0 \pmod{N};\ i \leftarrow i - 0$	0	0
	$R_0 \leftarrow R_0\, R_1 \pmod{N};\ i \leftarrow i - 1$	1	1
$(d_i = 0)$	$R_0 \leftarrow R_0\, R_0 \pmod{N}\ ;\ i \leftarrow i - 1$	2	1

- We can choose for the common atomic block ( )

$$R_0 \leftarrow R_0\, R_{(r \bmod 2)} \pmod{N}; i \leftarrow i - s$$

and for the update

$$\begin{cases} r = (\neg s) \cdot (r + 1) + s \cdot f(d_i) & \text{with } f(d_i) = 2 \cdot (\neg d_i) \\ s = (r \bmod 2) + (r \operatorname{div} 2) \end{cases}$$

Chaining the Blocks (3/4)

- Resulting algorithm

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1;\ R_1 \leftarrow \dot{m};\ i \leftarrow k - 1;\ s \leftarrow 1$
2. While $(i \geqslant 0)$ do
  - $r \leftarrow (\neg s) \cdot (r + 1) + s \cdot 2(\neg d_i)$
  - $s \leftarrow (r \bmod 2) + (r \operatorname{div} 2)$
  - $R_0 \leftarrow R_0 \cdot R_{(r \bmod 2)};\ i \leftarrow i - s$

3. Return R_0

  – after simplification. . .

Chaining the Blocks (4/4)

- Atomic square-and-multiply algorithm

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow k-1$; $b \leftarrow 0$
2. While $(i \geqslant 0)$ do
  - $R_0 \leftarrow R_0\, R_b \pmod N$
  - $b \leftarrow b \oplus d_i$; $i \leftarrow i - \neg b$

3. Return R_0

  - behaves regularly without dummy operations
  - only 2 temporary variables (R_0 and R_1)

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Further Atomic Algorithms (1/7)

- Right-to-left binary algorithm (classical)

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow 0$
2. While $(i \leqslant k-1)$ do
  - If $(d_i = 1)$ then $R_0 \leftarrow R_0\, R_1 \pmod N$
  - $R_1 \leftarrow {R_1}^2 \pmod N$; $i \leftarrow i + 1$

3. Return R_0

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Further Atomic Algorithms (2/7)

- Right-to-left binary algorithm (atomic)

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow 0$; $b \leftarrow 1$
2. While $(i \leqslant k-1)$ do
- $b \leftarrow b \oplus d_i$
- $R_b \leftarrow R_b R_1 \pmod{N}$; $i \leftarrow i + b$

3. Return R_0

Further Atomic Algorithms (3/7)

- ω-bit sliding window algorithm (classical)

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N, \omega > 1$
Output: $S = \dot{m}^d \bmod N$
Pre-comp.: $R_{j+1} \leftarrow \dot{m}^{2j+1} \pmod{N}$, $1 \leqslant j \leqslant 2^{\omega-1} - 1$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow k-1$
2. While $(i \geqslant 0)$ do
- If $(d_i = 0)$ then $R_0 \leftarrow {R_0}^2 \pmod{N}$; $i \leftarrow i - 1$
- Otherwise $(d_i \neq 0)$,
  1. find the longest string $(d_i, d_{i-1}, \ldots, d_\ell)_2$ s.t.
     (a) $i - \ell + 1 \leqslant \omega$ and (b) $e_\ell = 1$
  2. $j \leftarrow (d_i, d_{i-1}, \ldots, d_\ell)_2$
  3. $R_0 \leftarrow {R_0}^{2^{i-\ell+1}} \pmod{N}$; $i \leftarrow \ell - 1$

3. Return R_0

Further Atomic Algorithms (4/7)

- ω-bit sliding window algorithm (atomic)

Input:	$\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N, \omega > 1$
Output:	$S = \dot{m}^d \bmod N$
Pre-comp.:	$R_{j+1} \leftarrow \dot{m}^{2j+1} \pmod{N}$, $1 \leqslant j \leqslant 2^{\omega-1} - 1$

1. For $j = 1$ to $\omega - 1$ do $d_{-j} \leftarrow 0$
2. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $i \leftarrow k - 1$; $s \leftarrow 1$
3. While $(i \geqslant 0)$ do
   - $r \leftarrow (\neg s) \cdot (r + 1)$; $b \leftarrow 0$; $t \leftarrow 1$; $l \leftarrow \omega$; $u \leftarrow 0$
   - For $j = 1$ to ω do
     $b \leftarrow b \vee d_{i-\omega+j}$; $l \leftarrow l - (\neg b)$
     $u \leftarrow u + t \cdot d_{i-\omega+j}$; $t \leftarrow b \cdot (2t) + \neg b$
   - $l \leftarrow l \cdot d_i$; $u \leftarrow [(u + 1) \operatorname{div} 2] \cdot d_i$; $s \leftarrow (r = l)$
   - $R_0 \leftarrow R_0\, R_{u \cdot s} \pmod{N}$; $i \leftarrow i - u \cdot s - \neg d_i$
4. Return R_0

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Further Atomic Algorithms (5/7)

- (M, M^3) algorithm (classical)

Input:	$\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output:	$S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $R_2 \leftarrow \dot{m}^3 \pmod{N}$; $i \leftarrow k - 1$
2. While $(i \geqslant 0)$ do
   - $R_0 \leftarrow {R_0}^2 \pmod{N}$; $i \leftarrow i - 1$
   - If $(d_i = 1)$ then
     if $(d_{i-1} = 0)$ then $R_0 \leftarrow R_0\, R_1 \pmod{N}$
     else $(d_{i-1} = 1)$
       $R_0 \leftarrow {R_0}^2 \pmod{N}$; $R_0 \leftarrow R_0\, R_2 \pmod{N}$
       $i \leftarrow i - 1$
3. Return R_0

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Further Atomic Algorithms (6/7)

- (M, M^3) algorithm (atomic)

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1$; $R_1 \leftarrow \dot{m}$; $R_2 \leftarrow \dot{m}^3 \pmod{N}$; $i \leftarrow k-1$
2. While $(i \geqslant 0)$ do
   - $r \leftarrow (\neg s) \cdot (r+1)$; $s \leftarrow s \oplus d_i \oplus [d_{i-1} \wedge (r \bmod 2)]$
   - $R_0 \leftarrow R_0\, R_{r \cdot s} \pmod{N}$; $i \leftarrow i - r \cdot s - \neg d_i$
3. Return R_0

Further Atomic Algorithms (7/7)

- More involved algorithms
  - e.g., point multiplication on elliptic curves over $\mathbb{F}_p$
  - table-based methods apply

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Summary

- Side-channel atomicity
  - generic method to convert an algorithm into a SPA-protected algorithm
  - applies to a large variety of crypto-algorithms
  - can be combined with other techniques for preventing other classes of attacks

- Ex: atomic square-and-multiply algorithm
  - behaves regularly
  - complexity: $3k/2$ multiplications
    - as in the classical (i.e., unprotected) algorithm!

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Modes of Computation

- Setup:
  - $N = pq$ with p, q prime
  - (e, d) satisfying $e \cdot d \equiv 1 \pmod{\phi(N)}$
- Public parameters: $\{e, N\}$
- Private parameters:
  - standard mode: $\{d, N\}$
  - CRT mode: $\{p, q, d_p, d_q, i_q\}$ where $\begin{cases} d_p = d \bmod (p-1) \\ d_q = d \bmod (q-1) \\ i_q = q^{-1} \bmod p \end{cases}$

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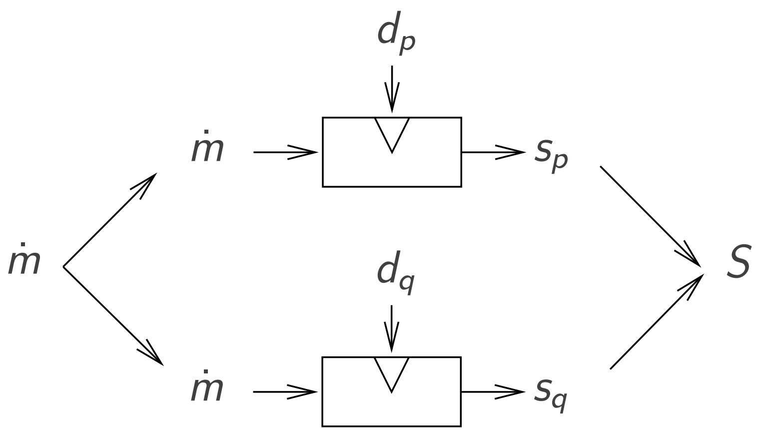
Chinese Remaindering

- Computation of a signature $S = \dot{m}^d \bmod N$ using CRT
  1. $s_p = \dot{m}^{d_p} \bmod p$
  2. $s_q = \dot{m}^{d_q} \bmod q$
  3. $S = \mathrm{CRT}(s_p, s_q) = s_q + q[i_q(s_p - s_q) \bmod p]$



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SPA-type Attacks

- Preventing SPA-like attacks
  - atomic algorithms
  - Montgomery ladder
  - ...
- Protections against SPA-type attacks are noit enough to thwart DPA-like attacks...

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DPA-type Attacks (1/3)

Recover secret d in the computation of $S = \dot{m}^d \bmod N$

- e.g., in the atomic square-and-multiply algorithm

Input: $\dot{m}, d = (d_{k-1}, \ldots, d_0)_2, N$
Output: $S = \dot{m}^d \bmod N$

1. $R_0 \leftarrow 1;\ R_1 \leftarrow \dot{m};\ i \leftarrow k-1;\ b \leftarrow 0$
2. While $(i \geqslant 0)$ do
- $R_0 \leftarrow R_0\, R_b \pmod N$
- $b \leftarrow b \oplus d_i;\ i \leftarrow i - \neg b$

3. Return R_0

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DPA-type Attacks (2/3)

- Let $d = (d_{k-1}, \ldots, d_0)_2$
- At step j, the attacker
  - already knows bits $d_{k-1}, d_{k-2}, \ldots, d_{j+1}$
  - guesses that next bit $d_j = 1$
  - chooses t [padded] messages $\dot{m}_1, \ldots, \dot{m}_t$ and computes

$$X_i = \dot{m}_i^{(d_{k-1}, d_{k-2}, \ldots, d_{j+1}, d_j)_2} \bmod N \quad \text{for } 1 \leqslant i \leqslant t$$

  - prepares two sets

$$\mathcal{S}_0 = \{\dot{m}_i \mid g(X_i) = 0\} \text{ and } \mathcal{S}_1 = \{\dot{m}_i \mid g(X_i) = 1\}$$

  - $\boxed{\text{if } \langle \mathcal{C}(i) \rangle_{\substack{1 \leqslant i \leqslant t \\ X_i \in \mathcal{S}_0}} - \langle \mathcal{C}(i) \rangle_{\substack{1 \leqslant i \leqslant t \\ X_i \in \mathcal{S}_1}} \begin{cases} \approx 0 & \text{then } d_j = 0 \\ \not\approx 0 & \text{then } d_j = 1 \end{cases}}$
- Iterate the attack to find $d_{j-1}, \ldots$

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DPA-type Attacks (3/3)

- The previous attack requires that
  1. the crypto device computes $S = \dot{m}^d \bmod N$ for known [padded] messages $\dot{m}$
     - does not apply to PSS-R (or OAEP in decryption)
  2. the attacker can evaluate

$$g(X_i) \quad \text{with } X_i = \dot{m}_i^{(d_{k-1}, \ldots, d_j)_2} \bmod N$$

Countermeasure

Randomize $\dot{m}$, d or N in the computation of $S = \dot{m}^d \bmod N$

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DPA-type Countermeasures (1/3)

Randomizing $\dot{m}$ (useless for probabilistic paddings)

- For a random r, compute
  1. $\dot{m}^* = r^e\,\dot{m} \bmod N$
  2. $S^* = (\dot{m}^*)^d \bmod N$
  3. $S = S^*\,r^{-1} \bmod N$
- If e is unknown, compute
  1. $\dot{m}^* = r\,\dot{m} \bmod N$
  2. $S^* = (\dot{m}^*)^d \bmod N$
  3. $S = S^*\,r^{-d} \bmod N$
- For a [short] random $r < 2^\ell$, compute
  1. $\dot{m}^* = \dot{m} + r\,N$ and $N^* = 2^\ell\,N$
  2. $S^* = (\dot{m}^*)^d \bmod N^*$
  3. $S = S^* \bmod N$

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DPA-type Countermeasures (2/3)

Randomizing d

- For a [short] random r, compute
  1. $d^* = d + r\,\phi(N)$
  2. $S = \dot{m}^{d^*} \bmod N$
- If $\phi(N)$ is unknown, compute
  1. $d^* = d + r(e\,d - 1)$
  2. $S = \dot{m}^{d^*} \bmod N$
- If e is unknown, for a random $r \in [0, d]$, compute
  1. $d^* = d - r$
  2. $S_1^* = \dot{m}^{d^*} \bmod N$ and $S_2^* = \dot{m}^r \bmod N$
  3. $S = S_1^*\,S_2^* \bmod N$

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DPA-type Countermeasures (3/3)

Randomizing N (combination with previous technique)

- For [short] randoms r_1 and $r_2 > r_1$, compute
  1. $\dot{m}^* = \dot{m} + r_1 N$ and $N^* = r_2 N$
  2. $S^* = (\dot{m}^*)^d \bmod N^*$
  3. $S = S^* \bmod N$

Fault Attacks

- Randomizing N also protects against fault attacks (when e is unknown)
  - For [short] randoms r_1 and $r_2 > r_1$, compute
    1. $\dot{m}^* = \dot{m} + r_1 N$ and $N^* = r_2 N$
    2. $S^* = (\dot{m}^*)^d \bmod N^*$
    3. $Y = (\dot{m}^*)^{d \bmod \phi(r_2)} \bmod r_2$
    4. $c = (S^* - Y + 1) \bmod r_2$
    5. $S = (S^*)^c \bmod N$
  - (when e is known, the correctness of the computation of S can be checked by verifying the validity of S)

Summary

How to implement the RSA

- Use a regular exponentiation algorithm to prevent SPA-like attacks
- Randomize the inputs to prevent DPA-like attacks
- Check the computations to prevent fault attacks

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Final Recommendations

- Consider side-channel attacks when implementing cryptographic routines
  - check that the countermeasures do not introduce new vulnerabilities
- Avoid decisional tests
- Randomize the execution
- Combine hardware and software protections
- Always prefer cryptographic standards