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Review article

A review on type-2 fuzzy logic applications in clustering, classification and pattern recognition

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ABSTRACT

In this paper a review of type-2 fuzzy logic applications in pattern recognition, classification and clustering problems is presented. Recently, type-2 fuzzy logic has gained popularity in a wide range of applications due to its ability to handle higher degrees of uncertainty. In particular, there have been recent applications of type-2 fuzzy logic in the fields of pattern recognition, classification and clustering, where it has helped improving results over type-1 fuzzy logic. In this paper a concise and representative review of the most successful applications of type-2 fuzzy logic in these fields is presented. At the moment, most of the applications in this review use interval type-2 fuzzy logic, which is easier to handle and less computational expensive than generalized type-2 fuzzy logic.

Contents

1. Introduction

The concept of information is inherently associated with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem-solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way. Uncertainty can be viewed as an attribute of information. The general framework of fuzzy reasoning allows handling much of this uncertainty and fuzzy systems can use type-1 fuzzy sets, which represent uncertainty by numbers in the range [0, 1]. When an entity is uncertain, like a measurement, it is difficult to determine its exact membership value, and of course type-1 fuzzy sets make more sense than sets. However, it is not reasonable to use an accurate membership function for something uncertain, so in this case what we is needed is another type of fuzzy sets, those which are able to handle these uncertainties, the so called type-2 fuzzy sets [14]. The amount of uncertainty in a system can be reduced by using type-2 fuzzy logic because this logic offers better capabilities to handle linguistic uncertainties by modeling vagueness and unreliability of information [5].

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Type-2 fuzzy models have emerged as an interesting generalization of fuzzy models based upon type-1 fuzzy sets. There have been a number of claims put forward as to the relevance of type-2 fuzzy sets being regarded as generic building constructs of fuzzy models [25]. Likewise, there is a record of some experimental evidence showing some improvements in terms of accuracy of fuzzy models of type-2 over their type-1 counterparts [5]. Unfortunately, no systematic and comprehensive design framework has been provided and while improvements over type-1 fuzzy models have been evidenced, it is not clear whether this effect could always be expected. Furthermore, it is not demonstrated whether the improvement is substantial enough and fully legitimized given the substantial optimization overhead associated with the design of this category of models. At this moment, most of type-2 fuzzy systems have been implemented as interval type-2 fuzzy systems, which are simpler and computationally less expensive. Basically, an interval type-2 fuzzy systems uses interval type-2 fuzzy sets, which assume a constant secondary membership degree and thus avoiding evaluating multiple degree values. There have been a lot of applications of interval type-2 fuzzy logic in intelligent control, pattern recognition, time series prediction, and others [5,14,25,26]. However, in this paper we will concentrate on applications in clustering, classification and pattern recognition.

The rest of the paper is structured as follows. Section 2 offers a brief overview of the basic concepts of type-2 fuzzy systems. Section 3 provides a concise review of type-2 fuzzy logic applications in clustering and classification. Section 4 presents a review of type-2 fuzzy logic applications in image processing and pattern recognition. Section 5 presents the future trend and direction in the area. Finally, Section 6 presents the conclusions.

2. Type-2 fuzzy systems

In this section, a brief overview of type-2 fuzzy systems is presented. This overview is intended to provide the basic concepts needed to understand the methods and algorithms presented later in the paper [10,13].

The structure of the type-2 fuzzy rules is the same as for the type-1 case because the distinction between type-2 and type-1 is associated with the nature of the membership functions [14]. Hence, the only difference is that now some or all the fuzzy sets involved in the rules are of type-2. In a type-1 fuzzy system, where the output sets are type-1 fuzzy sets, we perform defuzzification in order to get a number, which is in some sense a crisp (type-0) representative of the combined output sets. In the type-2 case, the output sets are of type-2; so we have to use extended versions of type-1 defuzzification methods [5].

If for a type-1 membership function, we blur it to the left and to the right, as illustrated in Fig. 1, then a type-2 membership function is produced. In this case, for a specific value x', the membership function (u'), takes on different values, which are not all weighted the same, so we can assign membership grades to all of those points.

By doing this for all $x \in X$, we form a three-dimensional membership function – a type-2 membership function – that characterizes a type-2 fuzzy set [13]. A type-2 fuzzy set $\tilde{A}$, is characterized by the membership function:

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \quad (1)$$

in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$. In fact $J_x \subseteq [0, 1]$ represents the primary membership of x, and $\mu_{\tilde{A}}(x, u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in [0, 1], the primary membership, and corresponding to each primary membership, there is a secondary membership (which can also be in [0, 1]) that defines the possibilities for the primary membership. Uncertainty is represented by a region, which is called the footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x, u) = 1, \forall u \in J_x \subseteq [0, 1]$ we have an interval type-2 membership function, as shown in Fig. 2. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\bar{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

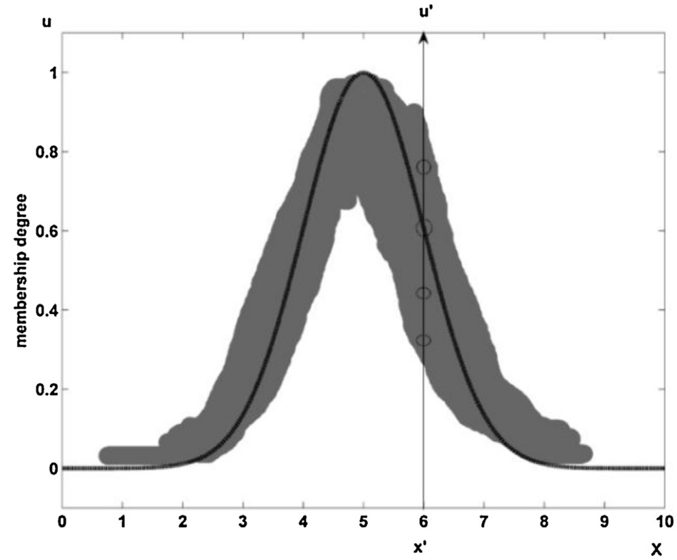

Fig. 1. Type-2 membership function as a blurred type-1 membership function.

An FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain, which are fully described by single numeric values. On the other hand, type-2 FLSs, are useful in circumstances where it is difficult to determine an exact numeric membership function, and there are measurement uncertainties.

A type-2 FLS is characterized by IF-THEN rules, where their antecedent or consequent sets are now of type-2. Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades such as when the training data is affected by noise. Similarly, to the type-1 FLS, a type-2 FLS includes a fuzzifier, a rule base, fuzzy inference engine, and an output processor, as we can see in Fig. 3 (in this case, a fuzzy system with two inputs and

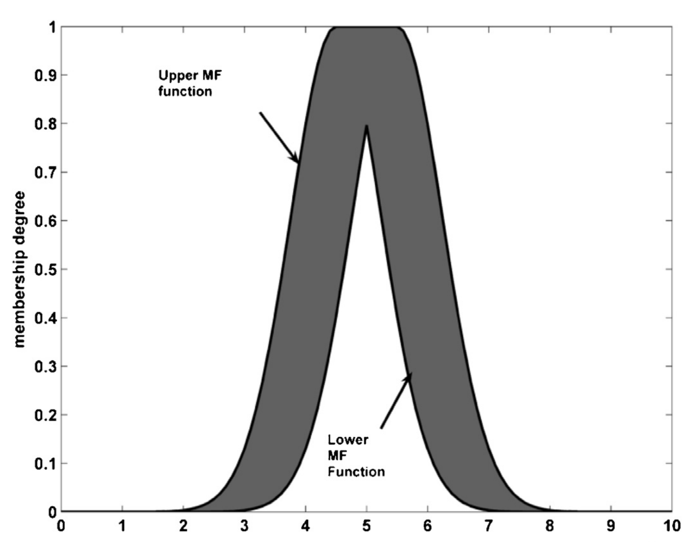

Fig. 2. Interval type-2 membership function.

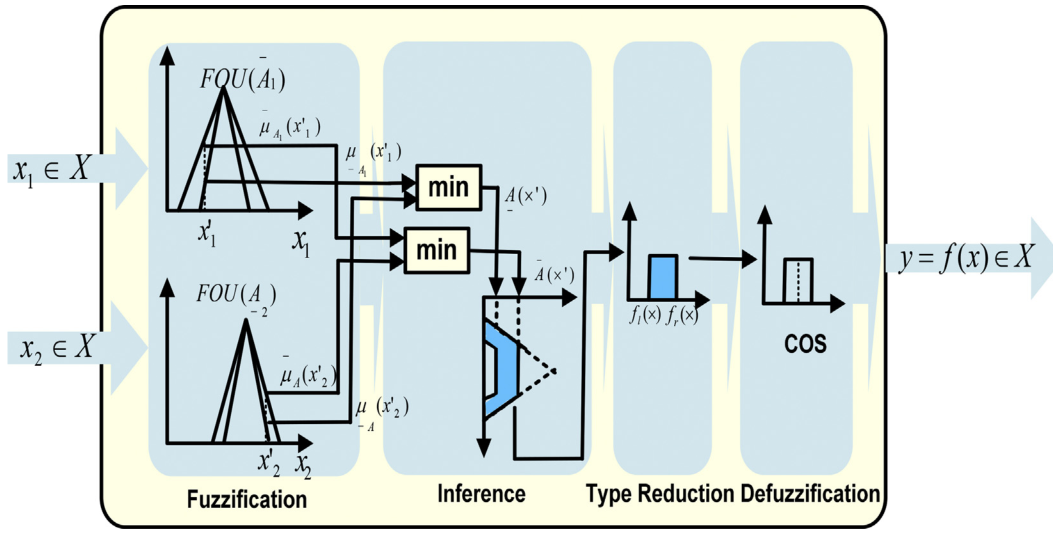

Fig. 3. Structure of a type-2 fuzzy logic system.

one output is used as illustration). The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (from the type-reducer) or a number (from the defuzzifier) [13]. Now we explain each of the blocks shown in Fig. 3.

2.1. Fuzzifier

The fuzzifier maps a numeric vector $\mathbf{x}=(x_1,\ldots,x_p)^{\mathrm{T}} \in X_1 \times X_2 \times \cdots \times X_p \equiv \mathbf{X}$ into a type-2 fuzzy set $\tilde{A}_x$ in $\mathbf{X}$ [13], an interval type-2 fuzzy set in this case. We use type-2 singleton fuzzifier, in a singleton fuzzification, the input fuzzy set has only a single point on nonzero membership. $\tilde{A}_x$ is a type-2 fuzzy singleton if $\mu_{\tilde{A}_x}(x) = 1/1$ for $\mathbf{x}=\mathbf{x}'$ and $\mu_{\tilde{A}_x}(x) = 1/0$ for all other $\mathbf{x} \neq \mathbf{x}'$.

2.2. Rules

The structure of rules in a type-1 FLS and a type-2 FLS is the same, but in the latter the antecedents and the consequents is represented by type-2 fuzzy sets. So for a type-2 FLS with p inputs $x_1 \in X_1,\ldots,x_p \in X_p$ and one output $y \in Y$, Multiple Input Single Output (MISO), if we assume there are M rules, the lth rule in the type-2 FLS can be written down as follows:

$$R^1: \quad \text{IF } x_1 \text{ is } \tilde{F}_1^l \text{ and} \cdots \text{and } x_p \text{ is } \tilde{F}_p^l, \quad \text{THEN } y \text{ is } \tilde{G}^l \quad l = 1, \ldots, M \tag{2}$$

2.3. Inference

In the type-2 FLS, the inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. It is necessary to compute the join $\sqcup$, (unions) and the meet $\prod$ (intersections), as well as the extended sup-star compositions (sup star compositions) of type-2 relations. If $\tilde{F}_1^l \times \cdots \times \tilde{F}_p^l = \tilde{A}^l$, (2) can be re-written as follows

$$R^l: \tilde{F}_1^l \times \cdots \times \tilde{F}_p^l \to \tilde{G}^l = \tilde{A}^l \to \tilde{G}^l \quad l = 1, \ldots, M \tag{3}$$

R^l is described by the membership function $\mu_{R^l}(\mathbf{x}, y) = \mu_{R^l}(x_1, \ldots, x_p, y)$, where

$$\mu_{R^l}(\mathbf{x}, y) = \mu_{\tilde{A}^l \to \tilde{G}^l}(\mathbf{x}, y) \tag{4}$$

can be written as [14]:

$$\begin{aligned}\mu_{R^l}(\mathbf{x}, y) &= \mu_{\tilde{A}^l \to \tilde{G}^l}(\mathbf{x}, y) = \mu_{\tilde{F}_1^l}(x_1) \prod \cdots \prod \mu_{\tilde{F}_p^l}(x_p) \prod \mu_{\tilde{G}^l}(y) \\ &= \left[\prod_{i=1}^{p} \mu_{\tilde{F}_i^l}(x_i)\right] \prod \mu_{\tilde{G}^l}(y)\end{aligned} \tag{5}$$

In general, the p-dimensional input to R^l is given by the type-2 fuzzy set $\tilde{A}_x$ whose membership function becomes

$$\mu_{\tilde{A}_x}(\mathbf{x}) = \mu_{\tilde{x}_1}(x_1) \prod \cdots \prod \mu_{\tilde{x}p}(x_p) = \prod_{i=1}^{p} \mu_{\tilde{x}i}(x_i) \tag{6}$$

where $\tilde{X}_i \quad (i = 1, \ldots, p)$ are the labels of the fuzzy sets describing the inputs. Each rule R^l determines a type-2 fuzzy set $\tilde{B}^l = \tilde{A}_x \circ R^l$ such that:

$$\mu_{\tilde{B}^l}(y) = \mu_{\tilde{A}_x \circ R^l} = \bigsqcup_{x \in \mathbf{X}} \left[\mu_{\tilde{A}_x}(\mathbf{x}) \prod \mu_{R^l}(\mathbf{x}, y)\right] \quad y \in Y \quad l = 1, \ldots, M \tag{7}$$

This dependency is the input/output relation shown in Fig. 3, which holds between the type-2 fuzzy set that activates a certain rule in the inference engine and the type-2 fuzzy set at the output of that engine.

In the FLS, we used interval type-2 fuzzy sets and intersection under product t-norm, so the result of the input and antecedent operations, which are contained in the firing set $\prod_{i=1}^{p} \mu_{\tilde{F}_{i_i}}(x'_i \equiv F^l(\mathbf{x}'))$, is an interval type-1 set,

$$F^l(\mathbf{x}') = [\underline{f}^l(\mathbf{x}'), \bar{f}^l(\mathbf{x}')] \equiv [\underline{f}^l, \bar{f}^l] \tag{8}$$

where

$$\underline{f}^l(\mathbf{x}') = \underline{\mu}_{\tilde{F}_1^l}(x'_1) * \cdots * \underline{\mu}_{\tilde{F}_p^l}(x'_p) \tag{9}$$

and

$$\bar{f}^l(\mathbf{x}') = \bar{\mu}_{\tilde{F}_1^l}(x'_1) * \cdots * \bar{\mu}_{\tilde{F}_p^l}(x'_p) \tag{10}$$

here * stands for the product operation.

2.4. Type reducer

The type-reducer generates a type-1 fuzzy set output, which is then converted in a numeric output through running the defuzzifier. This type-1 fuzzy set is also an interval set, for the case of our FLS we used center of sets (cos) type reduction, $Y_{\cos}$, which is expressed as

$$Y_{\cos}(\mathbf{x}) = [y_l, y_r] = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} \frac{1}{\sum_{i=1}^{M} f^i y^i / \sum_{i=1}^{M} f^i} \tag{11}$$

This interval set is determined by its two end points, y_l and y_r, which corresponds to the centroid of the type-2 interval consequent set $\tilde{G}^i$,

$$C_{\tilde{G}^i} = \int_{\theta_1 \in J_{y1}} \cdots \int_{\theta_N \in J_{yN}} \frac{1}{\sum_{i=1}^{N} y_i \theta_i / \sum_{i=1}^{N} \theta_i} = [y_l^i, y_r^i] \tag{12}$$

before the computation of $Y_{\cos}(\mathbf{x})$, we must evaluate Eq. (12), and its two end points, y_l and y_r. If the values of f_i and y_i that are associated with y_l are denoted f_l^i and y_l^i, respectively, and the values of f_i and y_i that are associated with y_r are denoted ${f_r}^i$ and ${y_r}^i$, respectively, from Eq. (13), we have

$$y_l = \frac{\sum_{i=1}^{M} f_l^i y_l^i}{\sum_{i=1}^{M} f_l^i} \tag{13}$$

$$y_r = \frac{\sum_{i=1}^{M} f_r^i y_r^i}{\sum_{i=1}^{M} f_r^i} \tag{14}$$

The values of y_l and y_r define the output interval of the type-2 fuzzy system, which can be used to verify if training or testing data are contained in the output of the fuzzy system.

2.5. Defuzzifier

From the type-reducer, we obtain an interval set $Y_{\cos}$, to defuzzify it we use the average of y_l and y_r, so the defuzzified output of an interval singleton type-2 FLS is

$$y(\mathbf{x}) = \frac{y_l + y_r}{2} \tag{15}$$

Eq. (15) produces a final crisp value output to the type-2 fuzzy system, that although it appears to be simply an average of two type-1 fuzzy systems, the output of (15) is more complex as described in [17].

3. Type-2 fuzzy logic in clustering and classification

In this section a representative account of the most successful applications of type-2 fuzzy logic in the fields of clustering and classification is presented. Type-2 fuzzy logic has been incorporated in methods of clustering and classification to allow handling higher levels of uncertainty in real world complex problems. In the applications presented in this section the superiority of type-2 over type-1 fuzzy logic has been shown to be significant. The applications considered in these papers are diverse, ranging from medicine to social sciences, which show the importance of the use of type-2 fuzzy logic for this kind of problems.

In the proposed approach of Sharma and Bajaj [41,42], an interval type-2 fuzzy system for vehicle classification is presented. The class is identified by checking the wheel base, ground clearance and body length of the vehicle, which are taken as axle distance, chassis height and body length, respectively. The problem of uncertainty and imperfection in the data was handled more effectively than with type-1 fuzzy or the adaptive neuro-fuzzy inference system (ANFIS).

In Hosseini et al. [12], a genetic type-2 fuzzy system for pattern recognition in computer aided detection systems is presented. A computer aided detection (CAD) system suffers from vagueness and imprecision in both medical science and image processing techniques. This work takes advantage of type-2 fuzzy sets as three-dimensional fuzzy sets with high potential for managing uncertainty issues in vague environments. Furthermore, the genetic algorithm is applied for tuning the MFs parameters and footprint of uncertainty. In order to assess the performance, the designed IT2FLSs are applied on a lung CAD application for classification of nodules. The results revealed that the Genetic IT2FLS classifier outperforms the equivalent type-1 FLS and is capable of capturing more degree of uncertainty.

In the proposed approach of Pimenta and Camargo [34], the design of interval type-2 fuzzy system classifiers using genetic algorithms is presented. An evolutionary architecture is proposed to generate the rule base and to optimize the membership functions of a type-2 fuzzy classification system. Some experiments using different datasets from the UCI Machine Learning Repository are presented in order to validate the proposed approach and to compare the results with the ones obtained with the Wang and Mendel method and a type-1 fuzzy classification system also generated by the proposed evolutionary architecture. The results demonstrated that the type-2 fuzzy classification system performed better than the other classifiers used in the study.

In the proposal of Chumklin et al. [8], an interval type-2 fuzzy system for micro-calcification detection in mammograms is presented. Mortality rate from this breast cancer is high and rapidly increasing. The detection at the earlier state can help to reduce the mortality rate. In this work, the application of interval type-2 fuzzy system with automatic membership function generation using the Possibilistic C-Means (PCM) clustering algorithm is presented. The results are compared with the ones from the interval type-2 fuzzy logic system with automatic membership function generation using the Fuzzy C-Means (FCM) clustering algorithm. The interval type-2 fuzzy system with PCM membership functions generation produced the best result.

In the approach of Sanz et al. [40], a genetic algorithm for tuning fuzzy ruled based classification systems is presented. In this work the concept of interval-valued fuzzy set was used to deal with this problem. The aim of this contribution was to show the improvement in the performance of linguistic fuzzy rule-based classification systems after the application of a cooperative tuning methodology between the tuning of the amplitude of the support and the lateral tuning applied to the linguistic labels modeled with interval-valued fuzzy sets.

In the proposal of Wu and Mendel [48], the classification of battlefield ground vehicles using type-2 fuzzy logic is presented. The uniqueness of the approach lies in the following. First, to facilitate prompt decision making, the acoustic features are extracted from short time (about 1 s) intervals in which the acoustic measurements can be assumed to be stationary. Second, the choice for the number of rules in the classifier is rationalized by the information inherent in the classification problem regarding the natural models of the vehicles and terrain conditions. And, third and finally, interval type-2 fuzzy rules are constructed to take advantage of the capabilities of interval type-2 fuzzy sets in modeling unknown time-variations and uncertainties.

In Phong and Thien [33], the classification of cardiac arrhythmias using interval type-2 Sugeno model is presented. This paper proposes a method to construct type-2 Takagi–Sugeno–Kang (TSK) fuzzy systems for electrocardiogram (ECG) arrhythmic classification. The method uses the fuzzy c-mean clustering algorithm and the back-propagation technique to find the parameters of the type-2 TSK fuzzy classifier. The generalized bell primary membership function was used to examine the performance of the classifier with different shapes of membership functions.

In the proposal of Yu et al. [50], the interval type-2 possibilistic c-means clustering and its applications to fuzzy modeling is presented. This paper describes a robust interval type-2 possibilistic c-means (IT2PCM) clustering algorithm, which is actually alternating cluster estimation, but membership functions are selected with interval type-2 fuzzy sets by the users. Excellent simulation results are obtained for the problems of classification and forecasting.

In the proposal of Santiago-Sanchez et al. [39], type-2 fuzzy sets are applied to the classification of cries from infants. Crying is an acoustic event that contains information about the functioning of the central nervous system, and the analysis of the infant's crying can be a support in the distinguishing diagnosis in cases like asphyxia and hyperbilirrubinemia. The classification of baby cry has been considered by the use of different types of neural networks and other recognition approaches. In this work a pattern classification algorithm based on type-2 fuzzy logic for the classification of infant cry is presented.

In the work of Herman et al. [10], a type-2 fuzzy logic system for a complex classification problem is presented. The practical applicability of brain–computer interface (BCI) technology is limited due to its insufficient reliability and robustness. One of the main problems in this regard is the extensive variability and inconsistency of brain signal patterns, observed especially in electroencephalogram (EEG). This work presents a fuzzy logic (FL) approach to the problem of handling of the resultant uncertainty effects. In particular, it outlines the design of a novel type-2 FL system (T2FLS) classifier within the framework of an EEG-based BCI.

In Chua and Tan [7], genetic evolved fuzzy classifiers for the problem of automotive classification are presented. This work was aimed at investigating if a type-2 fuzzy classifier can deliver a better performance when there exists an imprecise decision boundary caused by improper feature extraction method. Genetic Algorithm (GA) was used to tune the fuzzy classifiers under the Pittsburgh scheme. The proposed fuzzy classifiers have been successfully applied to an automotive application whereby the classifier needs to detect the presence of humans in a vehicle. Results reveal that the type-2 classifier has the edge over type-1 classifier when the decision boundaries are imprecise and the fuzzy classifier itself has not enough degrees of freedom to construct a suitable boundary.

In the approach of Lucas et al. [23], general type-2 fuzzy classifiers for land cover classification are presented. This work proposes a fuzzy classifier based on type-2 fuzzy sets to be applied in land cover classification. The classifier was built from the available data and considers the merging of information acquired from different experts. The new method proposed to design the classifier as well as the use of general type-2 fuzzy sets allows the modeling of input–output relations and minimize the effects of uncertainties in the usual fuzzy rule-based classifiers. The experiments carried out attest the efficiency of the proposed general type-2 fuzzy classifier.

In Starczewski et al. [43], modular type-2 neuro fuzzy systems are proposed. In this work a modular system, which can be converted into a type-2 neuro-fuzzy system was studied. The rule base of such system uses triangular type-2 fuzzy sets. The modular structure is trained using the backpropagation method combined with the AdaBoost algorithm. By applying the type-2 neuro-fuzzy system, the modular structure is converted into a compressed form. This allows overcoming the training problem of type-2 neuro-fuzzy systems. An illustrative example was given to show the efficiency of the proposed approach in problems of classification.

In Chen et al. [6], type-2 fuzzy logic based classifier fusion for support vector machines is presented. This paper proposes a fuzzy fusion model to combine multiple SVMs classifiers. To better handle uncertainties existing in real classification data and in the membership functions (MFs) in the traditional type-1 fuzzy logic system (FLS), interval type-2 fuzzy sets were applied to construct a type-2 SVMs fusion FLS. This type-2 fusion architecture takes considerations of the classification results from individual SVMs classifiers and generates the combined classification decision as the output. The experiments also show that the type-2 fuzzy logic-based SVMs fusion model is better than the type-1 based SVM fusion model in general.

In Aliev et al. [4], type-2 fuzzy neural networks with fuzzy clustering and differential evolution optimization are presented. An interesting alternative is to employ type-2 fuzzy sets, which augment fuzzy models with expressive power to develop models, which efficiently capture the factor of uncertainty. Type-2 fuzzy logic systems developed with the aid of evolutionary optimization forms a useful modeling tool subsequently resulting in a collection of efficient "If-Then" rules. The type-2 fuzzy neural networks take advantage of capabilities of fuzzy clustering by generating type-2 fuzzy rule base, resulting in a small number of rules and then optimizing membership functions of type-2 fuzzy sets present in the antecedent and consequent parts of the rules.

In the proposal of Abiyev et al. [2], a type-2 neuro-fuzzy system based on clustering was applied to system identification and channel equalization. This work describes the development of a novel type-2 neuro-fuzzy system for identification of time-varying systems and equalization of time-varying channels using clustering and gradient algorithms. It combines the advantages of type-2 fuzzy systems and neural networks. The type-2 fuzzy system allows handling the uncertainties associated with information or data in the knowledge base of the process. The proposed structure is used for identification and noise equalization of time-varying systems.

In Zheng et al. [53], a similarity measure between type-2 fuzzy sets was presented. The fuzzy similarity measure provides the similar degree of two fuzzy sets (FSs) and can be used in various areas. There are numerous studies as to it on type-1 fuzzy sets (T1 FSs), but little attention has been received on type-2 fuzzy sets (T2 FSs). In this work, a new similarity measure between interval type-2 fuzzy sets (IT2 FSs) is proposed. Several examples are presented to explain its calculation and combine it with Yang and Shih's clustering method for an application of clustering analysis of Gaussian IT2 FSs.

In Abiyev and Kaynak [1], a type-2 fuzzy neural structure for identification and control of time varying plants is presented. In such situations, the use of fuzzy approaches becomes a viable alternative. However, the systems constructed on the base of type 1 fuzzy systems cannot directly handle the uncertainties associated with information or data in the knowledge base of the process. In this paper, the structure of a type 2 Takagi–Sugeno–Kang fuzzy neural system is presented, and its parameter update rule was derived based on fuzzy clustering and gradient learning algorithm.

In the proposal of Zheng et al. [54], a similarity measure between general type-2 fuzzy sets and its application in clustering is proposed. The similarity measure between fuzzy sets is an important concept in fuzzy set theory, but little work as to it has been done on type-2 fuzzy sets. For that, a new similarity measure between general type-2 fuzzy sets was proposed in this paper.

In Qin et al. [35], sea surface clustering based on type-2 fuzzy theory was presented. Spatial data clustering is an effective method to find interesting spatio-temporal clustering patterns. There are many uncertainties in sea surface temperature (SST) clustering, so clustering methods with uncertainty must be used. Type-2 fuzzy theory takes into account the uncertainty of the membership grade while fuzzy c means (FCM) not. Based on the analysis of interval type-2 fuzzy c means (IT2FCM), the paper utilizes two normal cloud models to express fuzzifiers m_1 and m_2, and uses two cloud drops to substitute them. The paper applies the improved IT2FCM into global SST clustering, and discovers some interesting climate patterns.

In Albarracin and Melgarejo [3], an approach for channel equalization based on quasi type-2 fuzzy systems is presented. Basically, the quasi-type 2 fuzzy equalizer is tuned by clustering the output of the channel as it is proposed in previous reported works for other fuzzy equalizers. The proposed equalizer was compared with type-1 and interval type-2 equalizers. Although, simulation results show that the quasi type-2 fuzzy adaptive filter exhibits better performance for particular levels of uncertainty, it behaves similarly to the other equalizers in general terms.

In Ozkan and Turksen [31], a cluster validity index for type-2 fuzziness is proposed. Upper and lower values of the level of fuzziness for Fuzzy C-Mean (FCM) clustering methodology have been found as 2.6 and 1.4, respectively, in previous studies. This work concentrates on the usage of uncertainty associated with the level of fuzziness in determination of the number of clusters in FCM in any data set. A MiniMax ε-stable cluster validity index based on the uncertainty associated with the level of fuzziness within the framework of Interval Valued Type 2 fuzziness was proposed.

In Pedrycz [32], human centricity in computing with fuzzy sets is presented. The intent of this study was to investigate the capabilities of granular computing that are available in the currently existing framework to support the design of human-centric systems. Type-2 fuzzy sets can emerge as a result of linguistic interpretation of the original numeric membership grades. The study brings forward a detailed algorithmic framework leading to the determination of type-2 fuzzy sets: in the case of aggregation, the principle of justifiable granularity is a computational vehicle while in case of linguistic interpretation a certain optimization scheme minimizing entropy which associates with the interpretation of membership functions through a limited codebook of linguistic labels was introduced.

In Juang et al. [13], a recurrent self evolving interval type-2 fuzzy neural network for dynamic system processing is presented. This work proposes a recurrent self-evolving interval type-2 fuzzy neural network (RSEIT2FNN) for dynamic system processing. An RSEIT2FNN incorporates type-2 fuzzy sets in a recurrent neural fuzzy system in order to increase the noise resistance of a system. The antecedent parts in each recurrent fuzzy rule in the RSEIT2FNN are interval type-2 fuzzy sets, and the consequent part is of the Takagi–Sugeno–Kang (TSK) type with interval weights. The RSEIT2FNN initially contains no rules; all rules are learned online via structure and parameter learning. The RSEIT2FNN was applied to simulations of dynamic system identifications and chaotic signal prediction under both noise-free and noisy conditions.

In a paper by Türkşen [46], a review of fuzzy systems models with an emphasis on fuzzy functions is presented. In this paper, type 1 fuzzy system models known as Zadeh, Takagi–Sugeno and Turkşen models are first reviewed; then potentially future realizations of type 2 fuzzy systems again under the headings of Zadeh, Takagi–Sugeno and Turkşen fuzzy system models, in contrast to type 1 fuzzy system models are presented. Type 2 fuzzy system models have a higher predictive power. In data-driven FSM methods discussed here, a fuzzy c-means (FCM) clustering algorithm is used in order to identify the system structure, i.e., either the number of fuzzy rules or alternately the number of FFs.

In Ren et al. [37], high order type-2 Sugeno fuzzy models are presented. This work presents the generalized type-2 Takagi–Sugeno–Kang (TSK) fuzzy logic system (FLS) in which the antecedent or consequent membership functions are type-2 fuzzy sets and the consequent part a first or higher order polynomial function. The architecture of the generalized type-2 TSK FLS and its inference engine are based on the Mendel's first order type-2 TSK FLS. The design method of high order system is an extension of the subtractive clustering based type-2 TSK FLS identification algorithm.

In Zhang et al. [52], rule extraction of interval type-2 fuzzy logic systems based on fuzzy c means clustering is presented. An improved clustering algorithm is proposed in this work, which originates from Fuzzy c-Means Clustering (FCM). FCM is one of the algorithms commonly used to extract fuzzy rules from type-1 fuzzy logic system. However, its application is merely limited to dots set. This deficiency is improved in the new algorithm, Interval Fuzzy c-Means Clustering (IFCM), which is adequate to deal with interval sets. The enhanced algorithm was based on a new definition of distance between interval data. This work also focuses on extracting fuzzy rules from interval type-2 fuzzy systems.

In Tan et al. [44], a type-2 fuzzy system for ECG arrhythmic classification is presented. This work is aimed at assessing the feasibility of using a type-2 fuzzy system for ECG arrhythmic beat classification. Three types of ECG signals, namely the normal sinus rhythm (NSR), ventricular fibrillation (VF) and ventricular tachycardia (VT), are considered. Using a combination of the fuzzy c-means clustering algorithm and the amount of dispersion in each cluster, a method for designing the antecedent type-2 MFs of the classifier from a training data set is formulated.

In Qun et al. [36], type-2 Sugeno fuzzy logic system using subtractive clustering is presented. In this work, a subtractive clustering identification algorithm is introduced to model type-2 Takagi–Sugeno–Kang (TSK) fuzzy logic systems (FLS). The type-2 TSK FLS identification algorithm is an extension of the type-1 TSK FLS modeling algorithm. In the type-2 algorithm, subtractive clustering method is combined with least squares estimation algorithms to pre-identify a type-1 FLS form input/output data. Fuzzy modeling of type-2 TSK FLS is found to be more effective than that of type-1 TSK FLS.

In Liang and Mendel [17,18], the problem of overcoming time-varying co-channel interference type-2 fuzzy adaptive filters is solved. This work presented a method for overcoming time-varying co-channel interference (CCI) using type-2 fuzzy adaptive filters (FAF). The type-2 FAF is realized using an un-normalized type-2 Takagi–Sugeno–Kang fuzzy logic system. A clustering method is used to adaptively design the parameters of the FAF. Simulation results show that the equalizers based on type-2 FAFs perform better than the nearest neighbor classifiers or the equalizers based on type-1 FAFs when the number of co-channels is much large than 1.

In Table 1 a summary of the contributions where type-2 fuzzy systems have been used in clustering and classification is presented. The contributions are organized based on several factors; so that the reader can better appreciate what has been presented in these papers. The comparison shown in Table 1 is based on the domain of the problem where type-2 fuzzy logic has been applied, another attribute is to mention if a comparison with type-1 fuzzy logic is provided, and finally another attribute is to mention why type-2 fuzzy logic was used by the authors for solving the particular problem that was considered in their work.

4. Type-2 fuzzy logic in pattern recognition

In this section a representative account of the most successful applications of type-2 fuzzy logic in the field of pattern recognition is presented. Type-2 fuzzy logic has been incorporated in methods of pattern recognition to allow handling higher levels of uncertainty in complex recognition problems. In the applications presented in this section the superiority of type-2 over type-1 fuzzy logic has been shown to be significant. The diversity of applications, range from face recognition to tumor recognition in medicine.

In Hosseini et al. [12], a genetic type-2 fuzzy logic system for pattern recognition in computer aided detection systems is presented. A computer aided detection (CAD) system suffers from vagueness and imprecision in both medical science and image processing techniques. In this paper, an automatic optimized approach for generating and tuning type-2 Gaussian membership function (MF) parameters and their footprint of uncertainty is proposed. In this approach, two interval type-2 fuzzy logic system (IT2FLS) methods

Table 1
Type-2 fuzzy systems in clustering and classification.

Author(s) (pub. year)	Ref. no.	Domain of the problem	Comparison with type-1	Why type-2 is required for the problem?
Sharma and Bajaj (2009 and 2010)	[41,42]	Vehicle classification	Yes	Uncertainty and imperfection of data
Hosseini et al. (2010)	[12]	Computer aided detection	Yes	Imprecision in image processing
Pimenta and Camargo (2010)	[34]	Classification	Yes	Imprecision in classification
Chumklin et al. (2010)	[8]	Cancer detection	Yes	Uncertainty in medical classification
Sanz et al. (2010)	[40]	Classification	No	Imprecision in classification
Wu and Mendel (2010)	[48]	Vehicle classification	Yes	Uncertainty in information
Phong and Thien (2009)	[33]	Arrhythmia classification	No	Uncertainty in medical classification
Yu et al. (2009)	[50]	Classification and forecasting	No	Uncertainty in data
Santiago-Sanchez et al. (2009)	[39]	Classification of infant cries	No	Uncertainty in medical classification
Herman et al. (2008)	[10]	Brain signal classification	No	Uncertainty in medical classification
Chua and Tan (2008)	[7]	Automotive classification	Yes	Uncertainty in classification
Lucas et al. (2008)	[23]	Land cover classification	No	Uncertainty in classification
Starczewski et al. (2008)	[43]	Classification	No	Uncertainty in classification
Chen et al. (2008)	[6]	Classification	Yes	Uncertainty in classification
Aliev et al. (2011)	[4]	Clustering	Yes	Uncertainty in clustering
Abiyev et al. (2011)	[2]	Clustering	No	Uncertainty in clustering
Zheng et al. (2010)	[53]	Classification	No	Uncertainty in classification
Abiyev and Kaynak (2010)	[1]	Clustering	No	Uncertainty in clustering
Zheng et al. (2010)	[54]	Clustering	No	Uncertainty in clustering
Qin et al. (2010)	[35]	Climate pattern clustering	No	Uncertainty in clustering
Albarracin and Melgarejo (2010)	[3]	Signal clustering	Yes	Uncertainty in clustering
Ozkan and Turksen (2010)	[31]	Clustering	No	Uncertainty in clustering
Pedrycz (2010)	[32]	Clustering	Yes	Uncertainty in granulation
Juang et al. (2009)	[13]	Clustering	Yes	Uncertainty in clustering
Türkşen (2009)	[46]	Clustering	Yes	Uncertainty in clustering
Ren et al. (2010)	[37]	Clustering	No	Uncertainty in clustering
Zhang et al. (2007)	[52]	Clustering	No	Uncertainty in clustering
Tan et al. (2007)	[44]	Classification	No	Uncertainty in classification
Qun et al. (2010)	[36]	Clustering	Yes	Uncertainty in clustering
Liang and Mendel (2000)	[17,18]	Classification	Yes	Uncertainty in classification

based on the Mamdani rules model are presented for tackling the uncertainty issues in classification problems in pattern recognition. Furthermore, the genetic algorithm is applied for tuning of the MFs parameters and footprint of uncertainty. The results reveal that the Genetic IT2FLS classifier outperforms the equivalent type-1 FLS and is capable of capturing more uncertainties.

In Melin [26], interval type-2 fuzzy logic applications in image processing and pattern recognition are presented. Interval type-2 fuzzy logic is applied to perform image processing and pattern recognition. In this work a new type-2 fuzzy logic method is applied for edge detection in images and the results are compared with three different traditional techniques for the same goal with the type-2 edge detection outperforming the other techniques.

In Lopez et al. [22], a comparative study of feature extraction methods of type-1 and type-2 fuzzy logic for pattern recognition is presented. In this work a new approach for features extraction methods with type-1 and type-2 fuzzy logic for pattern recognition systems based on the pixels mean is presented. In this work, pattern recognition with fuzzy logic for feature extraction for ensemble neural networks for the case of fingerprints and using a fuzzy logic method for response integration is presented. An ensemble neural network of three modules was used. Each module is a local expert on person recognition based on their biometric measure.

In the approach of Li and Zhang [16], a hybrid learning algorithm based on additional momentum and self adaptive learning rate was proposed. An interval type-2 fuzzy neural network system (IT2FNN) was proposed to handle nonlinear and uncertain systems. The proposed IT2FNN is a combination of the interval type-2 fuzzy logic control (IT2FLC) and the neural network which inherits the benefits of these two methods. Applied in the pattern recognition of highway landscape, the simulation results show that the IT2FNN achieves the best tracking performance in comparison with classic backpropagation algorithm and structural equation modeling method.

In the work of Own [29], a switching between type-2 fuzzy sets and intuitionistic fuzzy sets with application to medical diagnosis is proposed. In this study, the advantage of type-2 fuzzy sets is applied, and the switching relation between type-2 fuzzy sets and intuitionistic fuzzy sets is defined axiomatically. The switching results are applied to show the usefulness of the proposed method in pattern recognition and medical diagnosis reasoning.

In Mendoza et al. [27], interval type-2 fuzzy logic and modular neural networks for face recognition is presented. In this work a method for response integration in multi-net neural systems using interval type-2 fuzzy logic and fuzzy integrals, with the purpose of improving the performance in the solution of problems with a great volume of information is presented. In this application, two interval type-2 fuzzy inference systems (IT2-FIS) were used; the first IT2-FIS was used for feature extraction in the face training data, and the second one to estimate the relevance of the modules in the multi-net system.

In Kim et al. [15], the design of optimized type-2 fuzzy neural networks and its application is presented. In order to develop reliable on-site partial discharge (PD) pattern recognition algorithm, Type-2 Fuzzy Neural Networks (T2FNNs) optimized by means of Particle Swarm Optimization (PSO) are introduced. T2FNNs exploit type-2 fuzzy sets which have a characteristic of robustness in the diverse area of intelligence systems.

In Hidalgo et al. [11], type-2 fuzzy inference systems as integration methods in modular neural networks for multimodal biometry are proposed. In this work a comparative study between fuzzy inference systems as methods of integration in modular neural networks for multimodal biometry is presented. First, the use of type-1 fuzzy logic and later the approach with type-2 fuzzy logic were considered. The fuzzy systems were developed using genetic algorithms to handle fuzzy inference systems with different membership functions. The comparative study of the type-1 and type-2 fuzzy inference systems was made to observe the behavior of the two different integration methods for modular neural networks for multimodal biometry.

In the work of Lopez and Melin [19,21], response integration in ensemble neural networks with type-2 fuzzy logic is proposed. This work describes a new approach for response integration in ensemble neural networks using interval type-2 fuzzy logic. When using ensemble neural networks it is important to choose a good method of response integration to obtain a better identification in pattern recognition. In this work a comparative analysis between interval type-2 fuzzy logic, type-1 fuzzy logic and the Sugeno Integral, as response integration methods, in ensemble neural networks was presented. Based on simulation results interval type-2 fuzzy logic is shown to be a superior method for response integration.

In Lopez et al. [20], optimization of response integration with fuzzy logic in ensemble neural networks using genetic algorithms is presented. In this work pattern recognition with ensemble neural networks for the case of fingerprints was considered. An ensemble neural network of three modules was used. The response integration method has the goal of combining the responses of the modules to improve the recognition rate of the individual modules. In this work the results of a type-2 approach for response integration that improves performance over the type-1 logic approaches are presented.

In Rhee and Choi [38], interval type-2 membership function design and its application to radial basis function neural networks is presented. In this work, an interval type-2 fuzzy membership design method and its application to radial basis function (RBF) neural networks was proposed. Type-1 fuzzy memberships which are computed from the centroid of the interval type-2 fuzzy memberships are incorporated into the RBF neural network. The proposed membership assignment was shown to improve the classification performance of the RBF neural network since the uncertainty of pattern data are desirably controlled by interval type-2 fuzzy memberships.

In Herman et al. [9], a support vector enhanced design of type-2 fuzzy logic approach to motor imagery related to EEG pattern recognition is presented. The significance of the initialization procedure in the development of type-2 fuzzy logic (T2FL) system-based classifiers should be highlighted considering their intrinsically non-linear nature. Initial structure identification has been recognized as a crucial stage in the design of an interval T2FL (IT2FL) classifier utilized in the framework of electroencephalogram (EEG)-based brain–computer interface (BCI). In conjunction with an efficient gradient-based learning algorithm it has allowed for robust exploitation of T2FL's capabilities to effectively handle uncertainties inherently associated with changing dynamics of electrical brain activity.

In the approach of Zeng and Liu [51], type-2 fuzzy hidden Markov models for phoneme recognition are proposed. This work presents a novel extension of Hidden Markov Models (HMMs): type-2 fuzzy HMMs (type-2 FHMMs). The advantage of this extension is that it can handle both randomness and fuzziness within the framework of type-2 fuzzy sets (FSs) and fuzzy logic systems (FLSs). Membership functions (MFs) of type-2 fuzzy sets are three-dimensional. Experimental results show that the type-2 FHMM has a comparable performance as that of the HMM but is more robust to noise.

In the work of Ozkan and Turksen [30], entropy assessment for type-2 fuzziness is presented. One of the sources of uncertainty, which perhaps is identified as parameter uncertainty, is the level of fuzziness in fuzzy system modeling. Given the optimum number of clusters and the cluster centers, one can explore type-2 membership values that capture the uncertainty of memberships. In this work, variations of type-2 membership values with the entropy measure for an artificially created 12 data sets are explored. Crisp to fuzzy data sets are constructed so that each data set has a different standard deviation within each cluster. Results are assessed by means of a particular entropy measure.

In Wang et al. [47], dynamical optimal training for interval type-2 fuzzy neural networks is presented. Type-2 fuzzy logic system (FLS) cascaded with neural network, type-2 fuzzy neural network (T2FNN), is presented in this work to handle uncertainty with dynamical optimal learning. A T2FNN consists of a type-2 fuzzy linguistic process as the antecedent part, and the two-layer interval neural network as the consequent part. A general T2FNN is computational-intensive due to the complexity of type 2 to type 1 reduction. Excellent results were obtained for the truck backing-up control and the identification of nonlinear system, which yield more improved performance than those using type-1 FNN.

In the approach of Mitchell [28], pattern recognition using type-2 fuzzy sets is proposed. A similarity measure for measuring the similarity, or compatibility, between two type-2 fuzzy sets was introduced. With this new similarity measure it is shown that type-2 fuzzy sets provide us with a natural language for formulating classification problems in pattern recognition.

In Madasu et al. [24], a novel approach for fuzzy edge detection using type-2 fuzzy sets is presented. A novel approach is presented for edge detection using the area feature at a pixel, since the area characterizes the structure of the edge present in the neighborhood of a pixel. The results of the proposed edge detector were compared with other well known edge detector like Canny, Gradient diffusion operator, etc. The edge detected image from the proposed approach seems to fare well over others.

In the approach of Tizhoosh [45], image thresholding using type-2 fuzzy sets is presented. In recent years, various researchers have introduced new thresholding techniques based on fuzzy set theory to overcome this problem. Regarding images as fuzzy sets (or subsets), different fuzzy thresholding techniques have been developed to remove the grayness ambiguity/vagueness during the task of threshold selection. In this work, a new thresholding technique was introduced which processes and uses thresholds as type 2 fuzzy sets.

In Table 2 a summary of the contributions where type-2 fuzzy systems have been used in pattern recognition is presented. The comparison shown in Table 2 is based on the domain of the problem, if a comparison with type-1 fuzzy logic is provided, and why type-2 fuzzy logic was used by the authors.

5. General overview of the area and future trends and directions

In this section a general overview of the area of type-2 fuzzy system in clustering, classification and pattern recognition is presented. Also, possible future trends that we can envision based on the review of this area are presented. It has been well-known for a long time that designing fuzzy systems is a difficult task, and this is especially true in the case of type-2 fuzzy systems. However, problems with a high level of uncertainty and/or noise are better handle by type-2 fuzzy systems, which is certainly the case for complex problems of clustering, classification and pattern recognition. The use of type-2 fuzzy systems in designing pattern recognition systems has become a more common practice in recent years, which has been accounted for with the review of papers presented in the previous sections of the paper. Also, it is now well recognized that in the case of designing type-2 fuzzy systems the problem is more complicated due to the higher number of parameters to consider, making it of upmost importance the use of bio-inspired optimization techniques for achieving the optimal designs of this sort of fuzzy systems. We have to mention that the search for the papers considered in this review has been done by using the search engine available in the Scopus online system of Elsevier, in which the papers can be searched for by subject or by author names. In this sense, an exhaustive search for papers on type-2 fuzzy systems for pattern recognition was done by using the following keywords: type-2 fuzzy pattern recognition, type-2 fuzzy system clustering,

Table 2
Type-2 fuzzy systems in pattern recognition.

Author(s) (pub. year)	Ref. no.	Domain of the problem	Comparison with type-1	Why type-2 is required for the problem?
Hosseini et al. (2010)	[12]	Computer aided detection	Yes	Uncertainty in pattern recognition
Melin (2010)	[26]	Edge detection	Yes	Uncertainty in edge detection
Lopez et al. (2010)	[22]	Fingerprint recognition	Yes	Uncertainty in fingerprint recognition
Li and Zhang (2010)	[16]	Pattern recognition	No	Uncertainty in pattern recognition
Own (2009)	[29]	Medical diagnosis	No	Uncertainty in diagnosis
Mendoza et al. (2009)	[27]	Face recognition	Yes	Uncertainty in face recognition
Kim et al. (2009)	[15]	Pattern recognition	No	Uncertainty in pattern recognition
Hidalgo et al. (2009)	[11]	Multimodal recognition	Yes	Uncertainty in multimodal pattern recognition
Lopez and Melin (2008)	[19,21]	Pattern recognition	Yes	Uncertainty in pattern recognition
Lopez et al. (2008)	[20]	Fingerprint recognition	Yes	Uncertainty in fingerprint recognition
Rhee and Choi (2007)	[38]	Pattern recognition	Yes	Uncertainty in pattern recognition
Herman et al. (2007)	[9]	EEC pattern recognition	No	Uncertainty in EEG pattern recognition
Zeng and Liu (2004)	[51]	Phoneme recognition	No	Uncertainty in pattern recognition
Ozkan and Turksen (2004)	[30]	Pattern recognition	Yes	Uncertainty in pattern recognition
Wang et al. (2004)	[47]	Pattern recognition	Yes	Uncertainty in pattern recognition
Mitchell (2005)	[28]	Pattern recognition	No	Uncertainty in pattern recognition
Madasu et al. (2008)	[24]	Edge detection	Yes	Uncertainty in edge detection
Tizhoosh (2005)	[45]	Image thresholding	No	Uncertainty in Image thresholding

and type-2 fuzzy system classification. It is also worth mentioning here that the Scopus database of Elsevier contains almost all the respected and relevant publications around the world, so the review that was formed based on the papers found by the search engine of Scopus can be considered representative of the publications in this area. Fig. 4 shows the number of type-2 fuzzy logic publications per year in the areas of clustering, classification and pattern recognition. An increasing trend in the number of publications can be noted in Fig. 4, although in 2013 a slight decrease is noted because at this moment of writing the paper, the year is not completed.

In general, based on the literature review that was performed we envision that the number of papers using type-2 fuzzy logic in the areas of clustering, classification and pattern recognition will continue to grow in the future years and the main reason for this statement is that real world problems are becoming more complex and require managing higher volumes of information. Type-2 fuzzy logic offers a more appropriate modeling approach to handle higher degrees of uncertainty in information and for this reason is better suited to manage these problems and people are realizing this fact, the popularity of type-2 fuzzy logic in these areas will increase.

Finally, we have to mention that most of the papers covered by this review are using interval type-2 fuzzy logic, which is a simplification of generalized type-2 fuzzy logic, and the main reason for this is the computational overhead required in generalized calculations. However, there are also ongoing works on developing more efficient algorithms for generalized type-2 fuzzy logic and we are expecting that in the future more applications in clustering, classification and pattern recognition will be made using generalized type-2 instead of interval type-2 fuzzy logic. Of course, theoretically the advantage of using generalized type-2 fuzzy logic is that higher degrees of noise or uncertainty would be handled and better results should be obtained.

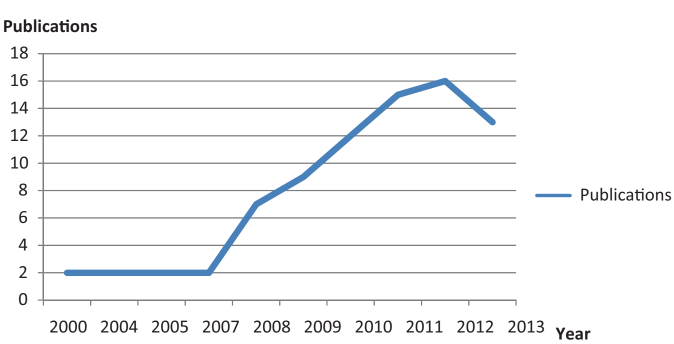

Fig. 4. Number of type-2 fuzzy logic publications per year in the areas of clustering, classification and pattern recognition.

6. Conclusions

In this paper a representative and concise review of type-2 fuzzy logic applications in pattern recognition, classification and clustering problems is presented. Recently, type-2 fuzzy logic has gained popularity in a wide range of applications due to its ability to handle higher degrees of uncertainty. In this paper a concise and representative review of the most successful applications of type-2 fuzzy logic in pattern recognition, classification and clustering problems is offered. We expect that in the future the number of applications of type-2 fuzzy logic in these areas will increase due to the complexity and higher degree of uncertainty of problems in pattern recognition, classification and clustering. In particular, we envision that in the near future the use of generalized type-2 fuzzy logic will be more common, but now at this moment most of the applications are treated using interval type-2 fuzzy logic, which is less computationally expensive.

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