Project 2 – Dynamic Programming

Optimal Airline Overbooking Policy

Group 29 - Optimization II

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Background

Our airline pricing department wants to determine the optimal amount of first class and coach tickets to sell in order to maximize expected discounted profit on a given flight. Profit is defined as revenue from ticket sales minus overbooking costs. There are 120 total seats on the plane, 20 first class seats and 100 coach seats. There are 365 days until this plane departs. On any given day we will sell 0 or 1 coach ticket, and 0 or 1 first class ticket. Coach tickets are allowed to be overbooked and in the event that more coach passengers arrive than there are seats available, the passenger will be moved to first class or given a voucher for a different plane ride. Our manager would like the analytical team to determine the optimal overbooking policy, how it impacts expected profits, and our recommendations.

First class tickets can be purchased at a low price of $425 or a high price of $500. The low price has a 8% chance of being sold, and the high price has a 4% chance of being sold. First class passengers show up for the flight 97% of the time. Coach tickets can be purchased at a low price of $300 or a high price of $350. The high price has a 65% chance of being sold, and the high price 30%. Coach passengers show up for the flight 95% of the time. Overbooking costs associated with coach tickets are $50 to bump to first class and $425 to bump off the plane. We assume that first class and coach tickets are purchased independently of each other.

Dynamic Program (DP) is an optimization technique that breaks the problem into subproblems that help us find the best overall solution. Maximizing profit depends upon making optimal decisions for each subproblem (daily pricing) leading up to the terminal value (departure day). We will formulate a Dynamic Program to maximize expected discounted profit given the prices above, probabilities above, the 15% annual discount rate (δ), and the terminal condition that on departure day only costs are incurred based on tickets sold. No additional revenue is made for that flight on or after the day of takeoff.

Dynamic Program Setup

Below we will define the six key components of a dynamic program, how we will use each to create an optimization algorithm, and the business implications that follow. State Variables are the information we know at each time period to help make optimal decisions:

$$f \; = \; number\ of\ first\ class\ tickets\ sold$$

$$c \ = \ number\ of\ coach\ tickets\ sold$$
$$t \ = \ time\ in\ days\ [0 \ - \ 365] \quad T \ = \ day\ 365$$

Choice Variables are what we will decide upon each time period, each day, to maximize the overall expected discounted profit. There are 2 possible price points for f first class tickets and c coach tickets, giving us 4 possible price combinations and their associated probabilities:

$$(\$500,\ \$350),\ (\$500,\ \$300),\ (\$425,\ \$350),\ (\$425,\ \$300)$$

The Dynamics of this problem are how the price point combinations we choose at each decision point will impact sales of f and c on a daily basis, t . As discussed above, the pricing of each ticket class will affect the demand distribution for each ticket class, meaning that our pricing decision impacts the probability of arriving in different states in the next period. A lower ticket price for coach seats in state(c,f,t) will mean we have a higher probability of ending up in state(c+1,f,t+1) than in state(c,f,t+1).

The Value Function is the discounted value of all future payoffs. This is a representation of our main overarching goal, to maximize expected discounted profit at v(0,0,0) where:

$$v(c, f, t) \ = \ max_{Pc,Pf} \ \left(E\left[\sum_{i=t}^{T-t} (profit\ at\ t \ + \ i\ )(\ \delta^{i})\right]\right)$$

The Bellman Equation helps us solve the problem step by step going backwards from the terminal value in order to maximize expected profit. At each time t we will choose the price combination that maximizes expected revenue for a given overbooking allowance (5 to 15 overbooked seats). The Bellman Equation at each time t will choose the price combination that maximizes expected revenue today plus the discounted expected profit.

$$\begin{aligned} v(c, f, t) \ = max\ (\ & E\left[profit_t + \ \delta v\ \right] for\ (\$500,\ \$350)\ , \\ & E\left[profit_t + \ \delta v\ \right] for\ (\$500,\ \$300)\ , \\ & E\left[profit_t + \ \delta v\ \right] for\ (\$425,\ \$350)\ , \\ & E\left[profit_t + \ \delta v\ \right] for\ (\$425,\ \$300)\ ) \end{aligned}$$

The terminal value is the expectation of overbooking costs. This is determined based on the number of coach and first class tickets sold. The number of coach and first class passengers to arrive are assumed to each follow an independent binomial distribution, with arrival probabilities as outlined in the background above.

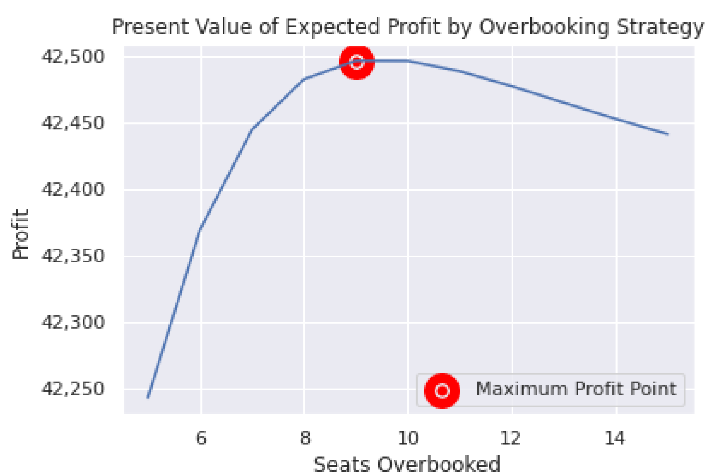
Implementation

Results Method 1 - Capacity for Tickets Sold:

This method follows the procedures outlined above, where each day we simply have four pricing possibilities, and we must offer each ticket for sale if it is not currently sold out. We solve the dynamic program repeatedly while varying the allowed amount of overbooked coach seats from a minimum of 5 to the max of 15.

Based on solving the dynamic program the expected discounted profit for the year with 5 overbooked seats is $42,242.86. When we explore different allowable amounts of overbooking seats, we observe that the best strategy when we establish a hard capacity on seats sold is to overbook by 9 seats. The various expected discounted profit amounts are shown below.

Seats	Discounted Profit
5	$42,502.67
6	$42,368.87
7	$42,444.24
8	$42,482.21
9	$42,496.11
10	$42,495.97
11	$42,488.26
12	$42,477.00
13	$42,464.60
14	$42,452.36
15	$42,440.94



Results Method 2 - Tickets Sold & Days to Departure:

The airline also wants to examine controlling oversold tickets based on both how many tickets we have sold and how many days are left until departure. Instead of a hard capacity of seats sold, the airline can force demand to zero on any given day for coach tickets by refusing to offer the tickets. There are now three daily choices for coach tickets: high price $350, low price $300, and no sale $0. There are still two options for first class tickets: high price $500, low price $425.

This changes our dynamic program setup. We now have 6 choice variables, and the Bellman Equation will choose the maximum of the 6 choice for each day t.

$$v(c, f, t) \; = \; max \;\; (E\left[profit_t + \; \delta v\right] for \; (\$500, \; \$350),$$

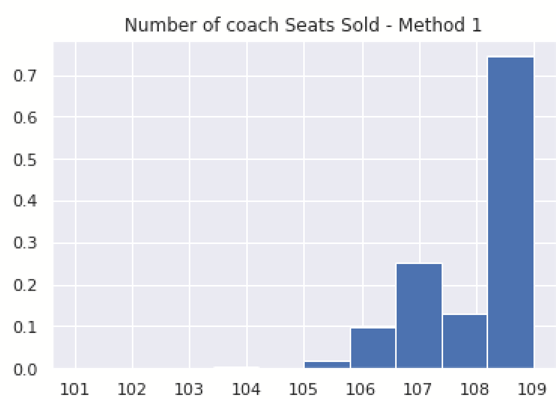
$$E\left[profit_t + \delta v\right] for (\$500, \$300),$$
$$E\left[profit_t + \delta v\right] for (\$500, \$0),$$
$$E\left[profit_t + \delta v\right] for (\$425, \$350),$$
$$E\left[profit_t + \delta v\right] for (\$425, \$300),$$
$$E\left[profit_t + \delta v\right] for (\$425, \$0))$$

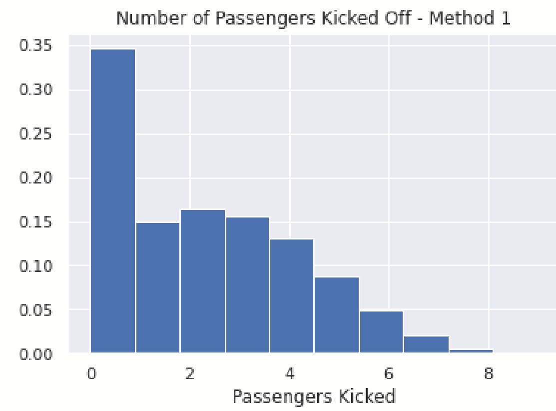
Controlling both tickets sold and days left until departure resulted in a better expected profit of $42,502.67, roughly $260 increase in expected discounted profit relative to the 5-seat overbooking policy. While this may seem like a minimal increase in profit, we are only examining one flight path in this analysis. If we increase profits by an average of $260 for all flights offered, this could make a significant impact on company profits. It was a simple change to our dynamic program, and would be easy to implement company wide to better capture expected profit for all flights. We worked backwards to solve the dynamic program, and will now use the algorithm to simulate using the DP in real time. We can estimate overbooking issues and the volatility of discounted profits for both methods.

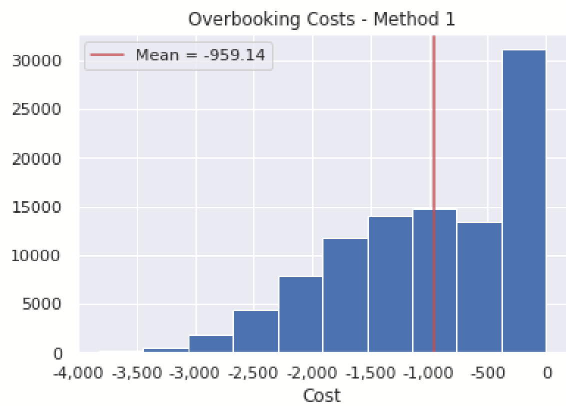
<u>Simulation & Comparison</u>

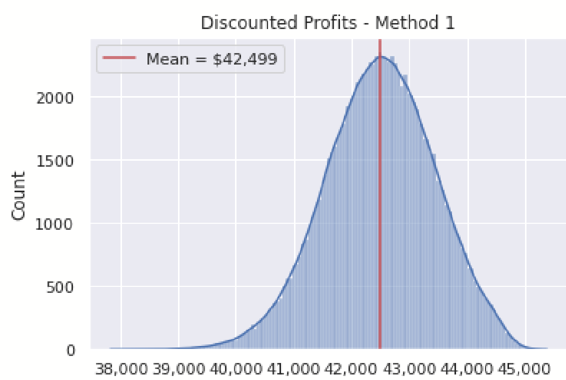
Method 1:

The graphs and conclusions below are based on Method One simulations, controlling tickets sold. With 100,000 simulations, the frequency of coach class overbooked is 100%, the frequency of passengers kicked off the plane is 68.8%, the average overbooking cost is $959.14, the average discounted profit is $42,498.56, and the standard deviation of discounted profits is $940.25.



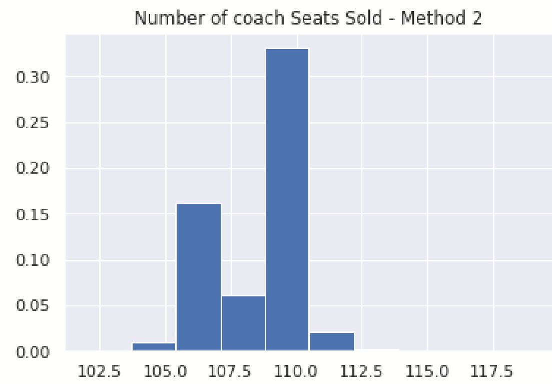


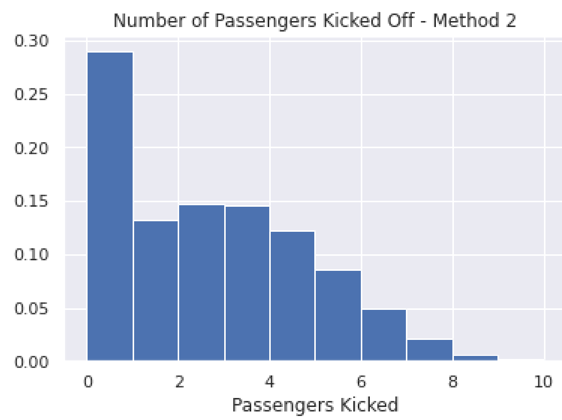


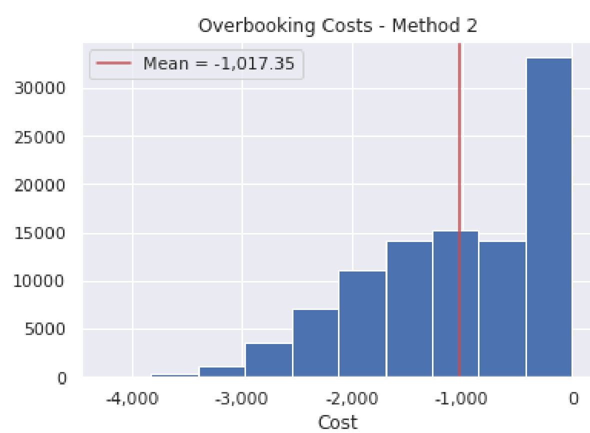


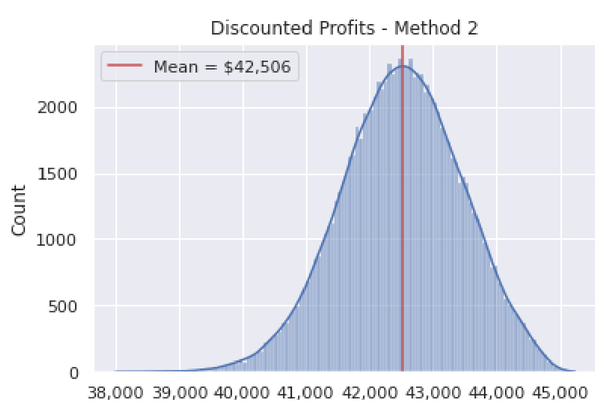
Method 2:

The graphs and conclusions below are based on Method Two simulations, controlling tickets sold and considering days until departure. With 100,000 simulations, the frequency of coach class overbooked is 100%, the frequency of passengers kicked off the plane is 71%, the average overbooking cost is $1,017.35, and the average discounted profit is $42,506 with a standard deviation of discounted profits is $948.61.

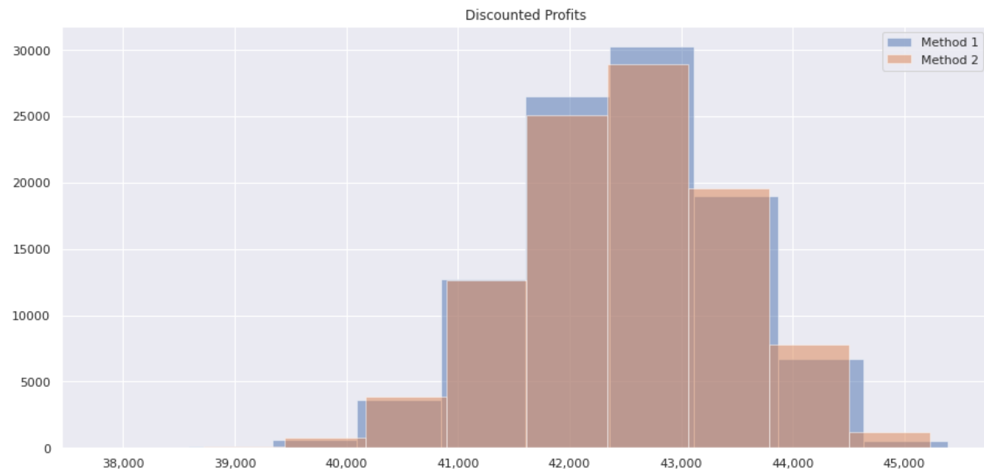

Number of coach Seats Sold - Method 2
0.30
0.25
0.20
0.15
0.10
0.05
0.00
102.5
105.0
107.5
110.0
112.5
115.0
117.5


Number of Passengers Kicked Off - Method 2
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
Passengers Kicked


Overbooking Costs - Method 2
Mean = -1,017.35
30000
25000
20000
15000
10000
5000
0
-4,000
-3,000
-2,000
-1,000
0
Cost


Discounted Profits - Method 2
Mean = $42,506
Count
2000
1500
1000
500
0
38,000
39,000
40,000
41,000
42,000
43,000
44,000
45,000

We see below that the two methods perform similarly to one another. Because performance is comparable, we think it is important to consider operational impacts and customer service rates under each method.



<u>Conclusion</u>

Although the overall profitability is slightly higher in the case of the ‘No Sale Flexibility’ strategy, the overbooking rates and the transfer rates are better in the ‘Optimal Overbooking’ strategy. The average profit for both simulations is roughly the same, while Method Two has a slightly broader standard deviation, higher overbooking cost, and more frequency of kicking customers off of flights. Customer satisfaction is competitive and highly important in the airline industry. Our analytical team recommends using Method One as less people are kicked off of flights leading to higher customer satisfaction and service levels. There can be hidden costs associated with customer service that are hard to measure quantitatively and as branding is very important in our competitive industry we want to prioritize happy customers over marginal profit increases.

Thus, from the customer retention standpoint, given that the overall profitability is quite similar, our recommendation is to go with the Optimal Overbooking strategy since overbooking rates and transfer rates are lower thereby providing a better customer experience. The two policies give similar results, with a minimal difference in profit. The executive team should not worry about implementing a new overbooking policy since the results are not overly sensitive or different between the two methods. This report verifies the impact of our companies overbooking policy, gives informed recommendations, and an overview of how our airline ticketing process impacts the bottom line, profit.