MTH 511 HW 1

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1. **Exercise 3.22:** Show that $\|x\|_\infty \leq \|x\|_2$ for any $x \in \ell_2$, and that $\|x\|_2 \leq \|x\|_1$ for any $x \in \ell_1$.
2. **Exercise 3.23:** The subset of ℓ_∞ consisting of all sequences that converge to 0 is denoted by c_0. (Note that c_0 is actually a linear subspace of ℓ_∞; thus c_0 is also a normed vector space under $\|\cdot\|_\infty$.) Show that we have the following proper set inclusions: $\ell_1 \subset ell_2 \subset c_0 \subset \ell_\infty$.
3. **Exercise 3.25:** State and prove lemma 3.7 and theorem 3.8 (also cover $p = 1, q = \infty$ for lemma 3.7).
4. **Exercise 3.36:** Given a metric space (M, d), prove a convergent sequence is Cauchy and a Cauchy sequence is bounded.
5. **Exercise 3.37:** A Cauchy sequence with a convergent subsequence converges.
6. **Exercise 3.39:** If every subsequence of (x_n) has a further subsequence that converges to x, then (x_n) converges to x.