Cauchy Residue Theorem

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First, we introduce the concept of a residue.

Theorem 1.1. *If f has a pole of order n at z_0, then*

$$_{z_0}(f) = \lim_{z \to z_0} \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} f(z)(z - z_0)^n.$$

□

Next, the theorem:

Theorem 1.2. *Suppose that f is holomorphic in an open set containing a circle C and its interior, except for a pole at z_0 inside C. Then*

$$\int_C f(z)dz = 2\pi i_{z_0} f.$$

□