(Q.) for the scalar f^n $f(x,y,z) = x^2+3y^2+2z^2$ than gradient at point $P(1,2,-1)$

$\Rightarrow \nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$\Rightarrow 2x\hat{i} + 6y\hat{j} + 4z\hat{k}$

$\nabla f|_{1,2,-1} = 2\hat{i} + 12\hat{j} - 4\hat{k}$

(Q.) $\vec{F} = x^3y\hat{i} - x^2y^2\hat{j} - x^2yz\hat{k}$ find $\nabla\cdot\vec{F}$

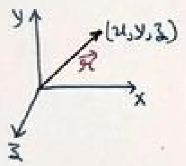
$\Rightarrow \nabla\cdot\vec{F} = \frac{\partial}{\partial x}x^3y - \frac{\partial}{\partial y}x^2y^2 + \frac{\partial}{\partial z}(-x^2yz)$

$\Rightarrow \nabla\cdot\vec{F} = 3\cancel{x^2}y - 2\cancel{x^2}y - \cancel{x^2}y = 0$

(Q.) find the curl $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$

$\Rightarrow \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \Rightarrow 0$

- Vorticity $= 2\omega = \nabla\cdot\vec{V}$
- Position vector :-



$\vec{r} = (x-0)\hat{i} + (y-0)\hat{j} + (z-0)\hat{k}$

$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$

length $= |\vec{r}| = \sqrt{x^2+y^2+z^2} = r$

① $\frac{\partial r}{\partial x} = \frac{x}{r}$ Proof:- $\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$

② $\frac{\partial r}{\partial y} = \frac{y}{r}$

③ $\frac{\partial r}{\partial z} = \frac{z}{r}$

④ $\nabla r = \frac{\vec{r}}{r}$

⑤ $\nabla\cdot\vec{r} = \text{Div}\,\vec{r} = 3$

⑥ $\nabla\times\vec{r} = \text{curl}\,\vec{r} = 0$

⑦ $\nabla f r = f'(r)\nabla r$

(Q.) $\nabla \ln r = ??$

$\Rightarrow \frac{1}{r}\times\nabla r \Rightarrow \frac{1}{r}\times\frac{\vec{r}}{r} \Rightarrow \frac{\vec{r}}{r^2}$

⑧ Laplace operator

$\nabla^2 = \nabla\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \Rightarrow \boxed{\nabla^2 f = 0}$

⑨ $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

⑩ If $|\vec{F}| = r^n$ then, $\vec{F} = r^{n-1}\vec{r}$

⑪ $\nabla\cdot r^n\vec{r} = 0$ than, $n = -3$ $\boxed{\nabla\cdot\frac{\vec{r}}{r^3} = 0}$

- Property :- [2 operator together]

① $\nabla\cdot(\nabla f) = \nabla^2 f$
Div(gradf)

② $\nabla\times(\nabla f) = 0$
curl(gradf)

③ $\nabla\cdot(\nabla\times\vec{F}) = 0$
div(curl $\vec{F}$)

- NOTE:-

① unit surface normal vector $(\hat{N}) = \frac{\nabla f}{|\nabla f|}$

(Q.) $f = x^3 - y^3 + x^2z$ than calculate surface normal vector at $(1,1,-2)$

② Angle b/n 2 surface $f(x,y,z) = c_1$ & $g(x,y,z) = c_2$ is given -

$$\boxed{\cos\theta = \frac{\nabla f\cdot\nabla g}{|\nabla f||\nabla g|}}$$

(Q.) Angle b/n $x^2+y^2+z^2 = 9$ & $x^2+y^2-z = 3$ at $(2,-1,2)$

- Dir. derivative of $f(x,y,z)$ in the dir^n of $\vec{a}$

$$\boxed{DD = \nabla f\cdot\frac{\vec{a}}{|a|}}$$

- $\boxed{DD_{max} = \nabla f\cdot\frac{\nabla f}{|\nabla f|} = \frac{|\nabla f|^2}{|\nabla f|} = |\nabla f|}$ in dir of ∇f

(Q.) The dir^n derivative of surface $f = \phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the dir^n of $2\hat{i} - 2\hat{j} - 2\hat{k}$ is – Ans – $37/3$

Multiple integration :- $\int_c^d\int_a^b f(x,y)\,dx\,dy$

① Const. limit :-

(Q.) $\int_0^2\int_0^1 xy\,dx\,dy$ (or) $\int_0^1\int_0^2 xy\,dy\,dx$

$\Rightarrow \int_0^2\left[\left(\frac{x^2}{2}\right)y\right]_0^1 dy$ $\Rightarrow$ 1 Ans

$\Rightarrow \int_0^2\frac{y}{2}\,dy \Rightarrow \left[\frac{y^2}{4}\right]_0^2 \Rightarrow 1$ Ans

$\Rightarrow$ When limit are const. the order of integration doesn't matter.

② Variable limit :-

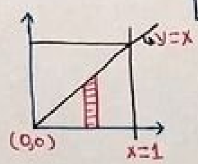
(Q.) $\int_0^1\int_0^x xy\,dy\,dx$ $\Rightarrow$ Initial limit $\Rightarrow$

$x=0$	$x=1$
$y=0$	$y=x$

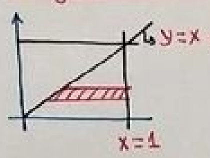
$\Rightarrow \int_0^1 x\left[\frac{y^2}{2}\right]_0^x dx$

$\Rightarrow \int_0^1 x\cdot\frac{x^2}{3}\,dx$

$\Rightarrow \left[\frac{x^4}{8}\right]_0^1 = \frac{1}{8}$ Ans



change of order :-



$\Rightarrow$ final limit $\Rightarrow$

$x=y$	$x=1$
$y=0$	$y=1$

$\int_0^1\int_y^1 xy\,dx\,dy \Rightarrow \frac{1}{8}$ Ans

"SUMIT KUMAR"