

(Q.) for the scalar fn $f(x,y,z) = x^2 + 3y^2 + 2z^2$ then gradient at point $P(1,2,-1)$

$$\Rightarrow \nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k}$$

$$\Rightarrow 2x\hat{i} + 6y\hat{j} + 4z\hat{k}$$

$$\nabla f|_{1,2,-1} = 2\hat{i} + 12\hat{j} - 4\hat{k}$$

(Q.) $\vec{F} = x^3y\hat{i} - x^2y^2\hat{j} - x^2yz\hat{k}$ find $\nabla \cdot \vec{F}$

$$\Rightarrow \nabla \cdot \vec{F} = \frac{\partial}{\partial x} x^3y - \frac{\partial}{\partial y} x^2y^2 + \frac{\partial}{\partial z} (-x^2yz)$$

$$\Rightarrow \nabla \cdot \vec{F} = 3x^2y - 2x^2y - x^2y = 0$$

(Q.) find the curl $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$

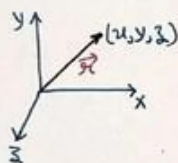
$$\Rightarrow \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \Rightarrow 0$$

• Vorticity $= 2\omega = \nabla \times \vec{v}$

• Position vector:-

$$\vec{r} = (x-0)\hat{i} + (y-0)\hat{j} + (z-0)\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



$$\text{length} = |\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

① $\frac{\partial r}{\partial x} = \frac{x}{r}$ Proof:- $\frac{\partial r}{\partial x} = \frac{\partial}{\partial x} \sqrt{x^2 + y^2 + z^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$

② $\frac{\partial r}{\partial y} = \frac{y}{r}$

⑤ $\nabla \cdot \vec{r} = \text{div } \vec{r} = 3$

⑥ $\nabla \times \vec{r} = \text{curl } \vec{r} = 0$

③ $\frac{\partial r}{\partial z} = \frac{z}{r}$

⑦ $\nabla f(x) = f'(x) \nabla x$

④ $\nabla r = \frac{\vec{r}}{r}$

⑧ $\nabla \ln r = ??$

$\Rightarrow \frac{1}{r} \times \nabla r \Rightarrow \frac{1}{r} \times \frac{\vec{r}}{r} \Rightarrow \frac{\vec{r}}{r^2}$

⑧ Laplace operator

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \Rightarrow \nabla^2 f = 0$$

⑨ $\nabla^2 f(x) = f''(x) + \frac{2}{x} f'(x)$

⑩ If $|\vec{F}| = x^n$ then, $\vec{F} = x^{n-1} \vec{r}$

⑪ $\nabla \cdot x^n \vec{r} = 0$ then, $n = -3$ $\nabla \cdot \frac{\vec{r}}{x^3} = 0$

• Property:- [2 operators together]

① $\nabla \cdot (\nabla f) = \nabla^2 f$ ② $\nabla \times (\nabla f) = 0$
 $\text{div}(\text{grad } f)$ $\text{curl}(\text{grad } f)$

③ $\nabla \cdot (\nabla \times \vec{F}) = 0$
 $\text{div}(\text{curl } \vec{F})$

• NOTE:-

① unit surface normal vector $(\hat{N}) = \frac{\nabla f}{|\nabla f|}$

(Q.) $f = x^3 - y^2 + x^2z$ then calculate surface normal vector at $(1,1,-2)$

② Angle b/n 2 surface $f(x,y,z) = c_1$ & $g(x,y,z) = c_2$ is given -

$$\cos \theta = \frac{\nabla f \cdot \nabla g}{|\nabla f| |\nabla g|}$$

(Q.) Angle b/n $x^2 + y^2 + z^2 = 9$ & $x^2 + y^2 - z = 3$ at $(2,-1,2)$

• Dir. derivative of $f(x,y,z)$ in the dirⁿ of \vec{a}

$$DD = \nabla f \cdot \frac{\vec{a}}{|\vec{a}|}$$

• $DD_{\max} = \nabla f \cdot \frac{\nabla f}{|\nabla f|} = \frac{|\nabla f|^2}{|\nabla f|} = |\nabla f|$
 in dir of ∇f

(Q.) The dirⁿ derivative of surface $f = \phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the dirⁿ of $2\hat{i} - 2\hat{j} - 2\hat{k}$ is - $\text{Ans} = 3\sqrt{3}$

Multiple integration:- $\int_c^d \int_a^b f(x,y) dx dy$

① Const. limit:-

(Q.) $\int_0^2 \int_0^1 xy dx dy$ (or) $\int_0^1 \int_0^2 xy dy dx$

$\Rightarrow \int_0^2 \left[\frac{x^2}{2} y \right]_0^1 dy$

$\Rightarrow 1 \text{ Ans}$

$\Rightarrow \int_0^1 \frac{y}{2} dy \Rightarrow \left[\frac{y^2}{4} \right]_0^2 \Rightarrow 1 \text{ Ans}$

\Rightarrow When limit are const. the order of integration doesn't matter.

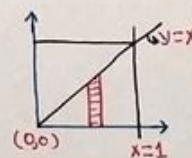
② Variable limit:-

(Q.) $\int_0^1 \int_0^x xy dy dx \Rightarrow$ Initial limit $\Rightarrow \begin{matrix} x=0 & x=1 \\ y=0 & y=x \end{matrix}$

$\Rightarrow \int_0^1 x \left[\frac{y^2}{2} \right]_0^x dx$

$\Rightarrow \int_0^1 x \cdot \frac{x^2}{2} dx$

$\Rightarrow \left[\frac{x^4}{8} \right]_0^1 = \frac{1}{8} \text{ Ans}$



change of order:-

\Rightarrow final limit $\Rightarrow \begin{matrix} x=y & x=1 \\ y=0 & y=1 \end{matrix}$



$\int_0^1 \int_y^1 xy dx dy \Rightarrow \frac{1}{8} \text{ Ans}$

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