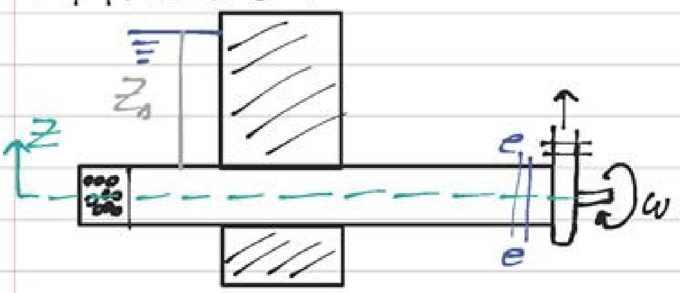
PHA2304



$$Z_1 + \left(\frac{P_1}{\rho g}\right)_{abs} + \frac{V_1^2}{2g} = Z_e + \frac{V_e^2}{2g} + \left(\frac{P_e}{\rho g}\right)_{abs} + \Delta H_1 \Leftrightarrow$$

$$\underbrace{\left[\left(\frac{P_e}{\rho g}\right)_{abs} + \frac{V_e^2}{2g} - \frac{P_v}{\rho g}\right]} = \left(\frac{P_{atm}}{\rho g}\right)_{abs} + Z_1 - \Delta H_1$$

$$NPSH_{disp.} = \left(\frac{P_{atm}}{\rho g}\right)_{abs} + \Delta Z - \Delta H_1 - \frac{P_v}{\rho g}$$

* Dicas: $(P_{abs})_{atm} = \rho_{Hg} \cdot g \cdot h_{Hg} = 13600 \cdot 9{,}81 \cdot 0{,}76 = 101361\ Pa$

$$\left(\frac{P_{atm}}{\rho g}\right)_{abs} = \frac{101361}{1000 \cdot 9{,}81} = 10{,}33\ m \quad ; \quad \left(\frac{P_{atm}}{\rho g}\right) = 10{,}33 - \frac{Z}{900}$$

$$\hookrightarrow \left(\frac{P_v}{\rho g}\right) = 0{,}077 \cdot e^{0{,}0558 \cdot \theta} \quad , \text{com } \theta \text{ em } ^\circ C$$

$\hookrightarrow$ Diâmetro ótimo da flange de sucção: $D_{s_{ótimo}} = 4{,}5 \cdot \sqrt[3]{\frac{Q}{n}}$
com Q em m^3/s; n em rpm; D em m

$$\hookrightarrow NPSH_{req} \cong 0{,}203 \cdot \omega_s^{4/3} \cdot H^* \quad ; \quad \omega_s = \omega \cdot \frac{\sqrt{Q^*}}{(gH^*)^{3/4}}$$

$\hookrightarrow \eta_{max}$

(Q.) for the scalar f^n $f(x,y,z) = x^2+3y^2+2z^2$ than gradient at point $P(1,2,-1)$

$\Rightarrow \nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$\Rightarrow 2x\hat{i} + 6y\hat{j} + 4z\hat{k}$

$\nabla f|_{1,2,-1} = 2\hat{i} + 12\hat{j} - 4\hat{k}$

(Q.) $\vec{F} = x^3y\hat{i} - x^2y^2\hat{j} - x^2yz\hat{k}$ find $\nabla\cdot\vec{F}$

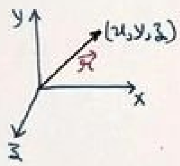
$\Rightarrow \nabla\cdot\vec{F} = \frac{\partial}{\partial x}x^3y - \frac{\partial}{\partial y}x^2y^2 + \frac{\partial}{\partial z}(-x^2yz)$

$\Rightarrow \nabla\cdot\vec{F} = 3\cancel{x^2}y - 2\cancel{x^2}y - \cancel{x^2}y = 0$

(Q.) find the curl $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$

$$\Rightarrow \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \Rightarrow 0$$

- Vorticity $= 2\omega = \nabla\cdot\vec{V}$
- Position vector:-



$\vec{r} = (x-0)\hat{i} + (y-0)\hat{j} + (z-0)\hat{k}$

$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$

length $= |\vec{r}| = \sqrt{x^2+y^2+z^2} = r$

① $\frac{\partial r}{\partial x} = \frac{x}{r}$ Proof:- $\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$

② $\frac{\partial r}{\partial y} = \frac{y}{r}$

③ $\frac{\partial r}{\partial z} = \frac{z}{r}$

④ $\nabla r = \frac{\vec{r}}{r}$

⑤ $\nabla\cdot\vec{r} = \text{Div}\,\vec{r} = 3$

⑥ $\nabla\times\vec{r} = \text{curl}\,\vec{r} = 0$

⑦ $\nabla f r = f(r)\nabla r$

(Q.) $\nabla \ln r = ??$

$\Rightarrow \frac{1}{r}\times\nabla r \Rightarrow \frac{1}{r}\times\frac{\vec{r}}{r} \Rightarrow \frac{\vec{r}}{r^2}$

⑧ Laplace operator

$\nabla^2 = \nabla\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \Rightarrow \boxed{\nabla^2 f = 0}$

⑨ $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

⑩ If $|\vec{F}| = r^n$ then, $\vec{F} = r^{n-1}\vec{r}$

⑪ $\nabla\cdot r^n\vec{r} = 0$ than, $n = -3$ $\boxed{\nabla\cdot\frac{\vec{r}}{r^3} = 0}$

- Property:- [2 operator together]

① $\nabla\cdot(\nabla f) = \nabla^2 f$
Div(grad f)

② $\nabla\times(\nabla f) = 0$
curl(grad f)

③ $\nabla\cdot(\nabla\times\vec{F}) = 0$
div(curl $\vec{F}$)

- NOTE:-

① unit surface normal vector $(\hat{N}) = \frac{\nabla f}{|\nabla f|}$

(Q.) $f = x^3 - y^3 + x^2 z$ than calculate surface normal vector at $(1,1,-2)$

② Angle b/n 2 surface $f(x,y,z) = c_1$ & $g(x,y,z) = c_2$ is given −

$\boxed{\cos\theta = \frac{\nabla f\cdot\nabla g}{|\nabla f||\nabla g|}}$

(Q.) Angle b/n $x^2+y^2+z^2 = 9$ & $x^2+y^2-z = 3$ at $(2,-1,2)$

- Dir. derivative of $f(x,y,z)$ in the dir^n of $\vec{a}$

$\boxed{DD = \nabla f\cdot\frac{\vec{a}}{|a|}}$

- $\boxed{DD_{max} = \nabla f\cdot\frac{\nabla f}{|\nabla f|} = \frac{|\nabla f|^2}{|\nabla f|} = |\nabla f|}$ in dir of ∇f

(Q.) The dir^n derivative of surface $f = \phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the dir^n of $2\hat{i} - 2\hat{j} - 2\hat{k}$ is − Ans − $37/3$

Multiple integration:− $\int_c^d\int_a^b f(x,y)\,dx\,dy$

① Const. limit:-

(Q.) $\int_0^2\int_0^1 xy\,dx\,dy$ (or) $\int_0^1\int_0^2 xy\,dy\,dx$

$\Rightarrow \int_0^2\left[\left(\frac{x^2}{2}\right)y\right]_0^1 dy$ $\Rightarrow$ 1 Ans

$\Rightarrow \int_0^2\frac{y}{2}\,dy \Rightarrow \left[\frac{y^2}{4}\right]_0^2 \Rightarrow 1$ Ans

$\Rightarrow$ When limit are const. the order of integration doesn't matter.

② Variable limit:-

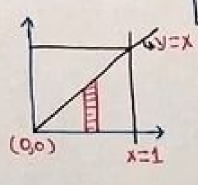
(Q.) $\int_0^1\int_0^x xy\,dy\,dx$ $\Rightarrow$ Initial limit $\Rightarrow$

$x=0$	$x=1$
$y=0$	$y=x$

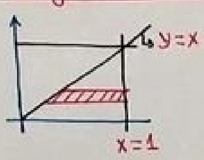
$\Rightarrow \int_0^1 x\left[\frac{y^2}{2}\right]_0^x dx$

$\Rightarrow \int_0^1 x\cdot\frac{x^2}{3}\,dx$

$\Rightarrow \left[\frac{x^4}{8}\right]_0^1 = \frac{1}{8}$ Ans



change of order:-



$\Rightarrow$ final limit $\Rightarrow$

$x=y$	$x=1$
$y=0$	$y=1$

$\int_0^1\int_y^1 xy\,dx\,dy \Rightarrow \frac{1}{8}$ Ans

"SUMIT KUMAR"