COMP226: Slides 10

Returns

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Overview

In these slides we will consider **returns**, which are building block for the measurement the performance of profit-seeking trading strategies

In contrast to the **execution algorithms**, **profit-seeking strategies** aim to generate a positive profit/return, rather than just execute a pre-defined large order an minimum cost

We study two types of returns: **simple** and **logarithmic** returns

- **Position (signal) vectors**
- **Equity curves**

Measuring performance

Relative returns are **building blocks** of performance measurement

There are **two types**:

- **simple returns**
- **log returns**

Straightforward to convert either one to the other

Definition

Suppose

$$P_{t-1}, \quad P_t$$

are consecutive observations of a **price** time series

The **simple return** is defined as:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

The **log return** (also called the continuously compounded return) is defined as:

$$r_t = \log(P_t/P_{t-1}) = \log(P_t) - \log(P_{t-1})$$

Simple correspondence

$$r_t = \log(1 + R_t) \quad R_t = e^{r_t} - 1$$

Log returns and simple returns are **essentially the same thing** just stated on a **different scale**

For **mathematical convenience** we often use **log returns**

Start Price	End Price	Simple Ret	Log Ret
100	0	-1.00	-Inf
100	75	-0.25	-0.288
100	90	-0.10	-0.105
100	92	-0.08	-0.083
100	94	-0.06	-0.062
100	96	-0.04	-0.041
100	98	-0.02	-0.020
100	100	0.00	0.000
100	102	0.02	0.020
100	104	0.04	0.039
100	106	0.06	0.058
100	108	0.08	0.077
100	110	0.10	0.095
100	125	0.25	0.223
100	200	1.00	0.693

Simple versus log returns

As the table shows:

- **For realistic simple returns** (smaller in aboslute value than 10%) **simple and log returns are very similar**
- **For larger simple returns they start to diverge**
- When a non-zero price goes down to zero
    - -1 is the **simple return** (**smallest possible**)
    - -Infinity is the **log return** (**smallest possible**)
- Both simple and log returns can be arbitrarily high, but log returns grow much more slowly than simple returns

Terminology for positions

For a given asset we can take three possible **positions**:

Position	Meaning
Long	We have **bought**
Flat	We have neither bought or sold
Short	We have **sold**

Those can be thought of as the **direction** of the position

In addition to the direction, there is the **size** of the position, e.g., the number of shares for equities, or the number of contracts (a.k.a lots) for futures

Reason for taking a position

Position	Expectation
Long	Prices will go **up**
Flat	Not sure (too risky)
Short	Prices will go **down**

Return of a short position

When we go short, the buying time is after the selling time

Log return

of a short position = negative of log return of the long position

Simple return

The relationship of the simple return of a short position relative to that of the long position is slightly more complicated:

$$-R_t/(R_t+1)$$

Simple Difference

The simple **price difference** (i.e. profit or loss)

$$P_t - P_{t-1}$$

is occasionally used.

But many applications require relative returns

Returns versus price differences

The benefit is **normalization**:

- measuring all variables in a comparable (dimensionless) way
- this allows the evaluation of relationships between asset prices of **different levels**
- relative returns is required for relevant statistical analysis

Example

```
> library(quantmod)
> getSymbols("AAPL") # daily Apple stock data
> prices  <- Ad(AAPL) # use adjusted price
> print(head(prices))
           AAPL.Adjusted
2007-01-03         80.54
2007-01-04         82.33
2007-01-05         81.75
2007-01-08         82.15
2007-01-09         88.97
2007-01-10         93.23
```

ROC function

Compute returns with function `ROC` from `TTR` package

`ROC` has parameter `type` which can either:

- `discrete` for **simple returns**
- `continuous` for **log returns**

Example

```
simple_diff <- diff(prices)

simple_ret  <- ROC(prices,type='discrete')

log_ret     <- ROC(prices,type='continuous')

prices <- cbind(prices,simple_diff,simple_ret,log_ret)

prices <- round(prices,3) # only show 3 decimal places

colnames(prices)[2:4] <- c("Simple Diff","Simple Ret","Log Ret")
```

Example

```
> print(head(prices))
           AAPL.Adjusted Simple Diff Simple Ret Log Ret
2007-01-03         80.54          NA         NA      NA
2007-01-04         82.33        1.79      0.022   0.022
2007-01-05         81.75       -0.58     -0.007  -0.007
2007-01-08         82.15        0.40      0.005   0.005
2007-01-09         88.97        6.82      0.083   0.080
2007-01-10         93.23        4.26      0.048   0.047

> prices[1,c(2:4)] <- 0
```

In all three cases we replaced the initial `NA` with a `0`.