# Solutions of the exercises on Turing Machines

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December, 2006

## Exercise 1 on slide 9

Find a sequence of configurations ending with an accepting configuration for $M_1$ on slide 6. Find a sequence leading to a rejecting configuration.

### Solution

1. Accepting configuration:
a) $q_0 \sqcup \rightarrow \sqcup q_{accept} \sqcup$

b) for $n \geq 1$ $q_0 0^n 1^n \rightarrow \sqcup q_1 0^{n-1} 1^n \rightarrow ... \rightarrow \sqcup 0^{n-1} 1^n q_1 \sqcup \rightarrow \sqcup 0^{n-1} 1^{n-1} q_3 1 \rightarrow$
(if $n \geq 1$) $\sqcup\, 0^{n-1} 1^{n-2} q_4 1 \sqcup \rightarrow ... \rightarrow q_4 \sqcup 0^{n-1} 1^{n-1} \sqcup \rightarrow q_0 0^{n-1} 1^{n-1} \rightarrow ... \rightarrow q_0 \sqcup \rightarrow \sqcup q_{accept}$

c) $q_0 0011 \rightarrow \sqcup q_1 011 \rightarrow \sqcup 0 q_1 11 \rightarrow \sqcup 01 q_1 1 \rightarrow \sqcup 011 q_1 \sqcup \rightarrow \sqcup 01 q_3 1 \rightarrow \sqcup 0 q_4 1 \sqcup \rightarrow \sqcup q_4 01 \sqcup \rightarrow$
$q_4 \sqcup 01 \sqcup \rightarrow q_0 01 \sqcup \rightarrow \sqcup q_1 1 \sqcup \rightarrow \sqcup 1 q_1 \sqcup \rightarrow \sqcup q_3 1 \sqcup \rightarrow q_4 \sqcup \sqcup \sqcup \rightarrow \sqcup q_0 \sqcup \rightarrow \sqcup q_{accept}$

2. Rejecting configuration:
a)$q_0 1 w \rightarrow 1 q_{reject} w$

b)$q_0 010 \rightarrow \sqcup q_1 10 \rightarrow \sqcup 1 q_1 0 \rightarrow \sqcup 10 q_1 \sqcup \rightarrow \sqcup 1 q_3 0 \rightarrow \sqcup q_{reject} 10$

## Exercise 2 on slide 9

Is $B = \{a^n b^n c^n : n = 0, 1, 2, ...\}$ Turing recognizable?

### Solution

Yes. TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where $Q = \{q_0, q_1, q_2, q_3, q_4, q_{accept}, q_{reject}\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{a, b, c, \sqcup\}$, $\delta$ is described as a state diagram (see Figure 1).

## Exercise 1 on slide 10

Argue why is the TM $M_1$ on slide 6 a decider?

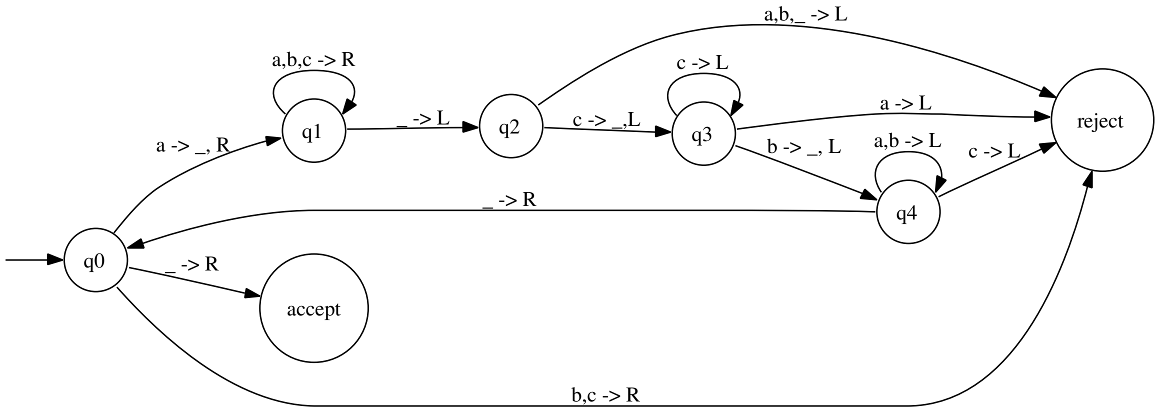


Figure 1. State diagram for TM that recognizes $\{a^n b^n c^n : n = 0, 1, 2, ...\}$

**Solution**

The TM $M_1$ is a decider because it never loops. And it is because all loops, namely $q_1 \overset{0,1\to R}{\longrightarrow} q_1$, $q_4 \overset{0,1\to L}{\longrightarrow} q_4$ and $q_0 \overset{0\to\sqcup,R}{\longrightarrow} q_1 \to ... \to q_1 \overset{\sqcup\to L}{\longrightarrow} q_3 \overset{1\to\sqcup,L}{\longrightarrow} q_4 \to ... \to q_4 \overset{\to R}{\longrightarrow} q_0$, will always end. The first two loops will end, since input, surrounded by $\sqcup$'s, is finite. Each round of the last loop replaces one $0$ and one $1$ with $\sqcup$, and so it ends when no 0's or 1's are left, or by going to *reject* or *accept* states.

# Exercise 2 on slide 10

Is $B = \{a^n b^n c^n : n = 0, 1, 2, ...\}$ decidable? Justify your answer.

**Solution**

$B = \{a^n b^n c^n : n = 0, 1, 2, ...\}$ is decidable, because there exists a TM $M$ from the exercise 2 on slide 9 (Figure 1.), that recognizes it. And this TM is a decider, due to the explanation similar to one in the exercise 1 on slide 10.

# Exercise 2 on slide 17

Define a TM with tape alphabet $0, 1$, and blank that given a string in $\{0, 1\}^*$ as input replaces all 0's on the tape with 1's and otherwise leaves the input unchanged. You are not supposed to give a high-level definition.

**Solution**

TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where $Q = \{q_0, q_{accept}, q_{reject}\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $\delta$ is described as a state diagram (see Figure 2).

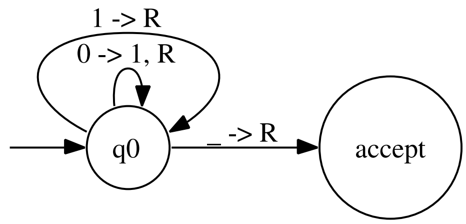


Figure 2. State diagram for TM that replaces all 0‘s with 1‘s.

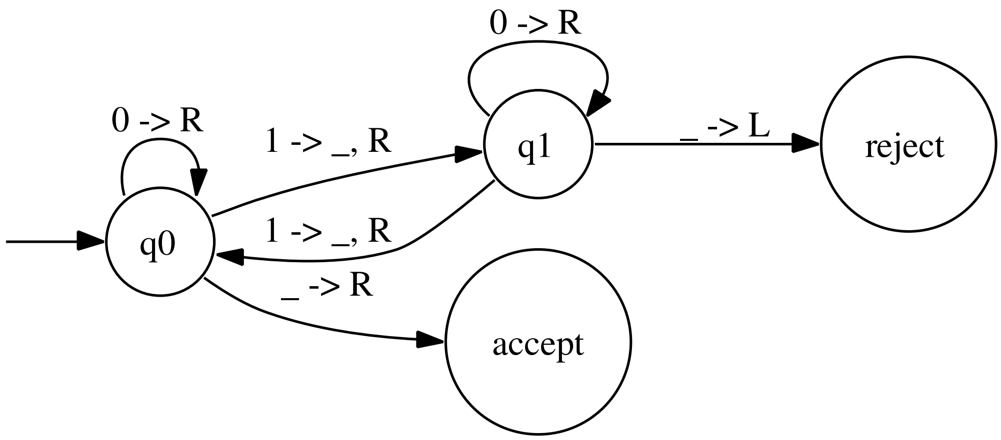


Figure 3. State diagram for TM that decides string containing even number of 1‘s

# Exercise 3 on slide 17

Define a TM that decide all strings in $\{0,1\}^*$ that contains an even number of 1‘s. You are not supposed to give a high-level definition.

### Solution

TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_1, q_{accept}, q_{reject})$, where $Q = \{q_0, q_1, q_{accept}, q_{reject}\}$, $\Sigma = \{0,1\}$, $\Gamma = \{0, 1, \sqcup\}$, $\delta$ is described as a state diagram (see Figure 3).

# Exercise 4 on slide 17

Give a high-level definition of a TM that recognizes the set $\{a^n b^{2n} : n = 0, 1, 2, ...\}$.

### Solution

1. Put the control on the leftmost tape symbol.
2. If the current tape symbol is not $a$ or $\sqcup$, reject.
3. If the current tape symbol is $\sqcup$, accept.
4. If the current tape symbol is $a$, replace it with $\sqcup$ and scan right to the next $\sqcup$, which is the end of the input.
5. As soon as the $\sqcup$ is reached, move one tape symbol to the left.
6. If this tape symbol is not $b$, reject, otherwise replace it with $\sqcup$ and move one tape symbol to the left.

7. If this tape symbol is not $b$, reject, otherwise replace it with $\sqcup$ and move to the left until $\sqcup$ is reached, so that control is set to the first tape symbol after the reached $\sqcup$.
8. Go to stage 2.