CS 281 – Homework 1
Solutions

1. **Exercise 8.2.2:** Design Turing machines for the following languages:

   (b) $\{a^n b^n c^n \mid n \geq 1\}$ (10 pts)

   (c) $\{ww^R \mid w$ is any string of 0's and 1's$\}$ (10 pts)

   <u>**Solution:**</u>

   (b) $\{a^n b^n c^n \mid n \geq 1\}$

$$M = (\{q_0, q_a, q_b, q_c, q_Y, q_Z, q_f\}, \{a, b, c\}, \{a, b, c, X, Y, Z\}, \delta, q_0, B, \{q_f\})$$

where δ is as follows:

State	a	b	c	X	Y	Z	B
q_0	(q_a, X, R)	-	-	-	(q_Y, Y, R)	-	-
q_a	(q_a, a, R)	(q_b, Y, R)	-	-	(q_a, Y, R)	-	-
q_b	-	(q_b, b, R)	(q_c, Z, L)	-	-	(q_b, Z, R)	-
q_c	(q_c, a, L)	(q_c, b, L)	-	(q_0, X, R)	(q_c, Y, L)	(q_c, Z, L)	-
q_Y	-	-	-	-	(q_Y, Y, R)	(q_Z, Z, R)	-
q_Z	-	-	-	-	-	(q_Z, Z, R)	(q_f, B, R)

Note: The above TM is the 3-character version of the same machine given in the textbook Example 8.2.

   (c) $\{ww^R \mid w$ is any string of 0's and 1's$\}$

$$M = (\{q_{init}, q_0, q_1, q_{0^R}, q_{1^R}, q_{back}, q_f\}, \{0, 1\}, \{0, 1\}, \delta, q_{init}, B, \{q_f\})$$

where δ is as follows:

State	0	1	B
q_{init}	(q_0, B, R)	(q_1, B, R)	(q_f, B, R)
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	(q_{0^R}, B, L)
q_1	$(q_1, 0, R)$	$(q_1, 1, R)$	(q_{1^R}, B, L)
q_{0^R}	(q_{back}, B, L)	-	-
q_{1^R}	-	(q_{back}, B, L)	-
q_{back}	$(q_{back}, 0, L)$	$(q_{back}, 1, L)$	(q_{init}, B, R)

2. **Exercise 8.2.3:** Design a Turing machine that takes as input a number N and adds 1 to it in binary. To be precise, the tape initially contains a \$ followed by N in binary. The tape head is initially scanning the \$ in state q_0. Your TM should halt with $N+1$, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state q_f. You may destroy the \$ in creating $N+1$, if necessary. For instance, $q_0\$10011 \vdash^* \$q_f10100$, and $q_0\$11111 \vdash^* q_f100000$.

   (a) Give the transitions of your Turing machine, and explain the purpose of each state. (10 pts)

   (b) Show the sequence of ID's of your TM when given input \$111. (5 pts)

   Solution:

   (a) Give the transitions of your Turing machine, and explain the purpose of each state.

State	\$	0	1	B	Explanation
q_0	$(q_R, \$, R)$	-	-	-	Initial \$ reader
q_R	-	$(q_R, 0, R)$	$(q_R, 1, R)$	(q_1, B, L)	Seeking end of string
q_1	$(q_B, 1, L)$	$(q_L, 1, L)$	$(q_1, 0, L)$	-	Carry bit $= 1$
q_L	$(q_f, \$, R)$	$(q_L, 0, L)$	$(q_L, 1, L)$	-	Carry bit $= 0$
q_B	-	-	(q_f, B, R)	-	Formatting

   (b) Show the sequence of ID's of your TM when given input \$111.

$$q_0\$111 \vdash \$q_R111 \vdash \$1q_R11 \vdash \$11q_R1 \vdash \$111q_R \vdash$$
$$\$11q_11 \vdash \$1q_110 \vdash \$q_1100 \vdash q_1\$000 \vdash q_BB1000 \vdash q_f1000$$

3. **Exercise 8.3.2:** A common operation in Turing-machine programs involves "shifting over." Ideally, we would like to create an extra cell at the current head position, in which we could store some character. However, we cannot edit the tape in this way. Rather, we need to move the contents of each of the cells to the right of the current head position one cell right, and then find our way back to the current head position. Show how to perform this operation. *Hint:* Leave a special symbol to mark the position to which the head must return. (10 pts)

   Solution:

   Assuming $\Sigma = \{0, 1\}$, `shift` converts an ID of the form wq_1v to ID wq_6Bv. `shift` is defined as follows:

State	0	1	B
q_1	(q_2, B, R)	(q_3, B, R)	(q_5, B, L)
q_2	$(q_2, 0, R)$	$(q_3, 0, R)$	$(q_4, 0, L)$
q_3	$(q_2, 1, R)$	$(q_3, 1, R)$	$(q_4, 1, L)$
q_4	$(q_4, 0, L)$	$(q_4, 1, L)$	(q_5, B, L)
q_5	$(q_6, 0, R)$	$(q_6, 1, R)$	(q_6, B, R)

   Note: There is no extra character, B is used to keep track of the location of call.