COMP226: Slides 13

Moving averages

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Using more historical data

- The copycat strategy we saw earlier used only **one day's history** to make a trading decision

- **Moving averages** are a useful way to incorporate more of the recent price history in trading decisions

Filtering time series

Two types of filter we consider:

- **low-pass filter**

  removes short-term fluctuations (noise); leaves trend

- **high-pass filter**

  removes trend; leaves short-term fluctuations

The terms low and high refer to **frequencies**

A rich theory of **filters** is central to **digital signal processing**

We will look at basic examples commonly used with time series

Causal filters

A filter is

- **causal** if its output depends only on past and present inputs
- **non-causal** if its output also depends on future inputs

> ***Warning***
>
> When developing trading systems we must avoid **look-ahead bias** (the system must only use data that is available at the time of a decision)
>
> This is why **we only consider causal filters**

Low-pass filter

- Often we are interested in the **trend** of a time series
- Short-term fluctuations obscure the trend
- To remove this **noise** we apply a low-pass filter
- We consider a simple implementation via moving averages

Moving average

Moving averages are a type of **digital** **low-pass filter**

Sometimes called a

rolling average or **rolling mean**

(later we consider rolling standard deviations and rolling medians)

We consider three types of moving average:

1. **Simple** (weights are all equal)
2. **Weighted** (weights form arithmetic progression)
3. **Exponential** (weights form geometric progression)

Notation

Notation	Meaning
t	current time period
x	time series
$x_1,\ x_2, \ldots,\ x_t,\ x_{t+1}, \ldots$	individual time series measurements
s_t	current moving average value

Simple moving average

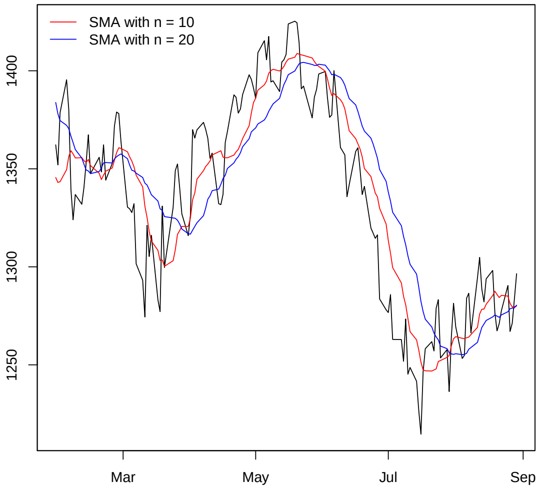
Parameter: n

- n is the number of periods to average over
- **Larger** n gives **more smoothing**

$$s_t = \frac{1}{n}(x_t + x_{t-1} + \ldots + x_{t-n+1}) = \frac{1}{n}\sum_{i=t-n+1}^{t} x_i$$

Easy update formula:

$$s_t = s_{t-1} + \frac{1}{n}(x_t - x_{t-n})$$



SMA with n = 10
SMA with n = 20
1250
1300
1350
1400
Mar
May
Jul
Sep

Exponential moving average

Parameter:

$$0 < \alpha < 1$$

- Weighted average of **all** previous values
- **Exponentially more weight on recent values**

Recursive definition:

$$s_1 = x_1$$

New value = **convex combination** of new input & old EMA value:

$$s_t = \alpha x_t + (1 - \alpha) s_{t-1}, \quad t > 1$$

Substituting

$$s_t = \alpha \cdot [x_t + (1-\alpha) x_{t-1} + \ldots + (1-\alpha)^{t-2} x_{2-(k+1)}] + (1-\alpha)^{t-1} x_1$$

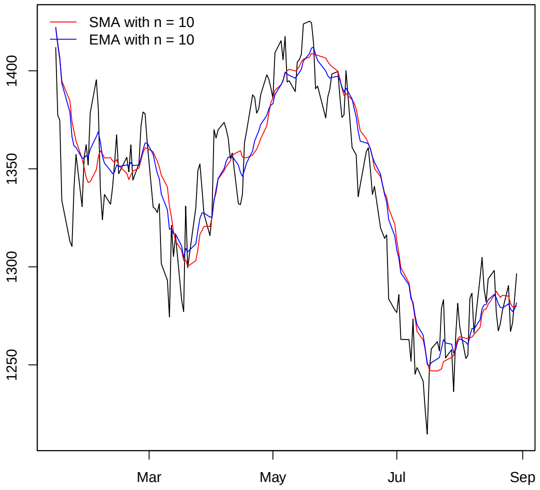
Simple versus Exponential MA

Similarities:

- **lagging** (they turn after the time series does)
- Roughly equal distribution of forecast error for alpha = 2/(n+1)

Differences:

- EMA responds more quickly to prices
- EMA takes into account all past data; SMA takes into account only n most recent data points
- EMA only needs the most recent EMA value to be kept; SMA requires all n most recent data points be kept


SMA with n = 10
EMA with n = 10
1250
1300
1350
1400
Mar
May
Jul
Sep

Comparison of weights

Let's compare the weights for the Simple, Weighted, and Exponential MAs

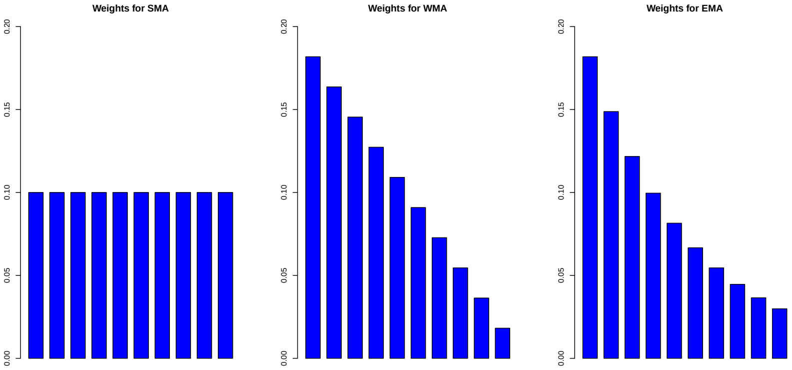
```
n <- 10

# simple
si <- rep(1/n,n)

# weighted
w <- seq(2/(n+1),by=-2/((n+1)*n),length.out=n)

# exponential
alpha <- 2/(n+1)
ex <- sapply(1:n,function(x) alpha*(1-alpha)^(x-1))
```

Comparison of weights

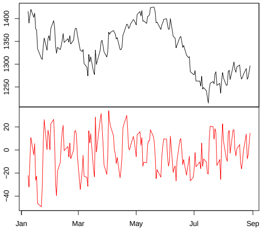


High-pass filter

Detrending filter: lets through high-frequency noise; rejects low-frequency trend - can be implemented as follows:

1. Apply low-pass filter to data

2. Subtract filtered data from original data



Summary on Moving Averages

- Moving averages are simple methods to **remove noise** from time series
- They can also be used to **de-trend** time series
- They are a building block for many technical analysis indicators that are used in trading strategies