COMP226: Slides 09

Execution algorithms

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Motivation

- Pension funds or other **buy-side firms often want trade a "large" volume** and will often task this to an agent
- **What is a large volume?** Depends on the market and order book. Large volume cannot be transacted as a market order without a significant price impact. Recall: **Market orders** incur **slippage**; **limit orders** may not get executed
- The agent typicallys uses an **execution algorithm** to automatically split the large volume and execute it incrementally (called **iceberging**) in an attempt to hide the existence of the large order and minimize price impact
- **Goal of the execution algorithm** - **minimize execution cost** in comparison to a given **execution benchmark**; we discuss 3 common benchmarks in these slides

Benchmarks

We study 3 standard **benchmarks** for **execution algorithms**:

- **VWAP** - Volume-Weighted Average Price
- **TWAP** - Time-Weighted Average Price
- **Implementation Shortfall**

Notation

We use the following notation for a sequence of n trades:

$$(t_1, p_1, v_1), (t_2, p_2, v_2), \ldots, (t_n, p_n, v_n)$$

where

Notation	Meaning
t_i	time of trade i
p_i	price of trade i
v_i	volume of trade i

Volume-Weighted Average Price

Benchmark:

Volume-Weighted Average Price of market over a fixed time period

$$\left(\sum_{i=1,\ldots,n} p_i \times v_i \right) / \sum_{i=1,\ldots,n} v_i$$

VWAP of the order is compared to the VWAP of the market

Strategy:

Goal is to do at least as well as VWAP of market

Concentrates on minimizing market impact by splitting the order into quantities based on prediction of volume distribution (usually just the historic volume profile)

Time-Weighted Average Price

Benchmark:

Time-Weighted Average Price of market over a fixed time period

$$\left(\sum_{i=2,\ldots,n} p_{i-1} \times \ (t_i - t_{i-1}) \right) / (t_n - t_1)$$

VWAP of the order is compared to the TWAP of the market

Strategy:

Goal is to do at least as well as TWAP of market

Trade based on a uniform time schedule, i.e., divide order into individual units and trade equi-spaced in time

Implementation Shortfall

Benchmark:

Implementation Shortfall is the difference between the price (midprice) of an asset at the time of a trading decision and the (average) execution price.

Thus it is a similar concept to "slippage".

Goal is to minimize the implementation shortfall

Strategy:

Seek to minimize both market impact and risk, often by determining optimal rate of trading

If the market is moving unfavourably one should execute quickly (e.g. for a long trade in a rising market); it is moving favourably one can hold off to try to achieve a favourable price.

Load in tick data

```
library(xts)

input <- read.csv("tick.csv",stringsAsFactors=FALSE)
```

```
> head(input)
           DateTime   Price Size
1 20040901 00:02:01 1105.25    1
2 20040901 00:02:02 1105.25   20
3 20040901 00:02:16 1105.25    5
4 20040901 00:03:33 1105.25    5
5 20040901 00:04:50 1105.25    1
6 20040901 00:04:58 1105.00    1
```

Convert to xts object

```
index <- as.POSIXct(input[,"DateTime"],format="%Y%m%d %H:%M:%S")

trades <- xts(input[,-c(1)],index)
```

```
> head(trades)
                     Price Size
2004-09-01 00:02:01 1105.25    1
2004-09-01 00:02:02 1105.25   20
2004-09-01 00:02:16 1105.25    5
2004-09-01 00:03:33 1105.25    5
2004-09-01 00:04:50 1105.25    1
2004-09-01 00:04:58 1105.00    1
```

Example: TWAP benchmark

Goal: compute a **time-weighted average price**:

Step 1: compute time differences that tell us how long each price level lasted

```
> head(index(x))
[1] "2004-09-01 00:02:01 BST" "2004-09-01 00:02:02 BST"
[3] "2004-09-01 00:02:16 BST" "2004-09-01 00:03:33 BST"
[5] "2004-09-01 00:04:50 BST" "2004-09-01 00:04:58 BST"

> timediffs <- diff(index(trades))
> timediffs <- c(timediffs,0) # add zero at end
> head(timediffs)
Time differences in secs
[1]   1  14  77  77   8 157
```

Example: TWAP benchmark

The first time difference should correspond to the first price:

```
> trades <- cbind(trades,timediffs)
> colnames(trades)[ncol(trades)] <- "TimeDiff"
> print(head(trades))
                     Price Size TimeDiff
2004-09-01 00:02:01 1105.25    1        1
2004-09-01 00:02:02 1105.25   20       14
2004-09-01 00:02:16 1105.25    5       77
2004-09-01 00:03:33 1105.25    5       77
2004-09-01 00:04:50 1105.25    1        8
2004-09-01 00:04:58 1105.00    1      157
```

Example: TWAP benchmark

Step 2: multiply time difference by corresponding price and sum

Step 3: divide by sum of time differences

```
> sum(timediffs*trades$Price)/sum(timediffs)
> twap
[1] 1109.713
```

Example: VWAP

Recall: R does multiplication **component-wise**, which is what we want here

```
> weighted_price <- sum(trades$Price*trades$Size)
> total_volume   <- sum(trades$Size)

> vwap <- weighted_price/total_volume
> vwap
[1] 1110.721
```

```
> library(TTR)
> vwap_ttr <- last(VWAP(trades$Price,trades$Size,n=nrow(trades)))
> vwap_ttr
                         [,1]
2004-09-03 16:15:01 1110.721
```

TWAP versus VWAP

The two will in general be quite different, since:

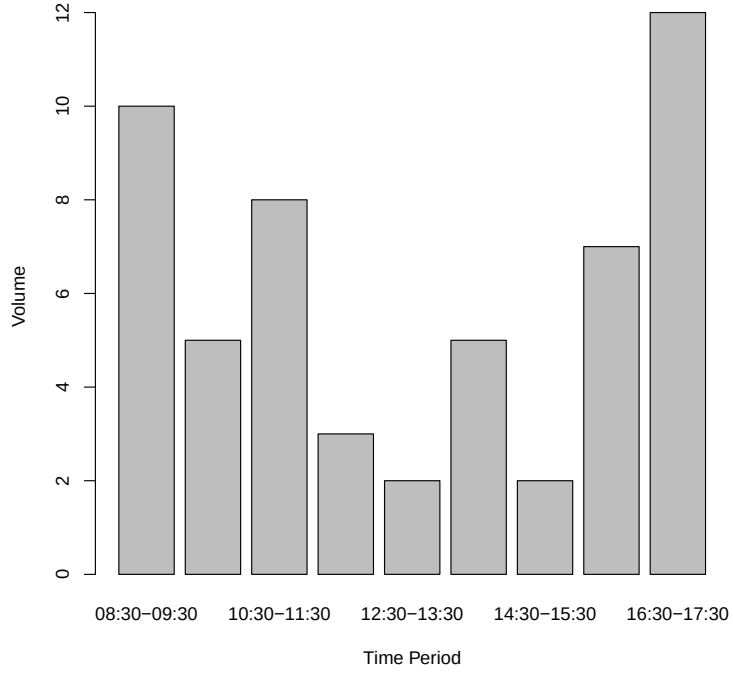
- VWAP will place
    - **low weight** on the prices during **time periods with low volume** and
    - **high weight** on **time periods with high volume**
- TWAP places **equal weight** on prices from all time periods

In our example, VWAP was 1110.721 and TWAP was 1109.713

The discrepency is evidence that the **volume curve is not flat**: normally there is more trading at the open and close

VWAP strategy

- Model the volume distribution using historical data
- Slice trades according to predicted distribution



TWAP strategy

Inputs: Order size, time period

Strategy:

- Split order into slices (e.g. 1 lots)
- Divide time equally to give the correct number of trading points for the size of each slice

TWAP versus VWAP

Notice that a TWAP strategy is like a VWAP one with a **uniform (flat) volume distribution**

Implementation shortfall strategy

Need to decide if price is going to move favourably or not

Trade off:

- **market impact** (place order now) with
- **volatility risk** (wait and risk adverse market moves)

References

Algorithmic trading & DMA : an introduction to direct access trading strategies, Barry Johnson.

TWAP: pages 120-123, 169, 200

VWAP: pages 123-127, 170, 200

Implementation Shortfall: pages 49, 133-138, 162, 200