COMP226 Assignment 2: Strategy Development

The latest version of this document can be found here:

https://www2.csc.liv.ac.uk/~rahul/teaching/comp226/_downloads/a2.pdf

Continuous Assessment Number	2 (of 2)
Weighting	10%
Assignment Circulated	Thursday 18 March 2020
Deadline	17:00 Friday 1st May 2020
Submission Mode	Electronic only
Learning Outcomes Assessed	This assignment will address the following learning outcomes: • Understand the spectrum of computer-based trading applications and techniques, from profit-seeking trading strategies to execution algorithms. • Be able to design trading strategies and evaluate critically their historical performance and robustness. • Understand the common pitfalls in developing trading strategies with historical data. • Understand methods for measuring risk and diversification at the portfolio level.
Summary of Assessment	The goal of this assignment is to implement and optimize a well-defined trading strategy within the `backtester_v5.3` framework. The assignment will be assessed via the testing of 6 functions that you need to implement. The input and output behaviour of each function is fully specified and a code template is provided as a starting point.
Marking Criteria	Individual marks are attributed for each of 6 functions that should be implemented. If all 6 function implementations pass all the automated tests then a mark of 100% will be achieved. Partial credit for a function may be awarded if some but not all automated tests for that function are passed. The marks available for each function are given below.
Submission necessary in order to satisfy module requirements	No
Late Submission Penalty	Standard UoL policy; **note that no resubmissions after the deadline will be considered**.
Expected time taken	Roughly 8 hours

Introduction: the backtester framework

You will write a strategy that should run in the backtester framework, which is available from

http://www2.csc.liv.ac.uk/~rahul/teaching/comp226/bt.html#backtester

The first thing you should do is download and unzip `backtester_v5.3.zip`, which will create a directory `backtester_v5.3` on your hard drive. Here is a listing of the zip file contents:

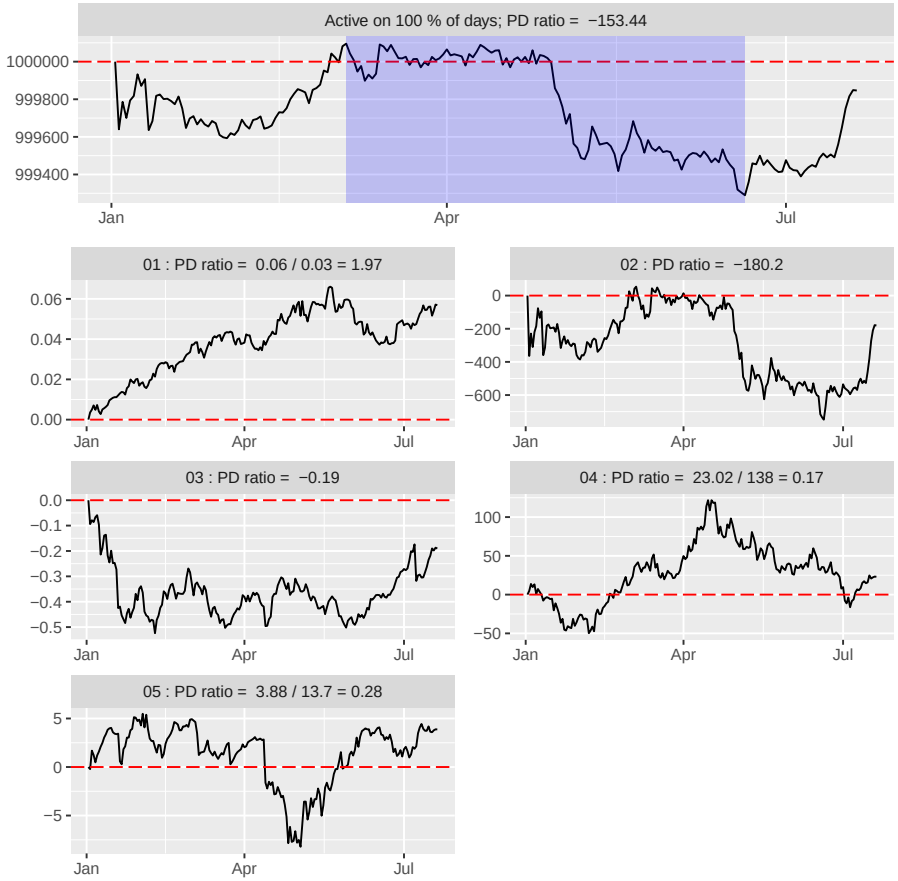
```
backtester_v5.3
├── DATA
│   ├── A2
│   │   ├── 01.csv
│   │   └── 02.csv
│   └── EXAMPLE
│       ├── 01.csv
│       ├── 02.csv
│       ├── 03.csv
│       ├── 04.csv
│       └── 05.csv
├── example_strategies.R
├── framework
│   ├── backtester.R
│   ├── data.R
│   ├── processResults.R
│   └── utilities.R
├── in-sample_period.R
├── main.R
├── main_optimize.R
├── main_template.R
└── strategies
    ├── a2_template.R
    ├── bankrupt.R
    ├── bbands_contrarian.R
    ├── bbands_holding_period.R
    ├── bbands_trend_following.R
    ├── big_spender.R
    ├── copycat.R
    ├── extreme_limit.R
    ├── fixed.R
    ├── random.R
    ├── rsi_contrarian.R
    └── simple_limit.R

5 directories, 28 files
```

Next you should open R and make sure that the working directory is the `backtester_v5.3` directory on your hard drive (you can use `setwd` if required). You can now try the example code as follows:

```
source('main.R')
```

If this doesn't work, first make sure you are have set the working directory correctly, and then make sure you have installed all the required packages (see the error messages you get to figure out what the problem is). When it works it will produce a plot like the following:



There is one equity curve for each series in the data (5 of them in this case), and one aggregate equity curve.

Let's go through `main.R` and see what the individual parts do.

Sourcing the framework and example strategies

First we source the framework itself.

```
source('framework/data.R')
source('framework/backtester.R')
source('framework/processResults.R')
source('framework/utilities.R')
```

Then we load `example_strategies.R`, which provides an easy way to run several examples, and which we will return to shortly.

```
source('example_strategies.R')
```

Loading data

Next, we load in data that we will use via the function `getData`, which is defined in `framework/data.R`. This function returns a list of xts objects. These will be passed to the function `backtester`, though we may first change the start and end dates of the xts objects (which you will need to do for assignment 2).

```
# load data
dataList <- getData(directory="EXAMPLE")
```

There are 5 series in the directory `backtester_5.3/DATA/EXAMPLE/`, and therefore the list `dataList` has 5 elements too.

```
> length(dataList)
[1] 5
```

Each element is an xts:

```
> for (x in dataList) print(class(x))
[1] "xts" "zoo"
[1] "xts" "zoo"
[1] "xts" "zoo"
[1] "xts" "zoo"
[1] "xts" "zoo"
```

All the series have the same start and end dates:

```
> for (x in dataList) print(paste(start(x),end(x)))
[1] "1970-01-02 1973-01-05"
[1] "1970-01-02 1973-01-05"
[1] "1970-01-02 1973-01-05"
[1] "1970-01-02 1973-01-05"
[1] "1970-01-02 1973-01-05"
```

The individual series contain Open, High, Low, Close, and Volume columns:

```
> head(dataList[[1]])
             Open   High    Low  Close Volume
1970-01-02 0.7676 0.7698 0.7667 0.7691   3171
1970-01-03 0.7689 0.7737 0.7683 0.7729   6311
1970-01-04 0.7725 0.7748 0.7718 0.7732   4317
1970-01-05 0.7739 0.7756 0.7739 0.7751   3409
1970-01-06 0.7760 0.7770 0.7754 0.7757   2904
1970-01-07 0.7738 0.7744 0.7728 0.7743   3514
```

Loading a strategy

We will now load a strategy using the `load_strategy` function that is defined in `example_strategies.R`. We can pick a strategy from a list of examples strategies that is specified at the start of `example_strategies.R`:

```
example_strategies <- c("fixed",
                        "big_spender",
                        "bankrupt",
                        "copycat",
                        "random",
                        "rsi_contrarian",
                        "bbands_trend_following",
                        "bbands_contrarian",
                        "bbands_holding_period",
                        "simple_limit")
```

Returning now to `main.R` we see that we have picked one of these (and then checked that it was a valid choice with)

```
# choose strategy from example_strategies
strategy <- "fixed"

# check that the choice is valid
is_valid_example_strategy <- function(strategy) {
    strategy %in% example_strategies
}
stopifnot(is_valid_example_strategy(strategy))
```

Now we actually "load the strategy".

```
# load in strategy and params
load_strategy(strategy) # function from example_strategies.R
```

We used the function `load_strategy` from `example_strategies.R`. This function sources the strategy file, in this case `backtester_v5.3/strategies/fixed.R`, and sets a variable `params` using `example_params` from `example_strategies.R`:

```
load_strategy <- function(strategy) {
    # load strategy
    strategyFile <- file.path('strategies', paste0(strategy,'.R'))
    cat("Sourcing",strategyFile,"\n")
    source(strategyFile) # load in getOrders
    # set params via global assignment
    params <<- example_params[[strategy]]
    print("Parameters:")
    print(params)
}
```

The structure of a strategy

Here is the contents of the strategy file `backtester_v5.3/strategies/fixed.R`:

```
# This strategy only uses market orders

# params$sizes specifies a fixed number of contracts per series

# We take the corresponding long/short position in each series
# by placing a market order on the 1st iteration

# No further orders are placed by getOrders

# The backtester automatically exits all positions
# as market orders at the end when the data runs out

getOrders <- function(store, newRowList, currentPos, info, params) {
    allzero      <- rep(0,length(newRowList))
    marketOrders <- allzero
    if (is.null(store)) {
        # take position during first iteration and hold
        marketOrders <- params$sizes
        store <- 1 # not null
    }
    return(list(store=store,marketOrders=marketOrders,
                            limitOrders1=allzero,
                            limitPrices1=allzero,
                            limitOrders2=allzero,
                            limitPrices2=allzero))
}
```

The backtester framework runs a strategy by calling `getOrders`. The arguments to `getOrders` are fixed, i.e., they are the same for all strategies. In the example strategy `fixed.R`, `getOrders` is the only function. The arguments to `getOrders` are as follows:

```
getOrders <- function(store, newRowList, currentPos, info, params) {
```

- `store`: contains all data you choose to save from one period to the next
- `newRowList`: new day's data (a list of single rows from the series)
- `currentPos`: the vector of current positions in each series
- `params`: a list of parameters that are sent to the function

Here's how the strategy `fixed.R` works. In the very first period the backtester always (for every strategy) passes `store` to `getOrders` with `NULL` as its value. Thus in the first period, and the first period only, the vector `marketOrders` will be set to the parameter `params$sizes`, which should be a vector of positions with length equal to the number of series, which is 5 in this case. In `example_strategies.R` we see this parameter `params$sizes`, which is the only parameter for `fixed.R`, set as follows:

```
list(sizes=rep(1,5))
```

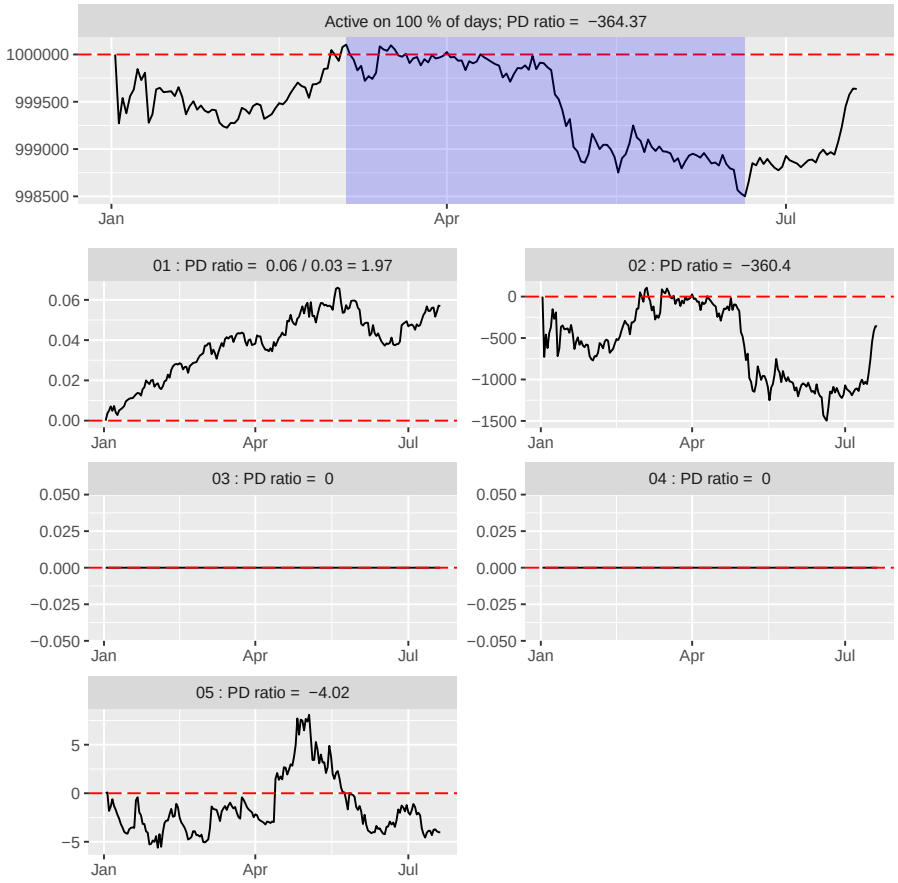
With these sizes, we buy and hold one unit in every series.

Changing the parameters

We can change the parameters and take positions in only some series and go short in some series, e.g., with:

```
params <- list(sizes=c(1,2,0,0,-1))
```

We can set this either in `example_strategies.R`, or in `main.R`, as long as it comes after we have called `load_strategy`.



Compare the new equity curves with the ones above. Note that for series 1 they are the same, for series 2 the new one is scaled by 2, for series 3 and 4 we no longer trade, and for series 5 we now take a short position, so the new series 5 is a reflection (in the profit and loss axis) of the old series 5 equity curve.

Market orders

The `fixed` example strategy enters position at the first opportunity using *market orders*. The backester framework supports market and limit orders and some of the example strategies use limit orders. However, we will **not** use limit orders for assignment 2, only market orders.

Recall that market orders specify volume and direction (but not price), and limit orders specify price, volume, and direction. In the backtester framework, trading decisions are made after the close of day `k`, and trades are executed on day `k+1`. For each day, the framework supports `one market order` for each series, and `two limit orders` for each series. These orders are returned from `getOrders` as follows:

```
return(list(store=store,marketOrders=marketOrders,
                    limitOrders1=limitOrders1,
                    limitPrices1=limitPrices1,
                    limitOrders2=limitOrders2,
                    limitPrices2=limitPrices2))
```

Market orders will be `executed at the open` on day `k+1`. The `sizes` and `directions` of market orders are encoded in the vector `marketOrders` of the return list of `getOrders`. For example, the vector

```
c(0,-5,0,1,0)
```

means place a market order for 5 units short in series 2, and 1 unit long in series 4.

To repeat, we will not use limit orders for assignment 2, so you can leave `limitOrders1`, `limitPrices1`, `limitOrders2`, `limitPrices2` as zero vectors when you do assignment 2.

Subsetting the data

The next thing in `main.R` is a subsetting of the time period for the backtest as follows:

```
inSampDays <- 200 # in-sample period 1:inSampDays
dataList <- lapply(dataList, function(x) x[1:inSampDays])
```

So we are only using the first 200 days.

> ***Hint***
>
> You should adapt this use of `lapply` on `dataList` in order to define the in-sample period in assignment 2.

Running the backtest

Finally we actually do the backtest and plot the results as follows:

```
# Do backtest
results <- backtest(dataList,getOrders,params,sMult=0.2)
pfolioPnL <- plotResults(dataList,results)
```

The arguments to the function `backtest` are the following:

- `dataList` - list of (daily) `xts` objects (with identical indexes)
- `getOrders` - the strategy
- `params` - the parameters for the strategy
- `sMult` - slippage multiplier (proportion of overnight gap)

Results for individual series are available in `results$pnlList`. The portfolio results are available in `pfolioPnL`, which is produced by `plotResults(dataList,results)`. This function also automatically plots individual and aggregate equity curves, and computes variant of the Calmar Ratio that we call the Profit Drawdown Ratio (PD ratio for short) - it is the final profit divided by the maximum drawdown in terms of profit and loss, or if the

strategy makes a loss overall the PD ratio is just that loss (which is negative). You do not need to write code to compute this, since it has already been done for you. In assignment 2 we will optimize the aggregate PD ratio - this value is stored in `pfolioPnL$fitAgg`, e.g., for our first example we have:

```
> print(pfolioPnL$fitAgg)
[1] -153.44
```

This matches up with the PD ratio that appears at the top of the aggregate equity curve produced by `plotResults`.

Parameter optimization

Before we move on to assignment 2, we will briefly look at an example of parameter optimization that will be useful for assignment 2. To make it easier to carry out parameter optimizations, `getOrders` takes an argument `params`. This can be used to pass a parameter combination to a strategy. This in turn can be used to do a parameter optimization as `main_optimize.R` demonstrates. Here is the source code for `main_optmize.R`, which uses the example strategy `bbands_contrarian`, which is an implementation of the "BBands Overbought/Oversold" strategy from the slides:

```
source('framework/data.R'); source('framework/backtester.R')
source('framework/processResults.R'); source('strategies/bbands_contrarian.R')

numOfDays <- 200
dataList <- getData(directory="EXAMPLE")
dataList <- lapply(dataList, function(x) x[1:numOfDays])
sMult <- 0.2 # slippage multiplier

lookbackSeq <- seq(from=20,to=40,by=10)
sdParamSeq  <- seq(from=1.5,to=2,by=0.5)
paramsList  <- list(lookbackSeq,sdParamSeq)
numberComb <- prod(sapply(paramsList,length))

resultsMatrix <- matrix(nrow=numberComb,ncol=3)
colnames(resultsMatrix) <- c("lookback","sdParam","PD Ratio")
pfolioPnLList <- vector(mode="list",length=numberComb)
count <- 1
for (lb in lookbackSeq) {
    for (sdp in sdParamSeq) {
        params <- list(lookback=lb,sdParam=sdp,series=1:5,posSizes=rep(1,5))
        results <- backtest(dataList, getOrders, params, sMult)
        pfolioPnL <- plotResults(dataList,results)
        resultsMatrix[count,] <- c(lb,sdp,pfolioPnL$fitAgg)
        pfolioPnLList[[count]]<- pfolioPnL
        cat("Just completed",count,"out of",numberComb,"\n")
        #print(resultsMatrix[count,])
        count <- count + 1
    }
}
print(resultsMatrix[order(resultsMatrix[,"PD Ratio"]),])
```

The code template and data for assignment 2

You are now ready to start working on assignment 2. To do so you should read and work through the rest of this document very carefully.

As a first step, try to run `main_template.R`, which is setup to use the right data for the assignment and to load in the template strategy code `strategies/a2_template.R`. If you try to source `main_template.R` you will get an error as follows:

```
Error in if (store$iter > params$lookbacks$long) { :
   argument is of length zero
```

If you read on you will see that the final strategy requires a parameter called `lookbacks`. Read on to see what form this parameter should take.

The code template contains templates for the 6 functions that you need to complete. These functions are:

1. `getTMA`
2. `getPosSignFromTMA`
3. `getPosSize`
4. `getOrders`
5. `getInSampleResult`
6. `getInSampleOptResult`

The rest of the document is split into two parts. The first part describes the function requirements and marking criteria for the first 4 functions, which relate to the strategy implementation. The second part describes the function requirements and marking criteria for the last 2 functions. Hints are given on how best to implement things, so read carefully. **For all 6 functions, example outputs are provided so that you can test whether you have implemented the functions correctly.**

> ***Note***
>
> You can develop the first three functions without running the backtester, which may be easier.

Part 1: strategy implementation

The overall goal of the assignment is the implementation and optimization of a triple moving average crossover (TMA) trading strategy. The specification of the strategy and the functions that it should comprise are given **in full detail**, so the correctness of your code can and will be checked automatically.

The TMA strategy you will implement is related to Example 1 in COMP226 slides 17. However, long and short positions are swapped as compared to that example (so you will here implement a mean-reversion as opposed to a trend following type strategy).

The strategy uses three moving averages with three different lookbacks (window lengths). The short lookback should be smaller than the medium window, which in turn should be smaller than the long lookback. In every trading period, the strategy will compute the value of these three moving averages. You will achieve this be completing the implementation of the function `getTMA`.

The following table indicates the position that the strategy will take depending on the relative values of the three moving averages (MAs). You will compute this position (sign, but not size) by completing the function `getPosSignFromTMA`. The system is out of the market (i.e., flat) when the relationship between the short moving average and the medium moving average does not match the relationship between the medium moving avergage and long moving average.

MA		MA		MA	Position
short	>	medium	>	long	short
short	<	medium	<	long	long

The function `getPosSignFromTMA` should use a function `getTMA`. The position size, i.e., the number of units to be long or short, will be determined by the function `getPosSize`. Finally, as usual in the backtester framework for COMP226 and COMP396, the position sizes are given to the backtester in the function `getOrders`. Here are the detailed specification and marks available for these first 4 functions.

Function name	Input parameters	Expected behaviour	Marks available for a correct implementation
`getTMA`	`close_prices`; `lookbacks`. The specific form that these arguments should take is specified in the template code via the 6 checks that you need to implement.	You should first implement the checks as described in the template. Hints of how to implement them are given below. The function should return a list with three named elements (named short, medium, and long). Each element should be equal to the value of a simple moving average with the respective window size as defined by `lookbacks`. The windows should all end in the same period, which should be the final row of `close_prices`.	18% (3% per check) for the checks; 12% for a correct return
`getPosSign FromTMA`	`tma_list` is a list with three named elements, short, medium, and long. These correspond to the simple moving averages as returned by `getTMA`. **Note**: You do not need to check the validity of the function argument in this case, or for the remaining functions either.	This function should return either 0, 1, or -1. If the short value of `tma_list` is less than the medium value, and the medium value is less than the long value, it should return 1 (indicating a long position). If the short value of `tma_list` is greater than the medium value, and the medium value is greater than the long value, it should return -1 (indicating a short position). Otherwise, the return value should be 0 (indicating a flat position).	15%
`getPosSize`	`current_close`: this is the current close for one of the series. `constant`: this argument should have a default value of 1000.	The function should return (`constant` divided by `current_close`) rounded **down** to the nearest integer.	5%
`getOrders`	The arguments to this function are always the same for all strategies used in the backtester framework.	This function should implement the strategy outlined above and again below in "Strategy specification".	20%

Strategy specification

The strategy should apply the following logic independently for both series.

The strategy does nothing until there have been `params$lookbacks$long`-many periods.

In the `(params$lookbacks$long+1)`-th period, and in every period after, the strategy computes three simple moving averages with window lengths equal to:

- params$lookbacks$short
- params$lookbacks$medium
- params$lookbacks$long

The corresponding windows always end in the current period. The strategy should in this period send **market orders** to assume a position (make sure you take into account positions from earlier) according to `getPosSignFromTMA` and `getPosSize`. (Limit orders are not required at all, and can be left as all zero.)

Hints

For the checks for `getTMA` you may find the following functions useful:

- The operator `!` means not, and can be used to negate a boolean.
- `sapply` allows one to apply a function element-wise to a vector or list (e.g., to a vector list `c("short","medium","long")`).
- `all` is a function that checks if all elements of a vector are true (for example, it can be used on the result of `sapply`).
- `%in%` can be used to check if a element exists inside a vector.

To compute the moving average in `getTMA` you can use `SMA` from the `TTR` package.

Note: The list returned by `getTMA` should work as input to the function `getPosSignFromTMA`.

For `getPosSize`, you can use the function `floor`.

As in the template, use the negative of `currentPos` summed with the new positions you want to take to make sure that you assume the correct position.

In order to help you check whether you have implemented the functions correctly, we next give some examples of how correct implementations of the functions will behave. These examples assume that you have correctly implemented the first 4 functions in `a2_template.R` and sourced the resulting code to make the functions available in the R environment.

Example output for `getTMA`

First you should make sure that you have correctly implemented all **6 checks** on the function arguments. Here are 3 examples of expected behaviour:

```
> close_prices <- c(1,2,3)
> lookbacks <- list(short=as.integer(5),medium=as.integer(10),long=as.integer(20))
> getTMA(close_prices,lookbacks) # bad close_prices
Error in getTMA(close_prices, lookbacks) :
  E04: close_prices is not an xts according to is.xts()

> dataList <- getData(directory="A2")
Read 2 series from DATA/A2
> close_prices <- dataList[[1]]$Close[1:19]
> getTMA(close_prices,lookbacks) # bad close_prices; too short
Error in getTMA(close_prices, lookbacks) :
  E05: close_prices does not enough rows

> lookbacks <- list(5,10,25)
> getTMA(close_prices,lookbacks) # bad lookbacks; list elements not named
Error in getTMA(close_prices, lookbacks) :
  E01: At least one of "short", "medium", "long" is missing from names(lookbacks)
```

Here is an example where we give the function valid arguments.

```
> lookbacks <- list(short=as.integer(5),medium=as.integer(10),long=as.integer(20))
> close_prices <- dataList[[1]]$Close[1:20]
> getTMA(close_prices,lookbacks)
$short
[1] 169

$medium
[1] 170.4

$long
[1] 171.05
```

Example output for `getPosSignFromTMA`

```
> getPosSignFromTMA(list(short=10,medium=20,long=30))
[1] 1
> getPosSignFromTMA(list(short=10,medium=30,long=20))
[1] 0
> getPosSignFromTMA(list(short=30,medium=20,long=10))
[1] -1
```

Example output for `getPosSize`

```
> current_close <- 100.5
> getPosSize(current_close)
[1] 9
> getPosSize(current_close,constant=100.4)
[1] 0
```

Example output for `getOrders`

To check your implementation of `getOrders`, see part 2 for examples of correct output for the function `getInSampleResult` below.

Part 2: in-sample tests

> ***Warning***
>
> The last two functions require a working implementation of `getOrders`. If your implementation of `getOrders` does not work, then you will receive 0 marks for these last two functions, even if they return the correct numbers. This is a simple protection again plagiarism and collusion.

There are two more functions that you need to implement: `getInSampleResult` and `getInSampleOptResult`. For both functions you will need to compute **your own** in-sample period, which is based on your MWS username. This ensures that for part 2 there are different answers for different students.

To get your in-sample period you should use `in-sample_period.R` as follows. Source it and run the function `getInSamplePeriod` with your MWS username as per the following example. Then use the first number in the returned vector as the start of the in-sample period and the second number as the end.

```
> source('in-sample_period.R')
> getInSamplePeriod('x4xz1')
[1]  230 644
```

So for this example username the start of the in-sample period is day 230 and the end is 644. **Note**: you may need to install the package `digest` to use this code.

Once you have your own in-sample period (and a correct implementation of `getOrders`), you are ready to complete the implementation of `getInSampleResult`.

Function name	Input parameters	Expected behaviour	Marks available for a correct implementation
`getInSampleResult`	None	This function should return the PD ratio that is achieved when the strategy is run with short lookback 10, medium lookback 20, and long lookback 30, on your username-specific in-sample period. The function should not contain **ANY** code except the return value; it should run and complete instantanously.	10% (0 marks will be given if the function contains any code other than the return statement)

To complete the final function `getInSampleOptResult` you need to do an in-sample parameter optimization using the following parameter combinations for the:

- short lookback
- medium lookback
- long lookback

You should **not** optimize the constant used with `getPosSize`, and leave it as 1000 as defined in the template code.

The parameter combinations are defined by two things: parameter ranges and a further restriction. Make sure you correctly use both to produce the correct set of parameter combinations. The ranges are:

Parameter	Minimum value	Increment	Maximum Value
short lookback	100	5	110
medium lookback	105	5	120
long lookback	110	5	130

The further restriction is the following:

Further restriction on parameter values

You should further restrict the parameter combinations as follows:

- The medium lookback should always be strictly greater than the short lookback.
- The long lookback should always be strictly greater than the medium lookback.

You need to find the best PD ratio that can be achieved one this set of parameter combinations for the in-sample period that corresponds to your username, and set it as the return value of `getInSampleOptResult`.

Hint

The correct resulting number of parameter combinations is 28.

You can adapt `backtester_v5.3/main_optimize.R`. It is probably easiest to use three nested for loops in order to ensure that you only check valid parameter combinations (where the short < medium < long for the respective window lengths).

Function name	Input parameters	Expected behaviour	Marks available for a correct implementation
`getInSampleOptResult`	None	This function should return the best PD ratio than can be achieved with the stated allowable parameter combinations on your username-specific in-sample period. The function should not contain **ANY** code except the return value; it should run and complete instantanously.	20% (0 marks will be given if the function contains any code other than the return statement)

Next we give some example output for these two functions.

Example output for `getInSampleResult`

Username	Correct return value

x1xxx	-747.6
x1yyy	-231.6
x1zzx	-639.8

Example output for `getInSampleOptResult`

Username	Correct return value
x1xxx	4.23
x1yyy	3.42
x1zzx	4.43

Marks summary

Function	Marks
`getTMA`	30
`getPosSignFromTMA`	15
`getPosSize`	5
`getOrders`	20
`getInSampleResult`	10 (0 if `getOrders` does not work)
`getInSampleOptResult`	20 (0 if `getOrders` does not work)

Submission

You need to submit **a single R file** that contains your implementation of 6 functions. The file shoud be called MWS-username.R where you should replace MWS-username by your MWS username. For example if your username is "abcd" then you should submit a file named "abcd.R".

Submission is via the department electronic submission system:

http://www.csc.liv.ac.uk/cgi-bin/submit.pl

> ***Warning***
>
> Your code will be put through the department's automatic plagiarism and collusion detection system. Student's found to have plagiarized or colluded will likely receive a mark of zero. Do not show your work to other students.