Homework 1 班号 学号 姓名

Give DFA's accepting the languages over the alphabet $\{0, 1\}$.

1. the set of all strings with at least one 0 and exactly two 1's.

2. The set of all strings such that each block of three consecutive symbols contains at least two 0's.

Design ε-NFA's for the following languages. Try to use ε-transitions to simplify your design.

3. The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

4. The set of strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

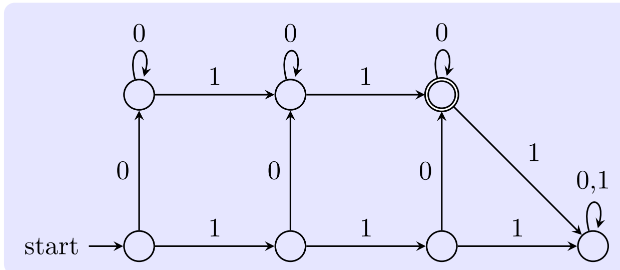
Design regular expression:

5. The set of all strings of 0's and l's not containing 101 as a substring.

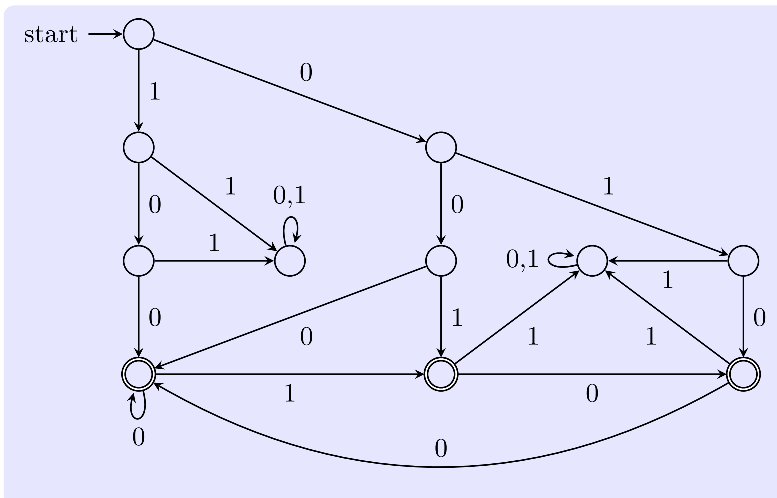
Homework 1　　　班号　　　学号　　　姓名

Give DFA's accepting the languages over the alphabet $\{0, 1\}$.

1. the set of all strings with at least one 0 and exactly two 1's.
   (含有至少一个 0 且只有两个 1 的所有字符串.)

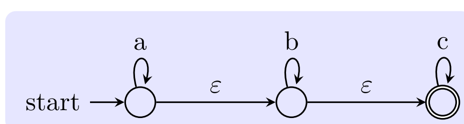


2. The set of all strings such that each block of three consecutive symbols contains at least two 0's.
   (字符串中任何连续的三字符种都至少有两个 0，答案中不接受任何长度小于 3 的字符串.)

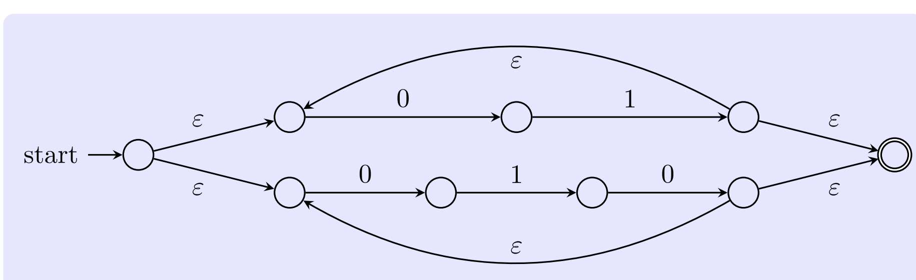


Design ε-NFA's for the following languages. Try to use ε-transitions to simplify your design.

3. The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.



4. The set of strings that consist of either 01 repeated one or more times or 010 repeated one or more times.
   (“01 的零或多次重复”与“010 的零或多次重复”两者中，只取其一)



Design regular expression:

5. The set of all strings of 0's and l's not containing 101 as a substring.

$$0^*(1 + 000^*)^*0^* \quad \text{or} \quad (0 + \varepsilon)(1 + 000^*)^*(0 + \varepsilon)$$

Homework 2 班号 学号 姓名

1. Prove that language $L = \{0^n \mid n \text{ is a power of } 2\}$ is not regular.

2. Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

   a) Give all the regular expressions $R_{ij}^{(0)}$. Note: Think of state q_i as if it were the state with integer number i.

   b) Give all the regular expressions $R_{ij}^{(1)}$. Try to simplify the expressions as much as possible.

   c) Give all the regular expressions $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible.

   d) Give a regular expression for the language of the automaton.

3. Design context-free grammars for the following languages:

   a) The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

   b) The set of all strings over $\{0, 1\}$ with twice as many 0's as 1's.

   c) The set of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

Homework 2 班号 学号 姓名

1. Prove that language $L = \{0^n \mid n \text{ is a power of } 2\}$ is not regular.

   证明思路：泵引理，反证法。取 $s = 0^{2^N}$, 则 $2^N < |xy^2z| < 2^N + N < 2^N + 2^N = 2^{(N+1)}$

2. Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

   a) Give all the regular expressions $R_{ij}^{(0)}$. Note: Think of state q_i as if it were the state with integer number i.

   b) Give all the regular expressions $R_{ij}^{(1)}$. Try to simplify the expressions as much as possible.

   c) Give all the regular expressions $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible.

	$k=0$	$k=1$	$k=2$	$k=2$ 等价于
R_{11}^k	$\varepsilon+1$	1^*	$1^*+1^*0(11^*0)^*11^*$	$(1+01)^*$
R_{12}^k	0	1^*0	1^*0	$(1+01)^*0$
R_{13}^k	$\varnothing$	$\varnothing$	$1^*0(11^*0)^*0$	$(1+01)^*00$
R_{21}^k	1	11^*	$(11^*0)^*11^*$	
R_{22}^k	ε	$\varepsilon+11^*0$	$\varepsilon+11^*0$	
R_{23}^k	0	0	$(11^*0)^*0$	
R_{31}^k	$\varnothing$	$\varnothing$	$1(11^*0)^*11^*$	
R_{32}^k	1	1	$1(11^*0)^*$	
R_{33}^k	$\varepsilon+0$	$\varepsilon+0$	$\varepsilon+0+1(11^*0)^*0$	$\varepsilon+0+10+11(1+01)^*00$

答案可能不对

$$R_{ij}^k = R_{ik}^{k-1}(R_{kk}^{k-1})^* R_{kj}^{k-1} \cup R_{ij}^{k-1} \quad (1)$$

$$R_{ij}^0 = \begin{cases} \{a \mid \delta(q_i,a)=q_j\} & i \neq j \\ \{a \mid \delta(q_i,a)=q_j\} \cup \{\varepsilon\} & i = j \end{cases} \quad (2)$$

等我有空更新

   d) Give a regular expression for the language of the automaton.

   $(1+01)^*00(0+10+11(1+01)^*00)^*$

1. Design context-free grammars for the following languages:

   a) The set $\{a^ib^jc^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

   $S \rightarrow A_1C|A_2C|AB_1|AB_2$
   $A_1 \rightarrow aA_1b|aA_1|a$
   $A_2 \rightarrow aA_2b|A_2b|b$
   $C \rightarrow Cc|\varepsilon$
   $B_1 \rightarrow bB_1c|bB_1|b$
   $B_2 \rightarrow bB_2c|B_2c|c$
   $A \rightarrow Aa|\varepsilon$

   (注意：$Cc|\varepsilon$ 若为 $Cc|c$ 则不能产生 a, c 同时为 0 个，或 b, c)

   b) The set of all strings over $\{0,1\}$ with twice as many 0's as 1's.

   $S \rightarrow S0S0S1S|S0S1S0S|S1S0S0S|\varepsilon$

   5. The set of all strings over $\{a,b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

   如果串长为奇数，显然不是 ww 形式 (对应下面文法中的 A 或 B)。而对于长度为偶数 $(2n)$ 的串，至少存在一对儿距离为 n(串长度的一半) 的两字符不相同。为了能够产生两个不相同字符的距离刚好是整个长度的一半，使用两个变元 A 和 B 分别产生基数长的串，然后合并即可。对 A 或 B，在为产生串时，如果增加了两字符间的字符数，那么也要增加两字符外的字符数。如 $aaaabbbb = \underline{aa\breve{a}ab}\ \underline{b\breve{b}b}$ 或 $aabaaa = \underline{aa\breve{b}aa}\ \underline{\breve{a}}$.

   $S \rightarrow A \mid B \mid AB \mid BA$
   $A \rightarrow XAX \mid a$
   $B \rightarrow XBX \mid b$
   $X \rightarrow a \mid b$

Homework 3 班号 学号 姓名

1. Design PDA for $L = \{w \in \{0,1\}^* \mid w = w^R \text{ and the length of } w \text{ is odd}\}$.

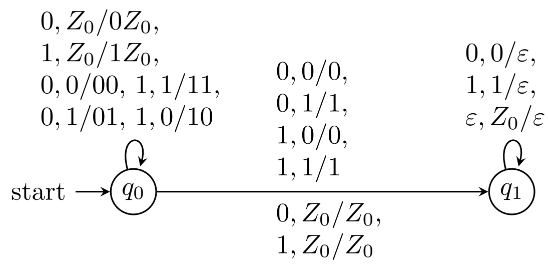
2. Give deterministic pushdown automata to accept $L = \{0^n 1^m 0^n \mid n \text{ and } m \text{ are arbitrary}\}$.

3. Proving that the language $L = \{a^i b^j c^k \mid i < j < k\}$ is not a CFL with the pumping lemma.

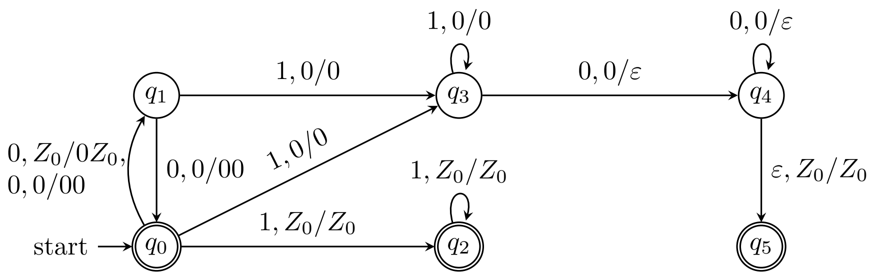
4. Design a Turing machine for language $L = \{a^i b^j c^k \mid k = i \times j \text{ and } k > 0\}$.

Homework 3 班号 学号 姓名

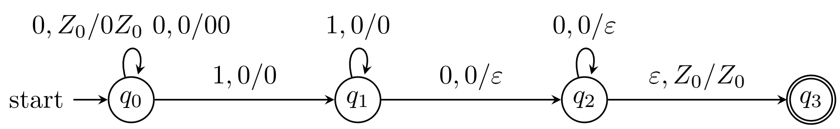
1. Design PDA for $L = \{w \in \{0,1\}^* \mid w = w^R \text{ and the length of } w \text{ is odd}\}$.



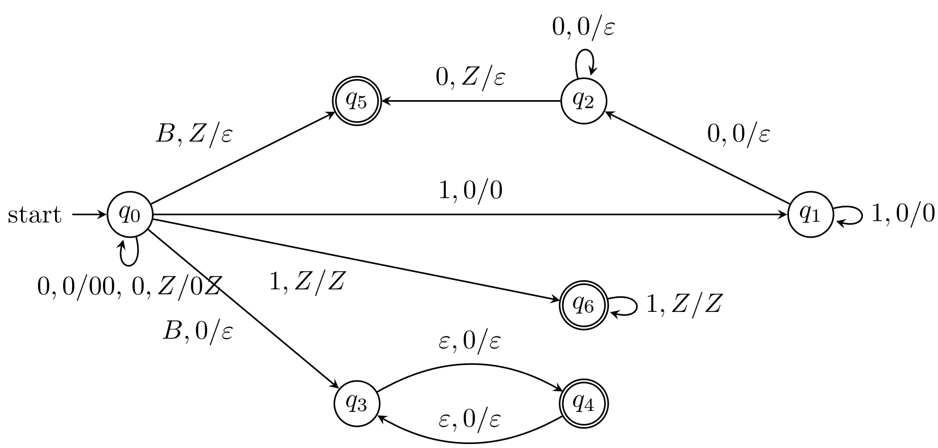
2. Give deterministic pushdown automata to accept $L = \{0^n 1^m 0^n \mid n \text{ and } m \text{ are arbitrary}\}$.
同学们很厉害，做出这个题的 DPDA 了



之前的答案:
如果 n 和 m 可以为 0，无法直接画出 DPDA，所以需要加个条件 $n, m \geq 1$.



但也可以增加个输入带上的空白符号比如 B，作为字符，就可以用 DPDA 处理 n 和 m 为零的情况了。
PDA $M = (Q, \Sigma = \{0, 1, B\}, \Gamma = \{0, Z\}, q_0, \delta, Z, F)$



	$0,0$	$0,Z$	$1,0$	$1,Z$	$B,0$	B,Z	$\varepsilon,0$
$\to q_0$	$q_0, 00$	$q_0, 0Z$	$q_1, 0$	q_6, Z	q_3, ε	q_5, ε	$-$
q_1	q_2, ε	$-$	$q_1, 0$	$-$	$-$	$-$	$-$
q_2	q_2, ε	q_5, ε	$-$	$-$	$-$	$-$	$-$
q_3	$-$	$-$	$-$	$-$	$-$	$-$	q_4, ε
$*q_4$	$-$	$-$	$-$	$-$	$-$	$-$	q_3, ε
$*q_5$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$*q_6$	$-$	$-$	$-$	q_6, Z	$-$	$-$	$-$

3. Proving that the language $L = \{a^i b^j c^k \mid i < j < k\}$ is not a CFL with the pumping lemma.

   反证法。假设 L=$\{a^i b^j c^k | i < j < k\}$ 是 CFL，由 CFL 泵引理，存在正整数 N，使长度超过 N 的串符合 CFL 泵引理。取 $s = a^N b^{N+1} c^{N+2}$ 则 $s = uvwxy$ 中，因为 $|vwx| \leq N$, vwx 可能几种情况:

   i) 都在 a 或 b 中，取 $i = 2$ 则 $s' = uv^i wx^i y$ 中 a 或 b 可能不小于 c

   ii) 在 c 中，取 $i = 0$

   iii) 在 ab 之间，取 $i = 2$

   iv) 在 bc 之间，取 $i = 0$

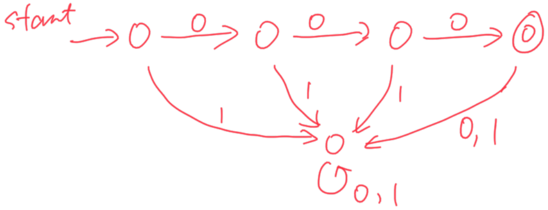
   无论何种情况，都存在 $uv^i wx^i y \notin L$ 的可能，与假设矛盾。得证。

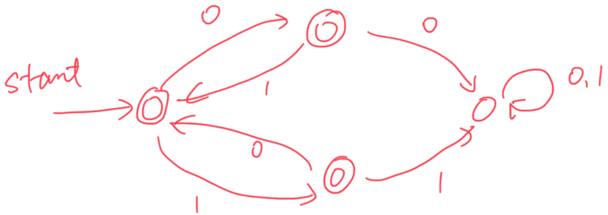
4. Design a Turing machine for language $L = \{a^i b^j c^k \mid k = i \times j \text{ and } k > 0\}$.

   见第 8 章习题 3

- 课堂答题 ch1-1.
  1. 字母表 Σ 的克林闭包 Σ^* 中一定有无穷多个元素吗? 正确, $\Sigma \neq \emptyset$
  2. 集合 A 的克林闭包 A^* 中一定有无穷多个元素吗? 不一定. A 可以为 $\emptyset$
- 课堂答题 ch1-2.
  任意字母表 Σ 的克林闭包 Σ^* 中都有无穷多个字符串，
  其中的字符串的长度：
  1. 是否是任意长都有? 是
  2. 是否有无穷长的? 否

- 课堂答题 ch2-1.
  Design DFA over $\Sigma = \{0, 1\}$ for the language with only one string 000.

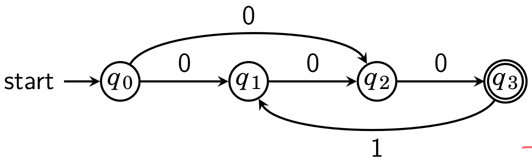




- 课堂答题 ch2-2.
  All strings $w \in \{0,1\}^*$ such that in *every prefix of* w, the number of 0s and 1s differ by at most 1.

- 课堂答题 ch2-3.
  The set of all strings over $\Sigma = \{0, 1\}$ that contain either 00 or 11 (or both) as a substring.

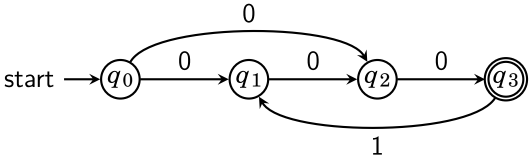
- 课堂答题 ch2-4.
  用子集构造法将该 NFA 转换为等价的 DFA.

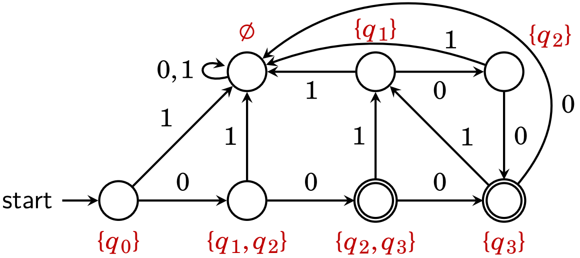


	0	1
$\rightarrow q_0$	$\{q_1, q_2\}$	$\varnothing$
q_1	$\{q_2\}$	$\varnothing$
q_2	$\{q_3\}$	$\varnothing$
$*q_3$	$\varnothing$	$\{q_1\}$

	0	1
$\rightarrow \{q_0\}$	$\{q_1, q_2\}$	$\varnothing$
$\{q_1, q_2\}$	$\{q_2, q_3\}$	$\varnothing$
$\varnothing$	$\varnothing$	$\varnothing$
$*\{q_2, q_3\}$	$\{q_3\}$	$\{q_1\}$
$*\{q_3\}$	$\varnothing$	$\{q_1\}$
$\{q_1\}$	$\{q_2\}$	$\varnothing$
$\{q_2\}$	$\{q_3\}$	$\varnothing$

- 课堂答题 ch2-4.
  用子集构造法将该 NFA 转换为等价的 DFA.





课堂练习.

Give regular expressions for each of the following languages over $\Sigma = \{0, 1\}$.

1. All strings containing the substring 000.

$$(0+1)^*000(0+1)^*$$

2. All strings *not* containing the substring 000.

$$(1+01+001)^*(\varepsilon+0+00)$$

- 课堂答题 ch4-1.
  Prove that $L = \{ a^i b^j c^k \mid i + j = k \}$ is not regular with pumping lemma.

$w = a^N b^N c^{2N}$ $\because$ $y = a^m$ $(m>0)$

$xy^2z = a^{N+m} b^N c^{2N} \notin L$.

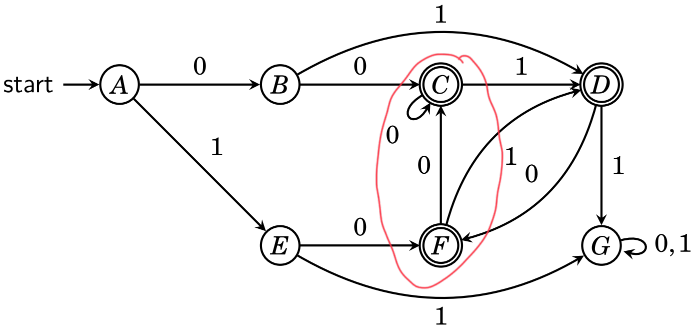
思考题

有语言 M, N 和 L, 满足

$$M \cap N = L$$

1. 若 M 和 N 都是正则语言, 由定理 9, 则 L 是一定正则语言.
2. 若 M 和 N 都不是正则语言, 则 L 一定不是正则语言吗? 不一定
3. 若 M 和 L 都是正则语言, 则 N 一定是正则语言吗? 不一定
4. 若 M 正则而 L 非正则, 则 N 一定非正则吗? 一定
5. 若 M 非正则而 L 正则, 则 N 呢? 不一定

- 课堂答题 ch4-6. Minimize the given DFA.



(C, F) 合并.

见第2章 习题 7

[Exercise 2.2.5]

答案.

课堂答题 ch5-1.

Design CFG for $L = \{w \in \{(,)\}^* \mid w$ is a string of balanced parentheses$\}$.

$S \to (S)S \mid \varepsilon$

课堂练习. Design PDA for $L = \{ a^i b^j c^k \mid i, j, k \geq 0,\ j = i + k \}$.