主管领导审核签字

哈尔滨工业大学 2022 学年 春 季学期

形式语言与自动机 试题 A

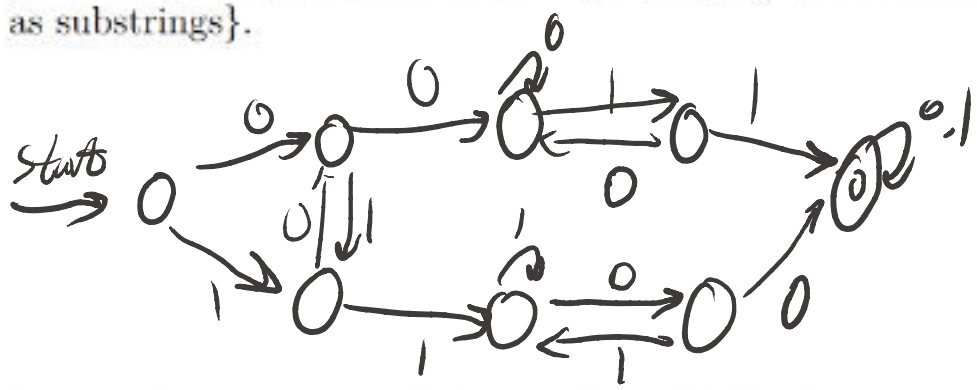
题号	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	总分
分数											
阅卷人											

密 封 线

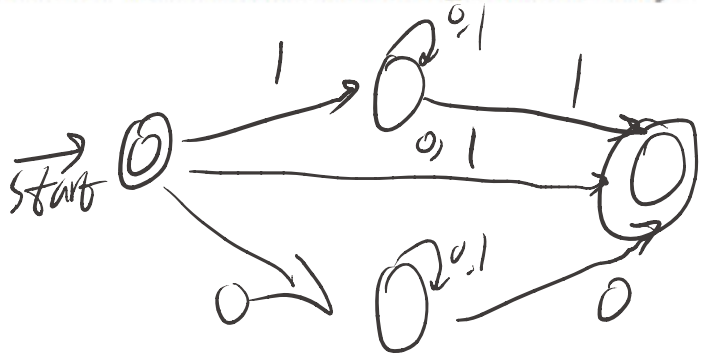
姓名 学号 班号

Note: please draw the diagram for automaton or Turing machine.

1. [10 points] Design a DFA for $L = \{w \in \{0,1\}^* \mid w \text{ contains both } 00 \text{ and } 11 \text{ as substrings}\}$.



2. [10 points] Design an NFA for $L = \{w \in \{0,1\}^* \mid w \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}$.



3. [10 points] Design regular expressions for languages over $\Sigma = \{a, b\}$:

(1) All strings having at least two occurrences of the substring aa.

$(a+b)^*aa(a+b)^*aa(a+b)^* + (a+b)^*aaa(a+b)^*$

(2) All strings that do not end with substrings aa or bb.

$(a+b)^*(ab+ba) + \varepsilon + a + b$

4. [10 points] Prove that the language L is not regular with pumping lemma

$$L = \{\, w \in \{0,1\}^* \mid w \text{ has the same number of substrings 00 and 11} \,\}.$$

$w = (00)^N (11)^N$

$|xy| < N.$

$0^{2N-m} 1^{2N}$

5. [10 points] Let $h : \Sigma \to \Sigma^*$ be a homomorphism: $\forall a \in \Sigma,\ h(a) = aa$. Please give a formal construction of the DFA for $h(L)$ from the DFA that accepts the regular language L over Σ.

$A = \{Q, \Sigma, \delta, q_0, F\}$

$A' = \{Q$

6. [10 points] Design a context-free grammar for the language

$$L = \{\, x \in \{0,1\}^* \mid x \text{ has just two nonempty blocks of 0s of the same length} \,\}.$$

of length

$S \to L0X0R$

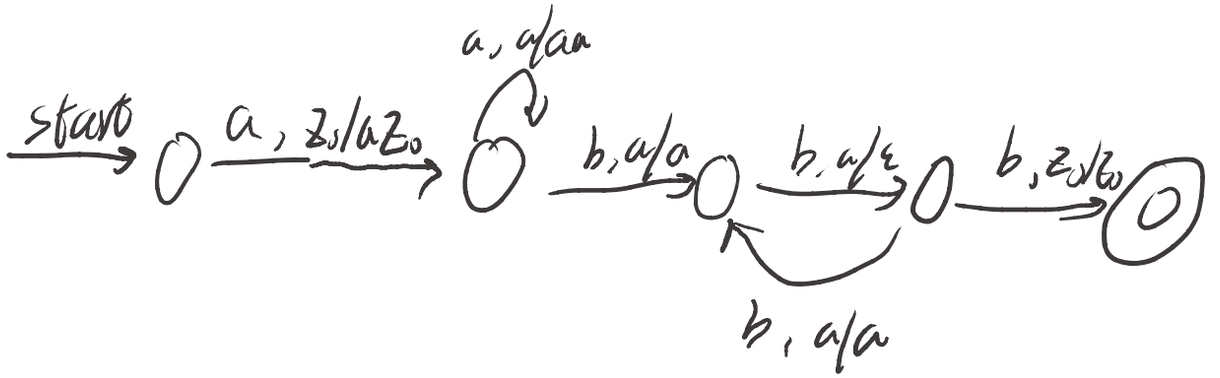
$L \to \varepsilon \mid 1 \mid B1$

$R \to \varepsilon \mid 1 \mid 1B$

$X \to 1 \mid 0X0 \mid 1B1$

$B \to \varepsilon \mid 1A \mid 0A$

7. [10 points] Design a deterministic PDA for $L = \{ a^n b^{2n+1} \mid n \geq 1 \}$.



8. [10 points] Begin with the grammar:

$$S \to ASA \mid A \mid \varepsilon$$
$$A \to 00 \mid \varepsilon$$

(1) Eliminate any ε-productions.

$S \to ASA \mid A \mid SA \mid AS \mid AA \mid S$

$A \to 00$

(2) Eliminate any unit productions in the resulting grammar.

$S \to ASA \mid SA \mid AS \mid AA \mid 00$

$A \to 00$

(3) Put the resulting grammar into Chomsky Normal Form.

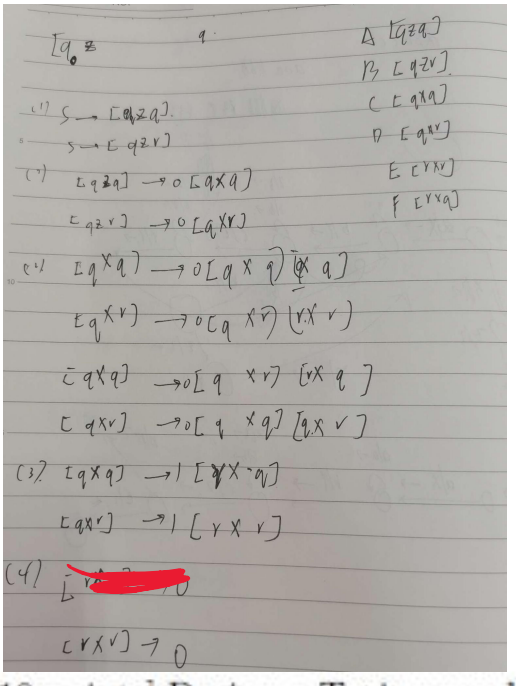
$D \to SA$

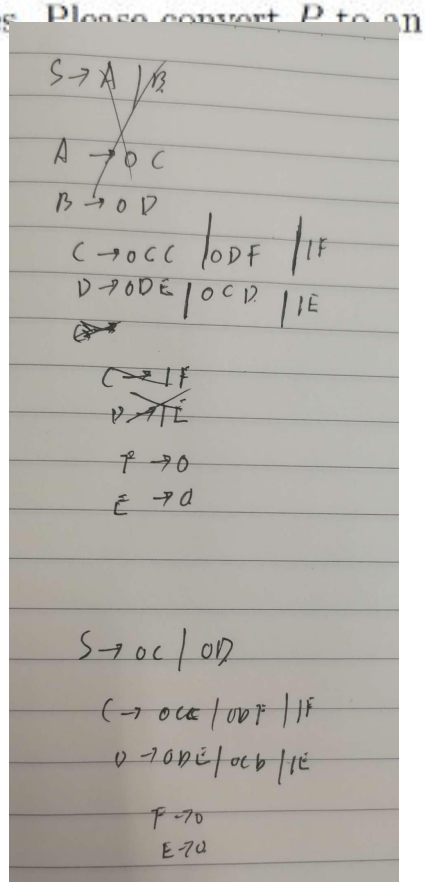
$A \to BB$

$S \to AD \mid SA \mid AS \mid AA \mid 00$

9. [10 points] Consider a PDA P with start state q, start symbol Z in the stack and the following transition rules. Please convert P to an equivalent CFG.

(1) $\delta(q, 0, Z) = \{(q, X)\}$
(2) $\delta(q, 0, X) = \{(q, XX)\}$
(3) $\delta(q, 1, X) = \{(r, X)\}$
(4) $\delta(r, 0, X) = \{(r, \varepsilon)\}$





$A \to 1 \mid 1A$

$S \to 0A0 \mid 00$

10. [10 points] Design a Turing machine for $L = \{ a^i b^j c^k \mid k = i \times j \text{ and } i, j > 0 \}$.