2020 春 形式语言与自动机 试题 C

1. [10 points] Design a DFA for $L = \{w \in \{0,1\}^* \mid w$ has exactly three 0s.$\}$

2. [10 points] Design an NFA for the language:

   $$L = \{w \in \{a, b, c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

3. [10 points] Design regular expressions for languages over $\Sigma = \{a, b\}$.

   (1) All strings that do not end with aba.

   (2) $L = \{w \mid w$ has no more than 5 a's. $\}$

4. [10 points] Prove that the language $L = \{w \in \{a, b\}^* \mid w = w^R\}$ is not regular with pumping lemma.

5. [10 points] Consider the following ε-NFA.

	ε	a	b	c
$\to p$	$\{q, r\}$	$\emptyset$	$\{q\}$	$\{r\}$
q	$\emptyset$	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

   (1) Compute the ε-closure of each state.

   (2) Give all the strings of length three or less accepted by the automaton.

   (3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

6. [10 points] Give a CFG for $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k\}$.

7. [10 points] Find a grammar equivalent to

   $$S \to AB \mid CA$$
   $$A \to a$$
   $$B \to BC \mid AB$$
   $$C \to aB \mid b$$

   with no useless symbols.

8. [10 points] Design a PDA for $L_{eq} = \{w \in \{0,1\}^* \mid w$ contains the same number of 0's and 1's $\}$.

9. [10 points] Prove or disprove: if L_1 is CFL and $L_1 \cup L_2$ is also CFL, then L_2 must be CFL.

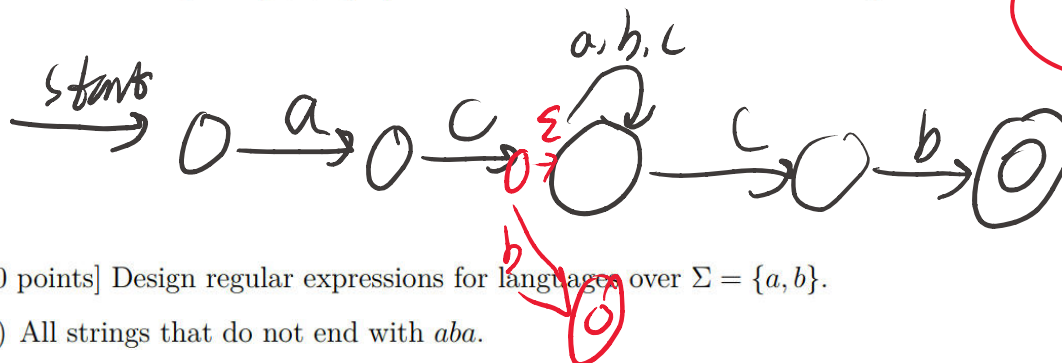10. [10 points] Design Turing machine for the language $\{0^{2n} 1^n \mid n \geq 0\}$.

1. [10 points] Design a DFA for $L = \{w \in \{0,1\}^* \mid w \text{ has exactly three 0s.}\}$

2. [10 points] Design an NFA for the language:

$$L = \{w \in \{a,b,c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

3. [10 points] Design regular expressions for languages over $\Sigma = \{a,b\}$.

(1) All strings that do not end with aba.

$\varepsilon + (a+b) + (a+b)(a+b) + (a+b)^*(aaa + baa + aab + abb + bab + bba + bbb)$

(2) $L = \{w \mid w \text{ has no more than 5 } a\text{'s. }\}$

$b^* + b^*ab^* + b^*ab^*ab^* + b^*ab^*ab^*ab^* + b^*ab^*ab^*ab^*ab^*$
$+ b^*ab^*ab^*ab^*ab^*ab^*$

4. [10 points] Prove that the language $L = \{w \in \{a,b\}^* \mid w = w^R\}$ is not regular with pumping lemma.

$a^N b^N a^N \qquad |xy| \leq N \qquad a^{N-m} b^N a^N$

6. [10 points] Give a CFG for $L = \{a^i b^j c^k \mid i,j,k \geq 0 \text{ and } i = j+k\}$.

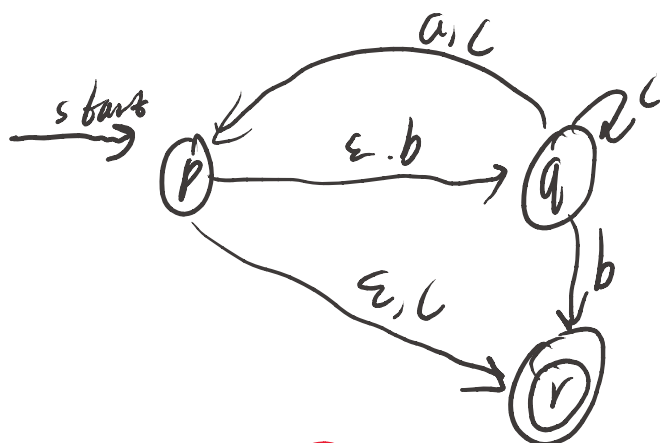$a^k a^j b^j c^k$

$B \to \varepsilon \mid aBb$

$S \to B \mid aSc$

5. [10 points] Consider the following ε-NFA.

	ε	a	b	c
→ p	{q, r}	∅	{q}	{r}
q	∅	{p}	{r}	{p, q}
*r	∅	∅	∅	∅

(1) Compute the ε-closure of each state.

$$\text{Eclose}(p) = \{p, q, r\} \qquad \text{Eclose}(q) = \{q\} \qquad \text{Eclose}(r) = \{r\}$$
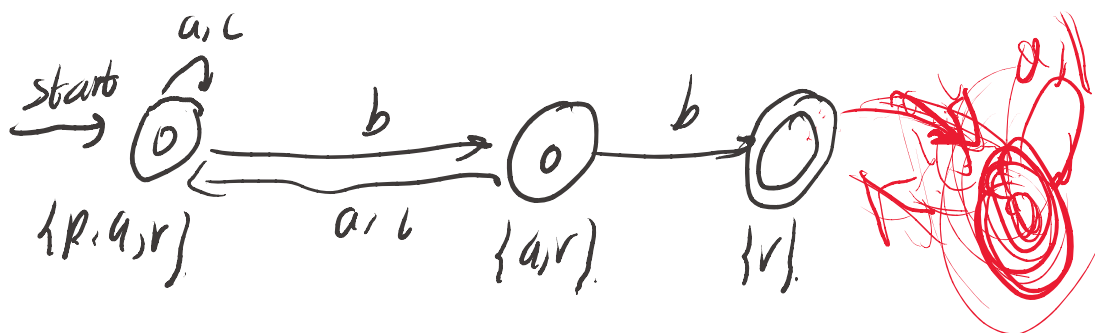
(2) Give all the strings of length three or less accepted by the automaton.

$$\{\varepsilon + c + (\varepsilon + b)(c + a + c)(\varepsilon + b)\}^*$$

$$(b + (a + c)(a + c))$$

(3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

	a	b	c
{p, q, r}	{p, q, r}	{q, r}	{p, q, r}
{q, r}	{p, q, r}	{r}	{p, q, r}

7. [10 points] Find a grammar equivalent to

$$S \to AB \mid CA$$
$$A \to a$$
$$B \to BC \mid AB$$
$$C \to aB \mid b$$

with no useless symbols.

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$$S \to CA$$
$$A \to a$$
$$C \to b.$$

8. [10 points] Design a PDA for $L_{eq} = \{w \in \{0,1\}^* \mid w$ contains the same number of 0's and 1's $\}$.

1, 0/ ε 0, 1/ ε
1, 1 /11   0, 0/00
0, Z₀/0Z₀   1, Z₀/1Z₀

ε ─

→◯↩ ε, ω/ Z_0

9. [10 points] Prove or disprove: if L_1 is CFL and $L_1 \cup L_2$ is also CFL, then L_2 must be CFL.

$$L_1 = \{ ww^R \mid w \in (0,1)^* \}$$

$$L_2 = (00)^*$$

对的

$$0^n 1^m 0^n$$

$$0^n 1^n 0^n$$

10. [10 points] Design Turing machine for the language $\{0^{2n}1^n \mid n \geq 0\}$.