Poisson distribution and Bayes's theorem

Here is assignment 3 for comp616/stat601/stat805. You are free to discuss the issues with your classmates but copying is not acceptable; see the AUT academic integrity guidelines.

This assessment has five sections. All have equal mark value.

Some sections below contain an excellence question. These are intended to challenge the very top students. Have a go at these, but be aware that they are intended as thought-provoking cues, rather than course material reproduction.

Poisson distribution part 1

Consider a random variable with a Poisson distribution with mean $\lambda = 6.5$.

- Plot the probability mass function for this random variable
- Verify that the probabilities sum to 1
- Sample from the distribution and plot a table of your results
- Excellence question: compare this random variable with $\mathrm{Bin}(20, 0.325)$ (which has the same mean). NB: for high marks you will have to interpret "compare' ' intelligently.

Poisson distribution part 2

A random variable X is known to be drawn from a Poisson distribution. An observation is made: $n = 6$ is obtained.

- State the precise definition of p-value and perform a one-sided test for the null that $\lambda = 3.3$.
- Calculate the likelihood ratio for $H_1 \colon \lambda = 4.1$ and $H_2 \colon \lambda = 2.2$.
- Plot a log-likelihood function in the range $2 \leq \lambda \leq 14$.
- Calculate a credible interval for λ, using a 2-units-of-support criterion [hint: follow page 67 of the course manual]. An approximate answer is fine
- Excellence question: calculate a confidence interval as per the end of chapter 3 in the course manual; compare the confidence interval with the credible interval above. One decimal place sufficient.

Poisson distribution part 3

Give an example drawn from your daily life of a random variable drawn from a Poisson distribution. You will recall examples from the manual and lectures: but do not use these examples! USe these examples to guide your understanding but I am looking for your own original examples.

- State *clearly* in what way it is a limiting form of a binomial distribution with large n and small p.
- Give rough estimates for n and p [an order-of-magnitude estimate is fine]
- Give a reasonable value for λ (a rough value is fine)
- Using your value for λ, sample from your Poisson distribution using `rpois(10,lambda)`. Interpret the meaning of the 10 random numbers. What does the first number mean? What does the second number mean?

- Excellence question: discuss whether your distribution really is a Poisson distribution. Are the statistical assumptions justified? Discuss.

Bayesian analysis part 1

A certain person buys a houseplant. She buys either a cactus or an orchid. From long experience I can say that it is a cactus with probability 0.4, or an orchid with probability 0.6. She then goes on holiday and does not water her plants. The plants get too dry and might die. Under these circumstances, a cactus will die with probability 0.05 and an orchid with probability 0.24.

The plant dies. Apply Bayes's theorem and show your working. What is the posterior probability that it was a cactus?

Excellence question: consider the probability of the plant being a cactus. Discuss how this probability changes from prior to posterior in the light of your observation and whether this is consistent with your intuition.

Bayesian analysis part 2

Give a situation from your everyday life and apply Bayes's theorem to it. Use two or three hypotheses. State your hypotheses *clearly*. State your prior probabilities *clearly*. State your data *clearly*. State your likelihood function *clearly*. Use Bayes's theorem to calculate posterior probabilities for your hypotheses and state your posterior probabilities *clearly*. For an excellence grade, interpret the difference between the prior and posterior probabilities using common-sense or intuitive reasoning.

Credit will be given for the use of a plausible and novel scenario, the inventiveness shown, and the appropriateness of Bayes's theorem.