The Journal of the Astronautical Sciences, Vol. 42, No. 2, April-June 1994, pp. 143–152

Determining Satellite Close Approaches, Part II

Salvatore Alfano[1]

Abstract

Improvements to the original Alfano/Negron Close Approach Software (ANCAS) tool are presented that increase accuracy and/or step size. Minimum spacing between two satellites is determined by creating a time-dependent third-order relative-velocity waveform produced from adjoining pairs of distances, velocities, and accelerations. Times of closest approach are obtained by extracting the real roots of the localized polynomial with the associated distances reconstructed from a set of fifth-order polynominals. Close approach entrance and exit times for an ellipsoidal quadric surface are found using a similar process. Both methods require a simplified computation of acceleration terms of the objects of interest. For this study a close approach *truth* table is constructed using a 0.1 second sequential step along the orbits and differencing the two position vectors. The simulation results show this algorithm produces close approach times almost identical to those of the *truth* model for larger time steps (up to 10 minutes), with a corresponding reduction in computer runtime. The results are created from real orbital data and include solution sets for three operational uses of close-approach logic. Satellite orbital motion is modeled by, but not limited to, first-order secular perturbations caused by mass anomalies.

Introduction

Evaluating the relative distance between two satellites is a routine requirement to support space missions. In close approach problems the satellite of interest is typically called the primary and the other satellite is called the secondary. The task is to predict the time the two objects are within a specified relative distance, as well as the duration of the encounter, on the time interval $t_n \leq t \leq t_{n+1}$. Upon predicting these encounters, orbital analysts can alert mission controllers of future close approaches, allowing them sufficient time to take corrective action. The extent of evaluating close approaches ranges from a primary versus a single satellite to the limiting case of a primary versus the entire US Space Command satellite catalog, which currently contains 7300 objects. For the latter, various pre-filter strategies exist to reduce processing time by reducing this population of artificial satellites to a candidate list of objects whose orbital geometries might create close encounters with the primary. Reduction strategies such as

[1]Chief, Space and Missiles Dynamics Division, Phillips Laboratory, PL/VTA, 3550 Aberdeen Ave. SE, Kirtland AFB, NM 87117-5776.

apoapsis/periapsis comparisons or geometric filtering should always be used to improve efficiency. Such strategies have been previously discussed [1,2] and are not addressed by this article; instead, the evaluation of the candidate list is explored.

The importance of determining satellite close approaches has been highlighted by the US Space Shuttle orbital maneuvers performed on missions 44 and 48. Orbital analysts make several hundred computer runs of the orbiter versus the tracked satellite population to identify future close approaches. With a tabulation of future close encounters, analysts alert NASA mission controllers of the conjunction times as well as their relative spatial separation. On two occasions already, debris was predicted to enter this ellipsoidal safety zone—called the *warning football* by orbital analysts—resulting in NASA's decision to command the Shuttle away from the approaching objects. More recently, preliminary analysis of a planned maneuver by Endeavour (EURECA retrieval on mission 57) predicted a close encounter with a spent Cosmos booster [3]; the data are included in this article as a test case.

Several authors have recently explored the satellite relative-distance problem [1, 2, 4–7]. An obvious solution to close approach problems is to step sequentially along the orbits of two satellites, then difference the position vectors to determine their relative distance; this approach is used to construct a *truth* table of encounters. Although straightforward, the approach is computationally burdensome, especially if the candidate satellite list becomes large. This article refines the method of Alfano and Negron [7] by replacing parabolic blending with other spline forms and defining new functions that improve accuracy and allow larger time steps. As in the previous work, this method is not limited to a specific analytical, numerical, or hybrid propagator. The satellite close approach algorithm outlined below does, however, require the simplified computation of acceleration terms; conjunction times are still found in closed form.

Curve Fitting

Curve fitting by cubic spline uses function values and their time derivatives at the beginning and end of a time interval to create a third-order polynomial [8]. This process is repeated until the set of sequential time intervals is exhausted, resulting in a first-order continuous curve created by piecing together numerous localized cubic polynomials.

Given a function $f(t)$ and its time derivative $\dot{f}(t)$ at t_n and t_{n+1}, define a cubic polynomial $C(\tau)$ as

$$C(\tau) = \gamma_3\tau^3 + \gamma_2\tau^2 + \gamma_1\tau + \gamma_0; \qquad (0 \le \tau \le 1) \tag{1}$$

where

$$\gamma_0 = f(t_n) \tag{1a}$$

$$\gamma_1 = \dot{f}(t_n)\Delta t \tag{1b}$$

$$\gamma_2 = -3f(t_n) - 2\dot{f}(t_n)\Delta t + 3f(t_{n+1}) - \dot{f}(t_{n+1})\Delta t \tag{1c}$$

$$\gamma_3 = 2f(t_n) + \dot{f}(t_n)\Delta t - 2f(t_{n+1}) + \dot{f}(t_{n+1})\Delta t \tag{1d}$$

$$\Delta t = t_{n+1} - t_n; \qquad (t_{n+1} > t_n) \tag{1e}$$

with τ uniformly spanning the interval $(0, 1)$. The τ coefficients are defined such that the polynomial C exactly represents the function and its time-scaled derivative at the interval boundaries.

An alternate formulation of the cubic spline involves fitting four data points on the τ interval [9]. Given four data points (P_1, P_2, P_3, P_4) occurring at the respective times $(0, \tau_1, \tau_2, 1)$ where $0 < (\tau_1, \tau_2) < 1$, the γ coefficients in equation (1) can be defined as

$$\gamma_0 = P_1 \tag{1f}$$

$$\begin{aligned}\gamma_1 = \{&(\tau_2^3 - \tau_2^2)(P_2 - P_1) + (\tau_1^2 - \tau_1^3)(P_3 - P_1) \\ &+ (\tau_1^3\tau_2^2 - \tau_1^2\tau_2^3)(P_4 - P_1)\}/D\end{aligned} \tag{1g}$$

$$\begin{aligned}\gamma_2 = \{&(\tau_2 - \tau_2^3)(P_2 - P_1) + (\tau_1^3 - \tau_1)(P_3 - P_1) \\ &+ (\tau_1\tau_2^3 - \tau_1^3\tau_2)(P_4 - P_1)\}/D\end{aligned} \tag{1h}$$

$$\begin{aligned}\gamma_3 = \{&(\tau_2^2 - \tau_2)(P_2 - P_1) + (\tau_1 - \tau_1^2)(P_3 - P_1) \\ &+ (\tau_1^2\tau_2 - \tau_1\tau_2^2)(P_4 - P_1)\}/D\end{aligned} \tag{1i}$$

$$D = \tau_1^3\tau_2^2 + \tau_1^2\tau_2 + \tau_1\tau_2^3 - \tau_1\tau_2^2 - \tau_1^3\tau_2 - \tau_1^2\tau_2^3 \tag{1j}$$

The γ coefficients are defined such that the polynomial C exactly represents the function at the four data points.

Extending the method outlined in Rogers and Adams [8], curve fitting by quintic spline uses function values and their first and second time derivatives at the beginning and end of a time interval to create a fifth-order polynomial. This process is repeated until the set of sequential time intervals is exhausted, resulting in a second-order continuous curve created by piecing together numerous localized quintic polynomials. Given $f(t)$, $\dot{f}(t)$, and $\ddot{f}(t)$ at t_n and t_{n+1}, define a quintic polynomial $Q(\tau)$ as

$$Q(\tau) = \alpha_5\tau^5 + \alpha_4\tau^4 + \alpha_3\tau^3 + \alpha_2\tau^2 + \alpha_1\tau + \alpha_0; \qquad (0 \le \tau \le 1) \tag{2}$$

where

$$\alpha_0 = f(t_n) \tag{2a}$$

$$\alpha_1 = \dot{f}(t_n)\Delta t \tag{2b}$$

$$\alpha_2 = 0.5\ddot{f}(t_n)\Delta t^2 \tag{2c}$$

$$\begin{aligned}\alpha_3 = &-10f(t_n) - 6\dot{f}(t_n)\Delta t - 1.5\ddot{f}(t_n)\Delta t^2 \\ &+ 10f(t_{n+1}) - 4\dot{f}(t_{n+1})\Delta t + 0.5\ddot{f}(t_{n+1})\Delta t^2\end{aligned} \tag{2d}$$

$$\begin{aligned}\alpha_4 = &\ 15f(t_n) + 8\dot{f}(t_n)\Delta t + 1.5\ddot{f}(t_n)\Delta t^2 \\ &-15f(t_{n+1}) + 7\dot{f}(t_{n+1})\Delta t - \ddot{f}(t_{n+1})\Delta t^2\end{aligned} \tag{2e}$$

$$\begin{aligned}\alpha_4 = &-6f(t_n) - 3\dot{f}(t_n)\Delta t - 0.5\ddot{f}(t_n)\Delta t^2 \\ &+ 6f(t_{n+1}) - 3\dot{f}(t_{n+1})\Delta t + 0.5\ddot{f}(t_{n+1})\Delta t^2\end{aligned} \tag{2f}$$

with τ uniformly spanning the interval $(0, 1)$. The α coefficients are defined such that the polynomial Q exactly represents the function and its time-scaled derivatives at the interval boundaries.

Distance and Range-Rate Functions

Let $\boldsymbol{r}_p$ and $\boldsymbol{r}_s$ be the Earth-Centered Inertial (ECI) position vectors of the primary and secondary satellites, respectively, at time t. The relative distance vector and its time derivatives become

$$\boldsymbol{r}_d = (\boldsymbol{r}_s - \boldsymbol{r}_p) \tag{3a}$$

$$\dot{\boldsymbol{r}}_d = (\dot{\boldsymbol{r}}_s - \dot{\boldsymbol{r}}_p) \tag{3b}$$

$$\ddot{\boldsymbol{r}}_d = (\ddot{\boldsymbol{r}}_s - \ddot{\boldsymbol{r}}_p) \tag{3c}$$

where the primary and secondary satellite vectors are provided by the user's orbit propagator or ephemeris. If the satellite acceleration vectors are not provided, the IJK components in the ECI frame can be simply approximated by

$$\ddot{r}_I = \frac{-\mu r_I}{r^3} + \frac{-3\mu J_2 r_e^2 r_I}{2r^5}\left(1 - \frac{5r_K^2}{r^2}\right) \tag{4a}$$

$$\ddot{r}_J = \frac{-\mu r_J}{r^3} + \frac{-3\mu J_2 r_e^2 r_J}{2r^5}\left(1 - \frac{5r_K^2}{r^2}\right) \tag{4b}$$

$$\ddot{r}_K = \frac{-\mu r_K}{r^3} + \frac{-3\mu J_2 r_e^2 r_K}{2r^5}\left(3 - \frac{5r_K^2}{r^2}\right) \tag{4c}$$

where r_e is the radius of the earth. The above simplification only includes J_2 perturbative forces.

The distance function $f_d(t)$ and its time derivatives are defined by dot products as

$$f_d(t) = \boldsymbol{r}_d \cdot \boldsymbol{r}_d \tag{5a}$$

$$\dot{f}_d(t) = 2(\dot{\boldsymbol{r}}_d \cdot \boldsymbol{r}_d) \tag{5b}$$

$$\ddot{f}_d(t) = 2(\ddot{\boldsymbol{r}}_d \cdot \boldsymbol{r}_d + \dot{\boldsymbol{r}}_d \cdot \dot{\boldsymbol{r}}_d) \tag{5c}$$

Satellite close approaches occur whenever $f_d(t)$ is at a local minimum; that is when $\dot{f}_d(t) = 0$ and $\ddot{f}_d(t) > 0$. To determine these times of closest approach, $\dot{f}_d(t)$ is represented by the range-rate polynomial equation $C_{\dot{f}_d}(\tau)$, where the subscript denotes the function being approximated. The corresponding polynomial coefficients, $\gamma_{\dot{f}_d0}$ through $\gamma_{\dot{f}_d3}$, are computed as

$$\gamma_{\dot{f}_d0} = \dot{f}_d(t_n) \tag{6a}$$

$$\gamma_{\dot{f}_d1} = \ddot{f}_d(t_n)\Delta t \tag{6b}$$

$$\gamma_{\dot{f}_d2} = -3\dot{f}_d(t_n) - 2\ddot{f}_d(t_n)\Delta t + 3\dot{f}_d(t_{n+1}) - \ddot{f}_d(t_{n+1})\Delta t \tag{6c}$$

$$\gamma_{\dot{f}_d3} = 2\dot{f}_d(t_n) + \ddot{f}_d(t_n)\Delta t - 2\dot{f}_d(t_{n+1}) + \ddot{f}_d(t_{n+1})\Delta t \tag{6d}$$

Extract the real, distinct root(s) $\tau_{d\mathrm{ROOT}}$ of $C_{\dot{f}_d}(\tau)$ on the interval $[0, 1)$. If no real root exists then no minimum occurred during the time interval. If a real root does exist and

$$\left.\frac{dC_{\dot{f}_d}(\tau)}{d\tau}\right|_{\tau=\tau_{d\mathrm{ROOT}}} > 0,$$

then a local minimum exists. To find the associated time and range, compute the α coefficients of equation (2) using the I component functions $(r_{dI}(t), \dot{r}_{dI}(t), \ddot{r}_{dI}(t))$ of equation (3) to determine $Q_{r_{dI}}(\tau)$. In a similar manner, find $Q_{r_{dJ}}(\tau)$ and $Q_{r_{dK}}(\tau)$. The minimum range is

$$r_{\text{MIN}} = \sqrt{Q^2_{r_{dI}}(\tau_{d\text{ROOT}}) + Q^2_{r_{dJ}}(\tau_{d\text{ROOT}}) + Q^2_{r_{dK}}(\tau_{d\text{ROOT}})} \tag{7}$$

and the associated time is

$$t(\tau_{d\text{ROOT}}) = t_n + \tau_{d\text{ROOT}}\Delta t \tag{8}$$

Because $C_{\dot{f}_d}(\tau)$ is a cubic equation defining the range rate between t_n and t_{n+1} more than one real root may be found in the time interval that satisfies the inequality constraint of the previous paragraph. This condition indicates that two minimums occurred in the interval and poses no problem computationally; just solve equations (7) and (8) for each root. The user is warned not to take too large a time step; in trial cases with this method roots were missed when an orbit had a true anomaly change of more than 40 degrees in a single time interval. For two minimums to occur in a single time interval, either the primary or secondary would most likely exceed this angle.

The Ellipsoidal Function

Identifying space objects that enter an ellipsoidal safety zone centered about manned spacecraft is a top priority. To do so, a *warning football*[2] is created with a and b denoting the major and minor axes, respectively. This football is created such that the major axis is colinear with the primary satellite's velocity vector $\dot{\boldsymbol{r}}_p$.

Projecting $\boldsymbol{r}_d$ onto the $\dot{\boldsymbol{r}}_p$ unit vector,[3] the ellipsoidal function $f_e(\tau)$, is

$$f_e(\tau) = \left\{\frac{(\boldsymbol{r}_d \cdot \dot{\boldsymbol{r}}_p)^2}{(\dot{\boldsymbol{r}}_p \cdot \dot{\boldsymbol{r}}_p)}\right\}\Big/ a^2 + \left\{(\boldsymbol{r}_d \cdot \boldsymbol{r}_d) - \frac{(\boldsymbol{r}_d \cdot \dot{\boldsymbol{r}}_p)^2}{(\dot{\boldsymbol{r}}_p \cdot \dot{\boldsymbol{r}}_p)}\right\}\Big/ b^2 - 1 \tag{9}$$

where

$$f_e(\tau)\begin{cases} > 0 \text{ out of range} \\ = 0 \text{ entrance or exit condition} \\ < 0 \text{ in range.}\end{cases}$$

Ellipsoidal entries and exits occur whenever $f_e(\tau) = 0$. To determine these times, the alternate γ_{f_e} coefficients from equations (1f–1j) are computed for the polynomial equation $C_{f_e}(\tau)$. To accomplish this, two endpoints (P_1, P_4) and two intermediate points (P_2, P_3) must be found for the interval. The endpoints are simply

$$\begin{aligned} P_1 &= f_e(0), \quad (t = t_n) \\ P_4 &= f_e(1), \quad (t = t_{n+1}) \end{aligned} \tag{10a}$$

[2]The *warning football* is needed only if one wishes to model the "safety zone" with an ellipsoid rather than a sphere. Typically, the error in modeling perturbed energy for orbital motion results in poor position knowledge in the along track direction; hence, the *warning football* for manned missions.

[3]Velocity for the primary satellite is needed to orient the *football* only and is not part of the overall logic. For a sphere, the minor and major axes are equal, thus the projections are not needed.

The intermediate points must be reconstructed from the distance functions of equation (3), assuming you cannot just get the function values by invoking the propagator. This reconstruction involves determining $Q_{r_{dI}}(\tau)$, $Q_{r_{dJ}}(\tau)$, and $Q_{r_{dK}}(\tau)$ as previously outlined. In addition, the primary velocity vector $\dot{\boldsymbol{r}}_p(\tau)$ must be reconstructed using $\dot{\boldsymbol{r}}_p(t_n)$, $\ddot{\boldsymbol{r}}_p(t_n)$, $\dot{\boldsymbol{r}}_p(t_{n+1})$, and $\ddot{\boldsymbol{r}}_p(t_{n+1})$ to produce $C_{\dot{r}_{pI}}(\tau)$, $C_{\dot{r}_{pJ}}(\tau)$, and $C_{\dot{r}_{pK}}(\tau)$ from equations (1a–1e). The intermediate points become

$$P_2 = f_e(\tau_1) \tag{10c}$$

$$P_3 = f_e(\tau_2) \tag{10d}$$

where τ_1 and τ_2 are evenly spaced on the interval: $\tau_1 = 1/3$ and $\tau_2 = 2/3$. Should a minimum distance occur in the interval, then $\tau_2 = \tau_{d\mathrm{ROOT}}$ and $\tau_1 = (1 + \tau_{d\mathrm{ROOT}})/2$ if $\tau_{d\mathrm{ROOT}} \leq 1/2$, else $\tau_1 = \tau_{d\mathrm{ROOT}}/2$. By including the minimum distance, the $C_e(\tau)$ curve is forced to pass through that point; this ensures an entry/exit will not be missed due to ill-conditioned endpoints provided the quintic functions are an accurate representation of relative distance components. Extract the real root(s), $\tau_{e\mathrm{ROOT}}$, of $C_e(\tau)$ on the interval $[0, 1)$.

A refining process is used to find the entry/exit time if a valid root exists. This step improves the accuracy of the $C_e(\tau)$ curve in the region of $\tau_{e\mathrm{ROOT}}$ by reconstructing the intermediate point P_2 at the root. If a second root is found P_3 is also recomputed, else it remains associated with the minimum distance. New γ coefficients are then computed for the $C_e(\tau)$ curve from equations (1a–1e). The real root(s), $\tau_{e\mathrm{ROOT}}$, of $C_e(\tau)$ on the interval $[0, 1)$ are recomputed and the time of entry/exit becomes

$$t(\tau_{e\mathrm{ROOT}}) = t_n + \tau_{e\mathrm{ROOT}} \Delta t \tag{11}$$

An exit exists if

$$\left. \frac{dC_e(\tau)}{dt} \right|_{\tau = \tau_{e\mathrm{ROOT}}} > 0,$$

else it is an entry.

Accelerated Search on the Interval [0, 1)

As was outlined in Alfano and Negron [7], the search for real unique roots on the interval $[0, 1)$ is accelerated by a simple examination of the third-order polynomial coefficients. Expressing the cubic equation with nested multiplication yields

$$((\gamma_3 \tau + \gamma_2)\tau + \gamma_1)\tau + \gamma_0 = 0. \tag{12}$$

Each subcomponent of the nested equation is linear, having a maximum or minimum at $\tau = 0$ or $\tau = 1$. No root exists on the interval if

$$\mathrm{MIN}(\gamma_1, \gamma_1 + \gamma_2, \gamma_1 + \gamma_2 + \gamma_3) > -\gamma_0, \qquad \gamma_0 > 0 \tag{13a}$$

or

$$\mathrm{MAX}(\gamma_1, \gamma_1 + \gamma_2, \gamma_1 + \gamma_2 + \gamma_3) < -\gamma_0, \qquad \gamma_0 < 0. \tag{13b}$$

Simulation Results

The satellite close approach times obtained from this method are compared to the results of a standard step-by-step general perturbations method [10] using Euler integration [11], which serves as the *truth*. For this study the primary and secondary satellites are advanced in 0.1 second intervals, with three sets of logic processed at each step: relative minimum, and sphere and ellipsoid entrance/exit time. Orbital motion is modeled using first-order secular rates caused by mass anomalies given by,

$$\bar{n} = n_0\left[1 + \frac{3}{2} J_2 \frac{\sqrt{1 - e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right] \tag{14}$$

$$\dot{\Omega} = -\left(\frac{3}{2}\frac{J_2}{p^2}\cos i\right)\bar{n} \tag{15}$$

$$\dot{\omega} = \left[\frac{3}{2}\frac{J_2}{p^2}\left(2 - \frac{5}{2}\sin^2 i\right)\right]\bar{n} \tag{16}$$

where: $\bar{n}$ is the anomalistic mean motion; n_0 is the mean motion at epoch; J_2 is the second harmonic coefficient; p is the semi-latus rectum; e is the eccentricity; i is the inclination; $\dot{\Omega}$ is the nodal rate; and $\dot{\omega}$ is the periapsis rate.

The classical orbital elements listed in Table 1a were converted from the US Space Command satellite catalog to analyze Intercosmos 25; they are identical to those used in Alfano and Negron [7] to illustrate the improved accuracy of this method. The simulation results are reported in Tables 1b through 1d with the format chosen to represent operational uses of satellite close-approach logic. Timing error magnitudes, when compared to the results of Alfano and Negron [7], were reduced approximately 94%. Examples of how the three reports are used include: reconstructing a satellite maneuver history given the pre- and post-maneuver element set as an application of a relative- or absolute-minimum table (Table 1b), generating a satellite volume-penetration report to identify the source of an uncorrelated observation as an application of a sphere entrance/exit table (Table 1c), and identifying objects that enter a *warning football* as an example of an ellipsoidal entrance/exit table (Table 1d).

The elements listed in Table 2a were obtained from the US Space Command as part of their predictive analysis for the US Space Shuttle Endeavour on mission 57. A planned maneuver was delayed 45 minutes to prevent a close approach with a spent Cosmos booster; the predicted conjunction is shown in Table 2b. It is interesting to note the timing accuracy of this method given the brevity of the closest encounter: 23 seconds within a 100 km sphere (Table 2c) with a close approach distance of 4.5 km. This brevity is due to the object's approach angle (110° off the primary velocity vector) with a closure rate of 8.7 km/s, where a timing error of 0.1 seconds can result in a range error of 870 m. Table 2d identifies the warning football entry and exit times of the Cosmos booster.

The accuracy of the curve fits used in this article is dependent on the time step and orbit propagator chosen. The user is urged to carefully evaluate these choices, perhaps through Monte Carlo analysis, to ensure desired accuracy.

TABLE 1a. Classical Orbital Elements

Satellite	MAGION III	Intercosmos 25
Eccentricity	0.1618711	0.1618918
Inclination	82.566°	82.5666°
Longitude of ascending node	145.6415°	145.6427°
Argument of perigee	59.8727°	59.8273°
Mean anomaly	315.2878°	315.6744°
Semi-major axis (DU)	1.27457712	1.27453329
Epoch time (Yr/Day/Hr/Min/Sec)	1992/031/20/34/00.284	1992/031/20/34/00.273

TABLE 1b. Relative Minima Solution Set

Truth		Cubic Spline (Δt = 600 s)		Absolute Difference	
Time[1]	Range (m)	Time[1]	Range (m)	Time (s)	Range (m)
13/22/41.4	4879.5	13/22/41.4	4879.7	0.0	0.2
15/24/28.1	7111.0	15/24/28.3	7111.2	0.2	0.2
17/26/12.3	9342.6	17/26/12.7	9342.8	0.4	0.2
19/27/55.1	11574.2	19/27/55.4	11574.2	0.3	0.0
21/29/36.9	13805.8	21/29/37.0	13805.8	0.1	0.0
23/31/18.1	16037.4	23/31/17.9	16037.5	0.2	0.1

[1]Hours/Minutes/Seconds for 1992 Day 30.

TABLE 1c. 10 km Sphere Entrance/Exit Solution Set

Truth[1]	Cubic Spline[1] (Δt = 600 s)	Absolute Difference
Entry 12/32/04.2	12/32/04.5	0.3
Exit 14/03/34.6	14/03/34.5	0.1
Entry 14/54/12.6	14/54/12.6	0.0
Exit 15/52/35.2	15/52/35.1	0.1
Entry 17/13/16.7	17/13/16.8	0.1
Exit 17/38/48.8	17/38/48.8	0.0

[1]Hours/Minutes/Seconds for 1992 Day 30.

TABLE 1d. 12 × 4 km Ellipsoid Entrance/Exit Solution Set

Truth[1]	Cubic Spline[1] (Δt = 600 s)	Absolute Difference
Entry 14/42/33.7	14/42/33.0	0.7
Exit 15/59/09.6	15/59/09.7	0.1
Entry 17/00/12.2	17/00/12.2	0.0
Exit 17/49/17.0	17/49/17.1	0.1
Entry 19/18/30.3	19/18/30.3	0.0
Exit 19/36/16.3	19/36/16.6	0.3

[1]Hours/Minutes/Seconds for 1992 Day 30.

TABLE 2a. Classical Orbital Elements

Satellite	Endeavour	Cosmos Booster
Eccentricity	0.003849877	0.001603018
Inclination	28.4446978°	64.99371631°
Longitude of ascending node	299.76179928°	207.22688939°
Argument of perigee	65.95283621°	264.83139957°
Mean anomaly	189.45294053°	64.46008215°
Semi-major axis (DU)	1.06945336	1.07404055
Epoch time (Yr/Day/Hr/Min/Sec)	1993/174/12/32/07.941	1993/174/12/32/07.941

TABLE 2b. Relative Minima Solution Set

Truth		Cubic Spline (Δt = 600 s)		Absolute Difference	
Time[1]	Range (m)	Time[1]	Range (m)	Time (s)	Range (m)
10/11/42.2	446064.3	10/11/42.3	446662.1	0.1	597.8
10/58/29.2	327301.7	10/58/29.2	326573.1	0.0	728.6
11/45/20.3	133258.1	11/45/20.3	133557.0	0.0	298.9
12/32/07.9	4445.1	12/32/08.0	4672.8	0.1	227.7
13/18/58.4	213053.0	13/18/58.4	213788.6	0.0	735.6

[1]Hours/Minutes/Seconds for 1993 Day 174.

TABLE 2c. 100 km Sphere Entrance/Exit Solution Set

Truth[1]	Cubic Spline[1] (Δt = 300 s)	Absolute Difference (s)
Entry 12/31/56.4	12/31/56.5	0.1
Exit 12/32/19.4	12/32/19.5	0.1

[1]Hours/Minutes/Seconds for 1993 Day 174.

TABLE 2d. 25 × 10 km Ellipsoid Entrance/Exit Solution Set

Truth[1]	Cubic Spline[1] (Δt = 600 s)	Absolute Difference(s)
Entry 12/32/06.4	12/32/06.4	0.0
Exit 12/32/09.0	12/32/09.0	0.0

[1]Hours/Minutes/Seconds for 1993 Day 174.

Conclusion

This article outlines an improved method to determine satellite close approaches with times found in closed form. Cubic and quintic splines are used as approximating functions to determine minimum distance and ellipsoidal entry/exit. Because the ephemeris generation process is not embedded in the close approach logic, this technique permits the user to choose the orbital-motion theory type. Orbit reconstruction is addressed if the orbital data generation is limited.

The method presented here can also be used to evaluate satellite insertion strategies versus the tracked satellite population. Typically, the ascent of a rocket is well understood with respect to the launch site; this position history lends itself readily to the close-approach logic just outlined. This method can also be used to quickly and accurately determine when a satellite is within some maximum range of another object.

Acknowledgments

David Negron, co-author of the original method and software tool ANCAS (Alfano/Negron Close Approach Software), is an Orbit Analyst of the United States Space Command and provided all the test cases. Cassandra Johnson is a member of the Technical Staff at the Aerospace Corporation and was the first to identify certain orbital geometries that require small time steps (on the order of 5 seconds) to maintain accuracy with the original method. John Gordeuk is a Software Engineering Specialist for LORAL Space Information Systems and suggested the investigation of dotting relative position and velocity to improve the search for minimum distances.

References

[1] HOOTS, F. R., CRAWFORD, L. L., and ROEHRICH, R. L. "An Analytic Method to Determine Future Close Approaches Between Satellites," *Celestial Mechanics,* Vol. 33, 1984, pp. 143–158.

[2] "Software Requirements Specification Technical Operating Report for the Collision Avoidance Hazard Evaluation Computer Software Configuration Item of the Initial Space Safety Systems," Contract No. F04701-89-C-0047, CDRL Sequence No. 603A2, Applied Technology Associates, Inc., Mountain View, California, August 13, 1993.

[3] KENNEDY, J. "Debris gets in your Space," *Guardian, The Magazine of Air Force Space Command,* Vol. 1, No. 1, August 1993, pp. 4–5.

[4] DYBCZYNSKI, P. A., JOPEK, T. J., and SERAFIN, R. A. "On the Minimum Distance Between Two Keplerian Orbits with a Common Focus," *Celestial Mechanics,* Vol. 38, 1986, pp. 345–356.

[5] BEERER, J., and BAUER, T. "Determination of Closest Approach and Duration of Encounter for Two Satellites in Circular Non-Coplanar Orbits," Paper No. 83-336, AAS/AIAA Astrodynamics Conference, Lake Placid, New York, August 1983.

[6] SCHINNERER, R. G. "Avoiding Collisions with Resident Space Objects," Paper No. 89-399, AAS/AIAA Astrodynamics Specialist Conference, Stowe, Vermont, August 1989.

[7] ALFANO, S., and NEGRON, D., JR. "Determining Satellite Close Approaches," *Journal of the Astronautical Sciences,* Vol. 41, No. 2, April-June 1993, pp. 217–225.

[8] ROGERS, D. F. and ADAMS, J. A. *Mathematical Elements for Computer Graphics,* second edition, McGraw-Hill, New York, 1990, pp. 250–277.

[9] MARON, M. J. *Numerical Analysis: A Practical Approach,* Macmillian, New York, 1982, pp. 200–203.

[10] ESCOBAL, P. R. *Methods of Orbit Determination,* Krieger, Malabar, Florida, 1985, pp. 360–369.

[11] MATHEWS, J. H. *Numerical Methods for Computer Science, Engineering, and Mathematics,* Prentice-Hall, Englewood Cliffs, New Jersey, 1987, pp. 402–408.