001 Microstructure Project

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Introduction

A market maker refers to a firm or individual who actively quotes both sides of a market in particular security by providing bids and offers. We assume that “individual who chooses to serve as a market-maker is assumed to optimize his position by setting a bid-ask spread which maximizes the difference between expected revenues received from liquidity-motivated traders and expected losses to information-motivated trader”. In this case we will value different kinds of scenarios to determine an optimal Bid/Ask price using Copelands & Galai´s model. We have a continuous probability distribution called “Weibull”.

Parameters

We have a continuous probability distribution called “Weibull”. Weibull is well known for analyzing data life, model failure times and access product reliability, and it can fit in many other fields, like in this case, in finance. There are two different parameters, lambda and K, which help us to graph our data. Lambda refers to the variability in our distribution, and the K means which shape our distribution will have, in this case K=10 which means our graph will have a bell shape. Our spot price in this example is 51.

P ~ Weibull (λ =50, K=10)

$$P_0 = 51$$

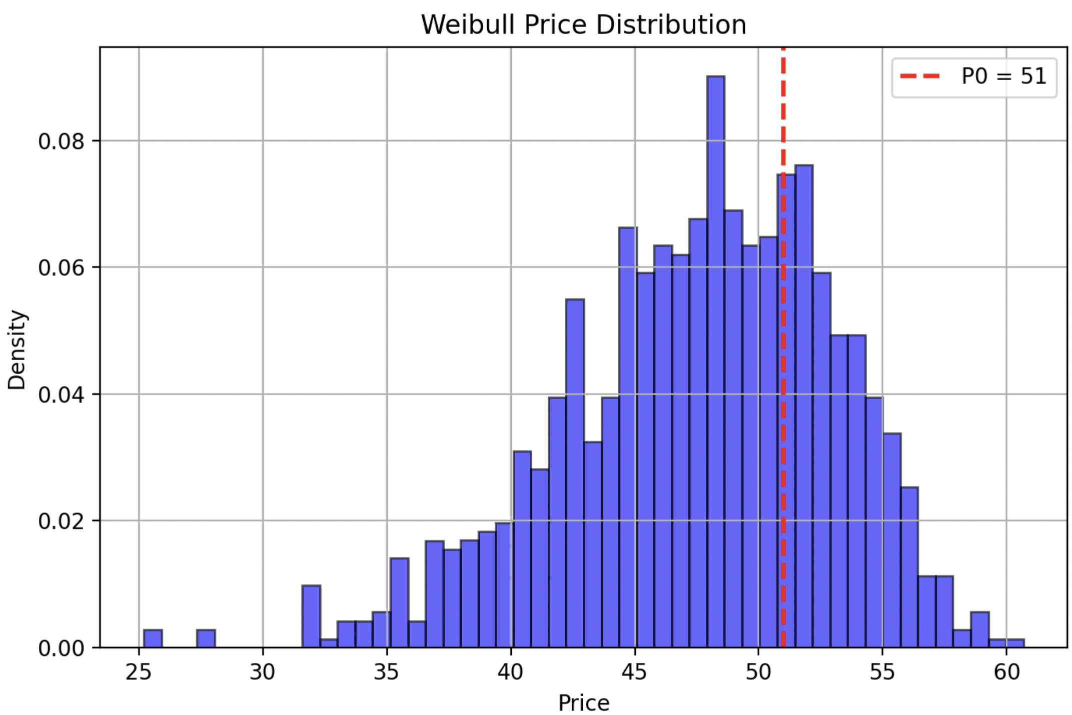
Price distribution

The first part of the project was to plot our price distribution with the parameters that were given to us. We used SciPy. Stats and we imported “Weibul_min”, which helped us make the density probability function on python. Also, to this function we added 3 different parameters: lambda, k and the size.

```
def generate_price_distribution():

    k = 10
    lambda_weibull = 50
    prices = weibull_min.rvs(c=k, scale=lambda_weibull, size=1000)
    return prices
```

After coding with the distribution and the parameters, we got the graphic below as the result. We can see clearly how the distribution is a bell-shaped graph thanks to our value of K. The blue line represents the Weibull probability function, which has its highest point is around 50, while our spot price is 51, which is represented with the red dash line.



Different scenarios

- **Expected revenue if all trades are liquidity motivated, $\Pi_I = 0$, or just $\Pi_{LS} = 0.5, \Pi_{LB} = 0.5$**

In this case we consider that the only movement the market has made is the liquidity traders, the informed traders don't make any movement in this scenario. This is represented by letter "Q", which is the 45° line.

- **Expected revenue if there is a 40% chance that an informed trade will be executed, keep the same liquidity probability as the previous point.**

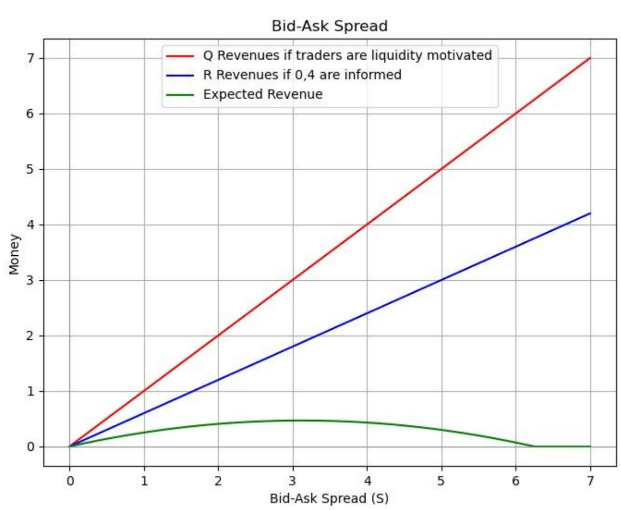
Th expected revenue for this is represented by the blue line in the graph below. In this case is the revenue if there is a 40% chance that an informed trade movement will be done, or in other words, if there is a 60% chance If a liquidity trade will be executed.

- **Expected revenue considering**
- $\Pi_I = 0.4$
- $\Pi_{LB}(S) = 0.5 - 0.08S, where\ \Pi_{LB}: [0,0.5]$
- $\Pi_{LS}(S) = 0.5 - 0.08S, where\ \Pi_{LS}: [0,0.5]$

To model expected revenue, we incorporated the provided linear probability functions for liquidity traders' demand:

Π_LB(S) = 0.5 − 0.08S and Π_LS(S) = 0.5 − 0.08S.

These functions reflect the decline in trading interest from liquidity traders as the bid-ask spread widens. Combined with the proportion of informed traders (Π_I = 0.4), we computed the expected revenue as a function of the spread and integrated this into our profit-maximization model to determine the optimal bid and ask prices.



Optimal Bid/Ask price using Copelands & Galai´s model

Copeland & Galai had some key points regarding the optimal bid and ask price model that were essential to create the project, first we started off by defining the expect cost that according to the paper the formula would look like this

$$\text{Expected Cost} = \text{PI} \times [\int \int_{KA}^{\infty} (\text{S} - \text{KA})\text{f}(\text{S})\text{dS} + \int \int_{-\infty}^{\text{KB}} (\text{KB} - \text{S})\text{f}(\text{S})\text{dS}\,]$$

Where:

- **PI** = Probability that the incoming trader is informed.
- **f(S)** = Probability distribution of the true price.
- **KA** = Ask price.
- **KB** = Bid price.
- **S** = True price after the trade.

Which we applied to the code so it would look something like the next formula:

$$\text{Expected Cost} \sim \text{PI} \times 2\Delta$$

Now we defined by the model what is the Expected revenue from Liquidity traders, which I would look something like this:

$$\text{Expected Revenue} = (1 - \text{PI}) \times [\text{PLB}(\text{KA} - \text{P0}) + \text{PLS}(\text{P0} - \text{KB})]$$

Where:

- **P0** = Dealer’s belief about the true price.
- **PLB** = Probability that a liquidity trader buys at the ask price.
- **PLS** = Probability that a liquidity trader sells at the bid price.

These **liquidity probabilities depend negatively on the spread (KA - KB)**. For instance:

$$\text{PLB} = 0.5 - \alpha(\text{KA} - \text{P0}), \text{PLS} = 0.5 - \alpha(\text{P0} - \text{KB})$$

Witch in this project the PLB would look like this:

$$\text{PLB} \ = \ 0.5 \ - \ 0.08\text{S}$$

At last, the profit is presented in this project as:

$$Expected\ Profit = Expected\ revenue - Expected\ cost$$

Results:

The result that our applied gave us is:

```
Optimal Delta: 0.5208
Optimal Bid: 50.48, Optimal Ask: 51.52
Optimal Spread: 1.04
Optimal Bid Price: 50.48, Optimal Ask Price: 51.52
```

Conclusions

The implementation of Copeland & Galai's model allowed us to understand the critical role of the bid-ask spread in financial markets. Through our simulations and revenue calculations, we observed that the optimal spread is not merely a function of transaction costs but is primarily driven by the dealer's need to balance potential losses to informed traders with gains from liquidity traders.

The use of a Weibull distribution to simulate price levels provided a flexible and realistic representation of price variability that overall, helped us demonstrated that market makers must carefully adjust their bid and ask prices to optimize profitability. The Copeland & Galai framework proved to be a valuable tool in modeling this trade-off, highlighting the fundamental link between market microstructure, liquidity, and information efficiency.

References

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