Exercises from Chapter 6 of Introduction to Stochastic Processes with R by Robert P. Dobrow

Ojeda Contreras Braulio Melquisedec

2022-11-11

6.41) Goals occur in a soccer game according to a Poisson process. The average total number of goals scored for a 90-minute match is 2.68. Assume that two teams are evenlymatched. Use simulation to estimate the probability both teams will score the same number of goals. Compare with the theoretical result.

```
trials <- 1000000
lambda <- 2.68
team1 <- rpois(trials, lambda / 2)
team2 <- rpois(trials, lambda / 2)
ssg = length(team1[team1 == team2])
prob_ssg = ssg / trials
cat("The probability that both teams score the same number of goals is", prob_ssg)</pre>
```

The probability that both teams score the same number of goals is 0.259397

6.42) Simulate the restaurant results of Exercise 6.12: Starting at noon, diners arrive at a restaurant according to a Poisson process at the rate of five customers per minute. The time each customer spends eating at the restaurant has an exponential distribution with mean 40 minutes, independent of other customers and independent of arrival times. Find the distribution, as well as the mean and variance, of the number of diners in the restaurant at 2 p.m.

```
diners_2pm <- numeric()
trials <- 1000000
for (i in 1:trials) {
    t <- 120
    lambda <- 5 * t
    no_diners <- rpois(1, lambda)
    arrival_times <- sort(runif(no_diners, 0, t))
    #print(arrival_times)
    rate_exp <- 1 / 40
    total_time <- arrival_times + rexp(arrival_times, rate_exp)
    #print(total_time)
    diners_2pm[i] <- sum(total_time > t)
}
```

```
#print(diners_2pm)
cat("The mean of the number of diners in the restaurant at 2pm", mean(diners_2pm))
### The mean of the number of diners in the restaurant at 2pm 190.0478
cat("The variance of the number of diners in the restaurant at 2pm", var(diners_2pm))
```

 $\mbox{\tt \#\#}$ The variance of the number of diners in the restaurant at $2pm\ 190.352$