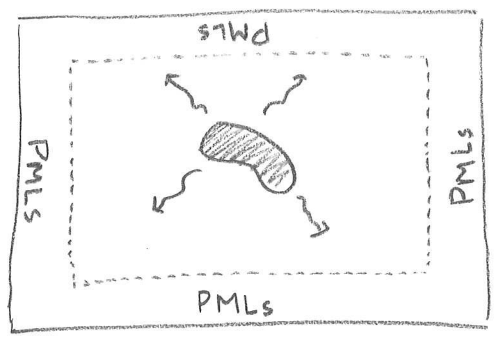
Perfectly Matched Layers (PMLs)

We use PMLs to mimic propagation into free space. They are a type of absorbing boundary condition, and are carefully designed to have nearly zero reflections.

Simulation Domain



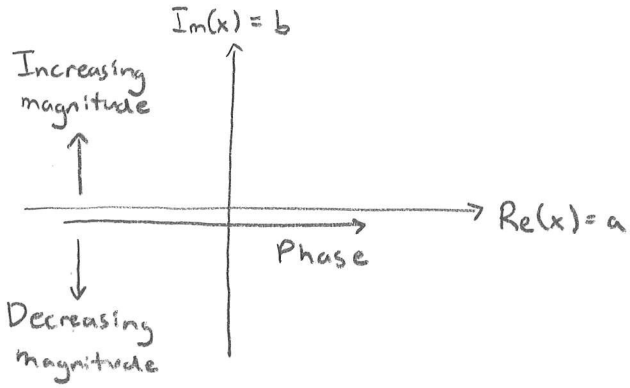
Suppose we have a wave moving in the +x direction:

$$u_+(x) = e^{-ikx}$$

Extending $x = a + ib$ to the complex plane:

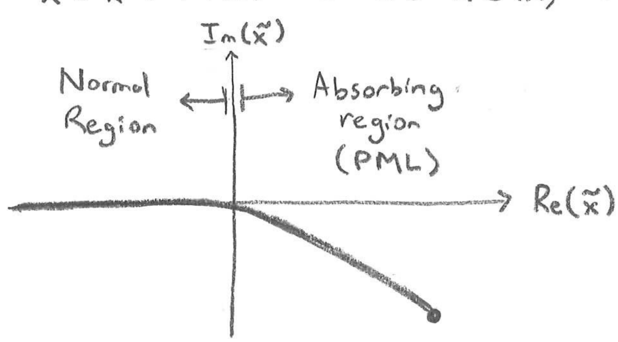
$$u_+(x) = e^{-ika} e^{kb}$$

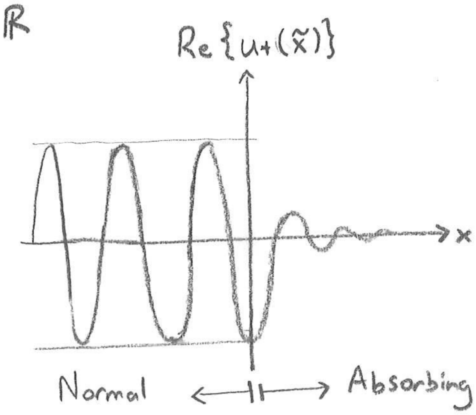
$$= \underbrace{(\cos ka + i \sin ka)}_{\text{phase}} \underbrace{e^{kb}}_{\text{amplitude}}$$



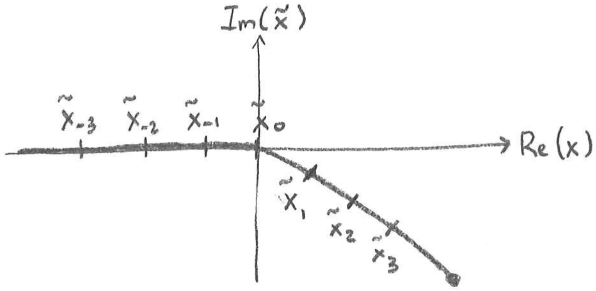
Normally, we are evaluating along the real axis. However, what if we evaluate $u_+(x)$ along some contour that extends into the complex plane?

Let $\tilde{x} = x + i f(x)$ where $x \in \mathbb{R}$, $f: \mathbb{R} \to \mathbb{R}$





To implement this "complex coordinate stretching", all we need to do is set the step size to be complex:



The step size is simply

$$\Delta x_j = \tilde{x}_j - \tilde{x}_{j-1}$$
$$= (x_j - x_{j-1}) + i(f(x_j) - f(x_{j-1}))$$

In our FDFD equations, all we need to do is set the step size Δx_j to be complex in the regions we want to be absorbing (PMLs):

$$\Delta x_j \longrightarrow \Delta x_j + i\alpha_j$$

Some additional notes:

1) We use a smooth transition (e.g. quadratic) from normal space to PML regions to minimize reflections. This is known as "grading".

2) We need to adjust the imaginary component α_j as a function of frequency to keep the decay length constant. This makes PMLs more difficult and complex to implement in FDTD since PMLs are necessarily dispersive.