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# APPENDIX 2

Some Introductory Notes on

# DIVERSITY TECHNIQUES

for the support of Experiment AM1

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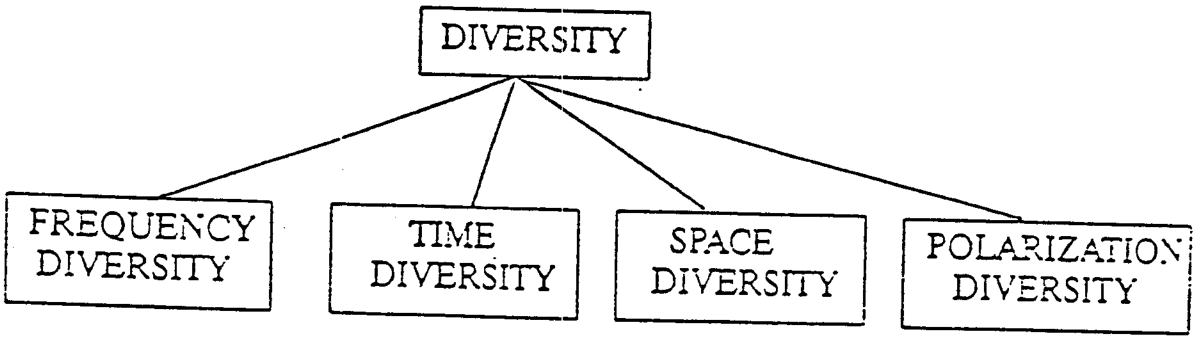
# 1. INTRODUCTION

Diversity may be defined as a general technique that utilizes two or more copies of a signal with varying degrees of noise effects to achieve, by a selection or a combination scheme, higher degree of message-recovery performance that is achievable from any one of the individual copies separately.

# 2. CLASSIFICATION OF DIVERSITY TECHNIQUES



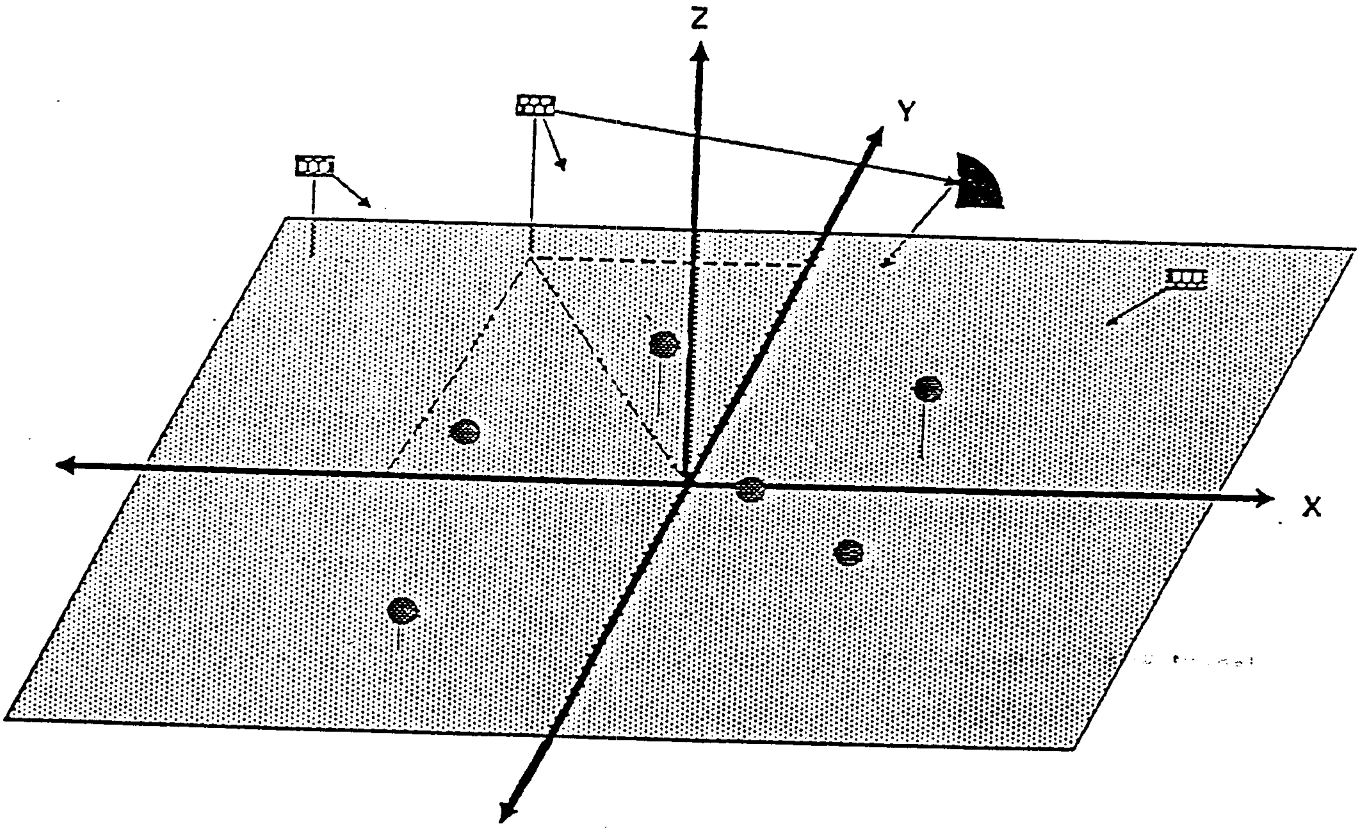

e.g. Frequ. Diver.: parallel channels may be used to transmit the same information simultaneously transmitted on 4, say, different carrier frequencies; then at the receivers the 4 signals are combined before a decision is made).

# 3. SPACE DIVERSITY: ARRAYS

By distributing a number of sensors (transducing elements, receivers, etc) in a 3-dimensional cartesian space, an array is formed; the region over which the sensors are distributed is called the aperture of the array. The general array processing problem is the obtaining of information about a signal environment from the waveforms received at the array elements (*Figure 1*), where the signal environment consists of a number of emitting sources plus noise. These emitting sources, in the case of radar-based systems, are often targets which either reflect transmitted signals (as in active radars) or emit their own signals, (as in passive radar). In situations involving the use of sonar and seismic signals the problems are essentially the same as those encountered in the case of radar.

# FIGURE– 1

ARRAY PROCESSING PROBLEM




: denotes an array element

: denotes an emitting source

: denotes a reflecting surface

**PROBLEM: USING A NUMBER OF SENSORS ESTIMATE SIGNAL ENVIRONMENT IN THE PRESENCE OF NOISE**

An important topic in the array processing problem is concerned with interference rejection. Since the emitting sources are distributed in space the array can perform both spatial and temporal filtering in order to optimize the reception of a signal from a desired source (desired signal). This can be achieved by using an array-pattern-forming-network (*Figure 2a*) so as to place relatively high gain in those directions and frequencies which contain the desired signal and at the same time place nulls in the directions and frequencies of the remaining unwanted (interfering) sources.

If the signal environment is known then the pattern-forming network can be fixed and the array response pre-determined. However, in practice the signal environment is often unknown and may vary with time or change its structure (i.e. the turning on and off of certain sources) and thus a very versatile scheme must be used which leads to the oncept of adaptive arrays. In adaptive arrays there is an adaptive processor which controls the pattern-forming network according to some performance criterion (*Figure 2b*).

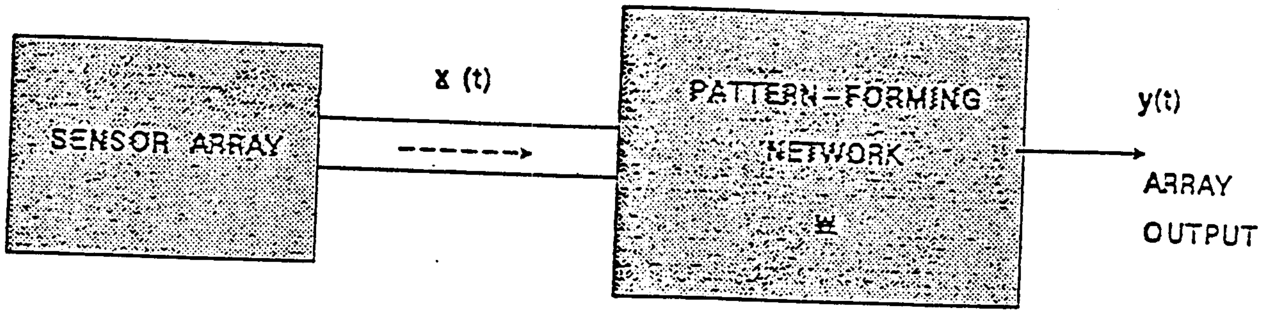
Another array processing problem is concerned with spatial spectrum estimation and identification. With problems of this type, the array detects the number of directional signals present in the array environment and estimates their parameters; such as location, power, cross correlation etc. Classical spatial spectral estimation techniques are based on the Fourier transform (Conventional Beamformer). The main drawback of the Fourier methods is that they offer limited resolving capabilities. Thus, in the last decade the so called *High Resolution Methods* have been introduced, their main object being to improve the resolving capabilities by using a model for the signals better than that used by Fourier methods. These methods have given fresh impetus to the array processing problem by dealing with the question of the resolution of the arrays in such a way that there is elimination of the effects of *Signal-to-Noise-plus-Interference Ratio (SNIR)* on resolution, in contrast to the conventional methods where the resolution is limited by noise.

Recently, the new class of processing techniques called *High Resolution Adaptive Array Processing* has been created by merging the objectives of both the above mentioned classes. The new class relies heavily on Adaptive Array Techniques, Modern Estimation Theory and Parallel Computer Processing. In general, the aim of *High Resolution Adaptive Array Processing* is to isolate one source and, in addition, to provide estimates of certain other parameters such as the number of interferences present and their directions etc. thus giving solutions to many physical problems arising in communications, radar, sonar, geophysics etc.

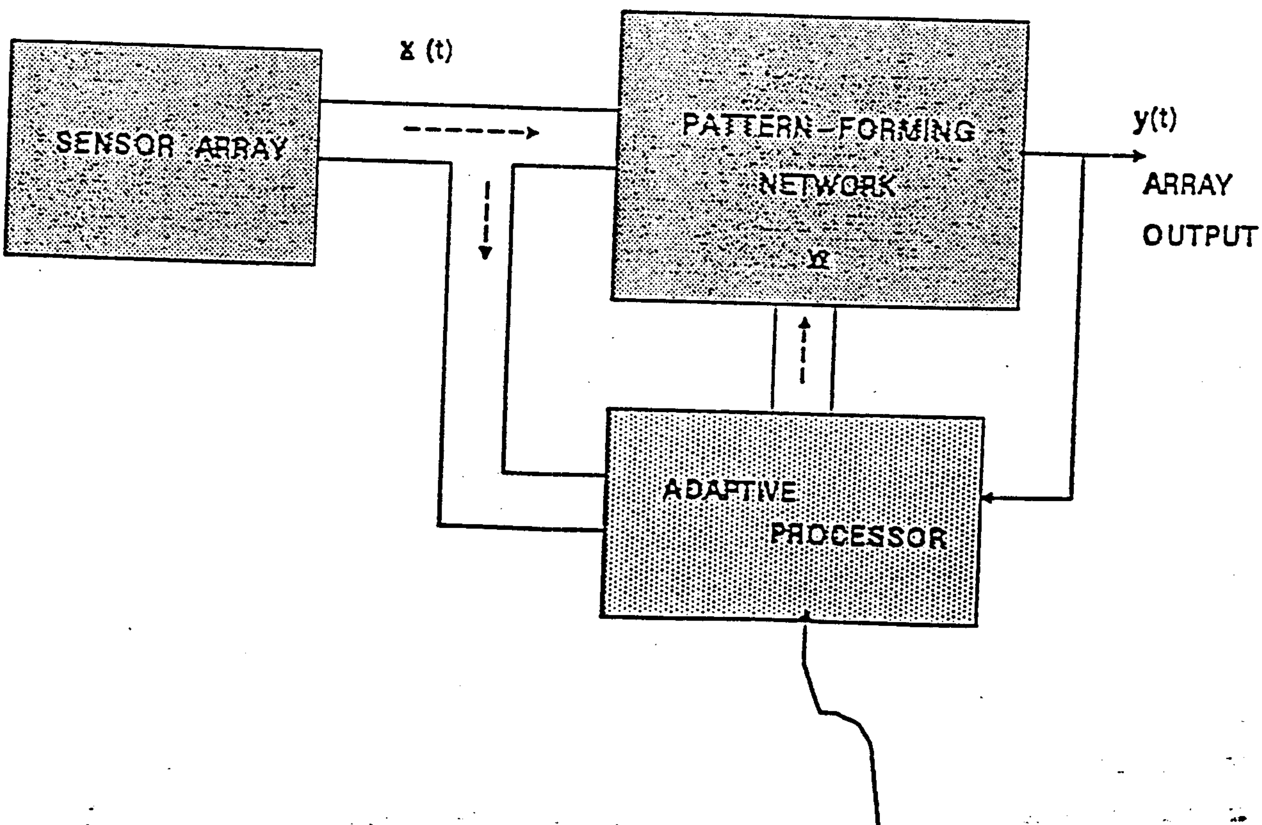
# FIGURE– 2

PATTERN FORMING BLOCK DIAGRAM

a)FIXED




b)ADAPTIVE



## 4. HISTORICAL PERSPECTIVES

Early work in array processing for interference suppression was carried out at the MIT Lincoln Laboratory in 1963. This work was concerned with a non-adaptive interference canceller which could handle one source at a time. The basic idea was to use a main antenna to look at the desired signal and a second antenna to look at the interference and then to subtract the output of the second antenna from the main antenna, with a proper phasing being employed.

Although the term *"adaptive arrays"* was first introduced by Van Atta in 1959, the first papers on adaptive arrays were probably those published in 1964 in a special *IEEE* issue on *Antenna and Propagation* 1964. In 1966 the foundations of array processing and particularly adaptive array processing were established in two papers: by Widrow [WIN-66] and Applebaum [APP-66]. The paper by Widrow introduced a new approach for controlling the weights of an adaptive filter and the paper by Applebaum presented a sidelobe canceller capable of handling multiple jamming sources by using the concept of a *correlation feedback loop* (known today as *Applebaum's loop*), for maximizing the signal-to-noise ratio (SNR). Applebaum's sidelobe canceller does, however, need prior knowledge of the signal directions and uses a high gain antenna as the main channel. In general, in adaptive arrays, there is however no need for prior knowledge of the directions of the sources, nor is there a need for use of a high gain antenna. The sidelobe canceller approach is thus not very general.

The first paper on *General Adaptive Arrays* was published the following year (1967) by Widrow et al who applied the ideas contained in his previous paper [WID-66] to develop an adaptive array system. Widrow's work was epoch making and it was based on the minimization of the mean square error between the desired signal and the array output. This approach has come to be known as the Least Mean Square (LMS) algorithm. Applebaum's loop and LMS algorithm have two common points: both use the array covariance matrix in order to derive their adaptive weights and both converge towards the same steady state weight vector which is the WIENER-HOPF solution.

An attractive alternative to the LMS algorithm was introduced in 1974 by Reed, Mallet and Brennan which overcomes the sensitivity of the LMS-type algorithm to eigenvalue spread. The Reed-Mallet-Brennan approach has come to be known as the Sample Matrix Inversion (SMI) algorithm. In the SMI algorithm the optimum weights are found by estimating the covariance matrix and then solving a linear system of equations.

One of the main problems with conventional adaptive arrays is related to their inability to resolve two sources which are positioned close together. This is illustrated in *Figure 3a*, which shows simulation results obtained with a linear array of 5 isotropic uniformly distributed elements. This inability is imposed by the fact that the resolution of adaptive arrays is limited by the SNR. This is demonstrated in *Figure 3b* in conjunction with *Figure 3a.*, which shows that if SNR is 10dB then the array is unable to resolve the two signals incident from directions 30° and 35° correspondingly (*Figure 3a*); with a SNR of approximately 30dB (*Figure 3b*) the two sources of *Figure 3a* can be resolved. The inability of an array to resolve sources that are close together when noise is present gave rise to a new class of techniques that have been used for the location of emitting sources. These techniques are called *High Resolution* (or *Superresolution*) techniques. Two popular methods belonging to this new class are the so called Maximum Likelihood Method (MLM) which is based on the work of Capon on frequency-wavenumber analysis, and the Maximum Entropy Method (MEM) of Burg. Capon's method is based on the minimization of the output power subject to the constraint that the inner product of the weight-vector and Source Position Vector is equal to 1. On the other hand, Burg's method is based on an iterative search technique which maximizes the entropy subject to a number of constraints.

Perhaps the most important High Resolution Techniques currently being examined are the so called *Signal Subspace* techniques. Aspects relating to techniques of this kind go back to 1795 when Baron de Prony published his work on the fitting of superimposed exponentials to data. The Signal Subspace approach was first introduced formally by Schmidt in narrow-band array processing problems and rediscovered independently by Bienvenu and Kopp.

The Signal Subspace approach to high resolution involves two main stages of processing. In the first stage a covariance matrix of the data at the sensors of the array is formed and in the second stage an eigenvector decomposition is performed (see for example MUSIC algorithm). By the eigenvector decomposition, the observation space is partitioned into two disjoint subspaces:

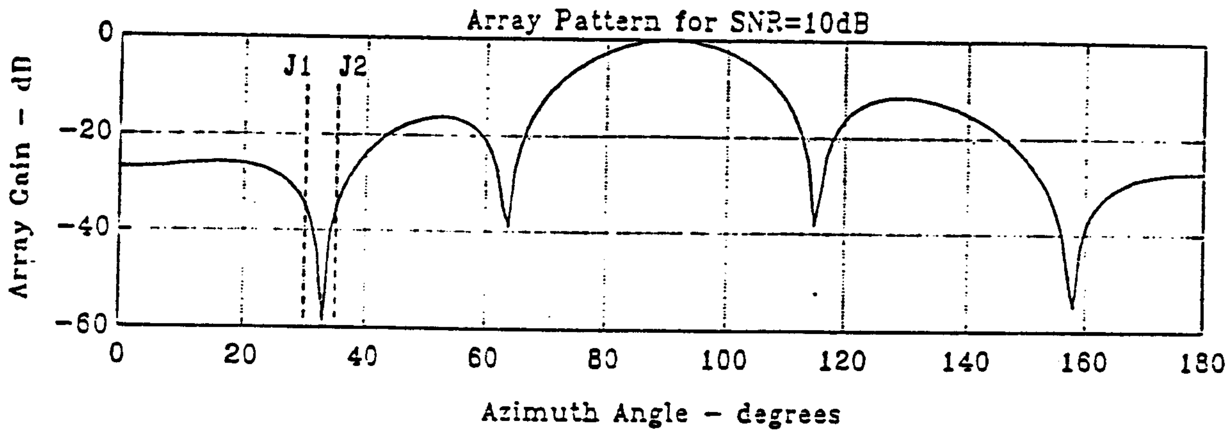
- **the Signal Subspace (SS), with dimension equal to the number of sources, spanned by the Source Position Vectors (SPV);**
- **the Noise Subspace (NS), with dimension equal to the number of sensors, minus the number of sources.**

FIGURE – 3 : ADAPTIVE ARRAY RESOLUTION LIMITATION DUE TO SNR

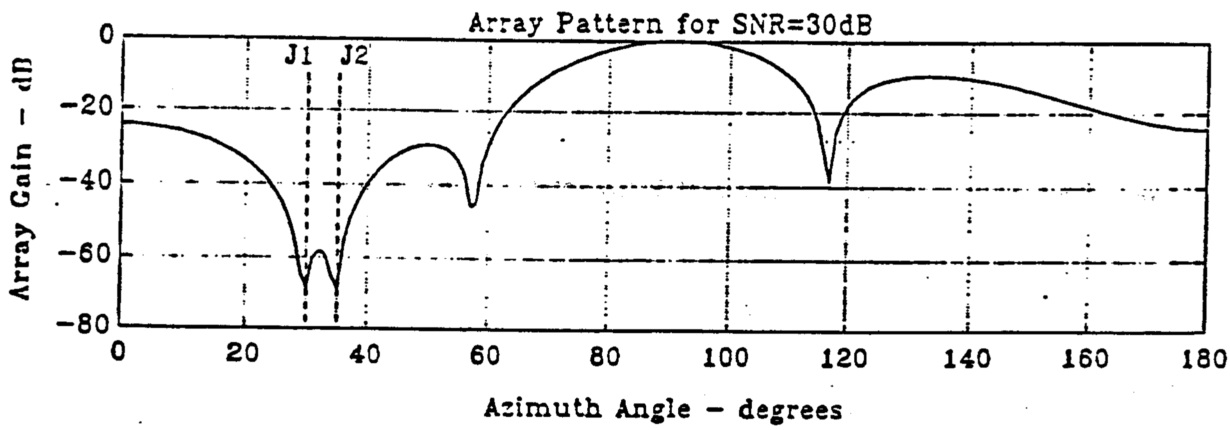
RELATIVE POWER TABLE :

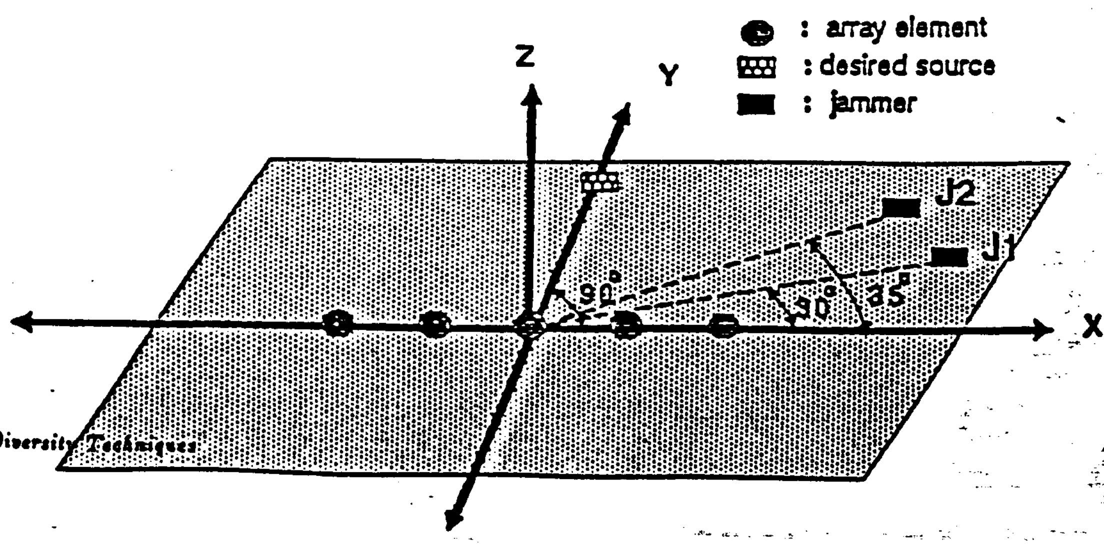
| TOP GRAPH: | | BOTTOM GRAPH: | |
|---|---|---|---|
| desired signal | :1.0 | desired signal | :1.0 |
| Jammer No.1 (J1) | :1.0 | Jammer No.1 (J1) | :1.0 |
| Jammer No.2 (J2) | :1.0 | Jammer No.2 (J2) | :1.0 |
| Noise | :0.1 | Noise | :0.001 |

a)




b)



Thus, every vector belonging to the NS is orthogonal to each SPV. The signal subspace approach offers higher resolving power and less ambiguity than other high resolution methods including the MLM technique of Capon and the MEM method of Burg. This can be seen, for example, in the comparative computer simulation studies carried out by Johnson and Miner where:

(*i*) the MLM was capable of resolving two sources with 10° separation, provided the SNR was sufficiently high, but it was unable to do so for sources with 5° separation;

(*ii*) the MEM produced no usable results and

(*iii*) the MUSIC algorithm (a signal subspace technique) produced results in every considered situation.

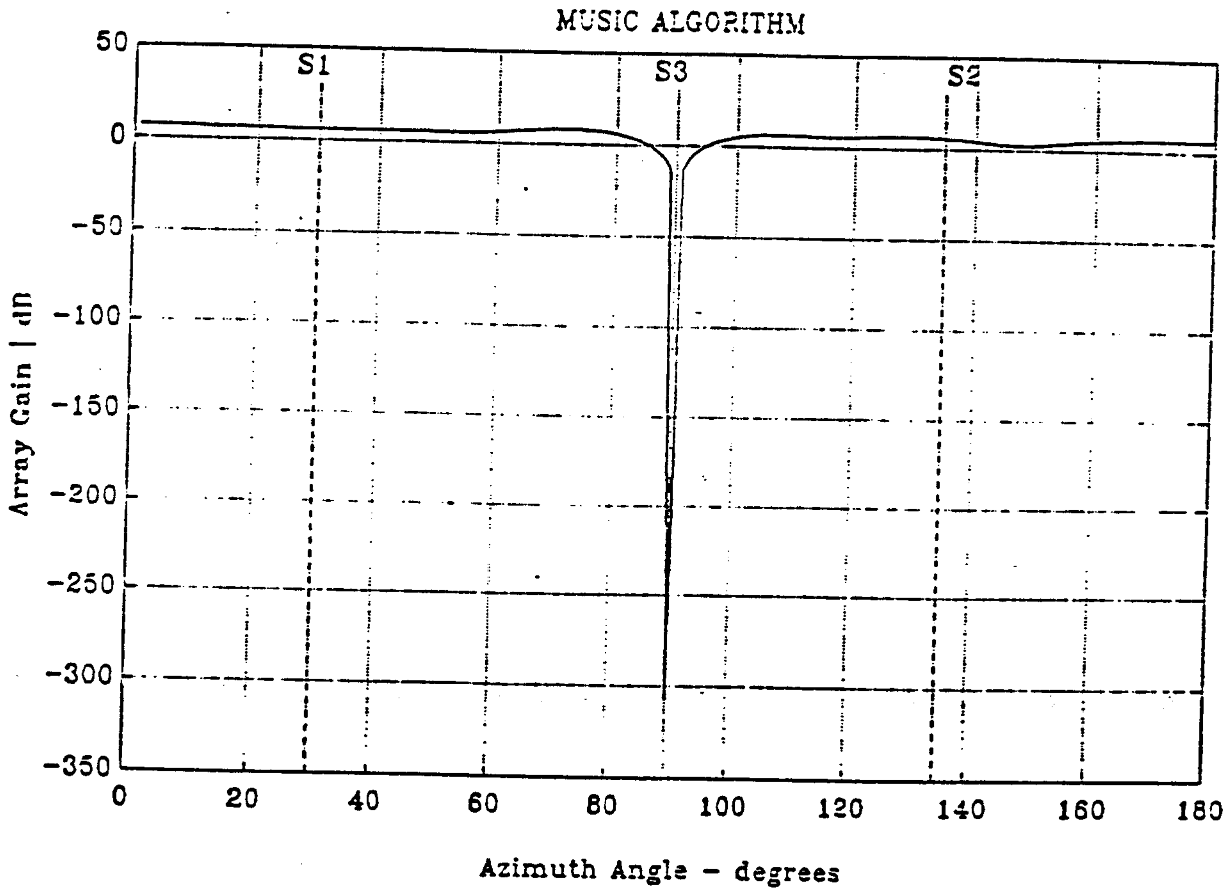
Thus, signal subspace techniques, which have been used mainly in estimating the directions of emitting sources by employing a spatial array, offer asymptotically "infinite" resolving power capabilities, with limitations imposed only by the limited observation time and the inaccurate modelling of the medium.

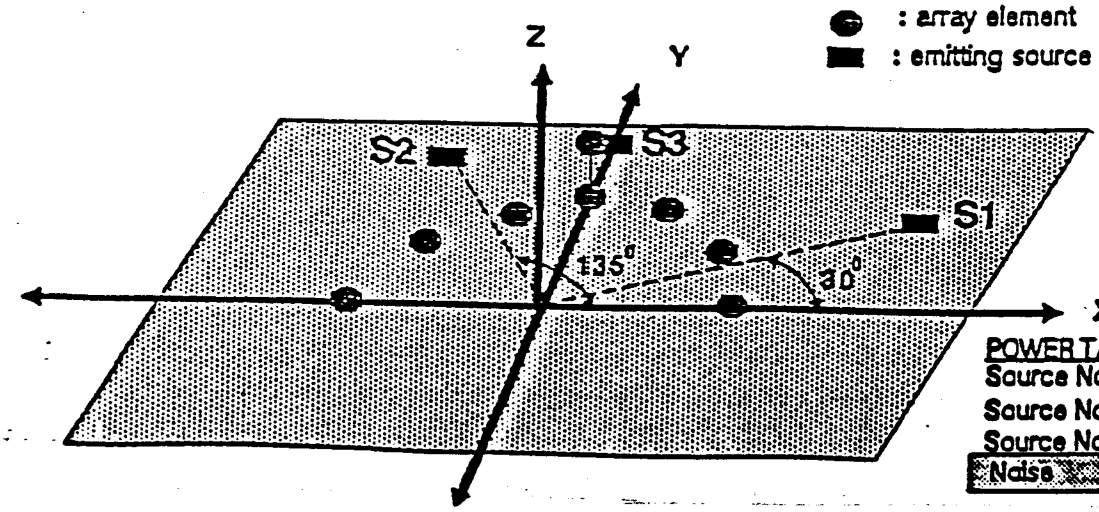
## 5. AN IMPORTANT PROBLEM

It has been mentioned that high resolution methods and particularly Signal Subspace techniques can handle the source location problem. This is true as long as the present signals are not correlated (coherent). If some of the incident signals are correlated (coherent), then all known existing signal subspace techniques <u>fail</u>. This is a serious limitation since there are many situations in which signal correlation exists. Two common examples are those where *smart* jamming exist and *multipath* propagation is present. The nature of the failure of existing techniques is illustrated in *Figure 4* where a linear array of 5 isotropic elements is considered and three signals, two of which are assumed correlated, are present. *Figure 4*, shows that although the Signal Subspace algorithm (MuSIC) correctly indicates the location (direction) of the uncorrelated source, it fails completely to indicate even the existence of the two correlated sources.

FIGURE – 4 (ASSUMING SOURCES S1 and S2 ARE FULLY CORRELATED)

| | TRUE directions | directions estimated by MUSIC |
|---|---|---|
| source No.1 (S1) | ( 0.0, 30.0) | Failure |
| source No.2 (S2) | ( 0.0, 135.0) | Failure |
| source No.3 (S3) | ( 0.0, 90.0) | ( 0.0000, 90.0000) |




POWER TABLE :
Source No.1 :1.0
Source No.2 :1.0
Source No.3 :1.0
Noise :0.1

In addition to their inability to operate with correlated signals, signal subspace algorithms also require prior knowledge of the number of signals present in order to be able to function correctly.

Thus, at this time much effort is being devoted to the question of handling correlated (multipath) sources. In order to rectify this 'breakdown' of high resolution and particularly signal subspace techniques when signal correlation (or coherence) is involved, a number of new techniques have been developed. The most significant of those new techniques are based on the idea of *subaperture sampling* or *spatial smoothing* discovered originally by Evans, Johnson and Sun and independently rediscovered and improved by Shan, Wax and Kailath. The *Spatial Smoothing* technique is based on defining a number of subarrays and for each subarray the covariance matrix $R_i$ is formed. Then the average covariance matrix $\widetilde{R}$ is estimated as follows:

$$\widetilde{R}=\frac{1}{No.\ of\ subarrays}\cdot\sum_i R_i \qquad (1)$$

After this, the next stage in the process is to use the previously developed MuSIC algorithm to provide the location of the sources.

However, this method can only be applied to linear arrays with uniformly spaced identical sensors. In addition, it provides a reduction of array aperture which implies a reduction in resolving power. Thus the Shan, Wax and Kailath technique is not very satisfactory.

## 6. ARRAY AMBIGUITIES

Source location ambiguity may arise whenever i) the array manifold repeats itself or ii) a point on the array manifold can be written as a linear combination of some other points. A simple example as a help in visualizing case (ii) is when a ray from the origin intersects the manifold at more than one point. A useful way of measuring the degree of ambiguity is by means of manifold dimensionality. Thus, consider that $M$ is the largest number, such that for every combination of $M$ distinct directions their corresponding Source Position Vector ($SPV$) can be a base of an $M$ dimensional subspace. Then $M$ is defined as the dimensionality of the array manifold. Once the array manifold, for a particular *visible area*, has been calculated and stored the array manifold dimensionality can be estimated and therefore becomes known for that particular array design. The dimensionality reflects the maximum number of signals which can be uniquely resolved by the array without any ambiguity arising.

It is important to point out that the array does not present the same resolution characteristics in the whole domain of directions.

## 7. SIGNAL SUBSPACE APPROACH

High-Resolution in array processing is taken to be the ability to distinguish the effects of two equal power sources located close together. Although the resolving power of an array can usually be improved by increasing the aperture of the array, this is not, in general, acceptable and an alternative for a given array aperture is to use High-Resolution or Superresolution Techniques.

Signal Subspace approach to high resolution involves two main stages of processing. In the first stage a covariance matrix of the data at the sensors of the array is formed and in the second stage an eigenvector decomposition is performed.

When the number of emitters is smaller than the number of sensors, the determinant of the covariance matrix is equal to zero in the absence of non-directional sources. This is due to the fact that the presence of an emitter increases the rank of the covariance matrix by one.Thus

$$rank(R_{xx})=M \tag{19}$$

If, however, in addition to the directional sources there are also non-directional sources present then the last equation is not quite true. The covariance matrix then is given by:

$$R_{xx}=R_{sig}+\sigma^2.I \tag{20}$$

In that case $R_{xx}$ has full rank (that is $rank(R_{xx})=N$) and the following relationship is valid:

$$rank\left(R_{xx}-\sigma^2.I\right)=M \tag{21}$$

However, since the presence of noise affects only the diagonal terms of the $R_{sig}$ covariance that means that

$$eig_i(R_{xx})=eig_i(R_{sig})+\sigma^2 \tag{22}$$

therefore

$$eig_{min}(R_{xx})=eig_{min}(R_{sig})+\sigma^2 \tag{23}$$

Now since $eig_{min}(R_{sig})=0$ with multiplicity $N-M$ that means

$$eig_{min}(R_{xx})=\sigma^2 \tag{24}$$

with multiplicity $N-M$ also.

Therefore $M$ can be determined by the eigenvalues of the covariance matrix and more specifically by the multiplicity of its minimum eigenvalue:

$$M=N-(multiplicity\ of\ min\text{-}eigenvalue) \tag{25}$$

Now let us consider that the eigendecomposition has been performed:

$$(\underline{v}_1,\lambda_1)\quad(\underline{v}_2,\lambda_2)\quad\ldots\ldots\ (\underline{v}_M,\lambda_M)\quad(\underline{v}_{M+1},\lambda_{M+1})\quad\ldots\ (\underline{v}_N,\lambda_N) \tag{26}$$

where $(\underline{v}_i,\lambda_i)$ represents the $i^{th}$ eigenvector and eigenvalue of $R_{xx}$. Let

$$\lambda_1\geq\lambda_2\geq\ \ldots\ \geq\lambda_M>\lambda_{M+1}=\ldots\ldots=\lambda_N=\sigma^2 \tag{27}$$

Then

$$R_{xx}.\underline{v}_i=\lambda_i.\underline{v}_i$$

$$(R_{sig}+\sigma^2 I).\underline{v}_i=\lambda_i.\underline{v}_i$$

$$R_{sig}.\underline{v}_i=(\lambda_i-\sigma^2).\underline{v}_i \qquad (28)$$

It is now clear from *Equation-27* in conjunction with *Equation-28* that the eigenvectors corresponding to the minimum eigenvalue of the $R_{xx}$ will satisfy:

$$R_{sig}.\underline{v}_i=\underline{0} \qquad \forall i\in[M+1,\dots,N]$$

$$i.e. \qquad R_{sig}.V_{noise}=O \qquad (29)$$

$$where\ V_{noise}=[\underline{v}_{M+1},\dots,\underline{v}_N]$$

That means that these eigenvectors will be orthogonal to the subspace spanned by the columns of $R_{sig}$, and because that subspace includes the *SPVs* corresponding to the directional sources, then, these eigenvectors will be orthogonal to them too. Thus, for instance, by forming the functional:

$$f(\theta)=\underline{S}(\theta)^H.V.V^H.\underline{S}(\theta) \qquad (30)$$

the well known as MuSIC algorithm is established.

Thus, for a linear array where the only parameter of interest is the azimuth angle, the above equation is evaluated for all SPV corresponding to angles from 0° to 180° and the directions where that equation becomes zero are the directions of the incident signals. A better reformulation of the Signal Subspace approach is to see the source locations as the intersection of the signal subspace with the set of all possible SPVs (array manifold).

Concluding, by the eigenvector decomposition the observation space is partitioned into two disjoint subspaces:

- the Signal Subspace (SS), with dimension equal to the number of sources, spanned by the Source Position Vectors (SPV);
- the Noise Subspace (NS), with dimension equal to the number of sensors minus the number of sources.

Thus, every vector belonging to the NS is orthogonal to each SPV.

## 8 STEERED VECTOR ADAPTIVE ARRAYS

The previous sections addressed the problem of locating a number of emitting sources. This section is concerned with the problem of isolating an emitting source in the presence of interference and noise and obtaining information about the unknown interference environment. An array scheme capable of isolating an emitting source is the so called *steered vector adaptive array*. In this array processing scheme, an array of $N$ sensors operates in a completely unknown interference environment. Its main function is the adjustment of the array pattern so as to receive a signal coming from a known direction, in the presence of $M$ unknown interferences or jammers (with $N \geq M$) which are spatially distributed in unknown directions. Its aim is the reception of the desired signal and maximum suppression of unwanted interferences (ideally to zero).

However, this array technique is willing to compromise on interference suppression and allow some interference to pass at the output of the array (contaminating the desired signal) in order to obtain maximization of the signal-to-noise ratio. That is, it does not provide complete suppression of unwanted interferences. In addition, the direction of arrival (DOA) of the desired signal should be known a priori as accurately as possible since the pointing errors affect significantly the performance of the array. Furthermore, the larger the power of the desired signal at the input of the array is the smaller the power of the desired signal is provided at the output of the array. This is known as the *power inversion problem* and it may result in desired-signal cancellation.

A partial solution to the above mentioned problems, is given by the modified Applebaum loop. This is based on the idea of filtering the desired signal. Thus, by forming a covariance matrix which does not include the desired signal one can make a new adaptive array which is more robust to pointing errors and overcomes the drawback of power inversion mentioned above.

All the same, the performance of adaptive arrays is significantly degraded as the number of jammers increases, with deteriorated results when some of them are close together or when they are located at less than half the array beam-width away from the desired signal. Thus, the performance of adaptive arrays is governed by the geometry involved in the array and signal environment, as well as by the powers of the directional sources.

All the above mensioned problems in conjunction with the resolving power limitations of conventional adaptive array techniques make this array scheme incapable of handling many real world problems.

In this section it will be shown how the concepts of signal subspace methods can be extended into conventional steered vector adaptive arrays in order

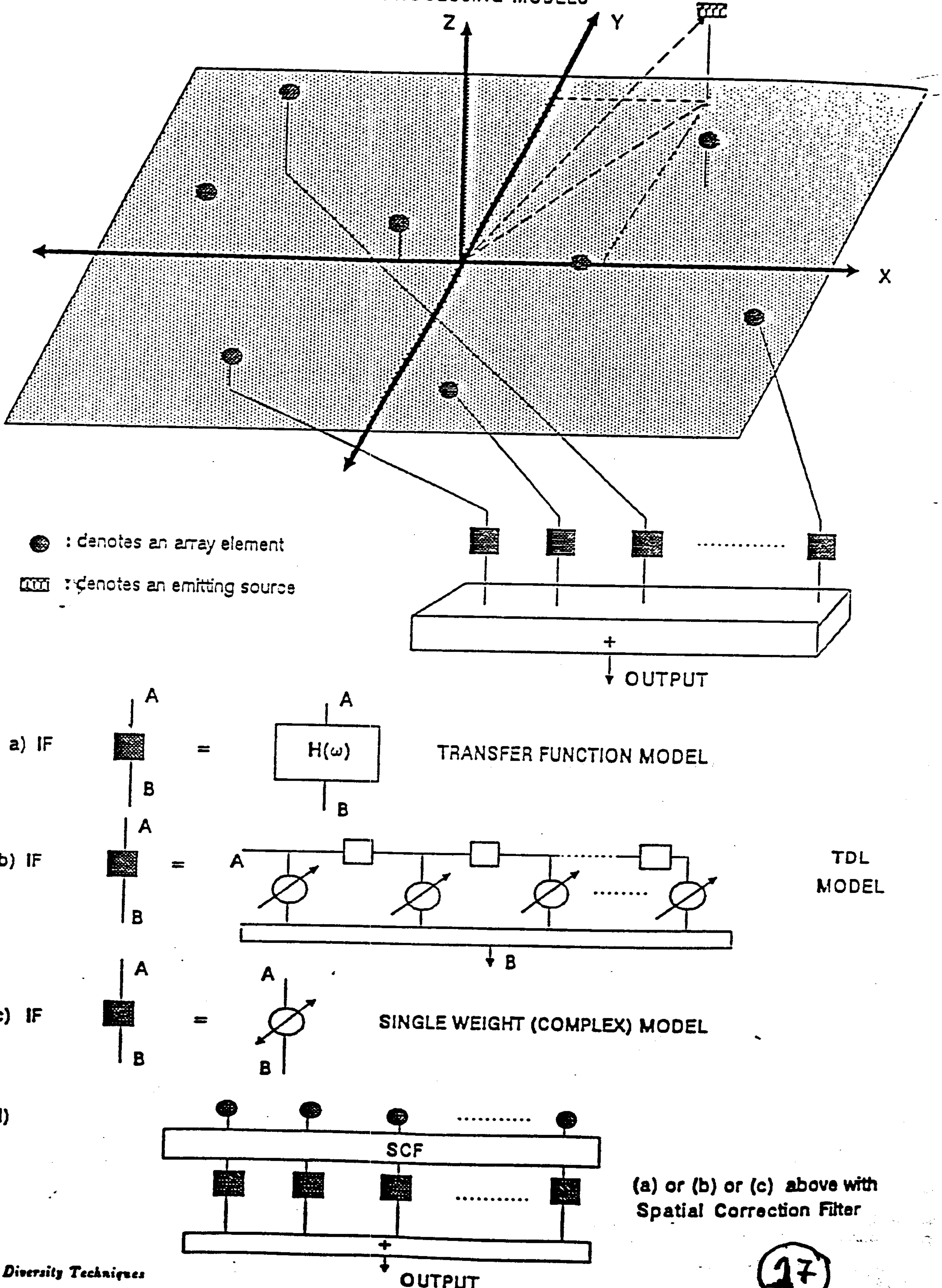
- to analyse their behavior and highlight the problems mentioned above;
- to present a new algorithm capable of receiving a desired signal in the presence of unknown non-coherent interferences and, which at the same time, is:
    - capable of providing completely interference cancellation
    - capable of locating the positions of interfering sources
    - less susceptible to pointing errors and without the disadvantage of power inversion problems.

## 9. ARRAY PROCESSING MODELS

When dealing with broadband signals the array processor can usually be represented by a transfer function as shown in *Figure 5a*. However, the transfer function is not generally suitable for adaptive processing and, therefore, discrete time approximations for this model can be established using *tapped delay line* (*TDL*) filters (*Figure 5b*), where each of the tap weights can be considered to be adjustable. The *TDL*, when it is used for narrowband applications, can be reduced to a *narrowband TDL* model which ideally can be represented by a single time delay per channel. This single time delay per channel can then be approximated by a complex weight in each channel and this last model is known as the *single complex weight model* (*Figure 5c*).

Finally, in many array applications the direction of arrival of the desired signal is known or can be measured. In such a case, the time delays needed to align the desired

FIGURE–5.
ARRAY PROCESSING MODELS



signal terms in each channel can be computed and this function can be modelled by a *spatial correction filter* as shown in *Figure 5d*. In the following subsections, the single complex weight model (with or without *spatial correction filter*) is going to be used.

In the following two subsections the basic concepts necessary to analyse the behaviour of an array, such as array output and array pattern, will be discussed.

## 10. ARRAY OUTPUT — SNIRoui

Consider the array as shown in *Figure 6* at some time $t$. At this time, the output $y(t)$ of the array will be the contribution of all the $x_k(t)$ 's weighted by the complex coefficients, $w_k|_{k=1,2,3,..N}$.
That is,

$$y(t)=\sum_{k=1}^{N} w_k . x_k(t) \tag{31}$$

or in more compact form:

$$y(t)=\underline{x}(t)^H \, \underline{w} \tag{32}$$

$$\text{where} \left[ \begin{array}{l} \underline{x}(t)=[x_1(t),\ x_2(t), \dots, x_N(t)]^T \\ x_i(t) \text{ is given by equation } 2.26 \\ \underline{w}=[w_1,\ w_2,\ \dots,\ w_N\ ]^T \end{array} \right.$$

Now the average output power from the array is given by:

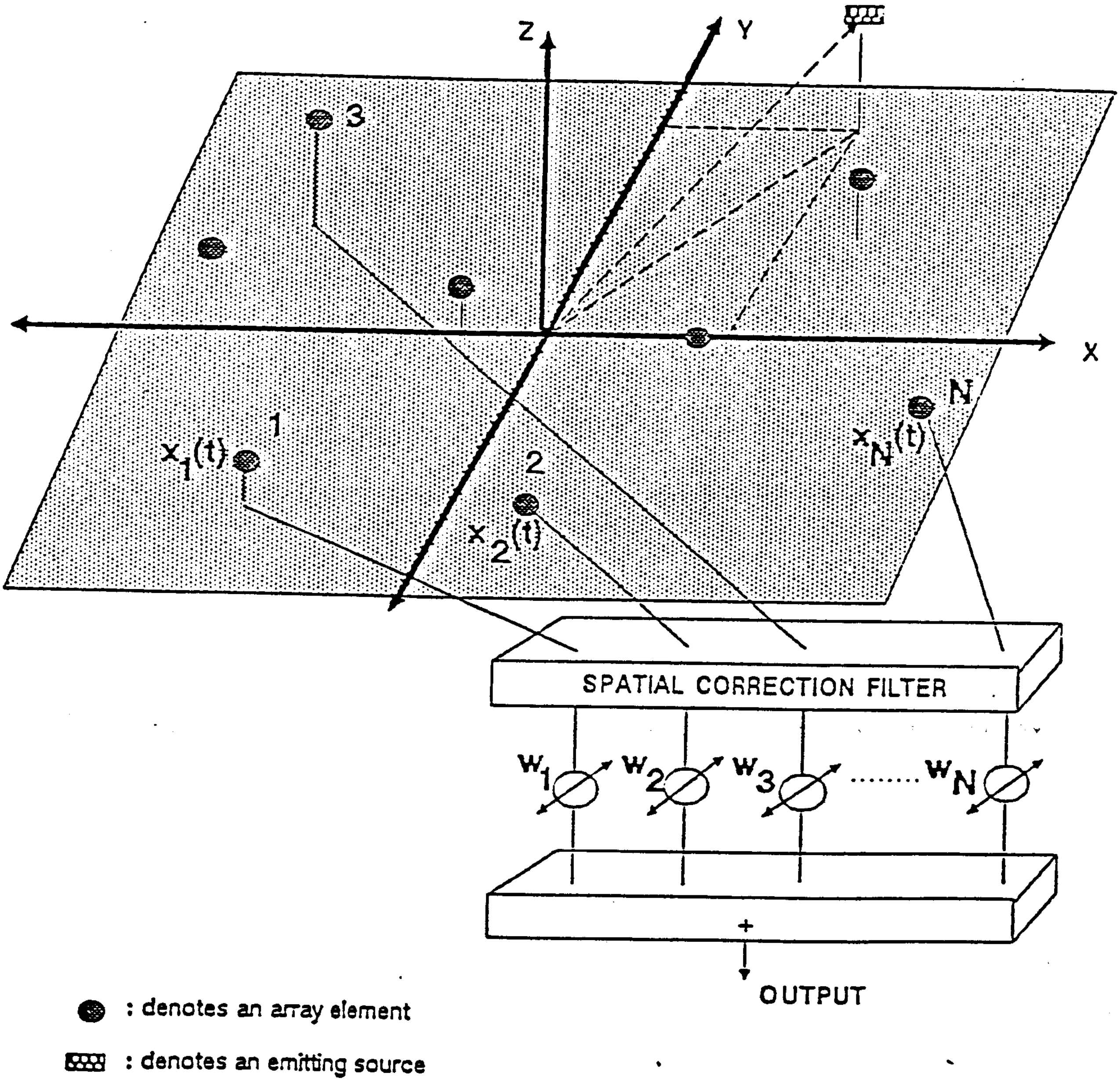
$$P_{out}=E[\ \|y(t)\|^2]=E[y(t)^H y(t)]$$
$$=E[\ \underline{w}^H\ \underline{x}(t).\underline{x}(t)^H\ \underline{w}]$$

*i.e.*

$$P_{out}=\underline{w}^H R_{xx}\ \underline{w} \tag{33}$$

If the incident signals are uncorrelated, then the output from the array can be expressed as the sum of powers of the separate sources, that is, FIGURE- 6

ARRAY PROCESSING MODEL (NARROW BAND)



$$P_{out}=P_{d\text{-}out}+P_{J\text{-}out}+P_{n\text{-}out} \tag{34}$$

$$\text{where}\quad \left[\begin{array}{l} P_{d\text{-}out}=\textit{desired output power}=\underline{w}^H \mathrm{R}_{dd}\underline{w} \\ P_{J\text{-}out}=\textit{total jammer output power}=\underline{w}^H \mathrm{R}_{JJ}\underline{w} \\ \qquad\quad =\sum_i P_i \quad \textit{where } i=\{1,\ldots,M\} \\ P_{n\text{-}out}=\textit{noise output power}=\underline{w}^H \mathrm{R}_{nn}\underline{w} \end{array}\right.$$

*Equation* 34 can also be written as follows:

$$P_{out}=\ \underline{w}^H\mathrm{R}_{dd}\underline{w}\ +\ \underline{w}^H\mathrm{R}_{JJ}\underline{w}\ +\ \underline{w}^H\mathrm{R}_{nn}\underline{w} \tag{35}$$

i.e.

$$P_{out}=\ P_d\cdot(\underline{w}^H\underline{S}_d)^2\ +\ \sum_{j=1}^{M}P_j\cdot(\underline{w}^H\underline{S}_j)^2+\ \sigma^2.\underline{w}^H\underline{w} \tag{36}$$

If the output signal to noise plus interference ratio ($SNIR$) is defined as the ratio of the wanted signal power to the total unwanted power then:

$$SNIR_{out}=\frac{\underline{w}^H\mathrm{R}_{dd}\underline{w}}{\underline{w}^H\mathrm{R}_{n+J}\cdot\underline{w}}=\frac{P_d\cdot(\underline{w}^H\underline{S}_d)^2}{\sum_{i=1}^{M}P_i\cdot(\underline{w}^H\underline{S}_i)^2+\sigma^2.\underline{w}^H\underline{w}} \tag{37}$$

In the next subsection the useful concept of array pattern will be presented.