On Spatial Smoothing for Direction-of-Arrival Estimation of Coherent Signals

TIE-JUN SHAN, MATI WAX, AND THOMAS KAILATH, FELLOW, IEEE

***Abstract*—We present an analysis of a "spatial smoothing" preprocessing scheme, recently suggested by Evans *et al.*, to circumvent problems encountered in direction-of-arrival estimation of fully correlated signals. Simulation results that illustrate the performance of this scheme in conjunction with the eigenstructure technique are described.**

I. Introduction

In recent years, there has been a growing interest in high resolution eigenstructure techniques for direction-of-arrival estimation. These methods, developed by Pisarenko [12], Ligget [9], Owsley [11], Schmidt [14], Reddi [13], Bienvenu and Kopp [1], Johnson and Degraff [8], and Wax *et al.* [18], are known to yield high resolution and asymptotically unbiased estimates, even in the case that the sources are *partially* correlated. Theoretically, these methods encounter difficulties only when the signals are *perfectly* correlated. In practice, however, significant difficulties arise even when the signals are *highly correlated*, as happens, for example, in *multipath propagation* or in military scenarios involving *smart jammers*. The perfect correlation case, referred to as the *coherent* case, serves as a good model for the highly correlated case.

In spite of its practical importance, the coherent case did not receive considerable attention until recently. Although a rather general solution was proposed by Schmidt [14], the high computational complexity involved makes it unattractive. Widrow *et al.* [19] and Gabriel [6], [7] described two similar approaches, both aimed at "decorrelating" the coherent signals. The scheme in Widrow *et al.*, called "spatial dither," is based on mechanical "dithering" of the array, while Gabriel's scheme is based on "Doppler smoothing." Recently, Evans *et al.* [4], [5], in an extensive study of direction-of-arrival estimation techniques, presented an attractive solution to the problem for the case of a uniform linear array. Their solution is based on a preprocessing scheme referred to as *spatial smoothing* that essentially "decorrelates" the signals and thus eliminates the special difficulties encountered with coherent signals.

In this paper, we present a more complete analysis of the spatial smoothing preprocessing scheme. We also present simulation results that illustrate its performance in conjunction with the eigenstructure technique.

Manuscript received June 29, 1983; revised February 27, 1985. This work was supported in part by the Air Force Office of Scientific Research, Air Force Systems Command under Contract AF49-620-79-C-0058, and by the Joint Services Program at Stanford University under Contract DAAG29-81-K-0057.

The authors are with the Information Systems Laboratory, Stanford University, Stanford, CA 94305.

II. Problem Statement

Consider a uniform linear array composed of p *identical* sensors. Let q ($q < p$) narrow-band *planewaves*, centered at frequency ω_0, impinge on the array from directions $\{\theta_1, \cdots, \theta_q\}$. Using complex (analytic) signal representation, the received signal at the ith sensor can be expressed as

$$r_i(t) = \sum_{k=1}^{q} a_k s_k(t) e^{-j\omega_0 (i-1)\sin\theta_k d/c} + n_i(t) \tag{1}$$

where, in fairly common notation, $s_k(\cdot)$ is the signal of the kth wavefront, a_k is the complex response of the sensor to the kth wavefront, d is the spacing between the sensors, c is the propagation speed of the wavefronts, and $n_i(\cdot)$ is the additive noise at the ith sensor.

We assume that the signals and noises are stationary and ergodic complex-valued random processes with zero mean. In addition, the noises are assumed to be uncorrelated with the signals and uncorrelated between themselves, and to have identical variance σ^2.

Rewriting (1) in vector notation, assuming for simplicity that the sensors are omnidirectional, i.e., $a_k \equiv 1$, we obtain

$$\boldsymbol{r}(t) = \sum_{i=1}^{q} \boldsymbol{a}(\theta_i)\, s_i(t) + \boldsymbol{n}(t) \tag{2a}$$

where $\boldsymbol{r}(t)$ is the $p \times 1$ vector

$$\boldsymbol{r}(t) = [r_1(t), \cdots, r_p(t)]^T \tag{2b}$$

and $\boldsymbol{a}(\theta_i)$ is the "steering vector" of the array in the direction θ_i:

$$\boldsymbol{a}(\theta_i) = [1\ e^{-j\omega_0\tau_i}, \cdots, e^{-j\omega_0(p-1)\tau_i}]^T,$$

$$\tau_i = \frac{d}{c}\sin\theta_i. \tag{2c}$$

To further simplify the notation, we rewrite (2) as

$$\boldsymbol{r}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{3a}$$

where $\boldsymbol{s}(t)$ is the $q \times 1$ vector

$$\boldsymbol{s}(t) = [s_1(t), \cdots, s_q(t)]^T \tag{3b}$$

and $\boldsymbol{A}$ is the $p \times q$ matrix

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_q)]. \tag{3c}$$

It follows from our assumptions that

$$Er(t)\, r^{\dagger}(t) := R = ASA^{\dagger} + \sigma^2 I,\; S := Es(t)s^{\dagger} \quad (4)$$

where $\dagger$ denotes the conjugate transpose. Notice that the S is diagonal when the signals are uncorrelated, nondiagonal and nonsingular when the signals are partially correlated, and nondiagonal but *singular* when some signals are fully correlated (or *coherent*).

Assuming that the spacing between the sensors is less than *half* a wavelength of the impinging wavefronts ($d <$ 1/2, λ_0 where $\lambda_0 = 2\pi c/\omega_0$), it follows that the columns of the matrix A are all different, and hence, because of their Vandermonde structure, *linearly independent*. Thus, if S is *nonsingular*, then the rank of $ASA^{\dagger}$ is q. If

$$\{\lambda_1 \geq \lambda_2 \cdots \geq \lambda_p\} \text{ and } \{v_1, v_2, \cdots, v_p\}$$

are the eigenvalues and the corresponding eigenvectors of R, then the above rank properties imply that

1) the minimal eigenvalue of R is equal to σ^2 with *multiplicity* $p - q$:

$$\lambda_{q+1} = \lambda_{q+2} = \cdots = \lambda_p = \sigma^2$$

2) the eigenvectors corresponding to the minimal eigenvalue are *orthogonal* to the columns of the matrix A, namely, to the "direction vectors" of the signals

$$\{v_{q+1}, \cdots, v_p\} \perp \{a(\theta_1), \cdots, a(\theta_q)\}.$$

We shall refer to the subspace spanned by the eigenvectors corresponding to the smallest eigenvalue as the "noise" subspace, and to its orthogonal complement, spanned by the "direction vectors" of the signals, as the "signal" subspace.

The high resolution eigenstructure techniques are based on the exploitation of properties 1) and 2) above. Unfortunately, these properties hold *only* when the covariance matrix of the sources S is *nonsingular*. Different relations hold when S is singular. Assume, for simplicity, that the rank of S is $q - 1$. This implies that two signals, say the first two, are *coherent*, i.e., $s_2(t) = \alpha s_1(t)$, with α denoting a complex scalar describing the gain and phase relationship between the two coherent signals. In this case, we can rewrite (2) as

$$r(t) = as(t) + n(t) \quad (5a)$$

where $s(t)$ is the $(q - 1) \times 1$ vector

$$s(t) = [(1 + \alpha)\, s_1(t), s_3(t), \cdots, s_q(t)]^T \quad (5b)$$

and A is the $(q - 1) \times m$ matrix

$$A = [a(\theta_1) + \alpha a(\theta_2), a(\theta_3), \cdots, a(\theta_q)].$$

From (5), it follows that the covariance matrix of $r(t)$ also can be written as

$$R = ASA^{\dagger} + \sigma^2 I. \quad (6)$$

Now $S = E[s(t)\, s(t)^{\dagger}]$, the covariance matrix of the modified signals, is a $(q - 1) \times (q - 1)$ *nonsingular* matrix and A is of full column rank. Therefore, in complete analogy to properties 1) and 2) above, we have 1) the multiplicity of the smallest eigenvalue is $p - (q - 1)$; 2) the eigenvectors corresponding to the minimal eigenvalue are orthogonal to the columns of the matrix A. Because of their Vandermonde structure, note that the first column of A in (5c) is no longer a legitimate steering vector since no linear combination of two "direction vectors" can yield another steering vector.

The results of a straightforward application of the eigenstructure technique to R can now be easily understood. First, because the multiplicity of the smallest eigenvalue of R is now $q - 1$, the detection step will give $q - 1$ as the number of signals. Second, since only the "direction vectors" corresponding to $\{\theta_3, \cdots, \theta_q\}$ are included in the "signal" subspace, only these directions-of-arrival will be resolved in the estimation step.

In general, if m out of the q wavefronts are coherent, the application of the conventional eigenstructure technique will result in an *inconsistency*: while the number of signals detected will be $q - m + 1$, only $q - m$ directions-of-arrival, corresponding to the the *incoherent* wavefronts, will be resolved.

Thus, if only one group of coherent signals exists, the difference between the number of signals detected and the number of signals resolved will be indicative of the size of the coherent group. Realizing this, Schmidt [15] proposed the following procedure: if a coherent group of size m is detected, search for the linear combination of m "direction vectors" that is included in the "signal" subspace or, equivalently, that is orthogonal to the "noise" subspace. Unfortunately, because of the high dimensionality of this search involved, this solution is computationally unattractive; in the next section, we present a different solution.

III. The Spatial Smoothing Preprocessing Scheme

As we have pointed out in the previous section, the nonsingularity of the covariance matrix of the signals is the key to a successful application of the eigenstructure technique. In this section, we present a preprocessing scheme, introduced by Evans *et al.* [5], that guarantees this property even when the signals are coherent.

Let a uniform linear array with L identical sensors $\{1, \cdots, L\}$ be divided into overlapping subarrays of size p, with sensors $\{1, \cdots, p\}$ forming the first subarray, sensors $\{2, \cdots, p\}$ forming the second subarray, etc. (see Fig. 1). Let $r_k(\cdot)$ denote the vector of received signals at the kth subarray. Following the notation of (3), we can write

$$r_k(t) = AD^{(k-1)}\, s(t) + n_k(t) \quad (7a)$$

where $D^{(k)}$ denotes the kth power of the $q \times q$ diagonal matrix

$$D = \text{diag}\, \{e^{-j\omega_0 \tau_1}, \cdots, e^{-j\omega_0 \tau_q}\}. \quad (7b)$$

The covariance matrix of the kth subarray is therefore given by

$$R_k = AD^{(k-1)}\, SD^{\dagger(k-1)}\, A^{\dagger} + \sigma^2 I \quad (8)$$

where S is the covariance matrix of the sources.

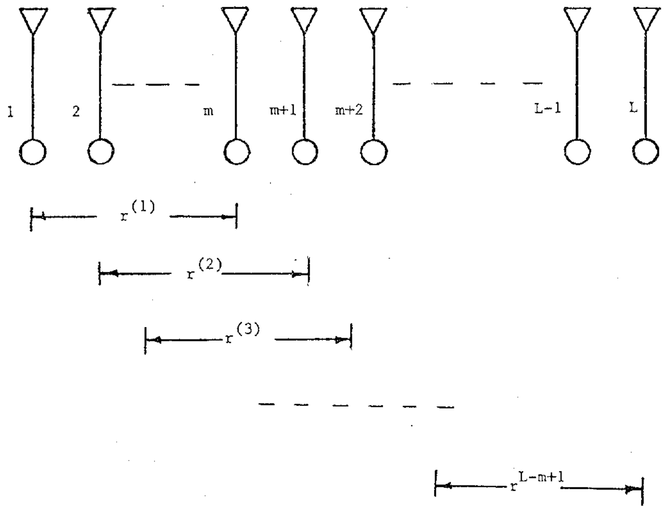


Fig. 1. Subarray spatial smoothing.

The *spatially smoothed covariance* matrix is defined as the sample means of the subarray covariances:

$$\bar{R} = \frac{1}{M} \sum_{k=1}^{M} R_k \tag{9}$$

where $M = L - p + 1$ is the number of subarrays.

Using (8), we can rewrite (9) as

$$\bar{R} = A \left(\frac{1}{M} \sum_{k=1}^{m} D^{(k-1)} S D^{\dagger(k-1)} \right) A^{\dagger} + \sigma^2 I \tag{10}$$

or more compactly as

$$\bar{R} = A \bar{S} A^{\dagger} + \sigma^2 I \tag{11a}$$

where S is the *modified* covariance matrix of the signals, given by

$$\bar{S} = \frac{1}{M} \sum_{k=1}^{M} D^{(k-1)} S D^{\dagger(k-1)}. \tag{11b}$$

We shall now prove that when $M \geq q$, the number of signal sources $\bar{S}$ will be *nonsingular* regardless of the coherence of the signals.

Theorem: If the number of subarrays is greater than or equal to the number of signals, i.e., if $M \geq q$, then the modified covariance matrix of the signals $\bar{S}$ is nonsingular.

Proof: First, note that we can rewrite $\bar{S}$ as

$$\bar{S} = [I D \cdots D^{(M-1)}] \begin{bmatrix} \frac{1}{M} S & & \\ & \ddots & \\ & & \frac{1}{M} S \end{bmatrix} \begin{bmatrix} I \\ D^{-1} \\ \vdots \\ D^{-(M-1)} \end{bmatrix} \tag{12}$$

which can be further simplified to

$$\bar{S} = G G^{\dagger} \tag{13a}$$

where G is the $q \times Mq$ block matrix

$$G = [C \; DC \cdots D^{M-1} C] \tag{13b}$$

with C denoting the Hermitian square root of $(1/M) S$:

$$C C^{\dagger} = \frac{1}{M} S. \tag{13c}$$

Clearly, the rank of $\bar{S}$ is equal to the rank of G. Thus, our task is to prove that G has rank q or, equivalently, using the rank operator ρ, to prove that $\rho\{G\} = q$. Recalling that the rank of a matrix is unchanged by a permutation of its columns, it can be easily verified that

$$\rho\{G\} = \rho \begin{bmatrix} c_{11} b_1 & c_{12} b_1 & \cdots & c_{1q} b_1 \\ \vdots & \vdots & \cdots & \vdots \\ c_{q1} b_q & c_{q2} b_q & \cdots & c_{qq} b_q \end{bmatrix} \tag{14a}$$

where c_{ij} is the ijth element of the matrix C and b_i $(i = 1, \cdots, q)$ is the $1 \times M$ row vector

$$b_i = [1 \; e^{-j\omega_i \tau_i}, \cdots, e^{-j_i (M-1)\tau_i}]$$
$$i = 1, \cdots, q. \tag{14b}$$

To show that the matrix G is of rank q, namely, is full row rank, it suffices to show that each row of the matrix C has at least one *nonzero* element and that the vectors $\{b_1, \cdots, b_q\}$ are *linearly independent*. The first fact follows by contradiction. Assume that a row of C, say the kth, is composed of *all zeros*. This implies, by (13c), that the kth signal has *zero* energy, in contradiction to the definition of S as the covariance matrix of the *nonvanishing* signals. The linear independence of the vectors $b_1, \cdots, b_q$ follows by observing that for $M \leq q$, these vectors can be embedded in a Vandermonde matrix, which is known to be nonsingular.

The above result is stated in Evans *et al.* [5, pp. 2–24]. Their proof, however, is incomplete; they show, correctly, that the matrix $\bar{S}(t) \triangleq 1/M \sum_{i=1}^{M} [D^{(i-1)} s(t)] \cdot [D^{(i-1)} s(t)]^{\dagger}$ is nonsingular. Notice that $E\bar{S} = 1/M \; E \cdot \sum_{i=1}^{M} [D^{(i-1)} s(t)] \, [D^{(i-1)} s(t)]^{\dagger} = \bar{S}$, that is, the expected value of $\bar{S}$ is equal to $\bar{S}$, the modified covariance matrix. Unfortunately, the nonsingularity of a random matrix *does not* imply the nonsingularity of its expected value. Thus, the nonsingularity of $\bar{S}$, the crucial element upon which the eigenstructure method hinges, does not follow from the nonsingularity of $\bar{S}$.

It can be shown that in the special case that the covariance matrix of the sources is block diagonal, i.e., when there are *several* groups of coherent signals that are uncorrelated with each other, the number of subarrays can be reduced to the size of the *largest* group of coherent signals.

Since the smoothed covariance matrix $\bar{R}$ has exactly the same form as the covariance matrix for the noncoherent case, one can successfully apply the eigenstructure methods to this smoothed covariance matrix *regardless* of the coherence of the signals.

However, this robustness comes at the expense of a *reduced* effective *aperture*. To see this more quantitatively, consider the number of sensors needed to cope with q coherent wavefronts. Recalling that the number of subar-

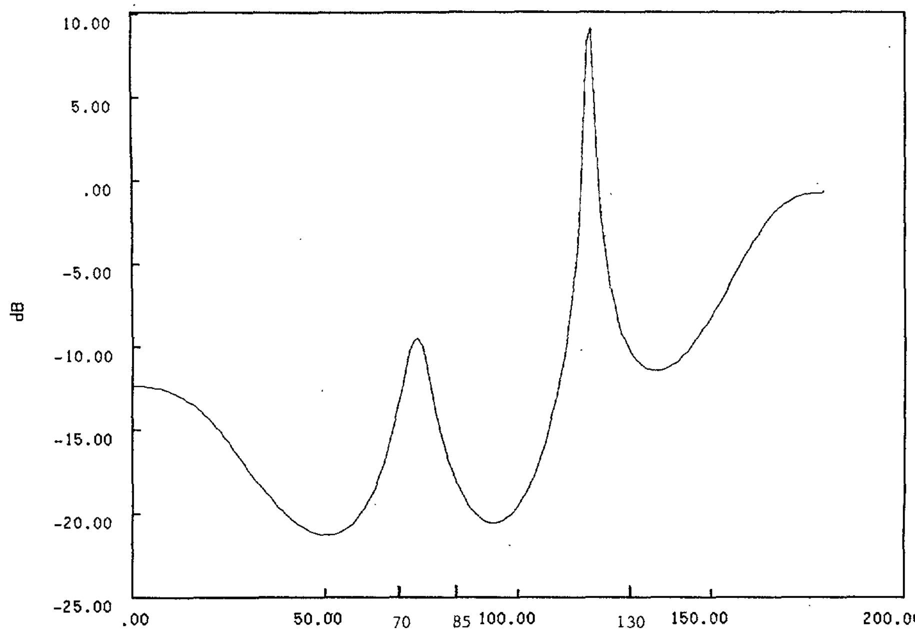


Fig. 2. Conventional beamforming method (with Hamming window) (six sensors; SNR = 3 dB; 500 "snapshots"; two coherent narrow-band sources from 85°, 130°, one incoherent source from 70°).

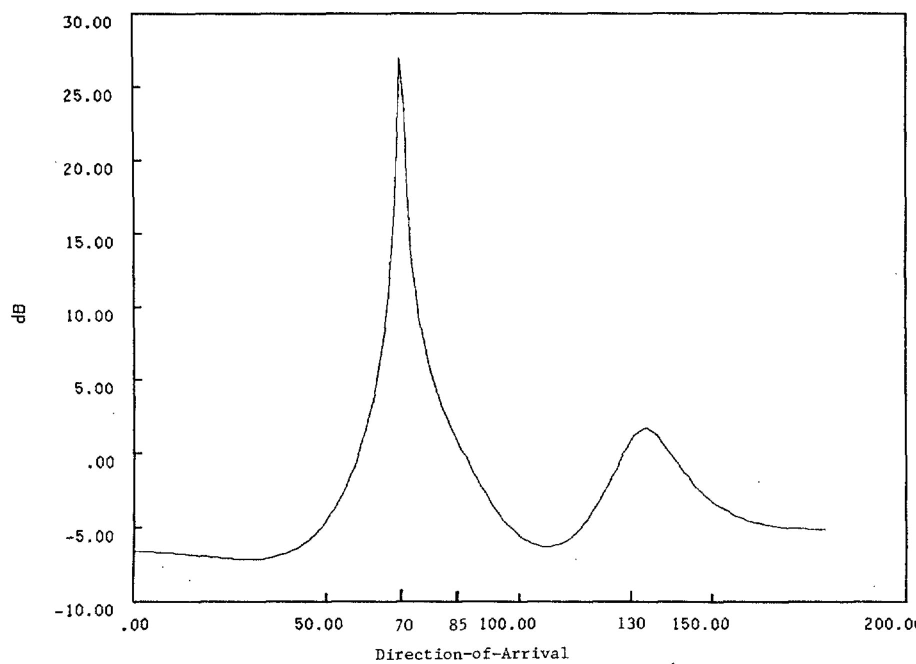


Fig. 3. Conventional MUSIC method (six sensors; SNR = 3 dB; 500 "snapshots"; two coherent narrow-band sources from 85°, 130°, one incoherent source from 70°)

rays, given by $M = p - m + 1$, must be greater than or equal to q, and that the size of each subarray m must be at least $q + 1$, it follows that the minimum number of sensors needed is $p = 2q$. Comparing this to $p = q + 1$ for the conventional case, it is clear that we trade off *half* the effective aperture.

IV. Simulation Results

In this section, we present simulation results that illustrate the performance of the spatial smoothing scheme in conjunction with the eigenstructure technique.

The example we considered had three ($q = 3$) planar wavefronts at directions-of-arrival 85°, 130°, and 70°. The first two signals were coherent, while the third signal was not correlated with the others. The array was uniform and linear, with six elements a third wavelength apart. The signal-to-noise ratio was 3 dB, and the number of samples ("snapshots") taken from the array was 500. Applying the conventional beamforming method and the eigenstructure method of Schmidt [14], we obtained the results shown in Figs. 2 and 3, respectively. Only one dominant peak corresponding to the direction-of-arrival of the third signal is

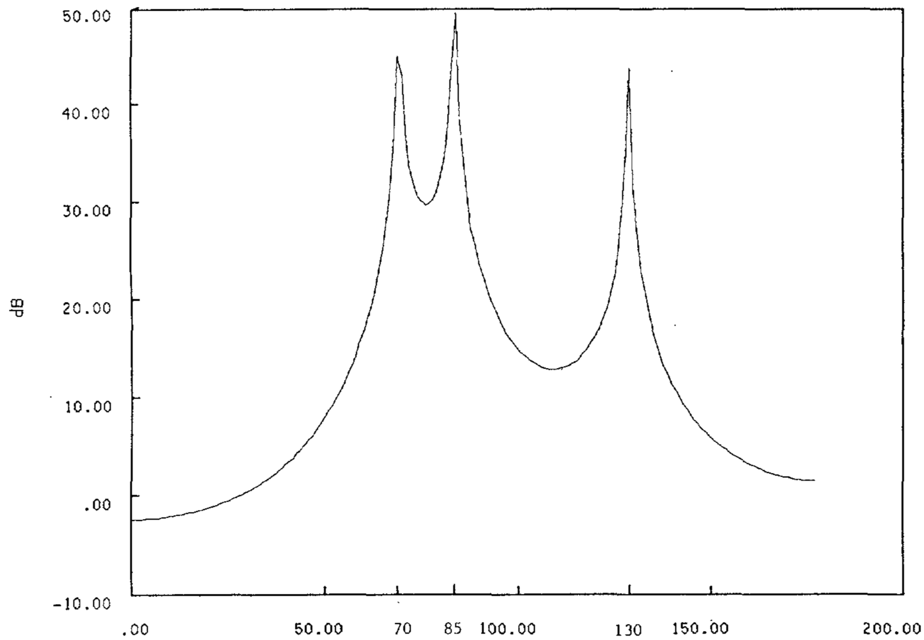


Fig. 4. New method (six sensors; subarray size = 4; SNR = 3 dB; 500 "snapshots"; two coherent narrow-band source from 85°, 130°; one incoherent source from 70°).

seen in both cases; the directions-of-arrival of the two coherent signals were not resolved. However, first applying the spatial smoothing preprocessing scheme with three (M = 3) subarrays of four (p = 4) sensors each, and then applying the eigenstructure method of Schmidt [14] to the spatially smoothed covariance matrix yielded the results shown in Fig. 4. In this case, the three peaks corresponding to the directions-of-arrival of all the three signals are clearly seen.

V. Concluding Remarks

A spatial smoothing scheme, introduced by Evans *et al.* [5] to circumvent problems encountered in the estimation of the directions-of-arrival of coherent signals, was more completely analyzed.

Our emphasis was on the use of the spatial smoothing scheme in conjunction with the eigenstructure technique. However, as pointed out by Evans *et al.*, this scheme can also be applied in conjunction with other processing techniques such as the minimum variance technique of Capon [3]. It is also interesting to note, as again pointed out by Evans *et al.*, that the linear prediction technique of Clayton and Nuttall [10], when used with a low-order predictor in the spatial domain, essentially performs the spatial smoothing implicitly. In fact, it is the improved performance observed for this method that apparently motivated Evans *et al.* to investigate the spatial smoothing scheme.

The extension of the spatial smoothing scheme to more difficult scenarios arising in array processing, e.g., to narrow-band signals with unknown center frequency and to wide-band signals, follows straightforwardly from Wax *et al.* [18]. A modification of the idea for adaptive beamforming in communication applications is described by Shan and Kalath [16].

Acknowledgment

The authors wish to thank the referees for their helpful comments.

References

[1] G. Bienvenu and L. Kopp, "Adaptivity to background noise spatial coherence for high resolution passive methods," in *Proc. IEEE ICASSP '80*, Denver, CO, pp. 307–310.

[2] J. P. Burg, "Maximum entropy spectral analysis," Ph.D. dissertation, Stanford Univ., Stanford, CA, 1975.

[3] J. Capon, "High resolution frequency-wavenumber spectral analysis," *Proc. IEEE*, vol. 57, pp. 1408–1418, Aug. 1969.

[4] J. E. Evans, J. R. Johnson, and D. F. Sun, "High resolution angular spectrum estimation techniques for terrain scattering analysis and angle of arrival estimation," in *Proc. 1st ASSP Workshop Spectral Estimation*, Hamilton, Ont., Canada, 1981, pp. 134–139.

[5] —, "Application of advanced signal processing techniques to angle of arrival estimation in ATC navigation and surveillance system," M.I.T. Lincoln Lab., Lexington, MA, Rep. 582, 1982.

[6] W. F. Gabriel, "Spectral analysis and adaptive array superesolution techniques," *Proc. IEEE*, vol. 68, pp. 654–666, 1980.

[7] —, "Adaptive superesolution of coherent RF spatial sources," in *Proc. 1st ASSP Workshop Spectral Estimation*, Hamilton, Ont., Canada, 1981, pp. 134–139.

[8] D. H. Johnson, and S. R. Degraff, "Improving the resolution of bearing in passive sonar arrays by eigenvalue analysis," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-30, pp. 638–647, 1982.

[9] W. S. Ligget, "Passive sonar: Fitting models to multiple time series," in *Signal Processing*, J. W. Griffith *et al.*, Eds. New York: Academic, 1973, pp. 327–345.

[10] A. H. Nuttall, "Spectral analysis of a univariate process with bad data points, via maximum entropy and linear prediction techniques," Naval Underwater Syst. Cen., New London, CT, Tech. Rep. 5303, 1976.

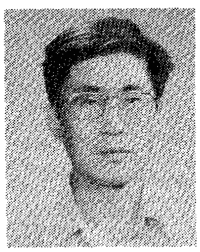
[11] N. L. Owsley, "Spectral signal set extraction," in *Aspects of Signal Processing, Part II*, G. Tacconi, Ed. Dordecht, The Netherlands: Reidel, 1977, pp. 469–475.

[12] V. F. Pisarenko, "The retrieval of harmonics from a covariance function," *Geophys. J. Roy. Astron. Soc.*, vol. 33, pp. 247–266, 1973.

[13] S. S. Reddi, "Multiple source location—A digital approach," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-15, pp. 95–105, 1979.

[14] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," in *Proc. RADC Spectral Est. Workshop*, 1979, pp. 243–258.

[15] ——, "A signal subspace approach to multiple source location and spectral estimation," Ph.D. dissertation, Stanford Univ., Stanford, CA, 1981.
[16] T. J. Shan, and T. Kailath, "Adaptive beamforming for coherent signals and interference," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, June 1985.
[17] T. J. Ulrych, and R. W. Clayton, "Times series and maximum entropy," *Phys. Earth Plan. Ins.*, vol. 12, pp. 188–200, 1976.
[18] M. Wax, T-J. Shan, and T. Kailath, "Spatio-temporal spectral analysis by eigenstructure methods," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. ASSP-32, pp. 817–827, 1984.
[19] B. Widrow, K. M. Duvall, R. P. Gooch, and W. C. Newman, "Signal cancellation phenomena in adaptive antennas: Causes and cures," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 469–478, 1982.



Tie-Jun Shan was born in Heilongjiang Province, China, on October 30, 1945. He graduated from the Harbin Institute of Technology, China, in 1968, and received the M.S.E.E. degree and the M.S. degree in statistics from Stanford University, Stanford, CA, in 1981 and 1982, respectively.

From 1969 to 1980 he conducted research in communication systems and central traffic control systems in China. Since 1984 he has been a Senior Research Engineer at TransImage Corporation, Santa Clara, CA. He is currently working towards the Ph.D degree in the Department of Electrical Engineering, Stanford University. His main research interests are statistical signal processing and its applications in communication and control, adaptive signal processing, sensor array processing, and pattern recognition.

Mati Wax was born in Brussels, Belgium, on March 24, 1947. He received the B.Sc. and the M.Sc. degrees from the Technion, Israel Institute of Technology, Haifa, in 1969 and 1975, respectively, and the Ph.D. degree from Stanford University, Stanford, CA, in 1985, all in electrical engineering.

From 1969 to 1973 he served in the Israel Defence Force, where he was involved in the development of communication systems. In 1974 he was with A.E.L Israel, where he developed microwaves components and subsystems. During 1975–1980 he was with RAFAEL, Israel, where he conducted research and development of communication systems, tracking system, and position location techniques. From 1981 to 1983 he was a Research Assistant in the Department of Electrical engineering, Stanford University, where he conducted research in the area of detection and estimation in sensor arrays. In 1984 he was a Visiting Scientist in IBM Research Laboratories, San Jose, where he was working on statistical modeling, pattern recognition, and image compression.

Thomas Kailath (S'57–M'62–F'70) was born in Poona, India, on June 7, 1935. He received the B.E. degree in telecommunications engineering from the University of Poona in 1956 and the S.M. and Sc.D. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1959 and 1961, respectively.

During 1961–1962 he worked at the Jet Propulsion Laboratories, Pasadena, CA, where he also taught part time at the California Institute of Technology. From 1963 to 1968 he was Associate Professor and from 1968 Professor of Electrical Engineering at Stanford University. He was Director of the Information Systems Laboratory from 1971 through 1980, and is presently Associate Chairman of the Department of Electrical Engineering. He has held shorter term appointments at several institutions around the world, including a Ford Fellowship in 1963 at U.C. Berkeley, a Guggenheim Fellowship in 1970 at the Indian Institute of Science, Bangalore, a Churchill Fellowship in 1977 at Cambridge University, England (where he is a Life Fellow of Churchill College), and a Michael Fellowship in 1984 at the Weizmann Institute of Science, Rehovot, Israel.

Dr. Kailath is on the Editorial Boards of several engineering and mathematics journals and is also the Editor of the Prentice-Hall Series on Information and System Sciences. He was on the IEEE Press Board for several years. From 1971 to 1978 he was a member of the Administrative Committees of the IEEE Professional Group on Information Theory and the IEEE Control Systems Society. During 1975 he served as President of the Information Theory Group. He is a member of the National Academy of Engineering and a Fellow of the Institute of Mathematical Statistics. He is also a member of the American Mathematical Society, the Society for Industrial and Applied Mathematics, the Society of Exploration Geophysicists, and several other scientific organizations. He is the author of *Linear Systems* (Prentice-Hall, 1980), *Lectures on Wiener and Kalman Filtering* (Springer-Verlag, 1981), Editor of *Benchmark Papers in Linear Least-Squares Estimation* (Academic, 1977), and of *Reviews of Modern Signal Processing* (Hemisphere-Springer-Verlag, 1985), and Coeditor of *VLSI and Modern Signal Processing* (Prentice-Hall, 1985).