STAR channel estimation in DS-CDMA communication systems

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Abstract: A novel blind DS-CDMA array channel estimator is presented which is capable of handling severe frequency-selective channels, where the multipath delay spread can be comparable with the data symbol period. By employing the concept of the spatiotemporal array (STAR) manifold, a subspace-type estimation algorithm is proposed, in conjunction with an one-dimensional temporal smoothing procedure, to jointly estimate the spatiotemporal multipath-channel parameters associated with the desired user. Close examination of the proposed STAR channel estimator reveals that it is 'near–far' resistant and the total number of signal paths can be more than the number of antenna elements. The effectiveness of the proposed approach is supported by computer simulation results.

List of symbols

a	=	scalar quantity
$\mathbb{A}$	=	matrix
$\boldsymbol{a}$	=	column vector
$\exp(\boldsymbol{a})$	=	element by element exponential
$\otimes$	=	Kronecker product
$\mathcal{L}(\mathbb{A})$	=	column space of matrix $\mathbb{A}$
$\boldsymbol{a}^b$	=	element by element power
$\boldsymbol{0}_N$	=	zero vector of N elements
$\text{diag}(\boldsymbol{a})$	=	diagonalisation of vector $\boldsymbol{a}$

1 Introduction

With DS-CDMA chosen as the air interface for the next generation of wireless communication networks, the intention is to support as many users as possible operating at very high data rates in the presence of severe channel distortion effects. In these systems, the multiple users transmit asynchronously, overlapping in both time and frequency resulting therefore, in multiple-access interference (MAI), which eventually limits the system performance [1–4]. The presence of multipath propagation effects not only increases the MAI by introducing extra interfering paths, but also distorts the desired signal component (frequency-selective fading). However, a well designed DS-CDMA system may exploit:

- the multipath channel by recombining the multipath rays in a positive way, thereby combating the multipath fading effects; and
- the highly structured signal format of the MAI to provide MAI cancellation,

thus significantly improving the system's overall performance. For this exploitation to be effective, channel parameters (such as multipath delays etc.) need to be estimated prior to the subsequent data symbol detection. Subspace-type techniques were derived for direction-finding array systems [5] and may be considered as a powerful type of parameter estimation methods. In [6, 7] these techniques were employed for single-antenna frequency-selective fading channel estimation, outperforming the conventional correlation channel estimation methods [8]. Signal-subspace techniques are superresolution-type techniques and subsequently have performance that is independent of the input signal-to-noise-plus-interference ratio and, therefore, provide 'near–far' resistance. Recently, these techniques have been employed in DS-CDMA systems using antenna arrays to handle multipath channels, e.g. [9, 10]. In [10], a reduced-dimension space–time RAKE receiver for DS-CDMA channels has been proposed, where a joint estimation procedure of the delays and directions-of-arrival (DOA) of the dominant paths is developed based on the estimated space–time channel response vector, of the desired user. Their proposed approach is a subspace-type approach employing the two-dimensional unitary ESPRIT algorithm as the estimator of the delays and DOAs, jointly. However, this approach restricts the multipath delay spread to be a fraction of the data symbol period, so that the RAKE fingers can be roughly located. It has been argued that joint spatiotemporal reception/processing is a breakthrough approach for future generations of wireless communications [8, 11, 12] and the estimation of the spatiotemporal channel parameters is central to the research work presented in this paper. However, in contrast to the previous approaches, the only assumption made in formulating the channel estimator proposed in this paper is that the whole set of space–time channel parameters is stationary during one coherence time of the channel.

2 Signal modelling

In an M-user asynchronous DS-CDMA system, the ith user's baseband transmitted signal may be written as

$$m_i(t) = \sum_{n=-\infty}^{\infty} a_i[n] c_{PN,i}(t - nT_{cs})$$

$$nT_{cs} \le t < (n+1)T_{cs} \tag{1}$$

where $\{a_i[n], \forall n \in \mathcal{N}\}$ is the ith user's data symbol sequence, T_{cs} is the data symbol period and $c_{PN,i}(t)$ is one period of the pseudorandom spreading waveform associated with the ith user, which is modelled as

$$c_{PN,i}(t) = \sum_{k=0}^{\mathcal{N}_c-1} \alpha_i[k] c_1(t - kT_c)$$
$$kT_c \le t < (k+1)T_c \quad (2)$$

In (2) $\{\alpha_i[k] \in \pm 1, 0 \le k \le \mathcal{N}_c - 1\}$ represents the ith user's PN sequence of period $\mathcal{N}_c = T_{cs}/T_c$ and $c_1(t)$ denotes the chip pulse shaping waveform of duration T_c.

Assume that an array of N antennas is used at the base station and the transmitted signal of the ith user arrives at the base station via K_i paths, $\forall i = 1, \ldots, M$. Consider that the jth path of the ith user arrives at the array from (azimuth, elevation) direction $(\theta_{ij}, \varphi_{ij})$ with channel propagation parameters β_{ij} and τ_{ij} representing the complex path gain and path delay, respectively. Let $S_{ij} \triangleq S(\theta_{ij}, \varphi_{ij})$ denote the array manifold vector (array response) associated with the jth path of the ith user, which has the form

$$S_{ij} = \exp(-j[r_1, r_2, \ldots, r_N]^T k_{ij}) \quad (3)$$

where r_k is a 3×1 real vector with elements the positions of kth antenna element (in half wavelengths) and k_{ij} is the wavenumber vector pointing towards the direction $(\theta_{ij}, \varphi_{ij})$ given by the expression

$$k_{ij} = \pi[\cos \theta_{ij} \cos \varphi_{ij}, \sin \theta_{ij} \cos \varphi_{ij}, \sin \varphi_{ij}]^T$$

Without loss of generality, $\varphi_{ij} = 0°$ is assumed in this paper.

Based on the above description, the received complex baseband signal vector at the antenna array can be represented as

$$x(t) = \sum_{i=1}^{M} \mathbb{S}_i \text{diag}(\beta_i) m_i(t) + n(t) \quad (4)$$

where $n(t)$ is a complex white Gaussian noise vector with covariance matrix $\sigma^2 \mathbb{I}_N$ and

$$\mathbb{S}_i = [S_{i1}, S_{i2}, \ldots, S_{iK_i}] \qquad \in \mathcal{C}^{N \times K_i}$$
$$\beta_i = [\beta_{i1}, \beta_{i2}, \ldots, \beta_{iK_i}]^T \qquad \in \mathcal{C}^{K_i \times 1}$$
$$m_i(t) = [m_i(t - \tau_{i1}), m_i(t - \tau_{i2}), \ldots, m_i(t - \tau_{iK_i})]^T \in R^{K_i \times 1}$$

The N-dimensional received array signal vector $x(t)$ is sampled at chip rate and then passed through a bank of N tapped-delay lines (TDL) each of length $2\mathcal{N}_c$. This is to ensure that one whole data symbol of the desired user and the corresponding multipath components are captured within this $2\mathcal{N}_c$ interval, when the multipath delay spread of the channel is less than one data symbol period. Upon concatenating the outputs of the TDLs, the $2N\mathcal{N}_c$-dimensional discretised signal vector is thus formed as

$$x[n] = [x_1[n]^T, \ldots, x_k[n]^T, \ldots, x_N[n]^T]^T$$
$$\in \mathcal{C}^{2\mathcal{N}_c N \times 1} \quad (5)$$

where $x_k[n]$ represents the contents of the TDL at the kth antenna associated with the nth data-symbol period. Note that, owing to the lack of synchronisation, the tapped-delay lines contain contributions from not only the current but also the previous and next symbols.

To model such contributions and thus to further model the received signal vector $x[n]$, the array manifold vector $S(\theta_{ij})$, given by (3), is expanded to the vector

$$S(\theta_{ij}) \otimes (\mathbb{J}^{l_{ij}} c_i) \triangleq h_{ij} \in \mathcal{C}^{2\mathcal{N}_c N \times 1} \quad (6)$$

which is defined as the spatiotemporal array (STAR) manifold vector for the jth path of the ith user. In (6), $l_{ij} = \lceil \tau_{ij}/T_c \rceil$ is the discretised path delay with $\lceil \bullet \rceil$ being the roundup-to-integer operator and the vector c_i is defined as

$$c_i = [\underbrace{[\alpha_i[0], \alpha_i[1], \cdots, \alpha_i[\mathcal{N}_c - 1]]}_{i\text{th users PN code}}, \quad \mathbf{0}_{\mathcal{N}_c}^T]^T$$
$$\in \mathcal{R}^{2\mathcal{N}_c \times 1} \quad (7)$$

representing one period of the PN sequence of the ith user padded with zeros to a length of $2\mathcal{N}_c$. This corresponds to a zero path delay situation. The matrix $\mathbb{J}$ is a $2\mathcal{N}_c \times 2\mathcal{N}_c$ matrix defined as follows:

$$\mathbb{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{2\mathcal{N}_c-1}^T & 0 \\ \mathbb{I}_{2\mathcal{N}_c-1} & \mathbf{0}_{2\mathcal{N}_c-1} \end{bmatrix} \quad (8)$$

having the property that every time the matrix $\mathbb{J}$ (or $\mathbb{J}^T$) operates on a column vector, it downshifts (or upshifts) the elements of the vector by one. For instance, $\mathbb{J}^l c_i$ is a version of c_i downshifted by l elements, while $(\mathbb{J}^T)^l c_i$ is a version of c_i upshifted by l elements.

Using the STAR manifold vector, $x[n]$ can be rewritten in a compact format as

$$x[n] = \sum_{i=1}^{M} [\mathbb{H}_{i,\text{prev}} \beta_i, \mathbb{H}_i \beta_i, \mathbb{H}_{i,\text{next}} \beta_i]$$
$$\times \begin{bmatrix} a_i[n-1] \\ a_i[n] \\ a_i[n+1] \end{bmatrix} + n[n] \quad (9)$$

where $n[n] \in \mathcal{C}^{2\mathcal{N}_c N \times 1}$ is the sampled noise vector and, for the ith user, the matrix $\mathbb{H}_i \in \mathcal{C}^{2\mathcal{N}_c N \times K_i}$ has as columns the spatiotemporal manifold vectors h_{ij}, $\forall j = 1, \ldots, K_i$, while $\mathbb{H}_{i,\text{prev}}$ and $\mathbb{H}_{i,\text{next}}$ are expressed as functions of $\mathbb{H}_i$ as follows:

$$\begin{cases} \mathbb{H}_{i,\text{prev}} = \left(\mathbb{I}_N \otimes (\mathbb{J}^T)^{\mathcal{N}_c}\right) \mathbb{H}_i \in \mathcal{C}^{2\mathcal{N}_c N \times K_i} \\ \mathbb{H}_{i,\text{next}} = \left(\mathbb{I}_N \otimes \mathbb{J}^{\mathcal{N}_c}\right) \mathbb{H}_i \in \mathcal{C}^{2\mathcal{N}_c N \times K_i} \end{cases}$$

and contain the contributions of the previous and next data symbols.

The array received signal-vector modelled by (9) can be decoupled into four terms representing the desired, ISI, MAI and noise terms, as follows (assuming that user 1 is the desired user):

$$x[n] = a_1[n]\mathbb{H}_1 \beta_1 + I_{ISI}[n] + I_{MAI}[n] + n[n] \quad (10)$$

where

$$\begin{cases} \mathbb{H}_1 \beta_1 a_1[n] \text{ is the desired signal component} \\ I_{ISI}[n] = [\mathbb{H}_{1,\text{prev}} \beta_1, \mathbb{H}_{1,\text{next}} \beta_1] \begin{bmatrix} a_1[n-1] \\ a_1[n+1] \end{bmatrix} \\ I_{MAI}[n] = \sum_{i=2}^{M} [\mathbb{H}_{i,\text{prev}} \beta_i, \mathbb{H}_i \beta_i, \mathbb{H}_{i,\text{next}} \beta_i] \begin{bmatrix} a_i[n-1] \\ a_i[n] \\ a_i[n+1] \end{bmatrix} \end{cases}$$

In Section 3, by exploiting the above modelling, a novel spatiotemporal array (STAR) CDMA channel-estimation approach is proposed.

3 Spatiotemporal-array (STAR) channel estimation

Define the $(2\mathcal{N}_c \times 2\mathcal{N}_c)$ Fourier transform matrix $\mathbb{F}$:

$$\mathbb{F} = [\phi^0, \phi^1, \phi^2, \ldots, \phi^{(2\mathcal{N}_c-1)}] \quad (11)$$

where

$$\boldsymbol{\phi} = \left[1, \phi^1, \phi^2, \cdots, \phi^{(2\mathcal{N}_c-1)}\right]^T$$

with

$$\phi = \exp(-j\tfrac{2\pi}{2\mathcal{N}_c}).$$

The overall transformation proposed for use in this paper is

$$\mathbb{Z}_1 = (\mathbb{I}_N \otimes \mathrm{diag}(\mathbb{F}\boldsymbol{c}_1))^{-1}(\mathbb{I}_N \otimes \mathbb{F}) \qquad (12)$$

or, equivalently, $\mathbb{Z}_1 = \mathbb{I}_N \otimes (\mathrm{diag}(\mathbb{F}\boldsymbol{c}_1)^{-1}\mathbb{F}) \qquad (13)$

which is related to the desired user (user 1). Note that $\boldsymbol{c}_1$ is the PN sequence of the desired user padded with zeros to a length of $2\mathcal{N}_c$. The proposed transformation $\mathbb{Z}_1$ will directly affect the matrix $\mathbb{H}_i = [\boldsymbol{h}_{i1}, \boldsymbol{h}_{i2}, \cdots, \boldsymbol{h}_{iK_i}]$. If this transformation is applied to the manifold vector $\boldsymbol{h}_{ij}$ then it can easily be proven that

$$\mathbb{Z}_1\boldsymbol{h}_{ij} = \boldsymbol{S}(\theta_{ij}) \otimes (\mathrm{diag}(\mathbb{F}\boldsymbol{c}_1)^{-1}\mathrm{diag}(\mathbb{F}\boldsymbol{c}_i)\boldsymbol{\phi}^{l_{ij}});$$
$$\forall i = 1, \ldots, M \qquad (14)$$

It is now obvious that, if and only if $i=1$ (i.e. only for the desired user), (14) can be simplified to $\boldsymbol{S}(\theta_{1j}) \otimes \boldsymbol{\phi}^{l_{1j}}$. We call the vector $\boldsymbol{S}(\theta_{1j}) \otimes \boldsymbol{\phi}^{l_{1j}}$ the 'transformed-STAR manifold vector' associated with the jth path of the desired user, i.e.

$$\mathbb{Z}_1\boldsymbol{h}_{ij} = \boldsymbol{S}(\theta_{1j}) \otimes \boldsymbol{\phi}^{l_{ij}}$$

while the transformation $\mathbb{Z}_1 = \mathbb{I}_N \otimes (\mathrm{diag}(\mathbb{F}\boldsymbol{c}_1)^{-1}\mathbb{F})$ is a 'STAR preprocessor' associated with the desired (first) user. Applying the pre-processor $\mathbb{Z}_1$ to the signal $\boldsymbol{x}[n]$ given by (10), we have

$$\begin{aligned}\boldsymbol{z}_1[n] &= \mathbb{Z}_1\boldsymbol{x}[n] \\ &= \bar{\mathbb{H}}_1\boldsymbol{\beta}_1\mathrm{a}_1[n] + \mathbb{Z}_1\boldsymbol{I}_{ISI}[n] \\ &\quad + \mathbb{Z}_1\boldsymbol{I}_{MAI}[n] + \mathbb{Z}_1\boldsymbol{n}[n] \end{aligned} \qquad (15)$$

where $\bar{\mathbb{H}}_1$ is a matrix with columns the transformed STAR manifold vectors of the desired user, i.e.

$$\bar{\mathbb{H}}_1 = \mathbb{Z}_1\mathbb{H}_1 = $$
$$[\boldsymbol{S}_{11} \otimes \boldsymbol{\phi}^{l_{11}}, \boldsymbol{S}_{12} \otimes \boldsymbol{\phi}^{l_{12}}, \cdots, \boldsymbol{S}_{1K_1} \otimes \boldsymbol{\phi}^{l_{1K_1}}] \qquad (16)$$

having a Vandermode structure. Note that the preprocessor $\mathbb{Z}_1$ is a function of the PN sequence of the desired user, which is known to the receiver, with $\mathbb{Z}_1\mathbb{Z}_1^H$ being a known diagonal matrix.

It is clear from (15), (16) and the previous discussion that, after the transformation, only the multipath rays of the desired user (columns of $\bar{\mathbb{H}}_1$) are described by $\boldsymbol{S}(\theta_{1j}) \otimes \boldsymbol{\phi}^{l_{1j}}$. The locus of all vectors

$$\boldsymbol{S}(\theta) \otimes \boldsymbol{\phi}^l \quad (\forall \theta, \forall \ell) \in \Omega \text{ where } \Omega = \text{parameter space}$$

is a two-dimensional continuum (i.e. a surface) lying in an $2N\mathcal{N}_c$-dimensional complex space $\mathcal{C}^{2N\mathcal{N}_c}$ (observation space of the signal $\boldsymbol{z}_1[n]$). This surface is the transformed 'STAR manifold surface of the desired user' and does not include any contribution from the ISI or MAI-transformed manifold vectors (which are described by different equations). However, from the first term of (15), i.e. $\bar{\mathbb{H}}_1\boldsymbol{\beta}_1\mathrm{a}_1[n]$, it is clear that the columns of $\bar{\mathbb{H}}_1$ are linearly combined by the fading vector $\boldsymbol{\beta}_1$ and thus the contribution of the desired symbol $\mathrm{a}_1[n]$ to the 'overall signal subspace' will be a subspace (desired-signal subspace) of only one dimension. To restore the dimensionality of this 'desired' subspace to K_1, the Vandermode structure of the submatrices of $\bar{\mathbb{H}}_1$ must be exploited. This is achieved by employing 'temporal smoothing'. This approach is similar to spatial smoothing proposed in [13], but use it to extract a set of Q subvectors of length d at each antenna with the extraction procedure illustrated in Fig. 1. Note that all the corresponding subvectors from the N antenna elements are concatenated to form Q space–time subvectors of length dN where $d < 2\mathcal{N}_c$.

It is straightforward to prove, based on the desired signal components in the $(q+1)$th and the qth subvectors of $\boldsymbol{z}_1[n]$, that the following important relationship is valid between the associated submatrices $\bar{\mathbb{H}}_1^{(q+1)}$ and $\bar{\mathbb{H}}_1^{(q)}$ of the matrix $\bar{\mathbb{H}}_1$:

$$\bar{\mathbb{H}}_1^{(q+1)} = \bar{\mathbb{H}}_1^{(q)}\mathbb{D} \quad \forall q \in [1, Q-1] \qquad (17)$$

where $\mathbb{D} = \mathrm{diag}([\phi^{l_{11}}, \phi^{l_{12}}, \cdots, \phi^{l_{1K_1}}]) \in \mathcal{C}^{K_1 \times K_1}$

In (17), $q = 1$ is used to represent the transformed STAR manifold matrix (given by (16)) including the 'temporal-smoothing' operation, i.e.

$$\bar{\mathbb{H}}_1^{(1)} = $$
$$\left[\boldsymbol{S}_{11} \otimes \boldsymbol{\phi}_d^{l_{11}}, \boldsymbol{S}_{12} \otimes \boldsymbol{\phi}_d^{l_{12}}, \cdots, \boldsymbol{S}_{1K_1} \otimes \boldsymbol{\phi}_d^{l_{1K_1}}\right]$$

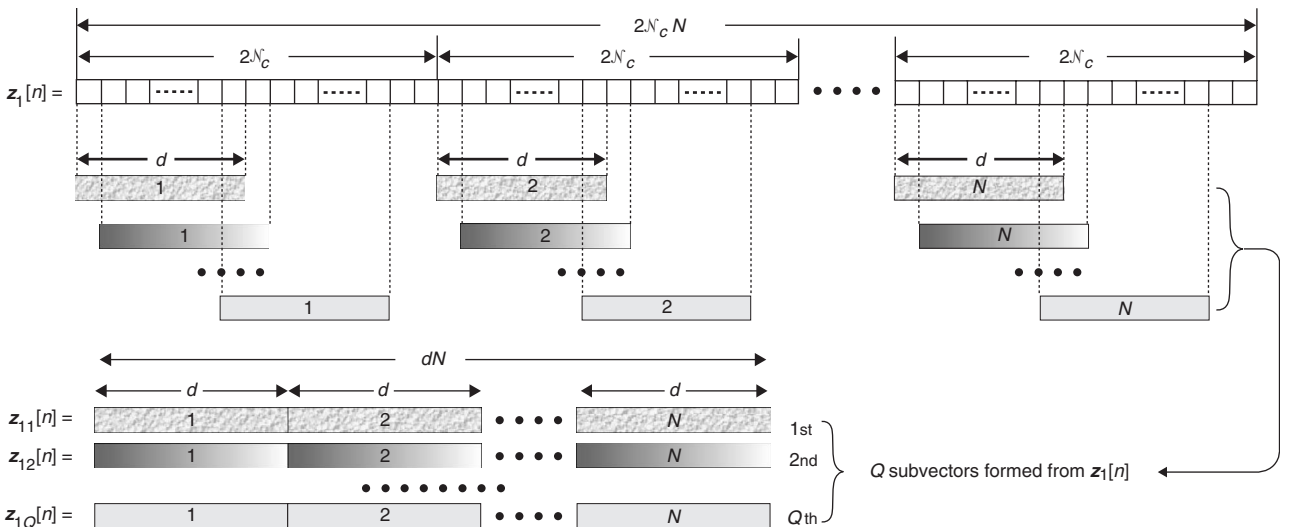where $\boldsymbol{\phi}_d = [1, \phi^1, \phi^2, \cdots, \phi^{d-1}]^T \in \mathcal{C}^{d \times 1}$

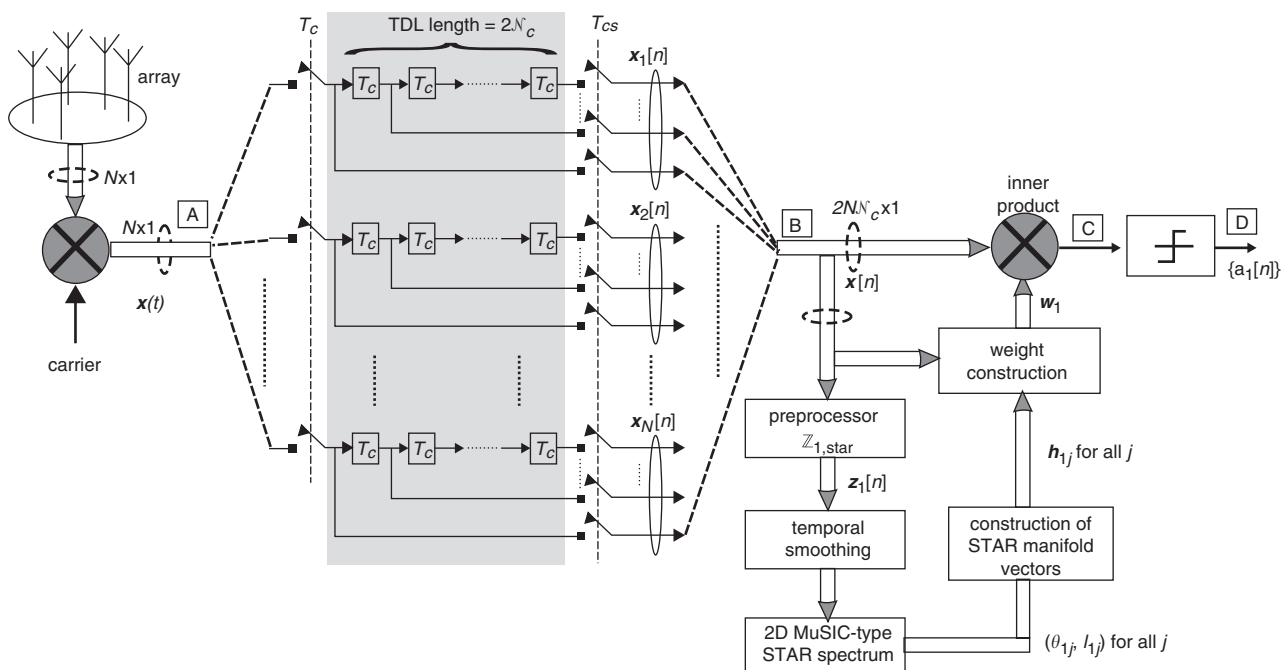**Fig. 1** *Forming the Q overlapping subvectors for temporal smoothing*

Based on (17), it can be proved in a similar fashion to the spatial smoothing algorithm [3], that the signal subspace spanned by the columns of $\bar{\mathbb{H}}_1^{(q)}$ is contained within the 'overall signal subspace' of the temporally smoothed covariance matrix $\mathbb{R}_{\text{smooth},dN}$, which is formed as follows:

$$\mathbb{R}_{\text{smooth},dN} = \frac{1}{Q}\sum_{q=1}^{Q}\mathbb{R}_{z_{1q}} \in \mathcal{C}^{dN \times dN} \qquad (18)$$

with $\mathbb{R}_{z_{1q}}$ representing the covariance matrix of the qth subvector $z_{1q}[n]$ of $z_1[n]$, formed over a frame of L_w data symbols (with this frame length smaller than the coherence time of the channel). The dimensionality of the desired signal component in the transformed domain is restored to K_1, provided that $Q \geq K_1$. However, although its dimensionality has been restored, the desired signal subspace cannot be distinguished (without using the 'manifold surface') from the 'overall signal-subspace', which includes also the contributions from the ISI and MAI terms. Indeed taking into account the 'temporal-smoothing' operation, this manifold surface is described by $\{S(\theta) \otimes \phi_d^l, (\forall\theta, \forall l) \in \Omega\}$ in which the desired signal components $S(\theta_{1j}) \otimes \phi_d^{l_{1j}}$, $\forall j$ (and not the MAI and ISI) belong to. However, the desired signal components $S(\theta_{1j}) \otimes \phi_d^{l_{1j}}$, $(\forall j)$ also belong to the 'overall signal subspace' of $\mathbb{R}_{\text{smooth},dN}$. Therefore, the intersection of the manifold surface with the 'overall signal-subspace' of $\mathbb{R}_{\text{smooth},dN}$ will provide only the desired signal components. Based on the above discussion and using the transformed STAR manifold vectors, the following two-dimensional MUSIC-type cost function can been constructed to search this manifold surface for every DOA θ and every delay l over their parameter space Ω:

$$\xi_{\text{star}}(\theta, l) = \frac{1}{(S(\theta) \otimes \phi_d^l)^H \mathbb{E}_n \mathbb{E}_n^H (S(\theta) \otimes \phi_d^l)} \qquad (19)$$

In (19), $\mathbb{E}_n$ is the matrix with columns the generalised noise eigenvectors of $(\mathbb{R}_{\text{smooth},dN}, \mathbb{M})$, with $\mathbb{M}$ denoting the diagonal matrix which is the smoothed version of the known matrix $\mathbb{Z}_1\mathbb{Z}_1^H$ (due to the noise term in (15)). The peaks of $\xi_{\text{star}}(\theta, l)$ provide the values of (θ_{1j}, l_{1j}) for $j = 1,\dots,$ K_1 associated with the spatiotemporal manifold vector of the desired user. Consequently, the matrix $\bar{\mathbb{H}}_1$ and, therefore, the channel matrix $\bar{\mathbb{H}}_1$ can be determined. Note that the proposed algorithm has the potential to resolve multipath rays of the desired user arriving from the same direction (codirectional paths) because the two paths will have two linearly independent spatiotemporal manifold vectors. If there are multipath rays that are arriving at the same time (travelling exactly the same distance) with different directions of arrival and the array is a uniform linear array, then by overlaying a spatial smoothing procedure on top of the proposed joint direction-of-arrival–time of arrival (DOA–TOA) estimator, the co-delay rays can also be resolved. The selection of parameters d and Q is related to the number of coherent paths to be identified and the resolution performance. Normally, the longer the subvector, the better the estimate, so d should be made as large as possible. However, d and Q must fulfil the condition that $d + Q - 1 \leq 2\mathcal{N}_c$.

4 Family of STAR receivers: a representative example

In Section 3 a novel antenna array CDMA channel estimation procedure has been proposed which, in this Section, is incorporated as an integral part of a receiver, the structure of which is shown in Fig. 2. This receiver provides (asymptotically) complete cancellation of both ISI and MAI effects and consequently, detects the data sequence $\{a_1[n]\}$, based on the estimated spatiotemporal manifold vectors of all paths associated with the desired user. Once the spatiotemporal parameters of the multipath rays have been identified, the path coefficient vector β_1 can be estimated using, for instance, the procedures described in [8] and [12].

The receiver, over an observation interval of L_w data symbols, initially estimates jointly the channel parameters $(\theta_{1j}, l_{1j}), \forall j$, and then forms a weight vector w_1 which is a

Fig. 2 *Framework of a STAR receiver*

function of the STAR manifold vectors of all paths associated with the desired user. This weight vector is capable of

- despreading and, at the same time, isolating the desired signal by removing the unwanted interference effects associated with the second and third terms of the right-hand side of (10), and
- coherently recombining the multipath components of the desired signal.

Then, the nth data symbol of the first (desired) user is easily detected assuming the following simple decision rule/device:

$$\hat{\mathrm{a}}_i[n] = \mathrm{sign}(\mathrm{Re}\{\boldsymbol{w}_1^H \boldsymbol{x}[n]\}) \qquad (20)$$

A representative example of such a weight vector is given as follows:

$$\boldsymbol{w}_1 = \kappa_1 \mathbb{P}_n \mathbb{H}_1 (\mathbb{H}_1^H \mathbb{P}_n \mathbb{H}_1)^{-1} \boldsymbol{\beta}_1 \qquad (21)$$

where κ_1 is a normalising factor such that $||\boldsymbol{w}_1|| = 1$.

In (21) $\mathbb{P}_n$ represents the projection-operator matrix onto the subspace spanned by the noise eigenvectors of the noise-plus-MAI/ISI matrix $\mathbb{R}_{n+\mathrm{MAI/ISI}}$ associated with the last three terms of the right-hand side of (10) and defined as follows:

$$\mathbb{R}_{n+\mathrm{MAI/ISI}} = \mathbb{R}_{xx} - \mathbb{H}_1 \boldsymbol{\beta}_1 \boldsymbol{\beta}_1^H \mathbb{H}_1^H \qquad (22)$$

$$\text{where } \mathbb{R}_{xx} = \mathrm{E}\{\boldsymbol{x}[n]\boldsymbol{x}^H[n]\}$$

Then, it can easily be proved that this weight vector, associated with the desired signal, is orthogonal to the interference subspace containing the contributions of the vectors $\boldsymbol{I}_{\mathrm{ISI}}[n]$ and $\boldsymbol{I}_{\mathrm{MAI}}[n]$. Using the weight vector of (21), the decision variable (at point C in Fig. 4) for the nth data symbol can be estimated as

$$\begin{aligned} d_1[n] &= \boldsymbol{w}_1^H \boldsymbol{x}[n] \\ &= \kappa_1 ||\boldsymbol{\beta}_1||^2 \mathrm{a}_1[n] + n_1[n] \end{aligned} \qquad (23)$$

Note that the power of the noise term $n_1[n] = \boldsymbol{w}_1^H \boldsymbol{n}[n]$ is equal to σ^2, while the amount of interference passed to the receiver output is determined by the angle between the subspace spanned by the actual MAI/ISI and the estimated noise subspace of $\hat{\mathbb{R}}_{n+\mathrm{MAI/ISI}}$.

5 Simulations

To evaluate the performance of the proposed approach consider a linear array of five antennas operating in the presence of multipath effects from a number of CDMA users with each user assigned a unique Gold sequence of length 31. Initially, it is assumed that the array operates in the presence of three users (including the desired user) plus noise. Table 1 provides the unknown channel parameters. It is clear from Table 1 that the desired user has 10 multipath rays, with the fourth path having one co-delay path ($10T_c$) and one co-directional path ($20°$). To handle the co-delay rays, the array is partitioned into two overlapping subarrays (spatial smoothing [13]), with each subarray having four antennas while 10 subvectors of length 51 (i.e. $Q = 10$, $d = 51$) are used at each antenna for temporal smoothing. The input signal-to-noise ratio associated with the desired user is assumed equal to 20 dB. The space–time parameters of the two additional users are also given in Table 1.

From Table 1 it can also be seen that

$$20 \log_{10} \left(\frac{||\boldsymbol{\beta}_1||}{||\boldsymbol{\beta}_i||} \right) = -20\mathrm{dB}, \text{ for } i = 2, 3$$

which implies that the interfering signals from the second and third users are much stronger than the received desired signal (near–far problem).

Figure 3 illustrates the STAR-channel-parameter estimation with the array operating in the presence of the previously described environment over an observation interval of 200 data symbols. It is clear from Fig. 3 that the two-dimensional MUSIC-type spectrum provides the correct directions and time-delays of the 10 multipath rays even in the case of co-delayed and co-directional paths.

Table 2 gives the estimated output SNIR of the proposed receiver (point C in Fig. 2) as well as of the decorrelating multiuser and RAKE receivers all operating under the conditions described above. It is clear from Table 2 that the proposed receiver has performance about 3 dB below that of the decorrelating receiver and outperforms the two-dimensional space–time RAKE receiver. This conclusion is further reinforced by adding more users to the array environment, with each additional user having three paths randomly generated and power 10 dB higher than the power of the desired user. (The TOA and DOA parameters of the MAIs are uniformly distributed over $0T_c \sim 20T_c$ and $0° \sim 120°$, respectively.)

Table 1: Channel parameters

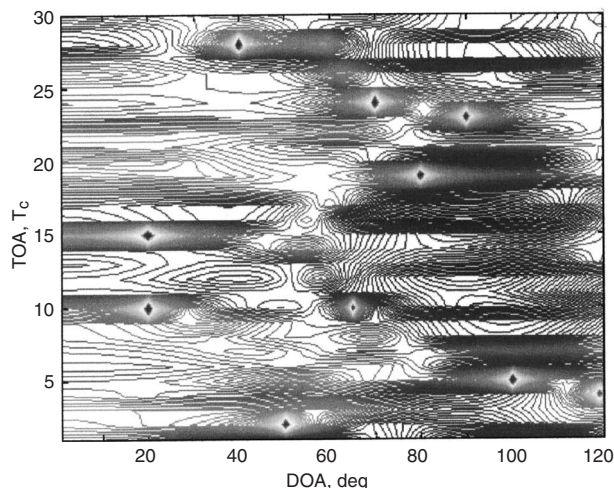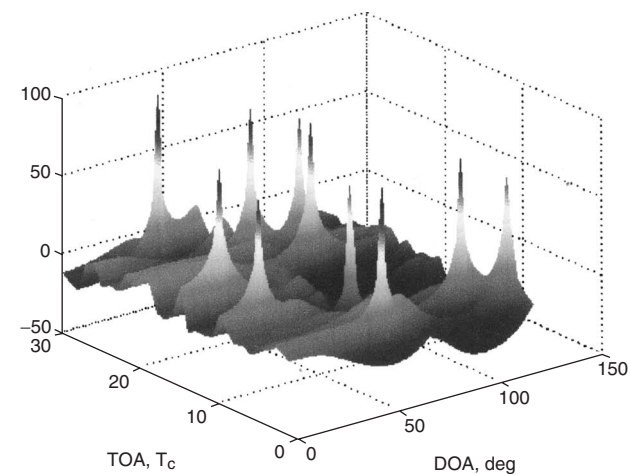|	Desired (first) user			Second user			Third user		
	TOA (T_c)	DOA	Path coefficients	TOA (T_c)	DOA	Path coefficients	TOA (T_c)	DOA	Path coefficients
Path	l_{1j}	θ_{1j}	$\beta_{1j}(||\boldsymbol{\beta}_1|| = 1)$	l_{2j}	θ_{2j}	$\beta_{2j}(||\boldsymbol{\beta}_2|| = 10)$	l_{3j}	θ_{3j}	$\beta_{3j}(||\boldsymbol{\beta}_3|| = 10)$
$j = 1$	2	50°	−0.081+j0.197	5	100°	−0.969+j3.836	3	50°	−1.362+j5.21
$j = 2$	4	119°	−0.122+j0.01	6	60°	−1.187−j2.945	4	10°	−1.067+j1.267
$j = 3$	5	100°	−0.009−j0.184	12	27°	3.799−j0.501	7	129°	−5.234+j2.883
$j = 4$	10	20°	−0.043−j0.022	15	85°	0.189−j0.467	8	15°	−0.845+j3.396
$j = 5$	10	65°	−0.234−j0.489	18	45°	−2.536+j0.721	11	25°	0.428+j3.396
$j = 6$	15	20°	0.315+j0.264	24	30°	−2.638−j1.343	12	70°	1.136−j3.580
$j = 7$	19	80°	0.107−j0.239	27	118°	4.072−j2.293	–	–	–
$j = 8$	23	90°	0.312−j0.168	29	105°	4.545−j1.048	–	–	–
$j = 9$	24	70°	−0.121−j0.327	–	–	–	–	–	–
$j = 10$	28	40°	−0.273−j0.222	–	–	–	–	–	–

Fig. 3 *Two-dimensional MUSIC-type STAR spectrum (and its contour diagram) associated with the desired user (see Table 1) for an array of five antennas operating in the presence of three CDMA users*

Table 2: SNIRout performance comparison

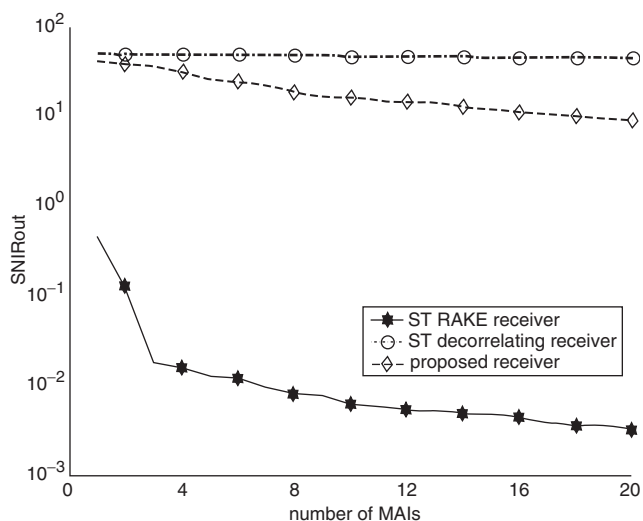|	RAKE receiver	Decorrelating receiver	Proposed receiver
SNIRout, dB	4.534	19.265	16.188

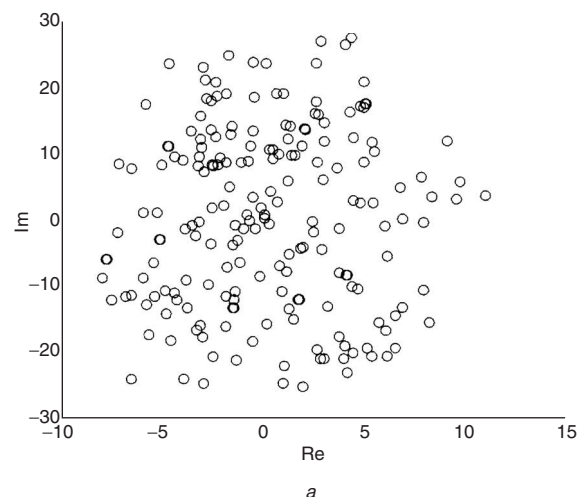**Fig. 4** *SNIRout-performance comparison as the number of users increases*
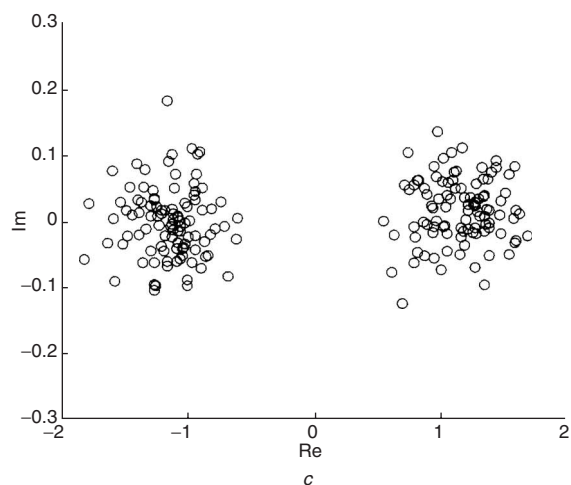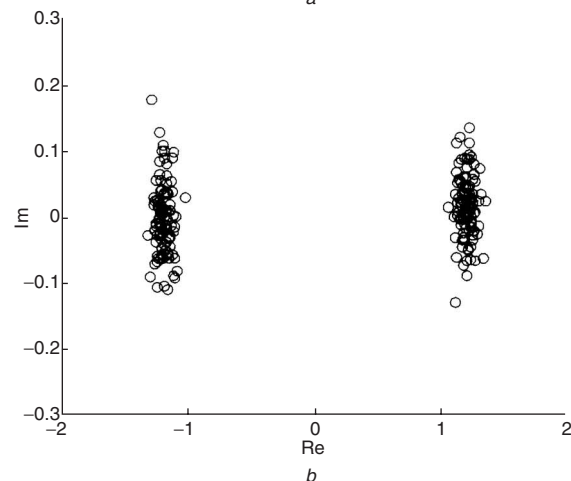
Fig. 5 *Decision variables for 200 data symbols for a single run of the simulation*
a ST RAKE receiver
b ST decorrelating receiver
c Proposed receiver

The results, over 100 runs, are shown in Fig. 4 while the decison variables (point C of Fig. 2) are also plotted for one of these runs with $M = 10$ (Fig. 5). In these Figures, the results of the decorrelating and two-dimensional space–time RAKE receivers are also shown for comparison. Note that the decorrelating receiver is a multiuser receiver requiring knowledge of the channel parameters of all users in advance. Similar conclusions can be drawn from Fig. 6, which shows the performance of the different receivers as a
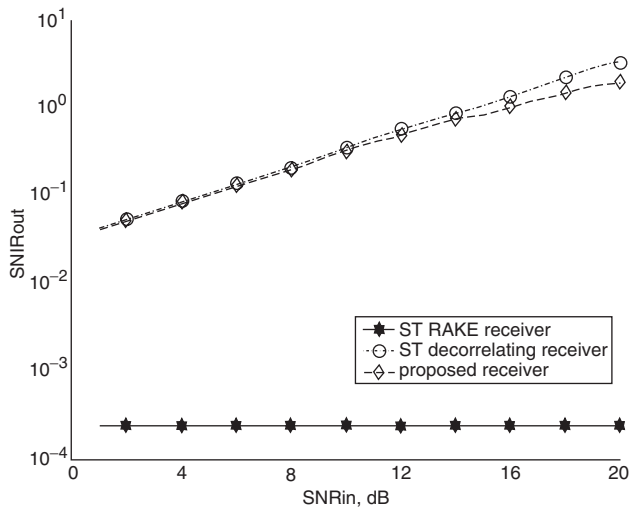
Fig. 6 *SNIRout against SNRin (10 MAIs with three paths and SIR = −10 dB)*
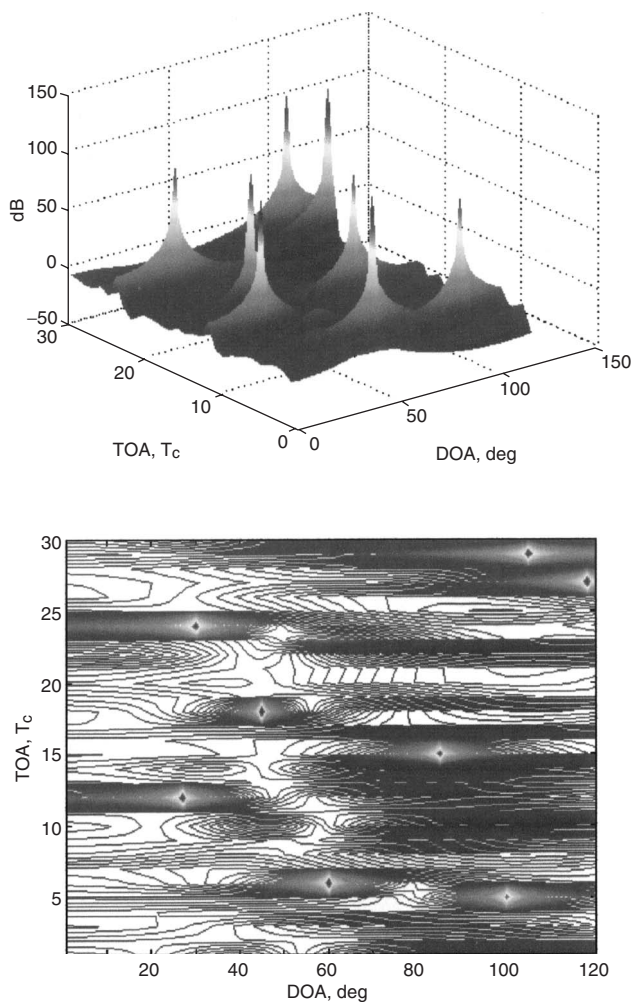

Fig. 7 *Two-dimensional MUSIC STAR spectrum (and contour diagram) associated with the second user (Table 1) as the desired user, for an array of two antennas operating in the presence of three CDMA users*

function of the input signal-to-noise-ratio for fixed number of users ($M = 10$). Overall, it has been observed through simulations that the proposed receiver achieves, asymptotically (infinite observation interval) and blindly, the same performance as the space–time decorrelating receiver.

Finally, it is clear from the previous Sections and the above simulation results that the number of identifiable paths is not limited by the number of antennas or MAI interference. Thus, it is possible to use the proposed STAR approach ever in a mobile unit, where the physical size limits the number of antennas to be used to no more than two or three. This characteristic (demonstrated in Fig. 7 for a two-antenna array) makes the proposed structure attractive not only for the uplink, but also for the downlink of a W-CDMA system where the processing gain is sufficiently high.

6 Conclusions

This paper has presented a joint space–time multipath channel estimator for frequency-selective DS-CDMA channels. The proposed estimation algorithm is a subspace-type technique and, owing to the degrees of freedom provided by this space–time structure, the number of identifiable paths is no longer limited by the number of antennas, and the multipath rays that overlay in time or in space can also be identified.

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