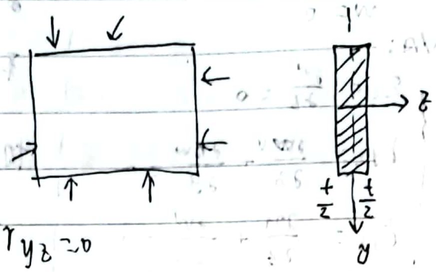
平面应力

① 

② $\sigma_z = \tau_{zx} = \tau_{zy} = 0$, $\gamma_{xz} = \gamma_{yz} = 0$

σ_x, σ_y, τ_{xy}, ε_x, ε_y, ε_z, τ_{xy} $\left(\frac{1}{2}\gamma_{ij} = \varepsilon_{ij}\right)$

物理方程
$$\begin{cases} \varepsilon_x = \cancel{-\frac{\mu}{E}(\sigma_y} \ \frac{1}{E}(\sigma_x - \mu\sigma_y) \\ \varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x) \\ \varepsilon_z = -\frac{\mu}{E}(\sigma_x + \sigma_y) \\ \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{yz} = \gamma_{zx} = 0 \end{cases}$$

几何方程: $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad \varepsilon_z = \frac{\partial w}{\partial z} \\ \cancel{\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \cancel{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}} \end{cases}$$

$\downarrow$

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \\ \varepsilon_y = \frac{\partial v}{\partial y} \\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{cases}$$

平衡方程: $\sigma_{ij,j} + F_i = 0$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0$$

$$\begin{cases} \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0 \\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + F_z = 0 \end{cases}$$

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + F_x = 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + F_y = 0 \end{cases}$$

协调方程

$$\frac{\partial^2 \varepsilon_x}{\partial x^2} + \frac{\partial^2 \varepsilon_y}{\partial y^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\left(\frac{\partial^2 \varepsilon_z}{\partial x^2} = 0 \quad \frac{\partial^2 \varepsilon_z}{\partial y^2} = 0 \quad \frac{\partial^2 \varepsilon_z}{\partial x \partial y} = 0\right)$$

平面应变：

$$\begin{cases} u = u(x,y) \\ v = v(x,y) \\ w = 0 \end{cases}$$

则有：

$$\begin{cases} \varepsilon_z = \frac{\partial w}{\partial z} = 0 \\ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \\ \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \end{cases}$$

几何：$\varepsilon_x = \frac{\partial u}{\partial x}$　$\varepsilon_y = \frac{\partial v}{\partial y}$

物理：　$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

$$\begin{cases} \sigma_z = \mu(\sigma_x + \sigma_y) \\ \varepsilon_x = \frac{1}{E}(\sigma_x - \mu(\sigma_y + \sigma_z)) \\ \varepsilon_y = \frac{1}{E}(\sigma_y - \mu(\sigma_x + \sigma_z)) \end{cases}$$

$\tau_{yz} = \tau_{xz} = 0$

$\varepsilon_y = \frac{1}{E_1}(\sigma_y - v_1 \sigma_x)$

$\sigma_x, \sigma_y, \sigma_z = v(\sigma_x + \sigma_y)$, $\underset{\tau_{xy}}{\wedge}$　ε_x，ε_y；γ_{xy}

$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0, \tau_{xz} = \tau_{yz} = 0$

平衡方程和平面应力的一样

协调方程：

$\nabla^2(\sigma_x + \sigma_y) = 0$

$\cancel{\tau_{xy} = \frac{1}{G}\gamma_{xy}}$

$\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+v_1)}{E_1}\tau_{xy}$

$$\begin{cases} \varepsilon_x = \frac{1}{E}(\sigma_x - \mu(\sigma_y + \mu(\sigma_x + \sigma_y))) = \frac{1}{E}(\sigma_x - \mu\sigma_y - \mu^2\sigma_x - \mu^2\sigma_y) \\ = \frac{1}{E}((1-\mu^2)\sigma_x - \mu^2\sigma_y - \mu\sigma_y) \end{cases}$$

$\downarrow E_1, v_1$　$E_1 = \frac{E}{1-\mu^2}$　$\mu_1 = \frac{\mu}{1-\mu}$

$\cancel{\varepsilon_x = \frac{1}{E_1}\sigma_x}$

$\frac{\mu_1}{E_1} = \frac{(1-\mu^2)\frac{\mu}{1-\mu}}{E}$

$\mu^2 + \mu = \frac{\mu_1^2}{(1+\mu_1)^2}$　$\frac{\mu_1 + \mu_1^2}{(1+\mu_1)^2}$

$\frac{\mu(1+\mu)}{E}$

$= \frac{2\mu_1 + \mu_1}{(1+\mu_1)}$

$\mu_1 - \mu\mu_1 = \mu$

$$\begin{cases} \mu = \frac{\mu_1}{1+\mu_1} \\ E = E_1(1-\mu^2) = E_1\left(1 - \left(\frac{\mu_1}{1+\mu_1}\right)^2\right) \end{cases}$$

$\varepsilon_x = \frac{\sigma_x}{E_1} - \frac{\mu + \mu^2}{E}\sigma_y = \frac{\sigma_x}{E_1} - \frac{\frac{\mu_1}{1+\mu_1} + \left(\frac{\mu_1}{1+\mu_1}\right)^2}{E_1\left(1 - \left(\frac{\mu_1}{1+\mu_1}\right)^2\right)}\sigma_y = \frac{1}{E_1}(\sigma_x - \mu$

在B站

~~致各位朋友们~~ ~~亲爱的朋友们~~ ~~你们~~ 优秀的工程师，科研学者或正成为以上人士的学生，技术爱好者，极客，你们 Hello world！~~……~~ 我是一个~~……研究者、爱好者~~ 从今天开始，我会给大家分享包括~~……~~ CAE技术、深度学习技术、力学和人工智能基础理论在内的 ~~……~~ 学习的乐趣，

研究

~~……~~ 力学和人工智能的~~研究~~研究者

人生态度等等，

板壳力学.

一、

① 直法线假设 —— 中面的法线
导出 $\gamma_{zx}=0$ $\gamma_{yz}=0$

②

①

二、

τ_{zx} τ_{yz} σ_z
σ_3 —— 影响不计

三.(~~板挠度~~)

$(u)_{z=0}=(v)_{z=0}=0$

薄板

+边界条件

几何方程：τ_{zx} τ_{zy} 的物理方程被放弃，用平衡方程推出 τ_{zx}, τ_{zy}
及 σ_z　　　　　　　　　　　　σ_z

薄壳（读

和薄板一样，τ_{z2} τ_{z1} 的物理方程被放弃。~~用平衡~~ 直接~~使用~~类比薄板
及 σ_3

引入 ~~τ_{zx} τ_{zy}~~ τ_{32} τ_{31} 公式，然后忽略了 σ_3

主要

很奇妙的一件事，薄板薄壳承受横向载荷，但 σ_z σ_3 却都是次要的因素

薄板、薄壳都服从了个物理方程.