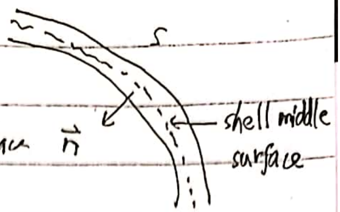
3.6.2

$x' = x + \lambda t \eta n$

$x(\theta^1, \theta^2)$ ~~是在~~ 在 reference surface $\vec{n}$



$n(\theta^1, \theta^2)$

λt strech of the shell

η

(θ^1, θ^2)

$\lambda t = \lambda t(\theta^1, \theta^2)$ only and $\frac{\partial \lambda t}{\partial \theta^1}$ $\frac{\partial \lambda t}{\partial \theta^2}$ 是小量.

$X' = X + \lambda t \eta N$

$$\frac{\partial x'}{\partial \theta^\alpha} \cdot \frac{\partial x'}{\partial \theta^\beta} = \left(\frac{\partial x'}{\partial \theta^\alpha} + \lambda t \eta \frac{\partial n}{\partial \theta^\alpha}\right)\left(\frac{\partial x'}{\partial \theta^\beta} + \lambda t \eta \frac{\partial n}{\partial \theta^\beta}\right)$$

$$= \frac{\partial x'}{\partial \theta^\alpha}\frac{\partial x'}{\partial \theta^\beta} + \lambda t \eta \frac{\partial n}{\partial \theta^\beta}\frac{\partial x'}{\partial \theta^\alpha} + \lambda t \eta \frac{\partial n}{\partial \theta^\alpha}\frac{\partial x'}{\partial \theta^\beta}$$

$$+ \lambda^2 t^2 \eta^2 \frac{\partial n}{\partial \theta^\alpha}\frac{\partial \eta}{\partial \theta^\beta}$$

$$\approx \frac{\partial x'}{\partial \theta^\alpha}\frac{\partial x'}{\partial \theta^\beta} + \lambda t \eta \left(\frac{\partial \eta}{\partial \theta^\beta}\frac{\partial x'}{\partial \theta^\alpha} + \frac{\partial n}{\partial \theta^\alpha}\frac{\partial x'}{\partial \theta^\beta}\right)$$

$$= g_{\alpha\beta} + \lambda t \eta\, b_{\alpha\beta}$$

$$g_{\alpha\beta} = \frac{\partial x'}{\partial \theta^\alpha}\frac{\partial x'}{\partial \theta^\beta}$$

~~$b_{\alpha\beta} =$~~

$$b_{\alpha\beta} = \frac{\partial \eta}{\partial \theta^\beta}\frac{\partial x'}{\partial \theta^\alpha} + \frac{\partial \eta}{\partial \theta^\alpha}\frac{\partial x'}{\partial \theta^\beta}$$

is an approximation to the curvature tensor (second fundamental ~~term~~ form) of the reference surface

$b_{\alpha\beta}$ 在 $n \cdot \frac{\partial x}{\partial \theta^\alpha} = 0$ 时将~~成大~~精确成为 curvature tensor

Finite-Strain Shell Element Formulation

3.6.5 Geo Description

$$x(s_i) = \bar{x}(s_\alpha) + \bar{f}_{33}(s_\alpha) t_3(s_\alpha) s_3$$

subscript i and other Roman subscripts 1-3

α, β : 1-2

f : 变形梯度张量

t_3 : is normal to the surface of the shell.

$$\begin{cases} \dfrac{\partial x}{\partial s_\beta} = \dfrac{\partial \bar{x}}{\partial s_\beta} + \bar{f}_{33} \dfrac{\partial t_3}{\partial s_\beta} s_3 \\ \dfrac{\partial x}{\partial s_3} = \bar{f}_{33} t_3 \end{cases}$$

$$\frac{\partial x}{\partial s_3} = \frac{\partial \bar{x}}{\partial s_3} + \bar{f}_{33} \frac{\partial t_3}{\partial s_3} s_3 + \bar{f}_{33} t_3 = 0 + 0 + \bar{f}_{33} t_3 = \bar{f}_{33} t_3$$

$t_i t_j = \delta_{ij}$ $\quad t_i t_i = I$: local, orthonormal shell ~~elements~~ dirs t_i directions

$$f_{\alpha\beta} = t_\alpha \frac{\partial x}{\partial s_\beta} = \bar{f}_{\alpha\beta} + B_{\alpha\beta} \bar{f}_{33} s_3$$

$$\bar{f}_{\alpha\beta} = t_\alpha \frac{\partial x}{\partial s_\beta}\Big|_{s_3=0}$$

$$B_{\alpha\beta} \overset{def}{=} t_\alpha \frac{\partial t_3}{\partial s_\beta}$$

$$X(s_i) = \bar{X}(s_\alpha) + T_3(s_\alpha) s_3$$

$$\begin{cases} \dfrac{\partial X}{\partial s_\beta} = \dfrac{\partial \bar{X}}{\partial s_\beta} + \dfrac{\partial T_3}{\partial s_\beta} s_3 \\ \dfrac{\partial X}{\partial s_3} = T_3 \end{cases}$$

$$\frac{\partial T_3}{\partial s_3} = T_3 \,; \quad T_3 \cdot s_3 = 0$$

in-plane

$$f^0_{\alpha\beta} = T_\alpha \cdot \frac{\partial X}{\partial s_\beta} = T_\alpha \left(\frac{\partial \bar{X}}{\partial s_\beta} + \frac{\partial T_3}{\partial s_\beta} s_3 \right)$$

$$\frac{\partial \bar{X}}{\partial s_\beta} = T_\beta$$

$$= T_\alpha \left(T_\beta + \frac{\partial T_3}{\partial s_\beta} s_3 \right)$$

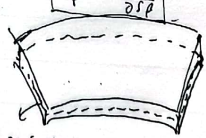
$$= \delta_{\alpha\beta} + T_\alpha \frac{\partial T_3}{\partial s_\beta} s_3$$

$$= \delta_{\alpha\beta} + B^0_{\alpha\beta} s_3$$

$$\boxed{s_\beta = \bar{X} \cdot T_\beta} \quad \boxed{T_\beta = \frac{\partial \bar{X}}{\partial s_\beta}}$$

$$T_\beta = \frac{\partial X}{\partial s_\beta}\Big|_{s_3=0} = \frac{\partial \bar{X}}{\partial s_\beta}$$

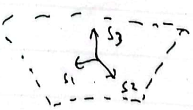
$$B^0_{\alpha\beta} \overset{def}{=} T_\alpha \frac{\partial T_3}{\partial s_\beta}$$



$$\frac{\partial \bar{X}}{\partial s_3} + \frac{\partial T_3(s_\alpha)}{\partial s_3} s_3 + T_3$$

$$= 0 + T_3 \cdot s_3 + T_3$$

$$= T_3.$$



s_1, s_2, s_3 只坐标.

s_1, s_2 在 Ref Plane 内且正交.

$T_1\ T_2\ T_3$ 是三个方向的单位方向向量.

扫描全能王 创建

Para interpolation:

对于 reference surface positions:

与知有限元形函数是一样的.

$$\bar{x}(\xi_\alpha) = N^I(\xi_\alpha)\,\bar{x}^I$$

$$\bar{X}(\xi_\alpha) = N^I(\xi_\alpha)\,\bar{X}^I.$$

The gradient of the position with respect to ξ_β are:

$$\frac{\partial \bar{x}}{\partial \xi_\beta} = \frac{\partial N^I}{\partial \xi_\beta}\,\bar{x}^I;$$

$$\frac{\partial \bar{X}}{\partial \xi_\beta} = \frac{\partial N^I}{\partial \xi_\beta}\,\bar{X}^I$$

I 代表 nodes of an element.

$$T_3 = \frac{\frac{\partial \bar{X}}{\partial \xi_1} \times \frac{\partial \bar{X}}{\partial \xi_2}}{\left|\frac{\partial \bar{X}}{\partial \xi_1} \times \frac{\partial \bar{X}}{\partial \xi_2}\right|}$$

$$\frac{\partial S_\alpha}{\partial \xi_\beta} = T_\alpha \frac{\partial \bar{X}}{\partial \xi_\beta} = T_\alpha \cdot \bar{X}^I \frac{\partial N^I}{\partial \xi_\beta}$$

$$\frac{\partial \xi_\alpha}{\partial S_\beta} = \left[\frac{\partial S_\alpha}{\partial \xi_\beta}\right]^{-1}$$

扫描全能王 创建

$$\frac{\partial N^I}{\partial s_\beta} = \frac{\partial N^I}{\partial \xi_\alpha} \frac{\partial \xi_\alpha}{\partial s_\beta}$$

$$B^o_{\alpha\beta} = T_\alpha T^I_3 \frac{\partial N^I}{\partial s_\beta}$$

T^I_3 : nodal normals.

Membrane deformation and curvature

$$[\bar{h}_{\alpha\beta}] = [\underbrace{\bar{f}_{\alpha\beta}}_{\text{reference surface deformation gradient}}]^{-1}$$

小s应该代表 ~~[redacted]~~ 当前.

$$\underset{小s}{\frac{\partial}{\partial s_\beta}} \stackrel{def}{=} \bar{h}_{\alpha\beta} \underset{大S}{\frac{\partial}{\partial S_\alpha}} \quad \text{or inverted} \quad \underset{大S}{\frac{\partial}{\partial S_\beta}} = \bar{f}_{\alpha\beta} \underset{小s}{\frac{\partial}{\partial s_\alpha}}$$

The gradient operator in the current state can also be seen as the derivative ~~of~~ with respect to ~~d~~ distance measuring coordinates s_α along the base vector t_α,

小s (~~[redacted]~~

since

$$t_\alpha \cdot \frac{\partial \bar{x}}{\partial s_\beta} = t_\alpha \cdot \frac{\partial \bar{x}}{\partial S_r} \bar{h}_{r\beta} = \bar{f}_{\alpha r} \bar{h}_{r\beta} = \delta_{\alpha\beta}$$

大S

and hence, $\overset{\partial s}{t_\alpha} = \frac{\partial \bar{x}}{\partial s_\alpha}$

$$\bar{h}_{\alpha\beta} = T_\alpha \frac{\partial \bar{x}}{\partial S_\beta}$$

扫描全能王 创建

$\bar{f}_{\alpha\gamma}\bar{f}_{\gamma\beta}$

$$\bar{f}_{\alpha\gamma}\bar{h}_{\gamma\beta} = t_\alpha \cdot \frac{\partial \bar{x}}{\partial \bar{X}} \cdot T_\gamma \cdot T_\gamma \frac{\partial \bar{X}}{\partial \bar{x}} \cdot t_\beta = t_\alpha \cdot \frac{\partial \bar{x}}{\partial \bar{X}} \cdot \frac{\partial \bar{X}}{\partial \bar{x}} \cdot t_\beta$$

$$= t_\gamma \cdot t_\beta = \delta_{\alpha\beta}$$

由 $t_\alpha = \dfrac{\partial \bar{x}}{\partial s_\alpha}$

$$\bar{h}_{\alpha\beta} = T_\alpha \frac{\partial \bar{X}}{\partial s_\beta}$$

$$\Delta \bar{f}_{\alpha\beta} = t_\alpha^{t+\Delta t} \frac{\partial \bar{x}^{t+\Delta t}}{\partial s_\beta^t}$$

$$\Delta \bar{h}_{\alpha\beta} = t_\alpha^t \frac{\partial \bar{x}^t}{\partial s_\beta^{t+\Delta t}}$$

$$b_{\alpha\beta} \overset{def}{=} t_\alpha \cdot \frac{\partial t_3}{\partial s_\beta} = B_{\alpha\gamma} \bar{h}_{\gamma\beta}$$

Orientation update $\qquad\qquad \hat{t}_\alpha$ 和 surface 正切.

$$\hat{f}_{\alpha\beta} = \hat{t}_\alpha \cdot \frac{\partial \bar{x}^{t+\Delta t}}{\partial s_\beta}$$

S 是 [illegible]

$\Delta R_{11} = \Delta R_{22} = \cos\Delta\psi$

$\Delta R_{21} = -\Delta R_{12} = \sin\Delta\psi$

$$\tan\Delta\psi = \frac{\hat{f}_{12} - \hat{f}_{21}}{\hat{f}_{11} + \hat{f}_{22}} \qquad \bar{t}_\alpha^{t+\Delta t} = \Delta R_{\alpha\gamma} \hat{t}_\gamma$$

缩并 ($\gamma = 1, 2, \ldots$)

扫描全能王 创建

Curvature Change $\Delta\vec{\phi}$: finite incremental rotation vector.

$$\Delta\vec{\phi} = \omega\Delta t = N^I(\xi_\alpha)\,\omega^I\Delta t = N^I(\xi_\alpha)\,\Delta\vec{\phi}^I.$$

$$\Delta\vec{\phi} = \Delta\phi\,\vec{p}$$

$\Delta\vec{\phi}$ 或 $\{\Delta\phi\}$ $\{p\}$ 或 $\vec{p}$ 均代表 vector.

$$\Delta\phi \overset{def}{=} |\Delta\phi|$$

$$\{p\} = \frac{\{\Delta\phi\}}{\Delta\phi}$$

$\{\Delta q\}$: the rotation quaternion (四元数)

$$\{\Delta q\} \overset{def}{=} \left(\cos\frac{\Delta\phi}{2},\ \sin\frac{\Delta\phi}{2}\{p\}\right)$$

注

An updated shell normal:

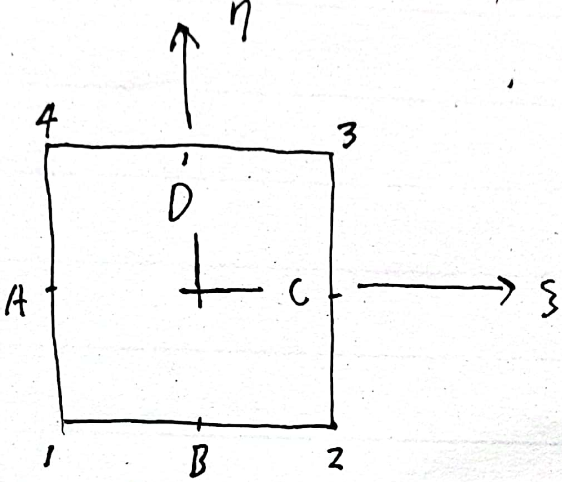
$$\{\tilde{t}_3\}^{t+\Delta t} = \{\Delta q\}\{t_3\}\{\Delta q\}^T$$

选读介质力学里的刚体旋转,客观性.

扫描全能王 创建

Transverse shear treatment.

$\hat{t}_3$: skew-symmetric tensor with ~~axis~~ axial vector t_3



$$\bar{\gamma}_1 = \frac{1}{2}[(1-\eta)\gamma_1^B + (1+\eta)\gamma_1^D]$$

$$\bar{\gamma}_2 = \frac{1}{2}[(1-\xi)\gamma_2^A + (1+\xi)\gamma_2^C]$$

$$\left.\begin{aligned}
\gamma_2^A &= t^A \cdot \bar{x}^A_{,2} - T^A \cdot \bar{X}^A_{,2} \\
\gamma_2^C &= t^C \cdot \bar{x}^C_{,2} - T^C \cdot \bar{X}^C_{,2} \\
\gamma_1^B &= t^B \cdot \bar{x}^B_{,1} - T^B \bar{X}^B_{,1} \\
\gamma_1^D &= t^D \cdot \bar{x}^D_{,1} - T^D \cdot \bar{X}^D_{,1}
\end{aligned}\right\} \text{剪应变}$$

x 代表 ~~sa~~ position

$\bar{x}$ 代表 reference surface

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$$\begin{cases} \bar{x}^{A}_{,2} = \frac{1}{2}(\bar{x}_4 - \bar{x}_1) \\ \bar{x}^{B}_{,1} = \frac{1}{2}(\bar{x}_2 - \bar{x}_1) \\ \bar{x}^{C}_{,2} = \frac{1}{2}(\bar{x}_3 - \bar{x}_2) \\ \bar{x}^{D}_{,1} = \frac{1}{2}(\bar{x}_3 - \bar{x}_4) \end{cases}$$

$\bar{x}_I$: for $I = 1,2,3,4$

由 $t_{t+\Delta t} = e^{\Delta\hat{\phi}} \cdot t_t \rightarrow \delta t = \delta\phi \times t$

从而 $\delta t^A = \frac{1}{2}(\delta\phi_4 + \delta\phi_1) \times t^A$

$\delta t^B = \frac{1}{2}(\delta\phi_2 + \delta\phi_1) \times t^B$

$\delta t^C = \frac{1}{2}(\delta\phi_2 + \delta\phi_3) \times t^C$

$\delta t^D = \frac{1}{2}(\delta\phi_3 + \delta\phi_4) \times t^D$

$$\bar{\gamma} = \begin{Bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{Bmatrix} \qquad \delta\bar{x} = \begin{Bmatrix} \delta\bar{x}_1 \\ \delta\bar{x}_2 \\ \delta\bar{x}_3 \\ \delta\bar{x}_4 \end{Bmatrix} \qquad \delta\phi = \begin{Bmatrix} \delta\phi_1 \\ \delta\phi_2 \\ \delta\phi_3 \\ \delta\phi_4 \end{Bmatrix}$$

$$\delta\bar{\gamma} = \begin{Bmatrix} \delta\bar{\gamma}_1 \\ \delta\bar{\gamma}_2 \end{Bmatrix} = \bar{B}_{sm}\,\delta\bar{x} + \bar{B}_{sb}\,\delta\phi$$

where

$$\bar{B}_{sm} = \frac{1}{4}\begin{bmatrix} -(1-\eta)t^{B^T} & (1-\eta)t^{B^T} & (1+\eta)t^{D^T} & -(1+\eta)t^{D^T} \\ -(1-\xi)t^{A^T} & -(1+\xi)t^{C^T} & (1+\xi)t^{C^T} & (1-\xi)t^{A^T} \end{bmatrix}$$

定义

$\eta^A_2 = t^A \times \bar{x}^A_{,2} \qquad \eta^B_1 = t^B \times \bar{x}^B_{,1}$

$\eta^C_2 = t^C \times \bar{x}^C_{,2} \qquad \eta^D_1 = t^D \times \bar{x}^D_{,1}$

the rotation/bending part of the strain/displacement operator is written

扫描全能王 创建

$$\bar{B}_{s_b} = \frac{1}{4}\begin{bmatrix} (1-\eta)\eta_1^{BT} & (1-\eta)\eta_1^{BT} & (1+\eta)\eta_1^{DT} & (1+\eta)\eta_1^{DT} \\ (1-\xi)\eta_2^{AT} & (1+\xi)\eta_2^{CT} & (1+\xi)\eta_2^{CT} & (1-\xi)\eta_2^{AT} \end{bmatrix}$$

- transverse shear force Q

transverse shear strains $\bar{\gamma}$

$$\begin{Bmatrix} Q^1 \\ Q^2 \end{Bmatrix} = C_s \begin{Bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{Bmatrix}$$

C_s: transverse shear stiffness.

$$C_s = \frac{5}{6} G s h \begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix}$$

$$[A^{\alpha\beta}] = [A_{\alpha\beta}]^{-1}.$$

$$A_{\alpha\beta} = \bar{x}_{,\alpha} \cdot \bar{x}_{,\beta}$$

Cauchy or true transverse shear force components in the shell orthonormal system $\{q^1, q^2\}^T$ are calculated with the coordinate transformation $f_\alpha^a = \frac{\partial \varsigma^a}{\partial \xi^\alpha}$ as

$$\{q\} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \frac{A}{a}\begin{bmatrix} f_1^1 & f_2^1 \\ f_1^2 & f_2^2 \end{bmatrix}\begin{Bmatrix} Q^1 \\ Q^2 \end{Bmatrix}$$

A: elements reference area.

a: current area.

Initial Stress stiffness

$$\delta u = \begin{Bmatrix} \delta\bar{x}_1 \\ \delta\phi_1 \\ \delta\bar{x}_2 \\ \delta\phi_2 \\ \delta\bar{x}_3 \\ \delta\phi_3 \\ \delta\bar{x}_4 \\ \delta\phi_4 \end{Bmatrix} \qquad du = \begin{Bmatrix} d\bar{x}_1 \\ d\phi_1 \\ d\bar{x}_2 \\ d\phi_2 \\ d\bar{x}_3 \\ d\phi_3 \\ d\bar{x}_4 \\ d\phi_4 \end{Bmatrix}$$

$$\{q\} d\delta\bar{\gamma} = \int \delta u \cdot K_s \cdot \Delta u \, da$$

I : 3x3 identity

$$A = Q^2(1-\xi)[\mathrm{sym}\{t^A(\bar{x}_{,2}^A)^T\} - \gamma_2^A I]$$

$$B = Q^1(1-\eta)[\mathrm{sym}\{t^B(\bar{x}_{,1}^B)^T\} - \gamma_1^B I]$$

$$C = Q^2(1+\xi)[\mathrm{sym}\{t^C(\bar{x}_{,2}^C)^T\} - \gamma_2^C I]$$

$$D = Q^1(1+\eta)[\mathrm{sym}\{t^D(\bar{x}_{,1}^D)^T\} - \gamma_1^D I]$$

Also define skew-symmetric matrices

$$\hat{a} = Q^2(1-\xi)\hat{t}^A$$

$$\hat{b} = Q^1(1-\eta)\hat{t}^B$$

$$\hat{c} = Q^2(1+\xi)\hat{t}^C$$

$$\hat{d} = Q^1(1+\eta)\hat{t}^D$$

K_s的量纲是[stress]应力

K_s is the transverse shear contribution to the initial stress 的量纲.

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