复合材料力学

$\varepsilon_x \to \varepsilon_1$ $\quad \gamma_{xy} \to \varepsilon_6 = 2\varepsilon_{xy}$

$\varepsilon_y \to \varepsilon_2$

$\varepsilon_z \to \varepsilon_3$ $\qquad [\varepsilon] = \{S\}[\sigma]$

$\gamma_{yz} \to \varepsilon_4 = 2\varepsilon_{yz}$ $\qquad [\sigma] = \{C\}[\varepsilon]$

$\gamma_{xz} \to \varepsilon_5 = 2\varepsilon_{xz}$

$$W = \frac{1}{2}\sigma_i\varepsilon_i = \frac{1}{2}\varepsilon^T C\,\varepsilon$$

$$= \frac{1}{2}C_{11}\varepsilon_1^2 + C_{12}\varepsilon_1\varepsilon_2 + C_{13}\varepsilon_1\varepsilon_3 + C_{14}\varepsilon_1\varepsilon_4 + C_{15}\varepsilon_1\varepsilon_5 + C_{16}\varepsilon_1\varepsilon_6$$

$$+ \frac{1}{2}C_{22}\varepsilon_2^2 + C_{23}\varepsilon_2\varepsilon_3 + C_{24}\varepsilon_2\varepsilon_4 + C_{25}\varepsilon_2\varepsilon_5 + C_{26}\varepsilon_2\varepsilon_6 + \frac{1}{2}C_{33}\varepsilon_3^2$$

$$+ C_{34}\varepsilon_3\varepsilon_4 + C_{35}\varepsilon_3\varepsilon_5 + C_{36}\varepsilon_3\varepsilon_6 + \frac{1}{2}C_{44}\varepsilon_4^2 + C_{45}\varepsilon_4\varepsilon_5$$

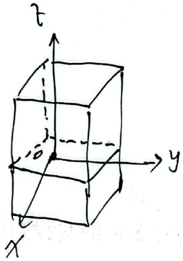
$$+ C_{46}\varepsilon_4\varepsilon_6 + \frac{1}{2}C_{55}\varepsilon_5^2 + C_{56}\varepsilon_5\varepsilon_6 + \frac{1}{2}C_{66}\varepsilon_6^2$$

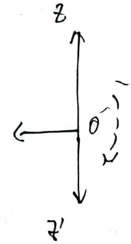
$$W = \frac{1}{2}S_{11}\sigma_1^2 + S_{12}\sigma_1\sigma_2 + S_{13}\sigma_1\sigma_3 + S_{14}\sigma_1\sigma_4 + S_{15}\sigma_1\sigma_5 + S_{16}\sigma_1\sigma_6$$

$$+ \frac{1}{2}S_{22}\sigma_2^2 + S_{23}\sigma_3\sigma_2 + S_{24}\sigma_2\sigma_4 + S_{25}\sigma_2\sigma_5 + S_{26}\sigma_2\sigma_6 + \frac{1}{2}S_{33}\sigma_3^2$$

$+ \ldots$

2.2.1 具有一个弹性对称平面的材料





$z' = -z$

$w' = -w$

$$\gamma_{yz'} = \frac{\partial w'}{\partial y} + \frac{\partial v}{\partial z'} = -\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = -\gamma_{yz} = -\varepsilon_4$$

$$\gamma_{z'x} = \frac{\partial u}{\partial z'} + \frac{\partial w'}{\partial x} = -\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = -\gamma_{zx} = -\varepsilon_5$$

那些含有z的一次项的要被消去

γ_{yz} ε_4

γ_{zx} ε_5

$C_{14} = 0$

$C_{15} = 0$

$C_{24} = 0$

$C_{25} = 0$

$C_{34} = 0$

$C_{35} = 0$

$C_{64} = 0$

$C_{65} = 0$

C的对称性：

当应力σ_i作用于应变增量$d\varepsilon_i$，单位体积外力功增量dA.

$$dA = dW = \sigma_i d\varepsilon_i$$

由(2-8)：

$$C_{ij} = \frac{\partial \sigma_i}{\partial \varepsilon_j} = \frac{\partial\left(\frac{\partial W}{\partial \varepsilon_i}\right)}{\partial \varepsilon_j} = \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} = \frac{\partial^2 W}{\partial \varepsilon_j \partial \varepsilon_i} = C_{ji}$$

求偏导.

由应变势能与加载过程无关：

$$dW = \frac{\partial W}{\partial \varepsilon_i} d\varepsilon_i$$

$$\frac{\partial W}{\partial \varepsilon_i} = \sigma_i = C_{ij}\varepsilon_j$$

沿整个加载中线性变形积分

$$W = \frac{1}{2} C_{ij}\varepsilon_i\varepsilon_j = \frac{1}{2}\sigma_i\varepsilon_i$$

即应变势能密度表示为应变分量的二次函数

$$\frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} = C_{ij}$$

~~(2-8)~~ 用齐次（二次）函数的欧拉定理（见知乎收藏）可以证明

$$W = \frac{1}{2} C_{ij}\varepsilon_i\varepsilon_j$$

$$W = \frac{1}{2} S_{ij}\sigma_i\sigma_j$$

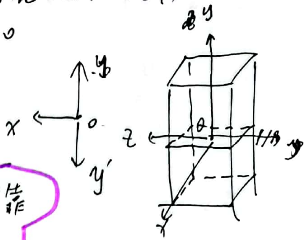
W是应变能密度

$C_{ij} = C_{ji}$ $S_{ij} = S_{ji}$

（因应变势能微分与次序无关）

2.2.2 正交各向异性材料：材料具有3个正交的弹性对称平面（或说 ~~2~~个 → 3个）

$y \to y'$ xOz 平面弹性对称，则同样可证明一些刚度系数 $=0$



$\gamma_{yx} = -\gamma_{yx}$ 含 ε 的一次项要为0

$\gamma_{yz} = -\gamma_{yz}$ ε_y 不变

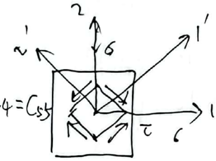
注意 $y \to y'$ 无法推 $C_{64}=0$，要靠 $z \to z'$ 推。

C_{61} C_{62} C_{63} $\cancel{C_{65}}$ $=0$

2.2.3 横观各向同性材料

若经过弹性体材料一轴线在垂直该轴线的平面内，各点的弹性性能在各方向上都相同，则此材料称为横观各向同性材料，此平面叫各向同性面

现取 1-2 坐标面为各向同性面，3轴垂直于1-2坐标面



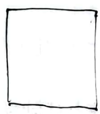
$S_{11}=S_{22}$ $S_{13}=S_{23}$ $S_{44}=S_{55}$ $C_{11}=C_{22}$ $C_{13}=C_{23}$ $C_{44}=C_{55}$

4: yz
5: xz
6: xy

$\sigma_1 = \sigma$ $\sigma_3 = 0$

$\sigma_2 = -\sigma$

$\sigma_4 = \sigma_5 = \sigma_6 = 0$



$W = \frac{1}{2}S_{11}\sigma^2 - S_{12}\sigma^2 + \frac{1}{2}S_{11}\sigma^2$

$= S_{11}\sigma^2 - S_{12}\sigma^2$

$\sigma_1' = 0 = \sigma_2' = \sigma_3'$

$\sigma_4' = 0$ $\sigma_5' = 0$ $\sigma_6' = -\sigma$

~~$\gamma_{xy} = \frac{1}{2}(\varepsilon_x - \varepsilon_y)$~~

$W = \frac{1}{2}S_{66}\sigma_6^2 = \frac{1}{2}S_{66}\sigma^2$

$S_{66} = 2(S_{11} - S_{12})$ $C_{66} = \frac{1}{2}(C_{11} - C_{22})$

2.2.4 各向同性材料

$S_{12} = S_{21}$

$S_{12} = -\frac{v_{12}}{E_2}$

$S_{21} = -\frac{v_{21}}{E_1}$

$\frac{v_{12}}{E_2} = \frac{v_{21}}{E_1}$

$|S_{12}| = \frac{v_{12}}{E_2} < \left(\frac{1}{E_1}\frac{1}{E_2}\right)^{\frac{1}{2}}$

$\frac{|v_{21}|}{E_1} < \left(\frac{1}{E_1}\frac{1}{E_2}\right)^{\frac{1}{2}}$

$|v_{21}| < \left(\frac{E_1}{E_2}\right)^{\frac{1}{2}}$

$|v_{12}| < \left(\frac{E_2}{E_1}\right)^{\frac{1}{2}}$

$|v_{23}|$

⋮

这些不等式可以用来检验实验数据是否合理

C3. 单层复合材料的 $\boxed{\text{宏观力学分析}}$

平面应力状态：$\sigma_3 = \tau_{13} = \tau_{23} = 0$ 单层复材（各向异性的平面应力状态）

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \\ S_{12} & S_{22} & \\ & & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = S\sigma.$$

$\gamma_{31} = \gamma_{23} = 0$

$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2 + S_{33}\tau_{12}$

$= -\frac{v_{31}\sigma_1}{E_1} - \frac{v_{32}\sigma_2}{E_2} + \frac{\tau_{12}}{E_3}$

对 S 求逆：

$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [Q]\{\varepsilon\}$ 　$[Q]$ 是 $\boxed{\text{折减}}$ 刚度矩阵. 　$C = S^{-1}$

C 是真实的

$\boxed{C_{ij} \neq Q_{ij}}$

（S 包括 S_{23}. S_{33}）

3.2 任意方向的(单层材料)应力-应变关系.

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=[T]^{-1}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}$$

应变转轴公式:

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \tau_{xy}\end{bmatrix}=T^{T}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \tau_{12}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=[T^{-1}][Q][T^{-1}]^{T}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=[Q]\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix}$$

↓

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=[T]^{-1}[Q]\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{bmatrix}$$

$$=[T]^{-1}[Q][T^{-1}]^{T}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}$$

C3. 正交各向异性(单层)材料的强度理论.

3.4.1 最大应力理论、3.4.2 最大应变理论 都不很适用.

3.4.3 Hill-蔡强度理论.

$$(G+H)\sigma_1^2+(F+H)\sigma_2^2+(F+G)\sigma_3^2-2H\sigma_1\sigma_2-2G\sigma_1\sigma_3-2F\sigma_2\sigma_3+2L\tau_{23}^2+2M\tau_{31}^2+2N\tau_{12}^2=1$$

F、G、H、L、M、N 为各向异性材料的破坏强度参数.

X, Y, S 来表示 F G H L M N.

X_t 纵向拉伸 X_c 纵向压缩 Y_t 横向拉伸 Y_c 横向压缩 S 剪切 (五大强度)

只有τ_{12}作用

$$2NS^2=1$$

$$2N=\frac{1}{S^2}.$$

只有σ_1作用

$$\begin{cases}(G+H)X^2=1\\(F+H)Y^2=1\\(F+G)Z^2=1\end{cases}$$

(用Z表示3方向强度)

$$\downarrow$$

$$\begin{cases}2F=-\frac{1}{X^2}+\frac{1}{Y^2}+\frac{1}{Z^2}\\2G=\frac{1}{X^2}-\frac{1}{Y^2}+\frac{1}{Z^2}\\2H=\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2}.\end{cases}$$

$$\downarrow$$

$$\frac{\sigma_1^2}{X^2}-\frac{\sigma_1\sigma_2}{X^2}+\frac{\sigma_2^2}{Y^2}+\frac{\tau_{12}^2}{S^2}=1.$$

$$\begin{cases}\sigma_1=\sigma_x\cos^2\theta\\\sigma_2=\sigma_x\sin^2\theta\\\tau_{12}=-\sigma_x\sin\theta\cos\theta\end{cases}$$

$$\frac{\sigma_x^2\cos^4\theta}{X^2}-\frac{\sigma_x^2\cos^2\theta\sin^2\theta}{X^2}+\frac{\sigma_x^2\sin^4\theta}{Y^2}+\frac{\sigma_x^2\sin^2\theta\cos^2\theta}{S^2}=1$$

$$\frac{\cos^4\theta}{X^2}+\frac{\sin^4\theta}{Y^2}+\left(\frac{1}{S^2}-\frac{1}{X^2}\right)\cos^2\theta\sin^2\theta=\frac{1}{\sigma_x^2}$$

复合材料学报.

Hill-蔡理论未考虑拉压性能不同的复合材料，Hoffman提出如下新的理论：

$$\frac{\sigma_1^2}{X_tX_c}-\frac{\sigma_1\sigma_2}{X_tX_c}+\frac{\sigma_2^2}{Y_tY_c}+\frac{X_c-X_t}{X_tX_c}\sigma_1+\frac{Y_c-Y_t}{Y_tY_c}\sigma_2+\frac{\tau_{12}^2}{S^2}=1$$

⑧ 拓扑优化与材料设计.

一、多材料的拓扑优化:

离散→连续. (连续化方法)

SIMP

p: 指示数

~~SIMP~~

ρ_{e1}: 有没有材料

ρ_{e2}: 哪种材料

$$E_{ijkl}(\rho_{e1},\rho_{e2}) = \rho_{e1}^{p}[\rho_{e2}^{\eta}E_{ijkl}^{(1)} + (1-\rho_{e2}^{\eta})E_{ijkl}^{(2)}].$$

$$E_{ijkl}^{*} = \rho_{e2}^{\eta}E_{ijkl}^{(1)} + (1-\rho_{e2})^{\eta}E_{ijkl}^{(2)}$$

DMO: 有多少试多少. 缺点: 变量太多难优化.

SFP: 改进DMO: 2个变量表示4个点,
采用等参元形函数优化插值函数

GSFP: $w_m(R_1 \ldots R_N) = \frac{1}{2^N}\prod(1+\xi_{mk}R_k)$ (老师组工作)

$R_k \in [-1,1]$

~~where~~

N个设计变量 $\xrightarrow{可表示}$ 2^N种候选材料

纤维增强复合材料角度选择

钢框架结构截面选型优化设计.(不同截面构型看成不同材料),
虽然不是拓扑优化,但也是可用拓扑优化的思想做.

二、多尺度分析与拓扑优化.

宏观 ~~结构~~ } 结构
微观

宏观结构 ~ 一类变量

微观结构 ~ 另一类变量

$\boxed{双尺度并发优化算法.}$ 程etc.

↓

如果有热载作用,优化容易出现多层级结构.

三

考虑应力约束的拓扑优化列式.

$(\sigma_e)_{vm}$: 单元 von Mises 应力.

不同提法: 让最大的$(\sigma_e)_{vm}$ < 总的应力约束.

应力约束的可微化处理

应力约束:

① 超大变量(约束数目太多) —— 约束暂时消除.
全局约束.(有课件拍照): p-norm 方法

② 有限元应力处理 ☆ 办法.

(1) 网格加密自适应

(2) 杂交元方法 位移场、应力场分别插值.

(3) 扩展有限元.