模式识别与机器学习作业

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1　第三章判别函数

1.1　题目1

在一个10类的模式识别问题中，有3类单独满足多类情况1，其余的类别满足多类情况2。问该模式识别问题所需判别函数的最少数目是多少？

1.1.1　解

因为多类情况1需要$M_1 = 3$个判别函数，多类情况2有$M_2 = 7$需要$\frac{M_2 \times (M_2 - 1)}{2} = 21$个判别函数。所以总得判别函数个数为：$3 + 21 = 24$个。

1.2　题目2

一个三类问题，其判别函数如下：

$$d1(x) = -x1, d2(x) = x1 + x2 - 1, d3(x) = x1 - x2 - 1$$

1. 设这些函数是在多类情况1条件下确定的，绘出其判别界面和每一个模式类别的区域。
2. 设为多类情况2，并使：d12(x)= d1(x), d13(x)= d2(x), d23(x)= d3(x)。绘出其判别界面和多类情况2的区域。
3. 设d1(x), d2(x)和d3(x)是在多类情况3的条件下确定的，绘出其判别界面和每类的区域。

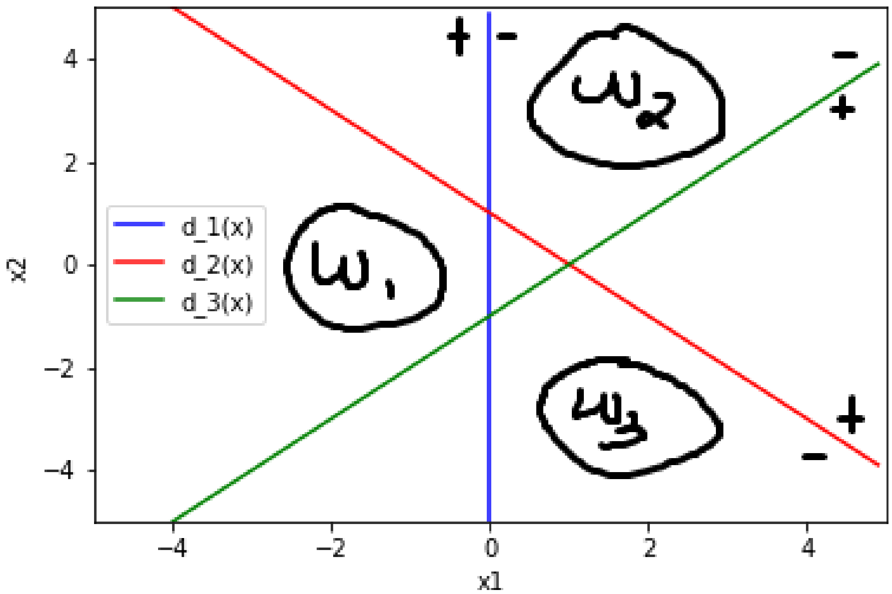

图 1: 多类情况判别函数1

1.2.1 解

判别函数和分别出来的区域如图Figure 1所示。

1.2.2 解

判别函数和分别出来的区域如图Figure 2所示。

1.2.3 解

当是多类情况3的时候，有3个判别函数为：

$$d_{12}(x) = d_1(x) - d_2(x) = -2x_1 - x_2 + 1$$
$$d_{13}(x) = d_1(x) - d_3(x) = -2x_1 + x_2 + 1$$
$$d_{23}(x) = d_2(x) - d_3(x) = 2x_2$$

判别函数和分别出来的区域如图Figure 3所示。

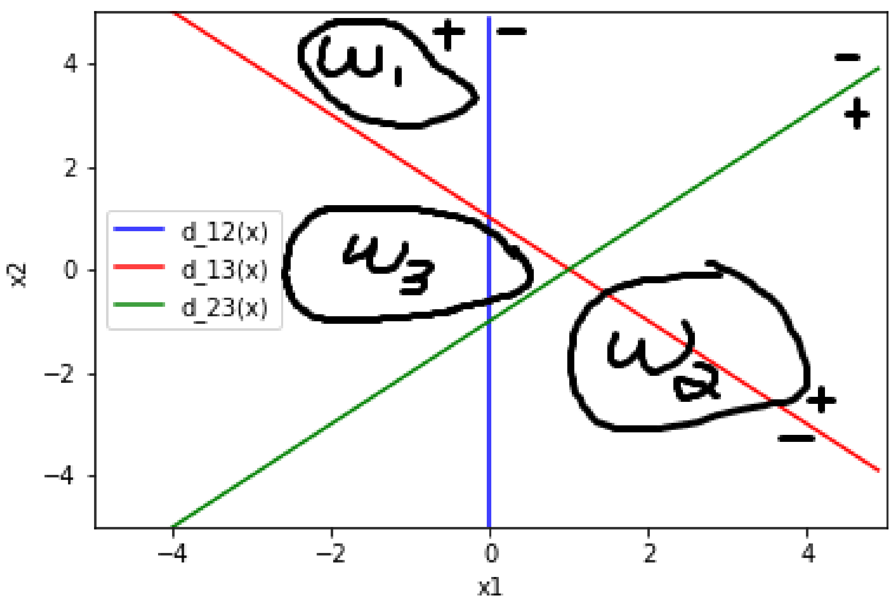

图 2: 多类情况判别函数2

1.3 题目3

两类模式，每类包括5个3维不同的模式向量，且良好分布。如果它们是线性可分的，问权向量至少需要几个系数分量？假如要建立二次的多项式判别函数，又至少需要几个系数分量？（设模式的良好分布不因模式变化而改变。）

1.3.1 解

当线性可分的时候，有判别函数$d(x) = W^{\intercal x} + W_{n+1}$因为x是3维的，所以n=3，权重向量为4维，所以最少需要4个系数分量。当为二次的时候根据公式需要$C_5^2 = 10$个系数分量。

1.4 题目4

- 用感知器算法求下列模式分类的解向量w:

$$\omega1 : (0, 0, 0)^T, (1, 0, 0)^T, (1, 0, 1)^T, (1, 1, 0)^T$$
$$\omega2 : (0, 0, 1)^T, (0, 1, 1)^T, (0, 1, 0)^T, (1, 1, 1)^T$$

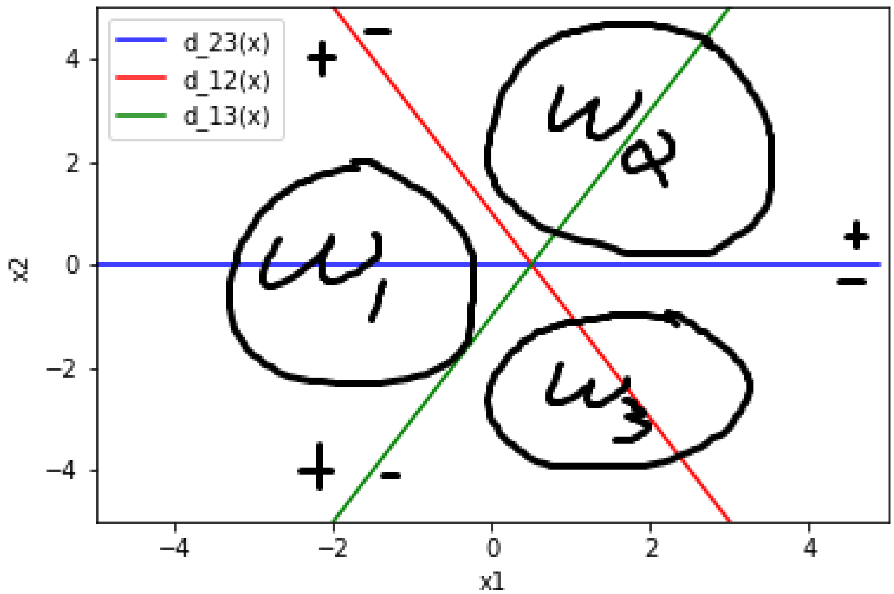

图 3: 多类情况判别函数3

- 编写求解上述问题的感知器算法程序。

1.4.1 解

将样本写成增广向量的模式，并将$\omega 2$的样本乘以-1进行规范化后有：

$\omega 1 : x_1 = (0,0,0,1)^T, x_2 = (1,0,0,1)^T, x_3 = (1,0,1,1)^T, x_4 = (1,1,0,1)^T$

$\omega 2 : x_5 = (0,0,-1,-1)^T, x_6 = (0,-1,-1,-1)^T, x_7 = (0,-1,0,-1)^T, x_8 = (-1,-1,-1,-1)^T$

第一轮迭代取$\eta = 1, W_1 = [0,0,0,0]^T$ 其中η为步长。

$$
\begin{aligned}
&W_1^T x_1 = 0 \leq 0 \qquad so: W_2 = W_1 + x_1 = (0,0,0,1)^T \\
&W_2^T x_2 = 1 > 0 \qquad so: W_3 = W_2 = (0,0,0,1)^T \\
&W_3^T x_3 = 1 > 0 \qquad so: W_4 = W_3 = (0,0,0,1)^T \\
&W_4^T x_4 = 1 > 0 \qquad so: W_5 = W_4 = (0,0,0,1)^T \\
&W_5^T x_5 = -1 \leq 0 \qquad so: W_6 = W_5 + x_5 = (0,0,-1,0)^T \\
&W_6^T x_6 = 1 > 0 \qquad so: W_7 = W_6 = (0,0,-1,0)^T \\
&W_7^T x_7 = 0 \leq 0 \qquad so: W_8 = W_7 + x_7 = (0,-1,-1,-1)^T \\
&W_8^T x_8 = 3 > 0 \qquad so: W_9 = W_8 = (0,-1,-1,-1)^T
\end{aligned}
$$

第二轮迭代有：

$$
\begin{aligned}
&W_9^T x_1 = -1 \leq 0 \qquad so: W_{10} = W_9 + x_1 = (0,-1,-1,0)^T \\
&W_{10}^T x_2 = 0 \leq 0 \qquad so: W_{11} = W_{10} + x_2 = (1,-1,-1,1)^T \\
&W_{11}^T x_3 = 1 > 0 \qquad so: W_{12} = W_{11} = (1,-1,-1,1)^T \\
&W_{12}^T x_4 = 1 > 0 \qquad so: W_{13} = W_{12} = (1,-1,-1,1)^T \\
&W_{13}^T x_5 = 0 \leq 0 \qquad so: W_{14} = W_{13} + x_5 = (1,-1,-2,0)^T \\
&W_{14}^T x_6 = 3 > 0 \qquad so: W_{15} = W_{14} = (1,-1,-2,0)^T \\
&W_{15}^T x_7 = 1 > 0 \qquad so: W_{16} = W_{15} = (1,-1,-2,0)^T \\
&W_{16}^T x_8 = 2 > 0 \qquad so: W_{17} = W_{16} = (1,-1,-2,0)^T
\end{aligned}
$$

第三轮迭代有：

$$W_{17}^T x_1 = 0 \le 0 \qquad so: W_{18} = W_{17} + x_1 = (1, -1, -2, 1)^T$$
$$W_{18}^T x_2 = 2 > 0 \qquad so: W_{19} = W_{18} + x_2 = (1, -1, -2, 1)^T$$
$$W_{19}^T x_3 = 0 \le 0 \qquad so: W_{20} = W_{19} + x_3 = (2, -1, -1, 2)^T$$
$$W_{20}^T x_4 = 3 > 0 \qquad so: W_{21} = W_{20} = (2, -1, -1, 2)^T$$
$$W_{21}^T x_5 = -1 \le 0 \qquad so: W_{22} = W_{21} + x_5 = (2, -1, -2, 1)^T$$
$$W_{22}^T x_6 = 2 > 0 \qquad so: W_{23} = W_{22} = (2, -1, -2, 1)^T$$
$$W_{23}^T x_7 = 0 \le 0 \qquad so: W_{24} = W_{23} = (2, -2, -2, 0)^T$$
$$W_{24}^T x_8 = 2 > 0 \qquad so: W_{25} = W_{24} = (2, -2, -2, 0)^T$$

第四轮迭代有：

$$W_{25}^T x_1 = 0 \le 0 \qquad so: W_{26} = W_{25} = (2, -2, -2, 1)^T$$
$$W_{26}^T x_2 = 3 > 0 \qquad so: W_{27} = W_{26} = (2, -2, -2, 1)^T$$
$$W_{27}^T x_3 = 1 > 0 \qquad so: W_{28} = W_{27} = (2, -2, -2, 1)^T$$
$$W_{28}^T x_4 = 1 > 0 \qquad so: W_{29} = W_{28} = (2, -2, -2, 1)^T$$
$$W_{29}^T x_5 = 1 > 0 \qquad so: W_{30} = W_{29} = (2, -2, -2, 1)^T$$
$$W_{30}^T x_6 = 3 > 0 \qquad so: W_{31} = W_{30} = (2, -2, -2, 1)^T$$
$$W_{31}^T x_7 = 1 > 0 \qquad so: W_{32} = W_{31} = (2, -2, -2, 1)^T$$
$$W_{32}^T x_8 = 1 > 0 \qquad so: W_{33} = W_{32} = (2, -2, -2, 1)^T$$

第五轮迭代有：

$$
\begin{aligned}
W_{33}^T x_1 &= 1 > 0 \qquad so: W_{34} = W_{33} = (2,-2,-2,1)^T \\
W_{34}^T x_2 &= 3 > 0 \qquad so: W_{35} = W_{34} = (2,-2,-2,1)^T \\
W_{35}^T x_3 &= 1 > 0 \qquad so: W_{36} = W_{35} = (2,-2,-2,1)^T \\
W_{36}^T x_4 &= 1 > 0 \qquad so: W_{37} = W_{36} = (2,-2,-2,1)^T \\
W_{37}^T x_5 &= 1 > 0 \qquad so: W_{38} = W_{37} = (2,-2,-2,1)^T \\
W_{38}^T x_6 &= 3 > 0 \qquad so: W_{39} = W_{38} = (2,-2,-2,1)^T \\
W_{39}^T x_7 &= 1 > 0 \qquad so: W_{40} = W_{39} = (2,-2,-2,1)^T \\
W_{40}^T x_8 &= 1 > 0 \qquad so: W_{41} = W_{40} = (2,-2,-2,1)^T
\end{aligned}
$$

因为所有的样本都大于0，结束迭代得到权向量为：$W = (2,-2,-2,1)^T$

1.4.2 解

```python
#Perception
import numpy as np
def perception(W, sample, step):
    flag = 1
    count = 0
    while(flag):
        count +=1
        print("iter count",count)
        flag = 0
        i = 1
        for item in sample:
            print("x:",i)
            print("sum:",sum(W*item))
            print("W old:",W)
            if(sum(W*item) <= 0):
                W += step*item
                flag += 1
            print("W new:",W)
            i += 1
    return W
W = np.array([0,0,0,0],dtype=float)
```

```
step = 1
x1 = np.array([0,0,0,1],dtype=float)
x2 = np.array([1,0,0,1],dtype=float)
x3 = np.array([1,0,1,1],dtype=float)
x4 = np.array([1,1,0,1],dtype=float)
x5 = np.array([0,0,-1,-1],dtype=float)
x6 = np.array([0,-1,-1,-1],dtype=float)
x7 = np.array([0,-1,0,-1],dtype=float)
x8 = np.array([-1,-1,-1,-1],dtype=float)
sample = [x1,x2,x3,x4,x5,x6,x7,x8]
print(perception(W,sample,step))
```

1.5 题目5

用多类感知器算法求下列模式的判别函数:

$$\omega_1 : (-1,-1)^T$$
$$\omega_2 : (0,0)^T$$
$$\omega_3 : (1,1)^T$$

1.5.1 解

将样本写成增广向量的模式，取步长$\eta = 1$，初试话权重为$W_1 = W_2 = W_3 = (0,0,0)^T$有：

$$\omega_1 : (-1,-1,1)^T$$
$$\omega_2 : (0,0,1)^T$$
$$\omega_3 : (1,1,1)^T$$

第一轮迭代有：

$$
\begin{aligned}
& d_1 = W_1 x_1 = 0 \\
& d_2 = W_2 x_1 = 0 \\
& d_3 = W_3 x_1 = 0 \\
& d_1 \leq d_2 \qquad d_1 \leq d_3 \\
so: & \\
& W_1 = W_1 + \omega_1 = (-1, -1, 1) \\
& W_2 = W_2 - \omega_1 = (1, 1, -1) \\
& W_3 = W_3 - \omega_1 = (1, 1, -1)
\end{aligned}
$$

第二轮迭代有：

$$
\begin{aligned}
& d_1 = W_1 x_2 = 1 \\
& d_2 = W_2 x_2 = -1 \\
& d_3 = W_3 x_2 = -1 \\
& d_2 \leq d_1 \qquad d_2 \leq d_3 \\
so: & \\
& W_1 = W_1 - \omega_2 = (-1, -1, 0) \\
& W_2 = W_2 + \omega_2 = (1, 1, 0) \\
& W_3 = W_3 - \omega_2 = (1, 1, -2)
\end{aligned}
$$

第三轮迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_3 = -2 \\
&d_2 = W_2 x_3 = 2 \\
&d_3 = W_3 x_3 = 0 \\
&d_3 > d_1 \qquad d_3 \leq d_2 \\
so: \\
&W_1 = W_1 = (-1, -1, 0) \\
&W_2 = W_2 - \omega_3 = (0, 0, -1) \\
&W_3 = W_3 + \omega_3 = (2, 2, -1)
\end{aligned}
$$

第四轮迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_1 = 2 \\
&d_2 = W_2 x_1 = -1 \\
&d_3 = W_3 x_1 = -5 \\
&d_1 > d_2 \qquad d_1 > d_3 \\
so: \\
&W_1 = W_1 = (-1, -1, 0) \\
&W_2 = W_2 = (0, 0, -1) \\
&W_3 = W_3 = (2, 2, -1)
\end{aligned}
$$

第五轮迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_2 = 0 \\
&d_2 = W_2 x_2 = -1 \\
&d_3 = W_3 x_2 = -1 \\
&d_2 \le d_1 \qquad d_2 \le d_3 \\
so: \\
&W_1 = W_1 - \omega_2 = (-1,-1,-1) \\
&W_2 = W_2 + \omega_2 = (0,0,0) \\
&W_3 = W_3 - \omega_2 = (2,2,-2)
\end{aligned}
$$

第六论迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_3 = -3 \\
&d_2 = W_2 x_3 = 0 \\
&d_3 = W_3 x_3 = 2 \\
&d_3 > d_1 \qquad d_3 > d_2 \\
so: \\
&W_1 = W_1 = (-1,-1,-1) \\
&W_2 = W_2 = (0,0,0) \\
&W_3 = W_3 = (2,2,-2)
\end{aligned}
$$

第七轮迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_1 = 1 \\
&d_2 = W_2 x_1 = 0 \\
&d_3 = W_3 x_1 = -6 \\
&d_1 > d_2 \qquad d_1 > d_3 \\
so:& \\
&W_1 = W_1 = (-1, -1, -1) \\
&W_2 = W_2 = (0, 0, 0) \\
&W_3 = W_3 = (2, 2, -2)
\end{aligned}
$$

第八轮迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_2 = -1 \\
&d_2 = W_2 x_2 = 0 \\
&d_3 = W_3 x_2 = -2 \\
&d_2 > d_1 \qquad d_2 > d_3 \\
so:& \\
&W_1 = W_1 = (-1, -1, -1) \\
&W_2 = W_2 = (0, 0, 0) \\
&W_3 = W_3 = (2, 2, -2)
\end{aligned}
$$

第九轮迭代有：

$$
\begin{aligned}
&d_1 = W_1 x_3 = -3\\
&d_2 = W_2 x_3 = 0\\
&d_3 = W_3 x_3 = 2\\
&d_3 > d_1 \qquad d_3 > d_2\\
so:&\\
&W_1 = W_1 = (-1,-1,-1)\\
&W_2 = W_2 = (0,0,0)\\
&W_3 = W_3 = (2,2,-2)
\end{aligned}
$$

1.6 题目6

- 编写求解上述问题的感知器算法程序，求下列模式分类的解向量w:

$$
\begin{aligned}
\omega 1 : (0,0,0)^T,(1,0,0)^T,(1,0,1)^T,(1,1,0)^T\\
\omega 2 : (0,0,1)^T,(0,1,1)^T,(0,1,0)^T,(1,1,1)^T
\end{aligned}
$$

- 尝试不同的初始值
- 尝试不同的迭代顺序

1.6.1 解

代码如1.4.1的代码所示。将迭代顺序固定为$x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8$ I表示迭代次数。步长设置为1，尝试权重向量的不同的初始值：

将W初始为$W=(0,0,0,0)$得到解为$W=(2,-2,-2,1), I=5$

将W初始为$W=(1,1,1,1)$得到解为$W=(2,-2,-2,1), I=6$

将W初始为$W=(1,0,2,4)$得到解为$W=(3,-3,-3,1), I=5$

将W初始为$W=(-1,1,-1,1)$得到解为$W=(3,-2,-3,1), I=4$

将权重初始化为$(0,0,0,0)$步长设置为1，尝试不同的迭代顺序：

迭代顺序$x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8$得到解为$W=(2,-2,-2,1), I=5$

迭代顺序$x_1,x_3,x_5,x_7,x_2,x_4,x_6,x_7$得到解为$W=(3,-2,-3,1), I=4$

迭代顺序$x_8,x_3,x_2,x_7,x_4,x_5,x_6,x_1$得到解为$W=(2,-2,-2,1), I=4$

1.7 题目7

采用梯度法和准则函数

$$J(w,x,b)=\frac{1}{8||x||^2}[(w^Tx-b)-|w^Tx-b|]^2$$

1.7.1 解

分别对w和b求偏导有：

$$\frac{\partial J(w,x,b)}{\partial w}=\frac{1}{4||x||^2}((w^Tx-b)-|w^Tx-b|)(x-x\times sign(w^Tx-b))$$
$$\frac{\partial J(w,x,b)}{\partial b}=\frac{1}{4||x||^2}((w^Tx-b)-|w^Tx-b|)(-1+sign(w^Tx-b))$$
$$sign(w^Tx-b)=\begin{cases}1 & w^Tx-b>0\\ -1 & w^Tx-b\le 0\end{cases}$$

所以当$W^Tx-b>0$的时候w和b无需更新。当$W^Tx-b\le 0$的时候按照：

$$w=w-\frac{1}{||x||^2}(w^Tx-b)x$$
$$b=b+\frac{1}{||x||^2}(w^Tx-b)$$

因此对于所有误分类的点，使用随机梯度下降的方法，按照上式进行更新w和b，直到最后没有误分类的点为止。

1.8 题目8

用二次埃尔米特多项式的势函数算法求解以下模式的分类问题

$$\omega_1:x_1=(0,1)^T,x_2=(0,-1)^T$$
$$\omega_2:x_3=(1,0)^T,x_4=(-1,0)^T$$

1.8.1 解

(1)选择合适的正交函数集$\{\phi_i(x)\}$
Hermite多项式前面3项的表达式为:

$$H_0(x) = 1 \qquad H_1(x) = 2x \qquad H_2(x) = 4x^2 - 2$$

(2)建立二维的正交函数集，利用前面两项构成：

$$\begin{aligned}
\phi_1(x) &= \phi_1(x_1, x_2) = H_0(x_1)H_0(x_2) = 1 \\
\phi_2(x) &= \phi_2(x_1, x_2) = H_0(x_1)H_1(x_2) = 2x_2 \\
\phi_3(x) &= \phi_3(x_1, x_2) = H_0(x_1)H_2(x_2) = 4x_2^2 - 2 \\
\phi_4(x) &= \phi_4(x_1, x_2) = H_1(x_1)H_0(x_2) = 2x_1 \\
\phi_5(x) &= \phi_5(x_1, x_2) = H_1(x_1)H_1(x_2) = 4x_1x_2 \\
\phi_6(x) &= \phi_6(x_1, x_2) = H_1(x_1)H_2(x_2) = 2x_1(4x_2^2 - 2) \\
\phi_7(x) &= \phi_7(x_1, x_2) = H_2(x_1)H_0(x_2) = 4x_1^2 - 2 \\
\phi_8(x) &= \phi_8(x_1, x_2) = H_2(x_1)H_1(x_2) = 2x_2(4x_1^2 - 2) \\
\phi_9(x) &= \phi_9(x_1, x_2) = H_2(x_1)H_2(x_2) = (4x_1^2 - 2)(4x_2^2 - 2)
\end{aligned} \tag{1}$$

(3) 生成势函数
按第一类势函数定义，得到势函数

$$K(x, x_k) = \sum_{i=1}^{4} \phi_i(x)\phi_i(x_k)$$

(4)通过训练样本逐步计算累积位势$K(x)$
第一步：带入x_1到$K(x, x_k)$有：

$$K_1(x) = K(x, x_1) = -15 + 20x_2 + 40x_2^2 + 24x_1^2 - 32x_1^2x_2 - 64x_1^2x_2^2$$

第二步：带入x_2到$K_1(x)$有：

$$\begin{aligned}
&K_1(x_2) = 5 > 0 \\
x_2 \in \omega_1, so \qquad &K_2(x) = K_1(x)
\end{aligned}$$

第三步：带入x_3到$K_2(x)$有：

$$K_2(x_3) = 9 < 0$$

$$x_3 \in \omega_2, so \qquad K_3(x) = K_2(x) - K(x, x_3) = 20x_2 + 16x_2^2 - 20x_1 - 16x_1^2$$

第四步：带入x_4到$K_3(x)$有：

$$K_3(x_3) = 4 > 0$$

$$x_4 \in \omega_2, so \qquad K_4(x) = K_3(x) - K(x, x_3) = 15 + 20x_2 - 56x_1^2 - 8x_2^2 - 32x_1^2x_2 + 64x_1^2x_2^2$$

第五步：带入x_1到$K_4(x)$有：

$$K_4(x_1) = 27 > 0$$

$$x_1 \in \omega_1, so \qquad K_5(x) = K_4(x)$$

第六步：带入x_2到$K_5(x)$有：

$$K_5(x_2) = -13 < 0$$

$$x_2 \in \omega_2, so \qquad K_6(x) = K_5(x) + K(x, x_2) = -32x_1^2 + 32x_2^2$$

第七步：带入x_3到$K_6(x)$有：

$$K_6(x_3) = -32 < 0$$

$$x_3 \in \omega_2, so \qquad K_7(x) = K_6(x)$$

第八步：带入x_4到$K_7(x)$有：

$$K_7(x_4) = -32 < 0$$

$$x_4 \in \omega_2, so \qquad K_8(x) = K_7(x)$$

第九步：带入x_1到$K_8(x)$有：

$$K_8(x_1) = 32 > 0$$

$$x_1 \in \omega_1, so \qquad K_9(x) = K_8(x)$$

第十步：带入x_2到$K_9(x)$有：

$$K_9(x_2) = 32 > 0$$
$$x_2 \in \omega_1, so \qquad K_{10}(x) = K_9(x)$$

在第七步到第十步的迭代中，4个训练样本都能正确分类，因此算法收敛于判别函数：

$$d(x) = K_{10}(x) = -32x_1^2 + 32x_2^2$$

1.9 题目9

用下列势函数

$$K(x, x_k) = e^{-\alpha||x-x_k||^2}$$

求题目8的模式分类问题

1.9.1 解

取$\alpha = 1$ 第一步：带入x_1到$K(x, x_k)$有：

$$K_1(x) = K(x, x_1) = e^{-x_1^2-(x_2-1)^2}$$

第二步：带入x_2到$K_1(x)$有：

$$K_1(x_2) = e^{-4} > 0$$
$$x_2 \in \omega_1, so \qquad K_2(x) = K_1(x)$$

第三步：带入x_3到$K_2(x)$有：

$$K_2(x_3) = e^{-2} > 0$$
$$x_3 \in \omega_2, so \qquad K_3(x) = K_2(x) - K(x, x_3) = e^{-x_1^2-(x_2-1)^2} - e^{-(x_1-1)^2-x_2^2}$$

第四步：带入x_4到$K_3(x)$有：

$$K_3(x_3) = e^{-2} - e^{-4} > 0$$
$$x_4 \in \omega_2, so \qquad K_4(x) = K_3(x) - K(x, x_3) = e^{-x_1^2-(x_2-1)^2} - e^{-(x_1-1)^2-x_2^2} - e^{-(x_1+1)^2-x_2^2}$$

第五步：带入x_1到$K_4(x)$有：

$$K_4(x_1) = 1 - we^{-2} > 0$$
$$x_1 \in \omega_1, so \qquad K_5(x) = K_4(x)$$

第六步：带入x_2到$K_5(x)$有：

$$K_5(x_2) = e^{-4} - 2e^{-2} < 0$$
$$x_2 \in \omega_2, so \qquad K_6(x) = K_5(x) + K(x, x_2) = e^{-x_1^2-(x_2+1)^2} + e^{-x_1^2-(x_2-1)^2} - e^{-(x_1-1)^2-x_2^2} - e^{-(x_1+1)^2-x_2^2}$$

第七步：带入x_3到$K_6(x)$有：

$$K_6(x_3) = 2e^{-2} - 1 - e^{-4} < 0$$
$$x_3 \in \omega_2, so \qquad K_7(x) = K_6(x)$$

第八步：带入x_4到$K_7(x)$有：

$$K_7(x_4) = 2e^{-2} - 1 - e^{-4} < 0$$
$$x_4 \in \omega_2, so \qquad K_8(x) = K_7(x)$$

第九步：带入x_1到$K_8(x)$有：

$$K_8(x_1) = -2e^{-2} + 1 + e^{-4} > 0$$
$$x_1 \in \omega_1, so \qquad K_9(x) = K_8(x)$$

第十步：带入x_2到$K_9(x)$有：

$$K_9(x_2) = -2e^{-2} + 1 + e^{-4} > 0$$
$$x_2 \in \omega_1, so \qquad K_{10}(x) = K_9(x)$$

在第七步到第十步的迭代中，4个训练样本都能正确分类，因此算法收敛于判别函数：

$$d(x) = K_{10}(x) = e^{-x_1^2-(x_2+1)^2} + e^{-x_1^2-(x_2-1)^2} - e^{-(x_1-1)^2-x_2^2} - e^{-(x_1+1)^2-x_2^2}$$