Dear Student and/or Teacher and/or Parent,

If a student is going to succeed in high school math and beyond, they need more than procedural diligence and a good memory. They need conceptual understanding. If they lack that understanding, the best time to find out is right now - or by the end of grade 8 at the latest - as it does not get easier as students progress to higher grades. Conceptual understanding is often weakest around arithmetic and introductory algebra. This assessment deals with arithmetic. [There is another for algebra.]

By excluding traditional procedures, this assessment should help detect shallow understanding where shallowness is most common. Such shallowness may have been hidden for years by procedural diligence and assessments that overemphasize that over mathematical abstraction.

The student may have many other areas of strength or weakness in math; this document only sheds light on whether or not the student struggles with general number sense and introductory algebra. Meltdowns and closed doors stemming from those topics are common; rarely do doors close because one can't determine the volume of a cylinder.

Please feel free to edit, use, and circulate as you like. I look forward to any feedback you can offer.

Andrew Uyeno
andrew.uyeno@gmail.com

Instructions

1. Expect significant challenge because this assessment is far more abstract and, in some cases, less structured than many other math assessments.
2. Students require only a pencil and an eraser.
3. No help should be required from any person, calculator, textbook, computer, etc. A teacher or parent should only ensure the student understands the instructions and offer no other assistance.
4. If the student determines that necessary calculations are too much of a pain, full credit will be awarded for correctly stating what should be typed into a calculator and what it means. For example, the student can write "$257 \times 1.135 =$ the total cost" as a final answer. This assessment is about concepts, not longhand calculations.
5. Take as much time as you need. This assessment is long and will likely take more than one sitting.
6. Answer to the best of your ability, as you would on a test in school. Even if you're not sure what a good answer is or how to express it using standard math symbols, an explanation with words might still be enough to earn full credit. Put *all* your knowledge on the page.
7. After the student is done, the student and teacher should discuss the student's thought process before completing the scorecard. [See next page.] This can help determine if the student has conceptual understanding that they can't express on paper. i.e. They may understand concepts but lack mastery of math notation and vocabulary.

The Arithmetic Concept Inventory

Name: _____

Date: _____

0 = No evidence or insufficient evidence.
1 = Not meeting expectations.
2 = Partially meeting expectations.
3 = Meeting expectations.
4 = Exceeding expectations.

Math Area	**0**	**1**	**2**	**3**	**4**
Integer Concepts: Determining & Relating Operations					
Integer Concepts: Determining Signs of Integer Expressions					
Rational Number Concepts: Magnitudes					
Rational Number Concepts: Notations & Conversions					
Rational Number Concepts: Estimates & Word Problems					

Comments

1 (Mostly) Integer Concepts

1.1 Determining and Relating Operations

1. A bus drove 86km at 110km per hour. How long did the trip take? [Remember that you can earn full credit on every exercise for writing what you would enter into a calculator, but estimate the result if you can.]

2. Years ago, Jay had \$1590 saved and he set a goal to triple his savings. Now, he has \$2345 and wants to achieve his goal in 4 years. What math should he do and what will that math tell him?

3. What would you type into a calculator to determine the unknown value in the following? Maximum credit if you can express your answer in a way that only requires you to press the equals button once.

   $3158 \times ? = 21556$ $\quad ? - 984 = 32588$ $\quad ? - 984 = 32588$

   $154 = ? \div 256$ $\quad 66784 \div ? = 780$ $\quad 95 \times 6 - ? = 3158$

   $9844 = ? + 354$ $\quad 2447 = 34708 - ?$

4.  Complete the face with smiling lips to indicate high confidence in this subsection, frowning lips for low confidence, and horizontal lips to indicate something in between.

1.2 Determining Signs of Integer Expressions

Without calculating exactly, are the following positive (+) or negative (−)?

1. $-31 + 59 =$ _____ $59 - (-31) =$ _____ $0 - 31 + +59 =$ _____

   $(-59)(-31) =$ _____ $(+59) \times (31) =$ _____ $31 + -59 =$ _____

   $+31 \div (-59) =$ _____ $(31) - (+59) =$ _____ $-59 - 31 =$ _____

2. $-31 \times 59 \times 924 =$ _____ $(924) \times (-31) + (-59)) =$ _____

   $-31 \times (59 - 924) =$ _____ $924/59 - 31 =$ _____ [not quite integers...]

   $-31 + 59 \times 924 =$ _____ $59/924 - 31 =$ _____ [not quite integers...]

3. 

2 Rational Numbers Concepts and Notation

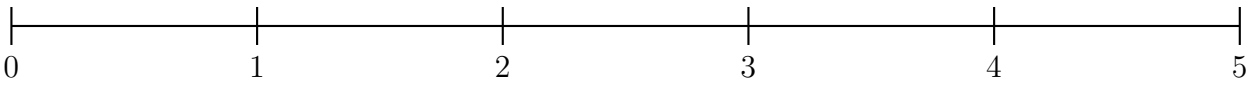
2.1 Division

Are $56 \div 3$ and $3 \div 56$ the same or different? If they're the same, explain why and create a single word problem that corresponds to both of them. If they are different, explain what that difference is and create a separate word problem for each.

2.2 Magnitudes

1. Approximate the locations of the following numbers on the number line.

$\frac{22}{9}$ 17% 3.2% 300.2% $3\frac{5}{6}$ $\frac{8}{2}$ $\frac{10}{19}$ 2.7 2.07 2.70 2.070 $\frac{156}{99}$



2. 

2.3 Rational Number Notations and Conversions

1. Complete the following chart.

Mixed or Whole	$3\frac{1}{2}$	$1000\frac{8}{9}$		
Improper			$\frac{29}{6}$	$\frac{397}{100}$

2. Pick any column of the chart and draw a diagram to show why your answer is correct.

3. Complete the following chart. Estimate to 2 decimal places as necessary.

Fraction or Whole	$\frac{1}{2}$	$\frac{9}{4}$	$\frac{9}{7}$						
Decimal				0.06	0.006	3.8			
Percent							400%	10.3%	0.2%

4. Circle all of the following expressions that mean the same thing as 20%.
   20 hundredths, 0.2, 0.200, 0.20, $\frac{20}{100}$, 0.020, 0.02, $\frac{1}{5}$, $\frac{200}{1000}$, $\frac{5}{1}$, 20, 20.0, one of five, $20 \div 100$, $\frac{2}{10}$

5. 

2.4 Rational Number Arithmetic & Word Problems

2.4.1 Word Problems and Short Answers

1. 31.5% of a farm is sold for $153,000.

   (a) Estimate the value of the whole farm. Use a diagram if it helps to explain your reasoning.

   (b) What would you type into a calculator to determine the exact value of the farm?

2. Rational Number Arithmetic Concepts

   (a) A class has 28 children. Approximately $\frac{2}{3}$ of them go on a field trip. Estimate how many children went on the field trip and how many did not.

   (b) Without calculating exactly, is the answer greater than 20? Explain.
   $\frac{6}{7} \times 19.01 =$ _____

   (c) Without calculating exactly, is the answer greater than 20? Explain.
   $21.29 \div \frac{1}{13} =$ _____

   (d) Evaluate exactly without longhand calculations. $285 - \frac{2}{9} =$ _____

3. Estimate. Then write what you would type into a calculator to get an exact answer.

   (a) Thirty five percent of sixty one is approximately _____.

   (b) _____ is about 90% of 827.

   (c) Two fifths of 81 is roughly _____.

   (d) 500 is about _____% of 409.

4. Joe thinks that 1.2 hours means either 1 hour and 20 minutes or 1 hour and 2 minutes, but isn't sure. What would you say to him?

5. Circle all of the following that are equal or approximately equal to "one third of 7.4".
   $7.4 \div 3$, $3 \div 7.4$, $\frac{1}{7.4} \times 3$, $\frac{7.4}{3}$, $33\% + 7.4$, $7.4 \times \frac{1}{3}$, $7.4 - (\frac{2}{3})(7.4)$, $7.4 \times 33\%$, $7.4 \div \frac{1}{3}$, $\frac{1}{3} \div 7.4$

6. 

2.4.2 Interpreting Arithmetic Expressions

Without performing any calculations, determine if the following expressions are relevant (R) to this situation. If yes, interpret them. If irrelevant (I), explain why.

1. An antique table was worth \$836. Now it's worth \$1154.

   (a) $836 + 1154$ R/I. _____

   (b) $1154 - 836$ R/I. _____

   (c) $1154 \div 836$ R/I. _____

   (d) $836 \div 1154$ R/I. _____

   (e) 836×1154 R/I. _____

   (f) $836 \div (836+1154)$ R/I. _____

   (g) $1154 \div 836 + 1154$ R/I. _____

   (h) $1 - \frac{836}{1154}$ R/I. _____

   (i) $836 - \frac{836}{1154}$ R/I. _____

2. Residents were polled for their opinion on new taxes to fund a new ambulance station. 836 people said they supported the resolution and 1154 said they opposed it.

   (a) $836 + 1154$ R/I. _____

   (b) $1154 - 836$ R/I. _____

   (c) $1154 \div 836$ R/I. _____

   (d) $836 \div 1154$ R/I. _____

   (e) 836×1154 R/I. _____

   (f) $836 \div (836+1154)$ R/I. _____

   (g) $1154 \div 836 + 1154$ R/I. _____

   (h) $1 - \frac{836}{1154}$ R/I. _____

   (i) $836 - \frac{836}{1154}$ R/I. _____

3. 

2.4.3 Open Analysis

1. In the 2015 Canadian federal election, about 68% of eligible citizens actually voted. About 19.7% of voters chose the New Democratic Party. Use these percents to state as many mathematically important facts about the 2015 election as you can. Then state other mathematically important facts about the election that you **cannot** determine from those percents.

   Remember that you can earn full credit for any number or expression as long as you state what it means and how you'd use a calculator to find it. You do *not* have to actually complete the calculation.

2.