# Dear Student and/or Teacher and/or Parent,

If a student is going to succeed in high school math and beyond, they need more than procedural diligence and a good memory. They need conceptual understanding. If they lack that understanding, the best time to find out is right now - or by the end of grade 8 at the latest - as it does not get easier as students progress to higher grades. Conceptual understanding is often weakest around arithmetic and introductory algebra. This assessment deals with algebra. [There is another for arithmetic.]

By excluding traditional procedures, this assessment should help detect shallow understanding where shallowness is most common. Such shallowness may have been hidden for years by procedural diligence and assessments that overemphasize that over mathematical abstraction.

The student may have many other areas of strength or weakness in math; this document only sheds light on whether or not the student struggles with general number sense and introductory algebra. Meltdowns and closed doors stemming from those topics are common; rarely do doors close because one can't determine the volume of a cylinder.

Please feel free to edit, use, and circulate as you like. I look forward to any feedback you can offer.

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# Instructions

1. Expect significant challenge because this assessment is far more abstract and, in some cases, less structured than many other math assessments.
2. Students require only a pencil and an eraser.
3. No help should be required from any person, calculator, textbook, computer, etc. A teacher should only ensure the student understands the instructions and offer no other assistance.
4. If the student determines that necessary calculations are too much of a pain, full credit will be awarded for correctly stating what should be typed into a calculator. For example, the student can write "$257 \times 1.135 =$ the total cost" as a final answer. This assessment is about concepts, not longhand calculations.
5. Take as much time as you need. This assessment is long and will likely take more than one sitting.
6. Answer to the best of your ability, as you would on a test in school. Even if you're not sure what a good answer is or how to express it using standard math symbols, an explanation with words might still be enough to earn full credit. Put *all* your knowledge on the page.
7. After the student is done, ideally, the student and teacher should discuss the student's thought process before completing the scorecard. [See next page.] This is crucial to ensure students get credit for conceptual understanding that they can express in conversation but not on paper. i.e. They may understand concepts but lack mastery of math notation and vocabulary.

# The Algebraic Concept Inventory

Name: _____

Date: _____

0 = No evidence or insufficient evidence.
1 = Not meeting expectations.
2 = Partially meeting expectations.
3 = Meeting expectations.
4 = Exceeding expectations.

| Math Area | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Algebra Concepts: Representing Relations | | | | | |
| Algebra Concepts: Preserving Equality & Structure | | | | | |

## Comments

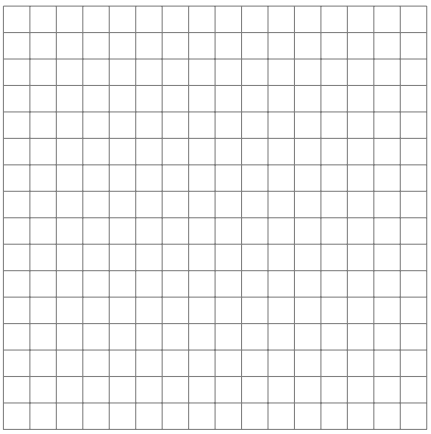
# 1 Representing Relations

1. Albert is 3 years older than his friend Ben.

   (a) Generate data in a table showing possible ages Ben and Albert could be.

   (b) In a variety of ways, use let statements and/or equations [or equations with entire words] to express the relationship of their ages. There are many possible answers. Maximum credit if you express both of their ages with the same letter variable. [Knowing this variety is necessary to get computers to do your calculations for you.]

   (c) Graph the relationship among their ages starting at either of their births. Label adequately.



   (d) 

2. Sue has \$1 in her bank account now. She saves \$2.25 more each day. [Reminder: If the numbers are a pain to calculate, simply write what you would enter into a calculator.]

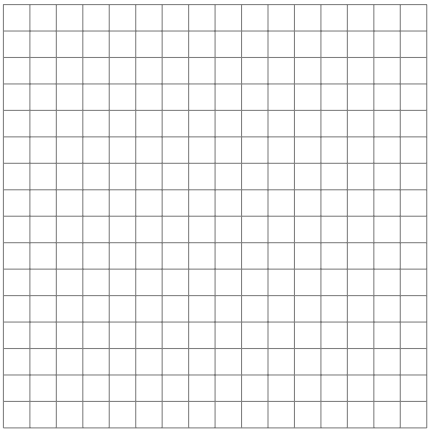
   (a) Generate data in a table for the first week of this situation. Begin the table at 0 days and \$1 to represent the present.

   (b) How much will she have in 5 weeks? In 50 weeks?

   (c) How long until she has \$337?

   (d) Create an equation relating the the number of days that pass to the amount of money in her account. How do you know that it works? [Even if an equation you create does not work, you are still demonstrating knowledge if you show how you know it does not work.]

   (e) Graph this situation. Label adequately.



(f) How could you determine if the point $(150, 400)$ is on your graph?

(g) 

# 2 Preserving Equality, Structure, and Function Notation

1. Which of the following mean the same thing as $a + b = c$?

| | | |
|---|---|---|
| $c = a + b$ | $b = c - a$ | $a + b - 2 = c - 2$ |
| $a + b + c = 0$ | $c - b = a$ | $a - c + b = 0$ |
| $b + a = c$ | $2 \times a + b = 2 \times c$ | $a + b^2 = c^2$ |
| $b - c = a$ | $c - a - b = 0$ | $a + b \div 2 = c \div 2$ |
| $-b = c - a$ | $a + b \div 2 = c \div 2$ | $b = c \div a$ |
| $0 = a + b - c$ | $2 \times a + b = 2 \times c$ | $(a + b)^3 = c \times c \times c$ |
| $-2 + a + b = c - 2$ | $\frac{a+b}{c} = 1$ | $-c + a + b = 0$ |

2. Are the equations following true or false? Explain. All letter variables represent non-zero real numbers.

   (a) $\sqrt{x^2 + y^2} = x + y$
   (b) $\frac{x+z}{y+z} = \frac{x}{y}$
   (c) $\frac{xz}{yz} = \frac{x}{y}$
   (d) $3n^2 = 3n \times 3n$
   (e) $-10^2 = 100$

3. Rewrite the equation from 2.(d) using function notation.

4.