The Topological Shiab: Replacing Algebraic Contraction with Persistent Homology in Geometric Unity

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February 2026

Abstract

Weinstein's Geometric Unity (GU) framework requires a "Shiab operator" $\urcorner$ that contracts curvature 2-forms on the total space Y^{14} into objects suitable for field equations, while preserving gauge covariance. Nguyen and Polya (2021) showed that this contraction is blocked by dimensionality constraints in the relevant Spin(14) representations. We propose replacing the algebraic Shiab operator with a topological one: a persistent homology filtration on gauge-invariant curvature data. The operator $\urcorner_{\mathrm{PH}} = \mathrm{PH} \circ \mathrm{Filt}$ builds a Rips filtration on plaquette action densities and extracts persistent homological features. We prove that this operator is (i) automatically gauge-invariant when built on closed-loop observables, (ii) free of dimensionality obstructions since persistence diagrams have no fixed target dimension, and (iii) stable under perturbation via the TDA stability theorem. We verify these claims computationally on SU(2) lattice gauge theory in $d = 2, 3, 4$ dimensions, demonstrating that $\urcorner_{\mathrm{PH}}$ distinguishes topologically distinct gauge field configurations (vacuum, instanton, disordered) with measurable bottleneck distances. The cost of this replacement is the loss of the Lagrangian framework: persistence diagrams are not differential forms. We discuss the implications for GU as either a physical theory or a structural theory of observation.

1 Introduction

Weinstein's Geometric Unity [1] begins with a 4-dimensional manifold X^4 and considers the bundle of metrics $Y^{14} \to X^4$, called the *Observerse*. The central innovation is that field content arises from the structure of Y rather than being imposed on X. An observer selects a section $\iota : X \to Y$, and this selection determines the physics they perceive.

The framework requires an operator—the *Shiab operator* $\urcorner$—that contracts curvature 2-forms in $\Omega^2(\mathrm{Ad})$ into objects that can appear in field equations, analogous to how the Ricci tensor contracts the Riemann tensor in general relativity. Critically, $\urcorner$ must preserve gauge covariance: the contraction must commute with gauge transformations.

Nguyen and Polya [2] identified a fundamental obstruction: for the structure group Spin(14), the required contraction is "impossible for basic dimensionality reasons." The representation theory of Spin(14) does not admit a map from 2-forms valued in the adjoint bundle to the target space needed for field equations. This is not a technical gap that more work might close; it is a structural impossibility in the algebraic framework as posed.

We propose a different kind of operator entirely. Rather than contracting tensor indices algebraically, we build a topological filtration on gauge-invariant curvature data and extract persistent homological features. The key insight is that gauge covariance, which must be *engineered* into an algebraic operator, comes *free* from a topological one: persistent homology is invariant under continuous deformations, and gauge transformations are continuous maps along the fiber.

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2 Background

2.1 The Shiab Operator in Geometric Unity

Let $Y^d \to X^n$ be the metric bundle with $d = n + n(n+1)/2$. For $n = 4$, $d = 14$. Let $\mathcal{A}$ denote the space of connections on the spinor bundle $S(Y)$, and let $F_A \in \Omega^2(\mathrm{Ad})$ denote the curvature of a connection A.

The GU field equations require a map

$$\daleth : \Omega^2(\mathrm{Ad}) \to \Omega^0(\mathrm{Ad}) \tag{1}$$

satisfying gauge equivariance: for any gauge transformation $g \in \mathcal{H}$,

$$\daleth(g \cdot F_A) = g \cdot \daleth(F_A). \tag{2}$$

Nguyen's obstruction: no such map exists as a linear algebraic contraction for the Spin(14) representations involved.

2.2 Lattice Gauge Theory

On a lattice Λ with spacing a, gauge fields are represented by *link variables* $U_\mu(x) \in G$ on each edge, where G is the gauge group. The discrete curvature is the *plaquette*:

$$P_{\mu\nu}(x) = U_\mu(x)\, U_\nu(x + \hat{\mu})\, U_\mu^\dagger(x + \hat{\nu})\, U_\nu^\dagger(x). \tag{3}$$

The local action density is

$$s(x, \mu, \nu) = 1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu\nu}(x). \tag{4}$$

Under a gauge transformation $G(x) \in G$ at each site, link variables transform as $U_\mu(x) \to G(x)\, U_\mu(x)\, G^\dagger(x + \hat{\mu})$, while plaquettes transform as $P_{\mu\nu}(x) \to G(x)\, P_{\mu\nu}(x)\, G^\dagger(x)$. Consequently, $\mathrm{Tr}\, P_{\mu\nu}$ and hence $s(x, \mu, \nu)$ are *exactly gauge invariant.*

2.3 Persistent Homology

Given a point cloud $\mathcal{P} \subset \mathbb{R}^n$, the Rips complex at scale ϵ is

$$\mathrm{Rips}_\epsilon(\mathcal{P}) = \{\sigma \subseteq \mathcal{P} : \mathrm{diam}(\sigma) \le \epsilon\}. \tag{5}$$

The nested family $\mathrm{Rips}_{\epsilon_1} \subseteq \mathrm{Rips}_{\epsilon_2} \subseteq \cdots$ is a *filtration.* Persistent homology tracks the birth and death of homological features (connected components, loops, voids) across the filtration, producing a *persistence diagram* PD_k for each dimension k.

The **stability theorem** [3]: if f, g are two functions inducing filtrations on the same space, then

$$d_B(\mathrm{PD}(f), \mathrm{PD}(g)) \le \|f - g\|_\infty, \tag{6}$$

where d_B is the bottleneck distance between persistence diagrams.

3 The Topological Shiab Operator

Definition 1 (Curvature Feature Map). *For a lattice gauge configuration on $\Lambda = \{1, \ldots, L\}^d$ with $\binom{d}{2}$ plaquette orientations, define the* curvature feature map

$$\phi : \Lambda \to \mathbb{R}^{\binom{d}{2}}, \quad \phi(x) = \big(s(x, \mu, \nu)\big)_{\mu < \nu}. \tag{7}$$

Each lattice site is mapped to a vector of its plaquette action densities across all orientations.

Definition 2 (Topological Shiab Operator)**.** *The* topological Shiab operator *is the composition*

$$\urcorner_{\mathrm{PH}} = \mathrm{PH}_k \circ \mathrm{Rips} \circ \phi \tag{8}$$

where ϕ extracts gauge-invariant curvature features, Rips *builds the Rips filtration on the image $\phi(\Lambda) \subset \mathbb{R}^{\binom{d}{2}}$, and PH_k extracts persistent homology in dimension k.*

The output is a persistence diagram $\mathrm{PD}_k \in \mathcal{D}$, where $\mathcal{D}$ is the space of persistence diagrams equipped with the bottleneck metric.

Remark 1. *The filtration parameter ϵ plays the role of a* scale coordinate along the fiber*. As ϵ increases, the Rips complex connects increasingly distant curvature configurations, analogous to probing larger-scale structure along the gauge orbit. This is the precise sense in which the filtration is "vertical"—it explores the space of curvature magnitudes rather than spatial proximity.*

4 Properties

Proposition 1 (Gauge Invariance)**.** *$\urcorner_{\mathrm{PH}}$ is exactly gauge invariant: for any gauge transformation g,*

$$\urcorner_{\mathrm{PH}}(g \cdot \{U_\mu\}) = \urcorner_{\mathrm{PH}}(\{U_\mu\}). \tag{9}$$

Proof. The plaquette action density $s(x, \mu, \nu) = 1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu\nu}(x)$ is gauge invariant because $\mathrm{Tr}(GPG^\dagger) = \mathrm{Tr}(P)$. Therefore $\phi(x)$ is identical before and after gauge transformation. Since the Rips complex and persistent homology are deterministic functions of the point cloud, $\urcorner_{\mathrm{PH}}$ produces identical output. □

Remark 2. *This is stronger than the equivariance condition* (2) *required by GU. The algebraic $\urcorner$ must* transform correctly *under gauge transformations; $\urcorner_{\mathrm{PH}}$ is* strictly invariant*—it does not transform at all. This is because it operates on gauge-invariant observables (closed loops) rather than on gauge-covariant objects (connections).*

Remark 3 (Critical constraint discovered computationally)**.** *The gauge invariance of $\urcorner_{\mathrm{PH}}$ depends on the input being restricted to gauge-invariant observables. In our initial implementation, we included link trace magnitudes $|\mathrm{Tr}\, U_\mu(x)|$ in the feature vector. These are* not *gauge invariant—under $U_\mu(x) \to G(x)U_\mu(x)G^\dagger(x+\hat{\mu})$ with $G(x) \neq G(x+\hat{\mu})$, the trace changes. Verification test V1 failed with feature discrepancies of order 1. Restricting to plaquette data (closed loops) restored exact invariance at machine precision ($\sim 10^{-16}$).*

This constraint has a geometric interpretation: $\urcorner_{\mathrm{PH}}$ must be built on holonomy *data (curvature), not on* connection *data (parallel transport). The filtration operates on the base of gauge-invariant observables, not on the total space of gauge-dependent fields.*

Proposition 2 (Dimensionality Freedom)**.** *$\urcorner_{\mathrm{PH}}$ has no fixed target dimension. It maps from any $\Omega^2(\mathrm{Ad})$ to the space of persistence diagrams $\mathcal{D}$, regardless of the dimension of X, the structure group G, or the representation.*

Proof. The Rips complex is defined for any point cloud in any $\mathbb{R}^n$. The number of curvature features per site is $\binom{d}{2}$, which varies with d but does not constrain the construction. Persistent homology is defined for simplicial complexes of any dimension. No index contraction or representation-theoretic compatibility is required. □

Corollary 1. *Nguyen's dimensionality obstruction does not apply to $\urcorner_{\mathrm{PH}}$. The impossibility of contracting Spin(14) 2-forms into the required target space is an obstruction specific to algebraic (linear) operators. Topological operators bypass this entirely.*

Proposition 3 (Stability). *$\urcorner_{\mathrm{PH}}$ is Lipschitz continuous: if two gauge configurations have curvature feature maps ϕ, ϕ' with $\|\phi - \phi'\|_\infty \leq \delta$, then*

$$d_B(\urcorner_{\mathrm{PH}}(\phi), \urcorner_{\mathrm{PH}}(\phi')) \leq C \cdot \delta \tag{10}$$

for a constant C depending on the lattice geometry.

Proof. By the stability theorem (6), the bottleneck distance between persistence diagrams is bounded by the L^∞ distance between the inducing functions. Here, the "inducing function" is the pairwise distance matrix on $\phi(\Lambda)$. A perturbation of δ in feature space produces a perturbation of at most $C\delta$ in pairwise distances (where C depends on the metric on $\mathbb{R}^{\binom{d}{2}}$), which bounds the bottleneck distance. □

5 Computational Verification

We implemented $\urcorner_{\mathrm{PH}}$ on SU(2) lattice gauge theory and ran five verification tests. SU(2) was chosen as the simplest non-abelian gauge group; the construction generalizes to any G.

5.1 Test V1: Gauge Invariance

Generate a random SU(2) configuration on $L = 4$, $d = 3$. Extract curvature features and compute persistence. Apply a random gauge transformation. Re-extract and re-compute. Measure bottleneck distance between pre- and post-transform diagrams.

Result: Over 5 trials, maximum feature discrepancy $< 10^{-15}$, maximum $d_B(H_0) < 10^{-15}$, maximum $d_B(H_1) < 10^{-15}$. Exact gauge invariance confirmed at machine precision. (Note: initial implementation with link traces failed at $\Delta \sim 1.7$; restricting to plaquettes resolved the failure.)

5.2 Test V2: Dimensionality Agnosticism

Run $\urcorner_{\mathrm{PH}}$ on lattices with $d = 2$ (1 plaquette orientation, $L = 4$), $d = 3$ (3 orientations, $L = 4$), and $d = 4$ (6 orientations, $L = 3$). All produced valid persistence diagrams with nontrivial features.

d	L	Sites	Features	β_0	β_1
2	4	16	1	11	0
3	4	64	3	64	13
4	3	81	6	81	36

Table 1: $\urcorner_{\mathrm{PH}}$ operates in any dimension with no obstruction.

5.3 Test V3: Topological Content Detection

Three SU(2) configurations on $L = 5$, $d = 3$: near-identity ("vacuum"), instanton-like (smooth winding), and hot (random). The persistence signatures are clearly distinguishable.

Bottleneck distances: $d_B(\text{vacuum}, \text{instanton}) = 0.024$, $d_B(\text{vacuum}, \text{hot}) = 0.189$, $d_B(\text{instanton}, \text{hot}) = 0.168$. The operator detects structure in gauge fields.

5.4 Test V4: Pipeline Composition

Gauge curvature $\to$ $\urcorner_{\mathrm{PH}}$ $\to$ persistence signature $\to$ schema check $\to$ spectral graph connectivity ($\Phi = 0.12 > 0$) $\to$ coherent configuration $C_t = \text{True}$. The operator composes into a downstream analysis pipeline without modification.

Config	β_0	β_1	max pers(H_1)	total pers(H_1)
Vacuum	4	0	0.006	0.052
Instanton	35	3	0.027	0.174
Disordered	125	35	0.195	2.771

Table 2: $\daleth_{\mathrm{PH}}$ discriminates topologically distinct gauge configurations.

5.5 Test V5: Stability

Perturbations of increasing magnitude $\epsilon \in \{0.001, 0.01, 0.05, 0.1, 0.3\}$ applied to link variables. Bottleneck distance remains bounded: $d_B \leq 0.07$ even at $\epsilon = 0.3$ (feature distance 0.65). Lipschitz stability confirmed.

6 What Is Lost

The replacement of $\daleth$ by $\daleth_{\mathrm{PH}}$ is not cost-free. Three consequences must be acknowledged.

6.1 Loss of the Lagrangian Framework

The algebraic $\daleth$ maps curvature forms to forms: $\Omega^2 \to \Omega^0$. This output can appear in a Lagrangian density $\mathcal{L}$, from which field equations follow via the variational principle. The topological $\daleth_{\mathrm{PH}}$ maps curvature data to persistence diagrams, which are not differential forms. They cannot be integrated over a manifold, varied, or inserted into an action functional.

This means: if GU is intended to recover the Einstein field equations and the Standard Model Lagrangian, $\daleth_{\mathrm{PH}}$ does not directly accomplish this. One would need an additional map from persistence diagrams back to differential-geometric objects—a "realization map" $\rho : \mathcal{D} \to \Omega^0(\mathrm{Ad})$. We do not construct such a map here.

6.2 Discretization Dependence

The lattice formulation introduces dependence on lattice spacing a and lattice size L. While the continuum limit $a \to 0$ is well-defined for standard lattice gauge theory, the persistence diagrams of the discretized theory need not converge to a well-defined continuum object. A rigorous continuum limit for $\daleth_{\mathrm{PH}}$ would require proving convergence of persistence diagrams as $a \to 0$, which is an open problem in computational topology.

6.3 SU(2) vs. the Standard Model

Our verification uses SU(2) on small lattices ($L \leq 5$). The Standard Model requires SU(3) $\times$ SU(2) $\times$ U(1) on lattices large enough to support physical observables. The construction generalizes formally, but computational feasibility and physical relevance at realistic scales are unverified.

7 Implications for Geometric Unity

The result forks depending on what GU is.

If GU is physics: The Shiab obstruction is circumvented but the Lagrangian is lost. This trades one problem for another. A program to recover field equations from topological data—perhaps via discrete Morse theory or combinatorial Hodge theory on the Rips complex—would be needed to make $\daleth_{\mathrm{PH}}$ physically productive.

If GU is a theory of observation: The Observerse $Y \to X$ as a model of observer-dependent reality, with sections ι as ontological selections, does not require a Lagrangian. It requires a way to extract invariant structure from observer-dependent data—which is exactly what $\urcorner_{\mathrm{PH}}$ does. The persistence diagram becomes the gauge-invariant "content" of a configuration, the Betti numbers become the structural signature, and the bottleneck metric provides a notion of similarity between configurations. Under this reading, $\urcorner_{\mathrm{PH}}$ resolves the Shiab problem without residual cost.

The question of which reading is correct is not mathematical. It is a question about the intended domain of the theory.

8 Conclusion

The algebraic Shiab operator required by Geometric Unity is blocked by representation-theoretic constraints. The topological replacement $\urcorner_{\mathrm{PH}} = \mathrm{PH} \circ \mathrm{Rips} \circ \phi$ circumvents this obstruction by changing the category of the operator from algebraic to topological. Gauge invariance follows from operating on closed-loop observables. Dimensionality freedom follows from persistence diagrams having no fixed target space. Stability follows from the TDA stability theorem.

The operator has been verified on SU(2) lattice gauge theory in $d = 2, 3, 4$, demonstrating gauge invariance at machine precision, discrimination of topologically distinct configurations, and Lipschitz continuity under perturbation.

The central trade-off is explicit: algebraic operators produce differential forms compatible with variational principles; topological operators produce persistence diagrams compatible with structural analysis. Whether this trade-off is acceptable depends on whether Geometric Unity is read as physics or as a theory of invariant structure extraction from observer-dependent data.

The construction is falsifiable. A lattice gauge theory group with access to production-scale SU(3) configurations with known instanton content could test whether $\urcorner_{\mathrm{PH}}$ recovers known topological charge and whether the persistence signature correlates with physical observables. We invite such tests.

References

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A Verification Suite Implementation

The complete verification suite is available as `shiab_verification.py`. It implements SU(2) lattice gauge theory via Pauli parameterization, plaquette computation, gauge transformation, and persistent homology via the GUDHI library. All tests are reproducible with `numpy` seed 42.

B The V1 Failure: A Constructive Constraint

The initial implementation included link trace magnitudes $|\mathrm{Tr}\, U_\mu(x)|$ alongside plaquette action densities in the curvature feature vector. This produced feature discrepancies of order $O(1)$ under gauge transformation, causing V1 to fail across all 5 trials.

The failure is instructive. Link traces are *open path* observables: the product $U_\mu(x)$ connects adjacent sites but does not close into a loop. Under $U_\mu(x) \to G(x)U_\mu(x)G^\dagger(x+\hat{\mu})$ with $G(x) \neq G(x+\hat{\mu})$, the trace is not preserved.

This establishes a constructive constraint on $\daleth_{\mathrm{PH}}$: **the filtration must be built on holonomy data (closed loops), not connection data (open paths)**. In the continuum limit, this corresponds to operating on curvature ($F_A = dA + A \wedge A$, which transforms homogeneously) rather than on the connection (A, which transforms inhomogeneously).

The constraint is not merely technical—it has geometric content. It says that $\daleth_{\mathrm{PH}}$ lives on the *reduced* configuration space $\mathcal{A}/\mathcal{H}$ (connections modulo gauge), accessed via gauge-invariant observables, rather than on the full configuration space $\mathcal{A}$.