BM3D : Python implementation

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Algorithm

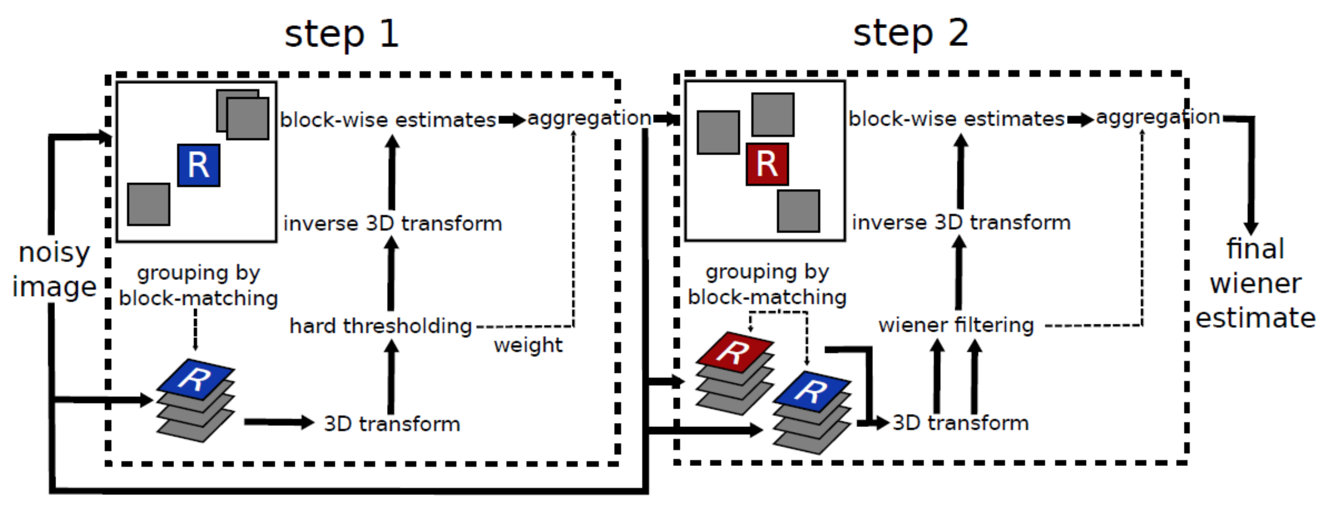

Figure 1: Scheme of the BM3D algorithm.

Step 1. Basic estimates
- Grouping
- Collaborative hard-thresholding
- Aggregation

Step 2. Final estimate
- Grouping
- Collaborative Wiener filtering
- Aggregation

Terminology

$$z(x) = y(x) + \eta(x),\ x \in X,$$

z(x) : noisy image
y(x) : true image
η(x) : noise $\eta(\cdot) \sim \mathcal{N}(0, \sigma^2)$

$N_1 \times N_1$: block size

Z_x : a block extracted from z; x is the coordinate of the top-left corner of the block

$\mathbf{Z}_S$: 3D array composed of blocks

Step 1. Collaborative Hard Thresholding

Grouping and Collaborative filtering – Grouping

Z_{x_R} currently processed image block, x_R ₩in X

$$d^{\text{ideal}}\left(Z_{x_R}, Z_x\right) = \frac{\left\| Y_{x_R} - Y_x \right\|_2^2}{\left(N_1^{\text{ht}}\right)^2}$$

ideal block distance, Y_x_R and Y_x are blocks of ideal image

$$d^{\text{noisy}}\left(Z_{x_R}, Z_x\right) = \frac{\left\| Z_{x_R} - Z_x \right\|_2^2}{\left(N_1^{\text{ht}}\right)^2}$$

block distance calculated from noisy blocks

If the blocks do not overlap, distance is a non-central chi-squared random variable

$$E\left\{d^{\text{noisy}}\left(Z_{x_R}, Z_x\right)\right\} = d^{\text{ideal}}\left(Z_{x_R}, Z_x\right) + 2\sigma^2$$

$$var\left\{d^{\text{noisy}}\left(Z_{x_R}, Z_x\right)\right\} = \frac{8\sigma^4}{\left(N_1^{\text{ht}}\right)^2} + \frac{8\sigma^2 d^{\text{ideal}}\left(Z_{x_R}, Z_x\right)}{\left(N_1^{\text{ht}}\right)^2}. \quad (3)$$

Grouping and Collaborative filtering – Grouping, Collaborative

$$d\left(Z_{x_R}, Z_x\right) = \frac{\left\| \Upsilon'\left(\mathcal{T}_{2D}^{ht}\left(Z_{x_R}\right)\right) - \Upsilon'\left(\mathcal{T}_{2D}^{ht}\left(Z_x\right)\right)\right\|_2^2}{\left(N_1^{ht}\right)^2}, \quad (4)$$

Υ' hard thresholding operator $\lambda_{2D}\sigma$ threshold $\mathcal{T}_{2D}^{ht}$ transform

$$S_{x_R}^{ht} = \left\{x \in X : d\left(Z_{x_R}, Z_x\right) \leq \tau_{match}^{ht}\right\}, \quad (5)$$

coordinate

τ_{match}^{ht} threshold $N_1^{ht} \times N_1^{ht} \times \left|S_{x_R}^{ht}\right|$ size $\mathbf{Z}_{S_{x_R}^{ht}}$ 3D block

$$\widehat{\mathbf{Y}}_{S_{x_R}^{ht}}^{ht} = \mathcal{T}_{3D}^{ht^{-1}}\left(\Upsilon\left(\mathcal{T}_{3D}^{ht}\left(\mathbf{Z}_{S_{x_R}^{ht}}\right)\right)\right), \quad (6)$$

Υ hard thresholding operator $\lambda_{3D}\sigma$ threshold $\mathcal{T}_{3D}^{ht}$ transform

Aggregation

$\sigma^2 N_{\text{har}}^{x_R}$ variance

$N_{\text{har}}^{x_R}$ number of non zero coefficients after ht

$$\widehat{\mathbf{Y}}_{S_{x_R}^{\text{ht}}}^{\text{ht}} = \mathcal{T}_{3D}^{\text{ht}^{-1}}\left(\Upsilon\left(\mathcal{T}_{3D}^{\text{ht}}\left(\mathbf{Z}_{S_{x_R}^{\text{ht}}}\right)\right)\right), \qquad (6)$$

aggregation weight

$$w_{x_R}^{\text{ht}} = \begin{cases} \frac{1}{\sigma^2 N_{\text{har}}^{x_R}}, & \text{if } N_{\text{har}}^{x_R} \geq 1 \\ 1, & \text{otherwise} \end{cases} \qquad (10)$$

global basic estimates

$$\widehat{y}^{\text{basic}}(x) = \frac{\sum_{x_R \in X} \sum_{x_m \in S_{x_R}^{\text{ht}}} w_{x_R}^{\text{ht}} \widehat{Y}_{x_m}^{\text{ht},x_R}(x)}{\sum_{x_R \in X} \sum_{x_m \in S_{x_R}^{\text{ht}}} w_{x_R}^{\text{ht}} \chi_{x_m}(x)}, \forall x \in X, \qquad (12)$$

$$\chi_{x_m} : X \rightarrow \{0,1\}$$

X(x) = 1 if and only if x ₩in Z,
0 otherwise.

Step 2. Collaborative Wiener Filtering

Grouping and Collaborative wiener filtering

set of coordinates

$$S_{x_R}^{\text{wie}} = \left\{ x \in X : \frac{\left\| \widehat{Y}_{x_R}^{\text{basic}} - \widehat{Y}_{x}^{\text{basic}} \right\|_2^2}{\left(N_1^{\text{wie}}\right)^2} < \tau_{\text{match}}^{\text{wie}} \right\}. \quad (7)$$

$\widehat{Y}_{x \in S_{x_R}^{\text{wie}}}^{\text{basic}}$ basic estimate blocks

group of the noisy blocks

$\mathbf{Z}_{S_{x_R}^{\text{wie}}}$ $Z_{x \in S_{x_R}^{\text{wie}}}$

empirical Wiener shrinkage coefficients

$$\mathbf{W}_{S_{x_R}^{\text{wie}}} = \frac{\left| \mathcal{T}_{3D}^{\text{wie}} \left(\widehat{\mathbf{Y}}_{S_{x_R}^{\text{wie}}}^{\text{basic}} \right) \right|^2}{\left| \mathcal{T}_{3D}^{\text{wie}} \left(\widehat{\mathbf{Y}}_{S_{x_R}^{\text{wie}}}^{\text{basic}} \right) \right|^2 + \sigma^2}. \quad (8)$$

group of block-wise estimates

$$\widehat{\mathbf{Y}}_{S_{x_R}^{\text{wie}}}^{wie} = \mathcal{T}_{3D}^{\text{wie}^{-1}} \left(\mathbf{W}_{S_{x_R}^{\text{wie}}} \mathcal{T}_{3D}^{\text{wie}} \left(\mathbf{Z}_{S_{x_R}^{\text{wie}}} \right) \right). \quad (9)$$

Aggregation

$\sigma^2 \left\| \mathbf{W}_{S^{\text{wie}}_{x_R}} \right\|_2^2$ variance $\mathbf{W}_{S^{\text{wie}}_{x_R}}$ wiener filter coefficients

$$\widehat{\mathbf{Y}}^{wie}_{S^{\text{wie}}_{x_R}} = \mathcal{T}_{3D}^{\text{wie}^{-1}} \left(\mathbf{W}_{S^{\text{wie}}_{x_R}} \mathcal{T}_{3D}^{\text{wie}} \left(\mathbf{Z}_{S^{\text{wie}}_{x_R}} \right) \right). \tag{9}$$

aggregation weight

$$w^{\text{wie}}_{x_R} = \sigma^{-2} \left\| \mathbf{W}_{S^{\text{wie}}_{x_R}} \right\|_2^{-2}, \tag{11}$$

final estimates

$$\widehat{y}^{\text{basic}}(x) = \frac{\sum_{x_R \in X} \sum_{x_m \in S^{\text{ht}}_{x_R}} w^{\text{ht}}_{x_R} \widehat{Y}^{\text{ht},x_R}_{x_m}(x)}{\sum_{x_R \in X} \sum_{x_m \in S^{\text{ht}}_{x_R}} w^{\text{ht}}_{x_R} \chi_{x_m}(x)}, \forall x \in X, \tag{12}$$

By replacing $w^{\text{ht}}_{x_R}$ with $w^{\text{wie}}_{x_R}$, and $\widehat{Y}^{\text{ht},x_R}_{x_m}(x)$ with $\widehat{\mathbf{Y}}^{wie}_{S^{\text{wie}}_{x_R}}$, we can get final estimates

	$\sigma \leq 40$	$\sigma > 40$
$N^{\mathbf{hard}}$	16	16
$N^{\mathbf{wien}}$	32	32
$\lambda_{3D}^{\mathbf{hard}}$	2.7	2.7
$\tau^{\mathbf{hard}}$	2500	5000
$\tau^{\mathbf{wien}}$	400	3500
$k^{\mathbf{hard}}$	8	8
$k^{\mathbf{wien}}$	8	8
$p^{\mathbf{hard}}$	3	3
$p^{\mathbf{wien}}$	3	3

Lebrun, M. (2012).

consts.py > ...

```python
sigma = 25
N_hard = 8 # block size
N_wien = 8 # block size
lambda_3d = 2.7 # 3d transform coefficient threshold
lambda_2d = 2.0 # 2d transform coefficient threshold
tau_hard = 2500 # 1st step distance threshold
tau_wien = 400 # 2nd step distance threshold
max_patch = 16
window_size = 39
speed_up = 3
```

Original

Noisy (σ=25)

Step 1.

Final

BM3D

Dabov, K., Foi, A., Katkovnik, V., & Egiazarian, K. (2007). Image denoising by sparse 3-D transform-domain collaborative filtering. *IEEE Transactions on image processing*, *16*(8), 2080-2095.

Lebrun, M. (2012). An analysis and implementation of the BM3D image denoising method. *Image Processing On Line*, *2012*, 175-213.