Ans1) $\int_0^{\pi/2} \sin^n x \, dx = \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdots \left(\frac{\pi}{2}\right)$

Put $n = 4$

$= \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$

$= \frac{3\pi}{16}$ (d)

Ans2) $\int_0^{\pi} \sin^4 x \cos^3 x \, dx$

$\int_0^{\pi} \sin^m x \cos^n x \, dx = 0$ if ~~odd~~ $n =$ odd

if $n =$ odd

$\therefore$ (a)

Ans3) $\int_0^{\pi/3} \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{(n-1)}{n} I_{n-2}$

$= \frac{\sqrt{3}}{n \cdot 2^n} + \frac{n-1}{n} I_{n-2}$ (b)

Ans9) $\int_0^{\pi} \sin^m x \cos^n x \, dx = \begin{cases} 2\int_0^{\pi/2} \sin^m x \cos^n x \, dx & \text{if } n = \text{even} \\ 0 & \text{if odd} \end{cases}$

since $n = 3$

$\therefore 0$

Ans5) $\int_0^{\pi}$ co

cos

Ans4)

Ans 5) $\int_0^{\pi} \cos^3 x \, dx$

$\cos(3x) = 4\cos^3 x - 3\cos x$

$\int_0^{\pi} \frac{3\cos x + \cos 3x}{4} \, dx$

$= \int_0^{\pi} \frac{3\cos x}{4} dx + \int_0^{\pi} \frac{\cos 3x}{4}$

$= \frac{3}{4} \sin x \Big]_0^{\pi} + \frac{\sin 3x}{12} \Big]_0^{\pi}$

$= 0$

(c)

Ans 4) $\int_0^{\infty} \frac{x^7}{7^x} \, dx$

let

$7^x = e^t$

$x \log 7 = t$

$dx = \frac{dt}{\log 7}$

$I = \int_0^{\infty} \left(\frac{t}{\log 7}\right)^7 e^{-t} \frac{dt}{\log 7}$

$= \frac{1}{(\log 7)^8} \Gamma 8 = \frac{7!}{(\log 7)^8} = \frac{5040}{(\log 7)^8}$

(d)

$\left(\frac{\pi}{2}\right)$

$n = $ odd

$\frac{(n-1)}{n} I_{n-2}$

$\cos^n x \, dx$

$n = $ even

odd

Ans 10) $erf(\infty) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2} du$

$= \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-y} \cdot y^{-1/2} dy$

$= \frac{1}{\sqrt{\pi}} \Gamma\frac{1}{2} = \frac{1}{\sqrt{\pi}} \sqrt{\pi} = 1$

(a)

Ans 6) $\int_0^1 x^{1/2} (1-x)^{5/2}$

Using

$\int_0^1 x^m (1-x)^n dx = \beta(m+1, n+1)$

$\therefore \left(\frac{1}{2}+1, \frac{5}{2}+1 \right) = \left(\frac{3}{2}, \frac{7}{2} \right)$

(A)

Ans 11) $e^{-u} = 1 - u + \frac{u^2}{2!} \cdots$

erf expansion

$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \left(1 - t^2 + \frac{t^4}{2!}\right)$

$= \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} \cdots \right)$

(B)

Ans 9)

Ans 12)

Ans 11) $I(\alpha) = \int_a^b f(x,a)$

$$= \int_a^b \frac{\partial (x,a)}{\partial \alpha}\, dx$$

$$= \lim_{\delta a \to 0} \frac{I(a+\delta a) - I(\alpha)}{\delta \alpha}$$

$$= \lim_{\delta \alpha \to 0} \frac{1}{\delta \alpha} \int_a^b [f(x, a+\delta a] - f(x,a)]\, dx$$

$$= \int_a^b \lim_{\delta \alpha \to 0} \left[\frac{f(x, a+\delta a) - f(x,a)}{\delta \alpha}\right] dx$$

$$\frac{dI(\alpha)}{d\alpha} = \int_a^b \frac{\partial}{\partial \alpha}(f(x,a))\, dx$$

(d)

Ans 12) $\frac{d}{dx}(\mathrm{erf}(ax)) = ?$

$$\frac{d}{dx}\left(\frac{2}{\sqrt{\pi}}\int_0^{ax} e^{-u^2}\, du\right)$$

$$= 0 + \frac{2}{\sqrt{\pi}}(a)\, e^{-a^2x^2} + 0$$

$$= \frac{2a\, e^{-a^2x^2}}{\sqrt{\pi}}$$

REDMI K20 PRO
AI TRIPLE CAMERA

Ans7) $\int_a^b (x-a)^m (b-x)^n \, dx$

comparing with

$$\int_0^1 t^{m-1}(1-t)^{n-1}\,dt = \beta(m,n)$$

Substituting

$(x-a) = (b-a)t$

we get same

$\therefore$ (A)

Ans8) $\int_0^\infty x^8 e^{-x^6}\,dx$

Gamma f^n : $\int_0^\infty x^{n-1} e^{-x}\,dx$

$= \Gamma n$

$x^6 \to t$

$6x^5\,dx \to dt$

$$\frac{1}{6}\int_0^\infty \sqrt{t}\, e^{-t}\,dt = \frac{1}{6}\int_0^\infty t^{1/2} e^{-t}\,dt$$

So, $n-1 = \frac{1}{2}$

$n = \frac{3}{2}$

$$= \frac{1}{6}\Gamma\frac{3}{2} = \frac{1}{6}\times\frac{1}{2}\Gamma\frac{1}{2} = \frac{1\sqrt{\pi}}{6\times 2} = \frac{\sqrt{\pi}}{12}$$

(B)

Ans 18) $\beta(m, n+1) + \beta(m+1, n) = ?$

$$\frac{\Gamma m\, \Gamma n+1}{\Gamma m+n+1} + \frac{\Gamma m+1\, \Gamma n}{\Gamma m+n+1}$$

$$\frac{\Gamma m\, n\Gamma n + m\Gamma m\, \Gamma n}{\Gamma m+n+1}$$

$$\frac{\Gamma m\, \Gamma n\, (m+n)}{\Gamma m+n+1} = \frac{\Gamma m\, \Gamma n\, (m+n)}{(m+n)\Gamma m+n}$$

$$\frac{\Gamma m\, \Gamma n}{\Gamma m+n} = \beta(m, n)$$

(B)

Ans 17) $erf(-x) + erfc(-x) = ?$

$\cancel{erf(-x)} + 1 - \cancel{erf(-x)}$

As, $erfc(-x) = 1 - erf(-x)$

0

(C)

Ans 4). $I_n = \frac{n(n-1)}{n^2+p^2} I_{n-2}$

$I_0 = \int_0^\infty e^{-px} \sin^6 x\, dx = 1/p$

$= \frac{n(n-1)}{n^2+p^2} I_0 = \frac{2\times 2}{4+4} \times \frac{1}{2} = \frac{1}{8}$ (B)

Page No.
Date

Ans 7) $\Gamma m \, \Gamma\left(m+\frac{1}{2}\right) = \dfrac{\sqrt{\pi}\,\Gamma(2m)}{2^{2m-1}}$

Put $m = \frac{1}{4}$

$$\therefore \Gamma\tfrac{1}{4}\,\Gamma\tfrac{3}{4} = \frac{\sqrt{\pi}\,\Gamma 2(1/4)}{2^{2(1/4-1)}} = \frac{\sqrt{\pi}\,\Gamma\frac{1}{2}}{2^{-1/2}}$$

$$= \pi\sqrt{2}$$

$\therefore$ (A)

Ans 18) $\displaystyle\int_0^{\infty} \frac{dx}{(1+x)^4}\, dx$

Put $x = \tan^2\theta$
$dx = 2\tan\theta \sec^2\theta \, d\theta$

$$= \int_0^{\pi/2} \frac{2\tan\theta\sec^2\theta}{\sec\theta} \, d\theta = \frac{[illegible]}{2}$$

Put $\sec\theta = y$
$\sec\theta\tan\theta \, d\theta = dy$

$$\int \frac{2\,dy}{y^7}$$

$$= \left[\frac{2y^{-6}}{-6}\right] = \frac{-1}{3y^6}$$

$$= \frac{-1}{3\sec\theta^6} = -\frac{1}{3}\cos^6\theta$$

Ans 8)

Ans 15)

$= \frac{\sqrt{\pi}\,\Gamma\frac{1}{2}}{2^{-1/2}}$

θ

$\frac{\theta}{2}$.

$+$

$\frac{1}{3y^6}$

$\log^6\theta$

$]$

Ans(8) $I = \int_0^\infty \frac{dx}{1+x^4}$

Put

Ans(5) $\phi(a) = \int_0^1 \frac{x^a - x^b}{\log x}\,dx$

$\phi'(x) = \int_0^1 \frac{d}{da}\frac{(x^a - x^b)}{\log x}\,dx.$

$\phi'(x) = \int_0^1 x^a \frac{\log x}{\log x}\,dx$

$\phi'(x) = \int_0^1 x^a\,dx$

(D)