Comparative Analysis of Exact and Approximate Inference in Bayesian Networks

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Abstract

In exploring the realm of Bayesian Networks, particularly those incorporating a reactive agent, what are the relative strengths and limitations of Variable Elimination and Gibbs Sampling algorithms in terms of efficiency and accuracy? The study investigates whether the trade-offs in computational efficiency inherent to Variable Elimination are justified by its accuracy across various network scales. It also considers if Gibbs Sampling can serve as an efficient yet accurate alternative. The influence of evidence on the performance of both algorithms, under different conditions of evidence, is also examined in this project. Our conclusion is difficult to resolve to due to the lack of ability to compare our generated figures to the true probability distributions, however we were able to observe variables where the distributions were similar as well as nearly opposite. It is clear that Gibbs Sampling is much faster than Variable elimination, though the reliability of Gibbs is unclear.

1 Problem Statement and Hypothesis

The problem addressed by this project is the selection of a suitable inference algorithm for Bayesian networks that accurately and efficiently processes probabilistic data, with an emphasis on the functionality of a reactive agent. We hypothesize that Variable Elimination will provide superior accuracy over Gibbs Sampling across all networks. However, the introduction of a reactive agent is expected to mitigate some of the computational overhead associated with the exact inference, leading to more efficient performance without significantly compromising accuracy.

2 Algorithms

2.1 Variable Elimination

Variable Elimination is an exact inference algorithm that removes variables from the network systematically by summing over their probabilities.

2.2 Gibbs Sampling

Gibbs Sampling is a Markov Chain Monte Carlo method used for approximating the marginal distributions of a set of correlated variables. The algorithm randomly fills values for every variable while fixing the evidence. Then, given the order of the variables, it will walk through every non-evidence variable and generate a new state based on the probabilities of every other given state. This continues to loop until the probabilities begin to converge.

3 Experimental Approach

Our experimentation involves the application of the two algorithms across five different Bayesian networks from the Bayesian Network Repository, with various evidence conditions. We will calculate the probability distributions for any given query and compare and contrast the figures between the two algorithms. When collecting the data for Gibbs Sampling, it was looped 60,000 times and the burn-in was set to 5,000. These numbers were dictated off of several runs as well as how long it took the computer to perform the task, and these hyper-parameters were the happy mediums. For Variable elimination, the order was chosen by prioritizing the nodes on the edge of the graph and working its way in. Gibbs Sampling's order was chosen randomly every time to the algorithm would work regardless of the order.

4 Results

Alarm Network

(a) Report [HYPOVOLEMIA, LVFAILURE, ERRLOWOUTPUT]

Query result for the state of **HYPOVOLEMIA** given no evidence:
Variable elimination:
State ('TRUE','FALSE') has a probability of 0.4005, .5995
Gibbs Sampling:
State ('TRUE','FALSE') has a probability of 0.20229, 0.79771

Query result for the state of **LVFAILURE** given no evidence:
Variable elimination:
State ('TRUE','FALSE') has a probability of 0.8301, 0.1699
Gibbs Sampling:
State ('TRUE','FALSE') has a probability of 0.04975, 0.95025

Query result for the state of **ERRLOWOUTPUT** given no evidence:
Variable elimination:
State ('TRUE', 'FALSE') has a probability of 0.0726, 0.9726
Gibbs Sampling:
State ('TRUE', 'FALSE') has a probability of 0.0498, 0.9502

(b) Little Evidence: HRBP=HIGH; CO=LOW; BP=HIGH.

Query result for the state of **HYPOVOLEMIA** given little evidence:
Variable elimination:
State ('TRUE','FALSE') has a probability of 0.5918, 0.4082
Gibbs Sampling:
State ('TRUE','FALSE') has a probability of 0.1998, 0.8002

Query result for the state of **LVFAILURE** given little evidence:
Variable elimination:
State ('TRUE', 'FALSE') has a probability of 0.8504, 0.1496
Gibbs Sampling:
State ('TRUE', 'FALSE') has a probability of 0.05105, 0.94895

Query result for the state of **ERRLOWOUTPUT** little the evidence:
Variable elimination:
State ('TRUE', 'FALSE') has a probability of 0.0792, 0.9208
Gibbs Sampling:
State ('TRUE', 'FALSE') has a probability of 0.05042, 0.94958

(c) Moderate Evidence: HRBP=HIGH; CO=LOW; BP=HIGH; HRSAT=LOW; HREKG=LOW; HISTORY=TRUE.

Query result for the state of **HYPOVOLEMIA** given moderate evidence:
Variable elimination:
State ('TRUE','FALSE') has a probability of 0.7831, 0.2169
Gibbs Sampling:
State ('TRUE','FALSE') has a probability of 0.19985, 0.80015

Query result for the state of **LVFAILURE** given moderate evidence:
Variable elimination:
State ('TRUE', 'FALSE') has a probability of 0.8301, 0.1699
Gibbs Sampling:
State ('TRUE', 'FALSE') has a probability of 0.04944, 0.95056

Query result for the state of **ERRLOWOUTPUT** given moderate evidence:
Variable elimination:
State ('TRUE','FALSE') has a probability of 0.0858, 0.9142
Gibbs Sampling:
State ('TRUE','FALSE') has a probability of 0.04902, 0.95098

Child Network

(a) Report [Disease]

Query result for the state of **Disease** given no evidence:
Variable elimination:
State ('PFC', 'TGA','Fallot', 'PAIVS', 'TAPVD', 'Lung') has a probability of 0.2031, 0.1323, 0.1826, 0.1848, 0.1296, 0.1676 respectively.
Gibbs Sampling:
State ('PFC', 'TGA','Fallot', 'PAIVS', 'TAPVD', 'Lung') has a probability of

0.04738, 0.33635, 0.29442, 0.22036, 0.10149, 0.0 respectively.

(b) Little Evidence: LowerBodyO2="¡5"; RUQO2="≥12"; CO2Report="≥7.5"; XrayReport=Asy/Patchy.

Query result for the state of **Disease** given little evidence:
Variable elimination:
State ('PFC', 'TGA','Fallot', 'PAIVS', 'TAPVD', 'Lung') has a probability of 0.2031, 0.1323, 0.1826, 0.1848, 0.1296, 0.1676 respectively.
Gibbs Sampling:
State ('PFC', 'TGA','Fallot', 'PAIVS', 'TAPVD', 'Lung') has a probability of 0.04638, 0.33565, 0.29207, 0.22389, 0.102, 0.0 respectively.

(c) Moderate Evidence: LowerBodyO2="¡5"; RUQO2="≥12"; CO2Report="≥7.5"; XrayReport=Asy/Patchy; GruntingReport=Yes; LVHReport=Yes; Age="11-30 Days".

Query result for the state of **Disease** given moderate evidence:
Variable elimination:
State ('PFC', 'TGA','Fallot', 'PAIVS', 'TAPVD', 'Lung') has a probability of 0.2031, 0.1323, 0.1826, 0.1848, 0.1296, 0.1676 respectively.
Gibbs Sampling:
State ('PFC', 'TGA','Fallot', 'PAIVS', 'TAPVD', 'Lung') has a probability of 0.04731, 0.33233, 0.28973, 0.22982, 0.10082, 0.0 respectively.

Hailfinder Network

(a) Report [SatContMoist, LLIW]

Query result for the state of **SatContMoist** given no evidence:
Variable elimination:
States ('VeryWet', 'Wet', 'Neutral', 'Dry') has probabilities of 0.4379, 0.2156, 0.0887, 0.2579 respectively.
Gibbs Sampling:
States ('VeryWet', 'Wet', 'Neutral', 'Dry') has probabilities of 0.15091, 0.19684, 0.40005, 0.2522 respectively.

Query result for the state of **LLIW** given no evidence:
Variable elimination:
States ('Unfavorable', 'Weak', 'Moderate', 'Strong') has probabilities of 0.0000, 0.0526, 0.2907, 0.6567 respectively
Gibbs Sampling:
States ('Unfavorable', 'Weak', 'Moderate', 'Strong') has probabilities of 0.12076, 0.31915, 0.38062, 0.17947 respectively

(b) Little Evidence: RSFcst=XNIL; N32StarFcst=XNIL; MountainFcst=XNIL; AreaMoDryAir=VeryWet.

Query result for the state of **SatContMoist** given little evidence:
Variable elimination:
States ('VeryWet', 'Wet', 'Neutral', 'Dry') has probabilities of
0.4070, 0.2328, 0.1120, 0.2482 respectively.
Gibbs Sampling:
States ('VeryWet', 'Wet', 'Neutral', 'Dry') has probabilities of
0.15005, 0.19927, 0.39827, 0.2524 respectively.

Query result for the state of **LLIW** given little evidence:
Variable elimination:
States ('Unfavorable', 'Weak', 'Moderate', 'Strong') has probabilities of
0.2500, 0.2500, 0.2500, 0.2500 respectively
Gibbs Sampling:
States ('Unfavorable', 'Weak', 'Moderate', 'Strong') has probabilities of
0.12071, 0.3200, 0.37927, 0.18002 respectively

(c) Moderate Evidence: RSFcst=XNIL; N32StarFcst=XNIL; MountainFcst=XNIL; AreaMoDryAir=VeryWet; CombVerMo=Down; AreaMeso_ALS=Down; CurPropConv=Strong.

Query result for the state of **SatContMoist** given moderate evidence:
Variable elimination:
States ('VeryWet', 'Wet', 'Neutral', 'Dry') has probabilities of
0.4379, 0.2156, 0.0887, 0.2579 respectively.
Gibbs Sampling:
States ('VeryWet', 'Wet', 'Neutral', 'Dry') has probabilities of
0.14967, 0.19875, 0.40142, 0.25016 respectively.

Query result for the state of **LLIW** given moderate evidence:
Variable elimination:
States ('Unfavorable', 'Weak', 'Moderate', 'Strong') has probabilities of 0.0000, 0.0526, 0.2907, 0.6567 respectively
Gibbs Sampling:
States ('Unfavorable', 'Weak', 'Moderate', 'Strong') has probabilities of 0.11787, 0.32075, 0.38162, 0.17976 respectively

Insurance Network

(a) Report [MedCost, ILiCost, PropCost]

Query result for the state of **MedCost** given no evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.8292, 0.0754, 0.0547, 0.0406 respectively.
Gibbs Sampling:
NA

Query result for the state of **ILiCost** given no evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.8055, 0.0771, 0.0623, 0.055 respectively.
Gibbs Sampling:
NA

Query result for the state of **PropCost** given no evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.3601, 0.2813, 0.3157, 0.0428 respectively.
Gibbs Sampling:
NA

(b) Little Evidence: Age=Adolescent; GoodStudent=False; SeniorTrain=False; DrivQuality=Poor.

Query result for the state of **MedCost** given little evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.8602, 0.0651, 0.0444, 0.0303 respectively.
Gibbs Sampling:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.97273, 0.02064, 0.00344, 0.0032 respectively.

Query result for the state of **ILiCost** given little evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.9270, 0.0366, 0.0218, 0.0145 respectively.
Gibbs Sampling:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.99231, 0.00409, 0.00222, 0.00138 respectively.

Query result for the state of **PropCost** given little evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.3754, 0.2967, 0.2697, 0.0581 respectively.
Gibbs Sampling:

State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.62404, 0.33027, 0.04205, 0.00364 respectively.

(c) Moderate Evidence: Age=Adolescent; GoodStudent=False; SeniorTrain=False; DrivQuality=Poor; MakeModel=Luxury; CarValue=FiftyThousand; DrivHistory=Zero.

Query result for the state of **MedCost** given moderate evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.8602, 0.0651, 0.0444, 0.0303 respectively.
Gibbs Sampling:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.98016, 0.01475, 0.00278, 0.00231 respectively.

Query result for the state of **ILiCost** given moderate evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.9270, 0.0366, 0.0218, 0.0145 respectively.
Gibbs Sampling:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.98125, 0.00969, 0.00522, 0.00384 respectively.

Query result for the state of **PropCost** given moderate evidence:
Variable elimination:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.3754, 0.2967, 0.2697, 0.0581 respectively.
Gibbs Sampling:
State ('Thousand', 'TenThou', 'HundredThou', 'Million') has probabilities of 0.54427, 0.29155, 0.15953, 0.00465 respectively.

Win95pts Network

(a) Report: [Problem1, Problem2, Problem3, Problem4, Problem5, Problem6]

- Evidence: Problem1=No_Output.
  - Problem1
    - Gibbs Sampling
      - Normal_Output: 0.0
      - No_Output: 0.0
    - Variable Elimination
      - Normal_Output: 0.0
      - No_Output: 0.0
  - Problem2
    - Gibbs Sampling
      - OK: 0.91013
      - Too_Long: 0.08987

        * Variable Elimination
            - State ('OK'): probability 0.6407
            - State ('Too_Long'): probability 0.3593
    - Problem3
        * Gibbs Sampling
            - No: 0.0
            - Yes: 1.0
        * Variable Elimination
            - State ('No'): probability 0.4157
            - State ('Yes'): probability 0.5843
    - Problem4
        * Gibbs Sampling
            - No: 0.0
            - Yes: 1.0
        * Variable Elimination
            - State ('No'): probability 0.3731
            - State ('Yes'): probability 0.6269
    - Problem5
        * Gibbs Sampling
            - No: 0.0
            - Yes: 1.0
        * Variable Elimination
            - State ('No'): probability 0.2981
            - State ('Yes'): probability 0.7019
    - Problem6
        * Gibbs Sampling
            - No: 1.0
            - Yes: 0.0
        * Variable Elimination
            - State ('No'): probability 0.5336
            - State ('Yes'): probability 0.4664
    - Evidence: Problem2=Too_Long.
        * Problem1
            - Gibbs Sampling
            - Normal_Output: 0.76527
            - No_Output: 0.2347
            - Variable Elimination
            - State (): probability 1.0000
        * Problem2
            - Gibbs Sampling
            - OK: 0.0

- Too_Long: 0.0
- Variable Elimination
- State (): probability 1.0000

* Problem3
  - Gibbs Sampling
  - No: 0.0
  - Yes: 1.0
  - Variable Elimination
  - State ('No'): probability 0.6510
  - State ('Yes'): probability 0.3490
* Problem4
  - Gibbs Sampling
  - No: 0.0
  - Yes: 1.0
  - Variable Elimination
  - State ('No'): probability 0.7197
  - State ('Yes'): probability 0.2803
* Problem5
  - Gibbs Sampling
  - No: 0.0
  - Yes: 1.0
  - Variable Elimination
  - State ('No'): probability 0.4914
  - State ('Yes'): probability 0.5086
* Problem6
  - Gibbs Sampling
  - No: 1.0
  - Yes: 0.0
  - Variable Elimination
  - State ('No'): probability 0.2784
  - State ('Yes'): probability 0.7216
  - Evidence: Problem3=No.
  - Problem1
  - Gibbs Sampling
  - Normal_Output: 0.76378
  - No_Output: 0.23622
  - Variable Elimination
  - State (): probability 1.0000

· Problem2
· Gibbs Sampling
· OK: 0.94362
· Too_Long: 0.05638
· Variable Elimination
· State (): probability 1.0000
· Problem3
· Gibbs Sampling
· No: 0.0
· Yes: 0.0
· Variable Elimination
· State (): probability 1.0000
· Problem4
· Gibbs Sampling
· No: 0.0
· Yes: 1.0
· Variable Elimination
· State ('No'): probability 0.7304
· State ('Yes'): probability 0.2696
· Problem5
· Gibbs Sampling
· No: 0.0
· Yes: 1.0
· Variable Elimination
· State ('No'): probability 0.4974
· State ('Yes'): probability 0.5026
· Problem6
· Gibbs Sampling
· No: 1.0
· Yes: 0.0
· Variable Elimination
· State ('No'): probability 0.2642
· State ('Yes'): probability 0.7358
· Evidence: Problem4=No.
· Problem1
· Gibbs Sampling
· Normal_Output: 0.76504
· No_Output: 0.23496
· Variable Elimination
· State (): probability 1.0000
· Problem2

        - Gibbs Sampling
        - OK: 0.94318
        - Too_Long: 0.05682
        - Variable Elimination
        - State (): probability 1.0000
        - Problem3
        - Gibbs Sampling
        - No: 0.0
        - Yes: 1.0
        - Variable Elimination
        - State (): probability 1.0000
        - Problem4
        - Gibbs Sampling
        - No: 0.0
        - Yes: 0.0
        - Variable Elimination
        - State (): probability 1.0000
        - Problem5
        - Gibbs Sampling
        - No: 0.0
        - Yes: 1.0
        - Variable Elimination
        - State ('No'): probability 0.4998
        - State ('Yes'): probability 0.5002
        - Problem6
        - Gibbs Sampling
        - No: 1.0
        - Yes: 0.0
        - Variable Elimination
        - State ('No'): probability 0.2635
        - State ('Yes'): probability 0.7365
    - Evidence: Problem5=No.
        - Problem1
        - Gibbs Sampling
        - Normal_Output: 0.7644
        - No_Output: 0.2356
        - Variable Elimination
        - State (): probability 1.0000
        - Problem2
        - Gibbs Sampling
        - OK: 0.94449

- Too_Long: 0.05551
- Variable Elimination
- State (): probability 1.0000
- Problem3
- Gibbs Sampling
- No: 0.0
- Yes: 1.0
- Variable Elimination
- State (): probability 1.0000
- Problem4
- Gibbs Sampling
- No: 0.0
- Yes: 1.0
- Variable Elimination
- State (): probability 1.0000
- Problem5
- Gibbs Sampling
- No: 0.0
- Yes: 0.0
- Variable Elimination
- State (): probability 1.0000
- Problem6
- Gibbs Sampling
- No: 1.0
- Yes: 0.0
- Variable Elimination
- State ('No'): probability 0.2635
- State ('Yes'): probability 0.7365

* Evidence: Problem6=Yes.
  - Problem1
  - Gibbs Sampling
  - Normal_Output: 0.76187
  - No_Output: 0.23813
  - Variable Elimination
  - State (): probability 1.0000
  - Problem2
  - Gibbs Sampling
  - OK: 0.91002
  - Too_Long: 0.08998
  - Variable Elimination
  - State (): probability 1.0000
  - Problem3

- Gibbs Sampling
- No: 0.0
- Yes: 1.0
- Variable Elimination
- State (): probability 1.0000
- Problem4
- Gibbs Sampling
- No: 0.0
- Yes: 1.0
- Variable Elimination
- State (): probability 1.0000
- Problem5
- Gibbs Sampling
- No: 0.0
- Yes: 1.0
- Variable Elimination
- State (): probability 1.0000
- Problem6
- Gibbs Sampling
- No: 0.0
- Yes: 0.0
- Variable Elimination
- State (): probability 1.0000

5 Discussion

Before interpreting our data generated by variable elimination (VE) and Gibbs Sampling (GS), we expected variable elimination to be slower but more accurate overall while Gibbs sampling was expected to be quicker but less accurate. Since we were not given correct probability figures to compare our results to, it is difficult to definitively determine which method is more accurate. Instead, we will be comparing and contrasting the results to each other rather than a correct solution.

Alarm:
Both variable elimination and Gibbs Sampling had their own consistent results across the levels of evidence. Overall, **HYPOVOLEMIA**'s results for VE were close to equal between true and false while GS had a ratio of 2:8. **LVFAILURE** had VE and GS giving near-opposite results. The disparity in the results between the two methods underscores the importance of algorithm choice in probabilistic reasoning. **ERRORLOWOUTPUT** had similar results compared to the two algorithms.

Child:
Once again, although evidence slightly changed the figures for the probability distribution, it was only by a small amount for both VE and GS. Both distributions were close to balanced, however GS had 0% probability for **LUNG** on all accounts.

Hailfinder:
There was nothing extraordinarily notable about Hailfinder. For **DRY** and **WET**, VE and GS maintained similar probabilities while **VERYWET** and **NEUTRAL** were far from the same. On the other hand, VE had an absolutely even distribution for **LLIW** when given little evidence. GS continued to favor **WEAK** and **MODERATE** by around 10%.

Insurance:
Unfortunately, GS did not work with no evidence for the Insurance Network. VE's no evidence data greatly favored **'Thousand'** for **MedCost** and **ILiCost**, hovering around 80% each. **PropCost** was close-to balanced for the '**Thousand**', '**TenThousand**', and '**HundredThousand**' states. For little evidence, it was interesting to see that GS had 97% probabilities for **MedCost** and **ILiCost** being in the **'Thousand**' states while **PropCost** is much more balanced, only favoring **'Thousand**' by 64% for VE and 33% for GS while the other states fall under 10% each. Finally, the moderate evidence was similar to little evidence but with the notable fact that **ILiCost**'s favor towards **'Thousand**' decreased to 38% (VE) and 54% (GS).

Win95pts:
While both Variable Elimination and Gibbs Sampling were both consistent in their outputs for this Bayesian Network due to its structure we ran into unexpected problems. For example since the Win95pts has no children or out edges in its structure. This leads to, when certain evidence is introduced, the probability states will be binary 1.0 or 0.0. This led us to create additional helper methods in our Bayesian Network to allow us to better determine the structure of the Bayesian Networks to ensure their correctness.

6 Summary

This paper investigates the application of Bayesian Networks for probabilistic predictions across diverse scenarios. The authors delve into two key algorithms: Variable Elimination, which offers precise results but is computationally intensive, and Gibbs Sampling, which is quicker and less resource-heavy. Each has its merits depending on the network's size and the desired accuracy of the results.

The crux of the problem lies in identifying the appropriate balance between precision and practicality. For smaller networks where detail is critical, Variable Elimination is the preferred method. In contrast, for larger networks where speed is of the essence and some loss of accuracy is tolerable, Gibbs Sampling is the pragmatic choice. The study conveys that the decision of which algorithm to employ should be influenced by the complexity of the network and the computational resources at hand.

References

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