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UNIVERSITY EXAMINATIONS 2021/2022

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN EDUCATION SCIENCE

SMA 3402: TOPOLOGY I

DATE: MAY 2022 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Given that A, B and C are subsets of a universal set u. prove that

i. $An(BuC) = (AnB)u(AnC)$. (3 Marks)

ii. $A - (BuC) = (A - B)n(A - C)$. (3 Marks)

b) Prove that $A = \{2,3,4,5\}$ is not a subset of $B = \{x: x \text{ is even}\}$. (2 Marks)

c) Find the power set $p(s)$ of $s = \{3, (1,4)\}$. (3 Marks)

d) Consider the topology $\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ on $X = \{a,b,c,d,e\}$. Determine the derived set of $A = \{c,d,e\}$. (5 Marks)

e) Prove that the intersection $\tau_1 n \tau_2$ of any two topologies τ_1 and τ_2 on set X is also a topology on X. (5 Marks)

f) Let $X = \{a,b,c,d,e\}$ and $A = \{\{a,b,c\},\{c,d\},\{d,e\}\}$. Find the topology on x generated by A. (4 Marks)

g) Consider the functions.

$f = \{(1,3),(2,5),(3,3),(4,1),(5,2)\}$

$g = \{(1,4),(2,1),(3,1),(4,2),(5,3)\}$ from

$X = \{1,2,3,4,5\}$ into X.

i. Determine the range of f and g. (2 Marks)

ii. Find the composition function $g$of. (3 Marks)

QUESTION TWO (20 MARKS)

a) Define the terms:

i. $T_1 -$ Space (2 Marks)

ii. $T_2 -$ Space (2 Marks)

b) Prove that the topology

$\tau = \{\phi, X, \{0\}\}$ on $X = \{0,1\}$ is a $T_1 -$ Space but not$T_2 -$ Space (6 Marks)

c) Prove that the property of being $\tau_2 -$ Space is hereditary. (10 Marks)

QUESTION THREE (20 MARKS)

a) Define a closed set. (1 Mark)

b) Consider the topology

$\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}\{a, b, e\}\}$ of $X = \{a, b, c, d, e\}$.

i. List the closed subsets of X. (3 Marks)

ii. Determine the closure of $\{c, e\}$. (3 Marks)

c) Using the subset $A = \{a, b, d\}$ of X, find the boundary of A. (8 Marks)

d) Let $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology on $X = \{a, b, c, d, e\}$.

Prove that;

i. $\{b, c, d, e\}$ is both open and closed. (2 Marks)

ii. $\{a, b\}$ is neither open nor closed. (3 Marks)

QUESTION FOUR (20 MARKS)

a) List all topologies on $X = \{a, b, c\}$ which consists of exactly four members. (8 Marks)

b) Prove that if A is a subset of B, then every accumulation point of A is also an accumulation point of B, ie if ACB then A^1CB^1. (6 Marks)

c) Study the following classes of subsets of $X = \{a, b, c, d, e\}$;

$\tau_1 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$

$\tau_2 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_3 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{a, b, d, e\}\}$

Determine whether each of them is a topology on X or not. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Define the terms;

i. First countable space. (2 Marks)

ii. Second countable space. (2 Marks)

iii. Separable space. (2 Marks)

b) Prove that any subspace (y, τ_y) of a first countable space (x, τ) is also first countable.

(6 Marks)

c) Show that every subspace of a second countable space is second countable. (4 Marks)

d) Show that the plane $\mathbb{R}^2$ with the usual topology satisfies the second axiom of countability.

(4 Marks)