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University Examinations 2021/2022

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN EDUCATION SCIENCE

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN EDUCATION ARTS

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 3402: TOPOLOGY 1

DATE: OCTOBER 2022 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) Let $A=\{x,y,z,w\}$ and $B=\{x,w,t\}$, find

i) A – B (2 marks)

ii) AxB (3 Marks)

b) Prove that for any sets A and B, $(A\cup B)^C = A^C \cap B^C$ (4 marks)

c) List all the possible topologies in the set $X = \{a,b\}$ (4 marks)

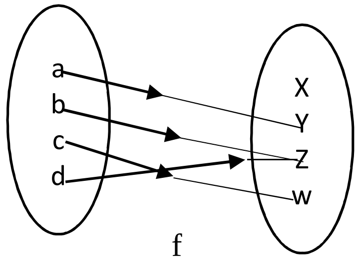
d) Consider the following topologies on $X = \{a,b,c,d\}$ and $Y = \{x,y,z,w\}$ respectively.

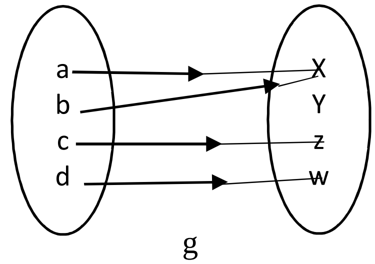
$$T = \{X,\emptyset,\{a\},\{a,b\}\{a,b,c\}\}$$

And

$$T^* = \{Y,\emptyset,\{x\}\,\{y\},\{x,y\}\{y,z,w\}\}$$

Given that $f: X \to Y$ and $g: X \to Y$ are defined as shown in the diagrams:-





determine whether f and g are continuous functions (4 marks)

e) Let $T = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ be a topology on X

(i) List the closed subsets of X (4 marks)

(ii) Determine the closure of the set $\{b\}$ (4 marks)

f) Show that the intersection $T_1 \cap T_2$ of any two topologies T_1 and T_2 on set x is also a topology on X (5 marks)

QUESTION TWO (20 MARKS)

a) Define a topological space X (4 marks)

b) Consider the following classes of subsets of $X = \{a,b,c,d,e\}$

$$T_1 = \{X, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d,e\}\}$$

$$T_2 = \{X, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$$

$$T_3 = \{X, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{a,b,d,e\}\}$$

Giving reasons determine which of the T's is/are topologies on X (9 marks)

c) Consider the topology $T_1\{X, \emptyset, \{a\}, \{a\ ,b\}, \{a,c,d, \{a,b,c,d\}, \{a,b,e\}\}$ on $X = \{a,b,c,d,e\}$. Let $A = \{a,b,c\}$ be a subset of X.

Determine: -

i) Interior points of A (3 marks)

ii) Exterior points of A (3 marks)

iii) The boundary of A (1 mark)

QUESTION THREE (20 MARKS)

a) Define a Hausdorff space X (2 marks)

b) Show that the property of a being Hausdorff space is hereditary (10 marks)

c) Show that any Hausdorff space is a T_1-space (4 marks)

d) Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$ be a base of a topology on X. find the topology on X generated by A (4 marks)

QUESTION FOUR (20 MARKS)

a) Define (2 marks)

(i) A limit point of a subset A

(ii) Closure of subset A

b) Let $T = \{x, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology on $X = \{a, b, c, \ d, e\}$, consider the subset $B = \{b, c, d, \}$ of X.

i) Find the derived set of B (4 marks)

ii) Find the closure of the subset B (5 marks)

c) Consider the following topology on $X = \{a, b, c, \ d, e\}$

$$T = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}\{a, b, e\}\}$$

List the neighborhood system of element c of X (ie N_c) (7 marks)

QUESTION FIVE (20 MARKS)

a) Define the terms:

i) First countable space (2 marks)

ii) Second countable space (2 marks)

iii) Separate space (2 marks)

b) Prove that any subspace (Y, T_Y) of a first countable space (X, T) is also first countable

(6 marks)

c) Show that every subspace of a second countable space is second countable (4 marks)

d) Show that the plane $\mathbb{R}^2$ with the usual topology satisfies the second axiom of countability

(4 marks)