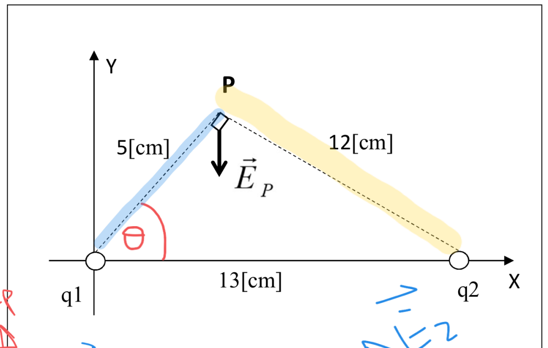
Ejercicio

Dos cargas se colocan como se muestra en la figura. La magnitud de q_1=3[μc], pero se desconoce su signo. Para la carga q_2, no se conoce su signo ni magnitud. La dirección del campo eléctrico resultante debido a estas dos cargas, en el punto "P" esta por completo en la dirección negativa del eje "Y".

a) Dibuje las 4 posibles configuraciones del campo eléctrico resultante.

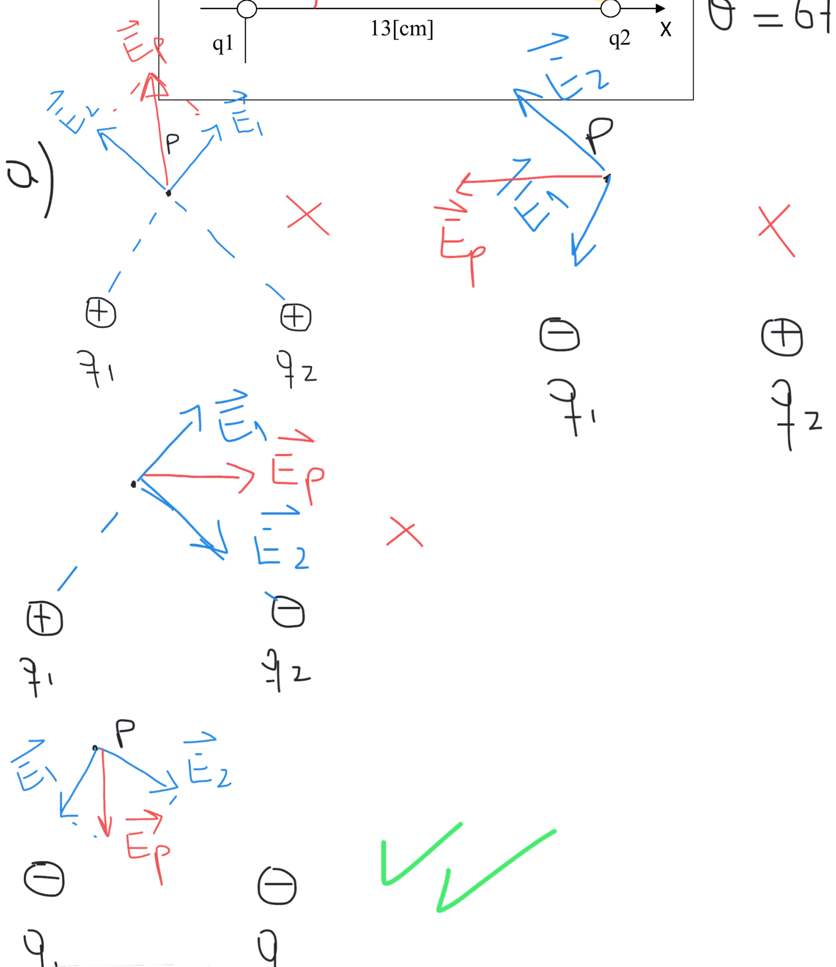
b) De acuerdo de la configuración que considere correcta, determine la magnitud del campo eléctrico resultante en el punto "P" y la carga q_2.



$\cos\theta = \frac{5}{13}$

$\theta = 67{,}4^\circ$

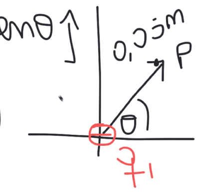
a)



b) $\vec E_p = \vec E_1 + \vec E_2$
(1) (2)

(1) $\vec E_1 = \frac{k q_1 (\vec r_p - \vec r_1)}{|\vec r_p - \vec r_1|^3}$

$\vec r_p = 0{,}05\cos\theta\,\hat i + 0{,}05\sin\theta\,\hat j$



$\vec r_p = (0{,}019\hat i + 0{,}046\hat j)$ (m)

$\vec r_1 = 0\hat i + 0\hat j$

$\vec r_p - \vec r_1 = \vec r_p$

$|\vec r_p - \vec r_1|^3 = 0{,}05^3$

$\rightarrow q_1$

$\vec E_1 = \frac{9\times10^{9} \cdot -3\times10^{-6}(0{,}019\hat i + 0{,}046\hat j)}{0{,}05^3}$

$\boxed{\vec E_1 = (-4104000\hat i - 9936000\hat j)\ (N/C)}$

(2) $\vec E_2 = \frac{k q_2 (\vec r_p - \vec r_2)}{|\vec r_p - \vec r_2|^3}$

$\vec r_p =$

$\vec r_2 = 0{,}13\hat i$ (m)

$\vec r_p - \vec r_2 = (-0{,}111\hat i + 0{,}046\hat j)$ (m)

$|\vec r_p - \vec r_2|^3 = \left(\sqrt{0{,}111^2 + 0{,}046^2}\right)^3 = 0{,}12^3$

$\vec E_2 = \frac{9\times10^{9} \cdot Q_2(-0{,}111\hat i + 0{,}046)}{0{,}12^3}$

$\boxed{\vec E_2 = (5{,}78\times10^{11} Q_2\hat i - 2{,}40\times10^{11} Q_2\hat j)\ (N/C)}$

$\vec E_1 + \vec E_2 = \vec E_p \qquad (-E_p\hat j)$

$-4104000\hat i - 9936000\hat j + 5{,}78\times10^{11} Q_2\hat i - 2{,}40\times10^{11} Q_2\hat j = -E_p\hat j$

$-4104000 + 5{,}78\times10^{11} Q_2 = 0$ (1)

$-9936000 - 2{,}40\times10^{11} Q_2 = -E_p$ (2)

$Q_2 = 7{,}1\times10^{-6}$ (C)

$\boxed{Q_2 = 7{,}1\ (\mu C)}$

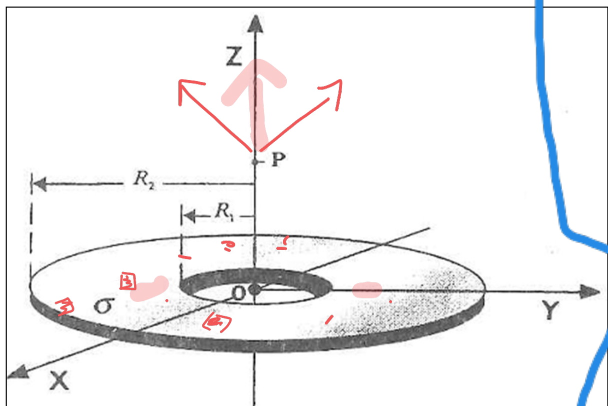
$\boxed{E_p = 1{,}164\times10^{7}\ (N/C)}$

$\vec E_p = (0\hat i - 1{,}16\times10^{7}\hat j)$ (N/C)

Ejercicio

La figura representa una corona de radios $R_1=4a$ y $R_2=\sqrt{27}\,a$

Si la carga de la corona esta uniformemente distribuida, calcule el campo eléctrico en el punto P(0,0,3a)



$$\vec{E}_p = K\int \frac{dq\,(\vec r - \vec r\,')}{|\vec r - \vec r\,'|^3}$$

① Vectores posición

$\vec r = 3a\,\hat k$

$\vec r\,' = r\cos\theta\,\hat\imath + r\,\text{sen}\,\theta\,\hat\jmath$

$\vec r - \vec r\,' = -r\cos\theta\,\hat\imath - r\,\text{sen}\,\theta\,\hat\jmath + 3a\,\hat k$

$|\vec r - \vec r\,'|^3 = (r^2+9a^2)^{3/2}$

② obt. dq

$$\left.\begin{aligned} dq &= \sigma\cdot ds \\ ds &= r\,dr\,d\theta \end{aligned}\right\}\ dq = \sigma\cdot r\,dr\,d\theta$$

③ límites

$R_1 \le r \le R_2$

$0 \le \theta \le 2\pi$

$$\vec E_p = K\sigma \int_{R_1}^{R_2}\int_0^{2\pi} \frac{r\,d\theta\,dr\,(\overset{A}{-r\cos\theta\,\hat\imath} \overset{B}{- r\,\text{sen}\,\theta\,\hat\jmath} \overset{C}{+ 3a\,\hat k})}{(r^2+9a^2)^{3/2}}$$

A $\displaystyle -\int_{R_1}^{R_2}\int_0^{2\pi} \frac{r^2\cos\theta\,d\theta\,dr}{(r^2+9a^2)^{3/2}}\,\hat\imath = -\int_{R_1}^{R_2}\frac{r^2\,dr}{(r^2+9a^2)^{3/2}}\Big[\cancel{\text{sen}\,2\pi} - \overset{0}{\text{sen}\,0}\Big]\hat\imath$

$= 0\,\hat\imath$

B $\displaystyle -\int_{R_1}^{R_2}\int_0^{2\pi} \frac{r^2\,\text{sen}\,\theta\,d\theta\,dr}{(r^2+9a^2)^{3/2}}\,\hat\jmath = -\int\frac{r^2\,dr}{(r^2+9a^2)^{3/2}}\Big(\cancel{-\cos 2\pi - \cos 0}\Big)\hat\jmath$

$-1+1 = 0 = 0$

$= 0\,\hat\jmath$

C $\displaystyle 3a\int_{R_1}^{R_2}\int_0^{2\pi} \frac{r\,d\theta\,dr}{(r^2+9a^2)^{3/2}}\,\hat k = 3a\int \frac{r\,dr}{(\quad)}\,(2\pi)\,\hat k$

$\displaystyle = 6\pi a\int_{4a}^{\sqrt{27}a} \frac{r\,dr}{(r^2+9a^2)^{3/2}}\,\hat k$

$\displaystyle = 6\pi a\left[-\frac{1}{\sqrt{r^2+9a^2}}\right]_{4a}^{\sqrt{27}a}\hat k$

$\displaystyle = 6\pi a\left(\frac{-1}{\sqrt{27a^2+9a^2}} + \frac{1}{\sqrt{16a^2+9a^2}}\right)\hat k$

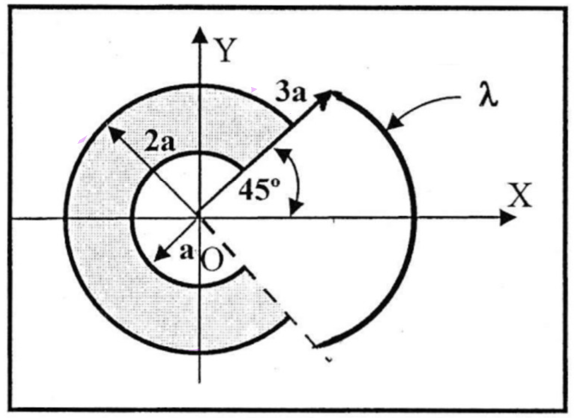
$\displaystyle = 6\pi a\left(-\frac{1}{6a} + \frac{1}{5a}\right)\hat k$

$\displaystyle = 6\pi \cancel{a}\left(\frac{1}{30\cancel{a}}\right)\hat k$

$\displaystyle = \frac{\pi}{5}\,\hat k$

$$\boxed{\vec E_p = \frac{K\sigma\pi}{5}\,\hat k\ (N/C)}$$

Una carga se encuentre uniformemente distribuida, formando ¾ (tres cuartos) de golilla de radios **“a”** y **“2a”**, siendo su densidad superficial **“σ”**. Otra carga está también uniformemente distribuida, formando ¼ (un cuarto) de anillo de radio **“3a”** y densidad lineal **“λ”**. Ambas configuraciones de carga están situadas como muestra la **Figura 2**. (a) Calcular el campo eléctrico resultante en el punto **“O”**. (b) Determinar la magnitud y signo de la densidad lineal de carga **“λ”** para que el campo anterior se anule en **“O”**. **(40 Puntos)**



a) $\vec{E}_0 = \vec{E}_\lambda + \vec{E}_\sigma$ (1) (2)

(1) $\vec{E}_\lambda = K\int \frac{dq(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

i) vectores posición

$\vec{r} = 0\hat{i} + 0\hat{j}$

$\vec{r}\,' = 3a\cos\theta\,\hat{i} + 3a\sin\theta\,\hat{j}$

$\vec{r}-\vec{r}\,' = -(\quad '' \quad)$

$|\vec{r}-\vec{r}\,'|^3 = (3a)^3 = 27a^3$

ii) obt. del dq

$\lambda = \frac{dq}{dl} \to dq = \lambda\, dl$

$dl = 3a\, d\theta$

$dq = \lambda \cdot 3a\, d\theta$

iii) límites $(7\pi/4 \le \theta \le \pi/4)$

$\vec{E}_\lambda = K\int_{7\pi/4}^{\pi/4} \lambda \cdot 3a\, d\theta \frac{(-3a\cos\theta\,\hat{i} - 3a\sin\theta\,\hat{j})}{27a^3}$

$= -\frac{K\lambda}{3a}\int_{7\pi/4}^{\pi/4} (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\,d\theta$ (A) (B)

(A) $\int_{7\pi/4}^{\pi/4} \cos\theta\, d\theta\,\hat{i} = \left(\sin\frac{\pi}{4} - \sin\frac{7\pi}{4}\right)\hat{i}$

$= \sqrt{2}\,\hat{i}$

(B) $\int_{7\pi/4}^{\pi/4} \sin\theta\, d\theta\,\hat{j} = \left(-\cos\frac{\pi}{4} + \cos\frac{7\pi}{4}\right)\hat{j}$

$= 0\,\hat{j}$

$$\boxed{\vec{E}_\lambda = -\frac{K\lambda\sqrt{2}}{3a}\hat{i}\ (N/C)}$$

(2) $\vec{E}_\sigma = K\iint \frac{dq(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

i) vectores posición

$\vec{r}\,' = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

$\vec{r}-\vec{r}\,' = -(\quad '' \quad)$

$|\vec{r}-\vec{r}\,'|^3 = r^3$

ii) obt. del dq

$\sigma = \frac{dq}{ds} \to dq = \sigma \cdot ds$

$ds = r\,d\theta\,dr$

$dq = \sigma \cdot r\,d\theta\,dr$

iii) límites

$a \le r \le 2a$

$\frac{\pi}{4} \le \theta \le \frac{7\pi}{4}$

$\vec{E}_\sigma = K\int_a^{2a}\int_{\pi/4}^{7\pi/4} \sigma \cdot r\,d\theta\,dr \frac{(-\cancel{r}\cos\theta\,\hat{i} - \cancel{r}\sin\theta\,\hat{j})}{r^{\cancel{3}}}$

$= -K\sigma\int_a^{2a}\int_{\pi/4}^{7\pi/4} \frac{(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\,d\theta\,dr}{r}$ (A) (B)

(A) $\int_a^{2a}\int_{\pi/4}^{7\pi/4} \frac{\cos\theta\,d\theta\,dr}{r}\hat{i} = \int_a^{2a}\frac{dr}{r}\left(\sin\frac{7\pi}{4} - \sin\frac{\pi}{4}\right)\hat{i}$

$-\sqrt{2}\int_a^{2a}\frac{dr}{r}\hat{i}$

$= -\sqrt{2}(\ln 2a - \ln a)\hat{i}$

$= -\sqrt{2}\ln 2\,\hat{i}$

b) $\int_a^{2a}\int_{\pi/4}^{7\pi/4} \frac{\sin\theta\,d\theta\,dr}{r}\hat{j} = \int\frac{dr}{r}\left(\cancel{-\cos\frac{7\pi}{4} + \cos\frac{\pi}{4}}\right)\hat{j} = 0\,\hat{j}$

$$\boxed{\vec{E}_\sigma = K\sigma(\sqrt{2}\ln 2)\hat{i}\ (N/C)}$$

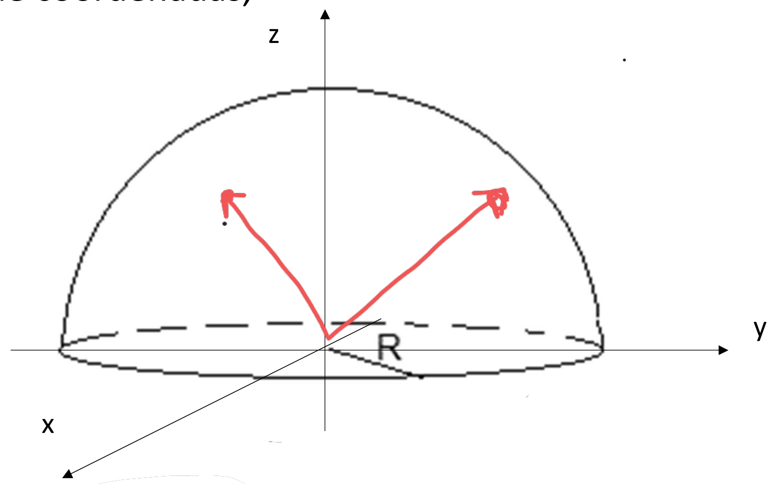
$$\boxed{\vec{E}_0 = \left[-\frac{K\lambda\sqrt{2}}{3a}\hat{i} + K\sigma(\sqrt{2}\ln 2)\hat{i}\right](N/C)}$$

b) $-\frac{\cancel{K}\lambda\sqrt{2}}{3a} + \cancel{K}\sigma(\sqrt{2}\ln 2) = 0$

$$\boxed{\lambda = 3a\sigma\ln 2\ (C/m)}$$

Ejercicio

Una semiesfera hueca dieléctrica de radio R, tiene en su superficie una distribución de carga eléctrica superficial σ uniforme. Determinar en campo eléctrico en el origen del sistema de coordenadas,



$$\vec{E}_0 = K\iint \frac{dq\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$$\vec{r} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

$$\vec{r}\,' = R\,sen\theta\cos\phi\,\hat{\imath} + R\,sen\theta\,sen\phi\,\hat{\jmath} + R\cos\theta\,\hat{k}$$

$$\vec{r}-\vec{r}\,' = -(\quad // \quad)$$

$$|\vec{r}-\vec{r}\,'|^3 = \left(\sqrt{R^2 sen^2\theta\cos^2\phi + R^2 sen^2\theta\,sen^2\phi + R^2\cos^2\theta}\right)^3$$

$$dq : = R^3$$

$$dq = \sigma \cdot ds$$

$$ds = R^2 sen\theta\,d\phi\,d\theta$$

$$dq = \sigma\,R^2 sen\theta\,d\phi\,d\theta$$

$$0 \le \theta \le \pi/2$$

$$0 \le \phi \le 2\pi$$

$$\vec{E}_0 = K\int_0^{\pi/2}\!\!\int_0^{2\pi} \frac{\sigma\,\cancel{R^2} sen\theta\,d\phi\,d\theta\,(\overbrace{-\cancel{R}\,sen\theta\cos\phi\,\hat{\imath}}^{A} \overbrace{-\cancel{R}\,sen\theta\,sen\phi\,\hat{\jmath}}^{B} \overbrace{-\cancel{R}\cos\theta\,\hat{k}}^{C})}{\cancel{R^3}}$$

A $\iint sen^2\theta\cos\phi\,d\phi\,d\theta\,\hat{\imath} = 0\,\hat{\imath}$

B $\iint sen^2\theta\,sen\phi\,d\phi\,d\theta\,\hat{\jmath} = 0\,\hat{\jmath}$

C $\int_0^{\pi/2}\!\!\int_0^{2\pi} sen\theta\cos\theta\,d\phi\,d\theta\,\hat{k} = 2\pi\int_0^{\pi/2} sen\theta\cos\theta\,d\theta$

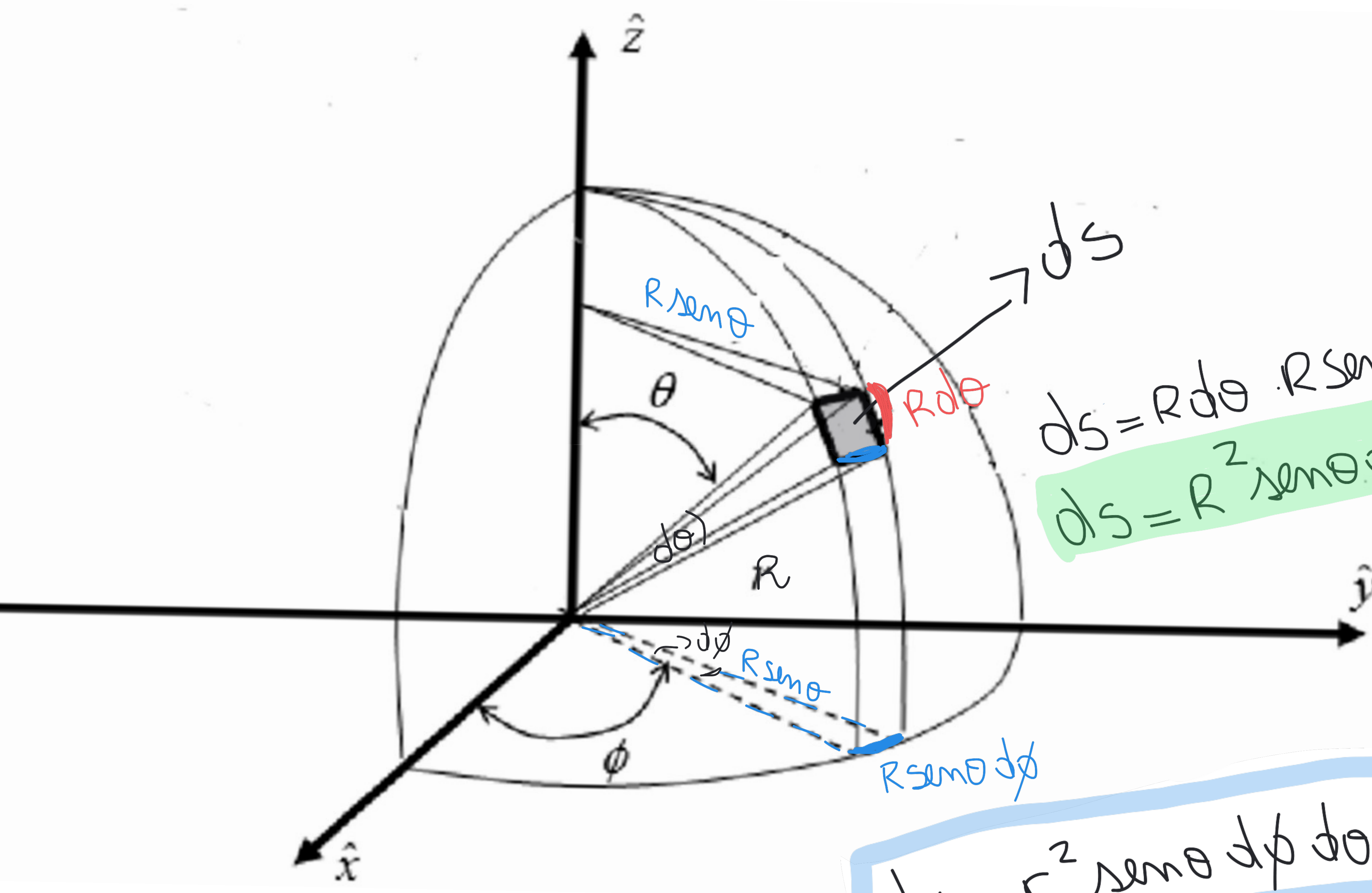
$$\left.\begin{aligned} u &= sen\theta \\ du &= \cos\theta\,d\theta \end{aligned}\right\} \quad \int u\,du = \frac{u^2}{2}$$

$$= \left.\frac{sen^2\theta}{2}\right|_0^{\pi/2} =$$

$$= \frac{1}{2}\left(\cancel{sen^2\frac{\pi}{2}}^{\,1} - \cancel{sen^2 0}^{\,0}\right)$$

$$= 2\pi \cdot \frac{1}{2} = \pi\,\hat{k}$$

$$\boxed{\vec{E}_0 = K\sigma\pi\,\hat{k}\ (N/C)}$$



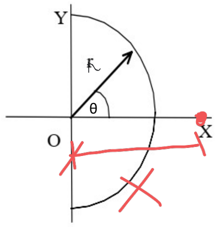
$$ds = R\,d\theta \cdot R\,sen\theta\,d\phi$$

$$ds = R^2\,sen\theta\,d\phi\,d\theta$$

$$\boxed{dv = r^2\,sen\theta\,d\phi\,d\theta\,dr}$$

Sobre una semicircunferencia se distribuye una densidad de carga lineal $\lambda = \lambda_0 cos\theta$

a) Calcular la carga total distribuida sobre la semicircunferencia.
b) Calcular el campo eléctrico en el origen.
c) En que punto del eje x debe situarse la carga calculada en a) para que el campo en el origen sea el mismo que el obtenido en b)



b) $\vec{E}_0$:

$\vec{r} = 0\hat{i} + 0\hat{j}$

$\vec{r}\,' = R\cos\theta\,\hat{i} + R\,\text{sen}\,\theta\,\hat{j}$

$(\vec{r} - \vec{r}\,') = (\quad // \quad)$

$|\vec{r} - \vec{r}\,'|^3 = R^3$

$$\left.\begin{array}{l} dq = \lambda\, dl \\ dl = R\, d\theta \end{array}\right\} dq = \lambda_0 \cos\theta\, R\, d\theta$$

$\frac{3\pi}{2} \leq \theta \leq \pi/2$

$$\vec{E}_0 = K \int_{3\pi/2}^{\pi/2} \frac{\lambda_0 \cancel{R} \cos\theta\, d\theta \,(-\cancel{R}\cos\theta\,\hat{i} - \cancel{R}\,\text{sen}\,\theta\,\hat{j})}{R^{\cancel{3}}}$$

$$= -\frac{K\lambda_0}{R} \int_{3\pi/2}^{\pi/2} (\underset{(A)}{\cos\theta\,\hat{i}} + \underset{(B)}{\text{sen}\,\theta\,\hat{j}}) \cos\theta\, d\theta$$

(A) $\int_{3\pi/2}^{\pi/2} \cos^2\theta\, d\theta\, \hat{i}$ $\quad \left[\cos^2\theta = \frac{\cos 2\theta + 1}{2}\right]$

$$\int_{3\pi/2}^{\pi/2} \frac{(\cos 2\theta + 1)}{2} d\theta\, \hat{i} = \frac{1}{2}\Big[\underset{(i)}{\cancel{\int \cos 2\theta\, d\theta}}^{\,0} + \underset{(ii)}{\int d\theta}\Big]\hat{i}$$

(i) $\int_{3\pi/2}^{\pi/2} \cos 2\theta\, d\theta$

$$\left.\begin{array}{l} u = 2\theta \\ \frac{du}{2} = d\theta \end{array}\right\} \int \frac{\cos u\, du}{2} = \frac{1}{2}(\text{sen}\, u)\Big|_{u_1}^{u_2}$$

$$= \frac{1}{2}(\text{sen}\, 2\theta)\Big|_{3\pi/2}^{\pi/2} = \frac{1}{2}\cancel{\left(\text{sen}\, 2\cdot\frac{\pi}{2} - \text{sen}\, 2\cdot\frac{3\pi}{2}\right)}^{\,0} = 0$$

(ii) $\int_{3\pi/2}^{\pi/2} d\theta = \left(\frac{\pi}{2} - \frac{3\pi}{2}\right) = \frac{-2\pi}{2} = -\pi$

(A) $= \frac{1}{2}(-\pi)\,\hat{i} = -\frac{\pi}{2}\,\hat{i}$

b) $\int_{3\pi/2}^{\pi/2} \text{sen}\,\theta \cos\theta\, d\theta\, \hat{j}$

$$\left.\begin{array}{l} u = \text{sen}\,\theta \\ du = \cos\theta\, d\theta \end{array}\right\} \int u\, du = \frac{u^2}{2}$$

$$\frac{\text{sen}^2\theta}{2}\Big|_{3\pi/2}^{\pi/2} \hat{j} = 0\,\hat{j}$$

$$\boxed{\vec{E}_0 = \frac{K\lambda_0\pi}{2R}\,\hat{i}\ (N/C)}$$

a) $\lambda = \frac{dq}{dl}$

$dq = \lambda\, dl \ // \int$

$\int dq = \int \lambda\, dl$

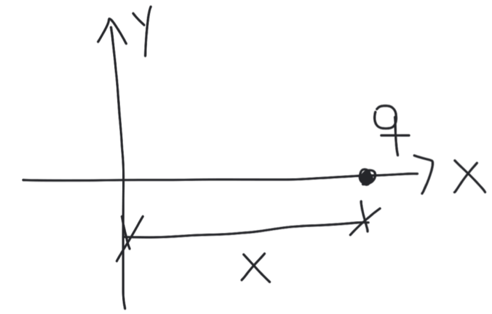
$q = \int_{3\pi/2}^{\pi/2} \lambda_0 \cos\theta\, R\, d\theta$

$q = \lambda_0 R \int_{3\pi/2}^{\pi/2} \cos\theta\, d\theta$

$= \lambda_0 R \underbrace{\left[\text{sen}\,\frac{\pi}{2} - \text{sen}\,\frac{3\pi}{2}\right]}_{2}$

$$\boxed{q = 2\lambda_0 R}$$

c)



$q = 2\lambda_0 R\ [C]$

$\vec{E}_0 = \frac{K\lambda_0\pi}{2R}\,\hat{i}\ (N/C)$

$E = \frac{KQ}{r^2}$

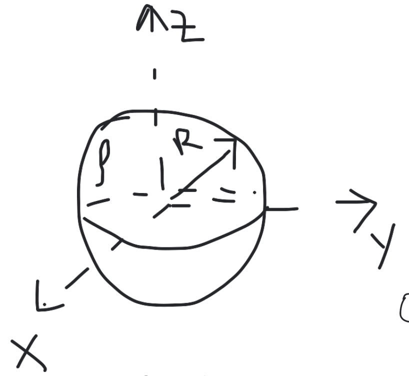
$E = \frac{Kq}{x^2} = \frac{K\lambda_0\pi}{2R}$

$\frac{\cancel{K}(2\cancel{\lambda_0}R)}{x^2} = \frac{\cancel{K}\cancel{\lambda_0}\pi}{2R}$

$\pi x^2 = 4R^2$

$x = \sqrt{\frac{4R^2}{\pi}} = \frac{2R}{\sqrt{\pi}}$

Una esfera dielectrica maciza de radio R tiene una distribuciónde carga volumetrica dada por: $\rho = \frac{A}{(1+r)}$
donde A es tna constanste y r es la distancia radial al centro de la esfera.
Determinar la carga total en la esfera.



$$\rho = \frac{dq}{dV}$$

$$dq = \rho\, dV \ //\int$$

$$q = \iiint \rho \cdot dV$$

$$q = \int_0^R \int_0^\pi \int_0^{2\pi} \frac{A}{(1+r)} \cdot \overbrace{r^2 \operatorname{sen}\theta \, d\phi \, d\theta \, dr}^{dV}$$

$$0 \le \theta \le \pi$$

$$0 \le \phi \le 2\pi$$

$$0 \le r \le R$$

$$q = A \int_0^R \int_0^\pi \frac{r^2}{(1+r)} \operatorname{sen}\theta \, d\theta \, dr \, [2\pi]$$

$$= 2\pi A \int_0^R \left(\frac{r^2}{1+r}\right) dr \, \underbrace{[-\cos\pi - -\cos 0]}_{2}$$

$$= 4\pi A \int_0^R \frac{r^2}{(1+r)} \, dr$$

$$u = (1+r) \rightarrow r = (u-1)$$

$$du = dr \qquad r^2 = (u-1)^2$$

$$\int \frac{(u-1)^2}{u} du = \int \left(\frac{u^2 - 2u + 1}{u}\right) du$$

$$= \int \left(u - 2 + \frac{1}{u}\right) du$$

$$= \left[\frac{u^2}{2} - 2u + \ln u\right]$$

$$= \left[\frac{(1+r)^2}{2} - 2(1+r) + \ln(1+r)\right]_0^R$$

$$\boxed{q = 4\pi A \left(\frac{R^2}{2} - R + \ln(1+R)\right) (c)}$$