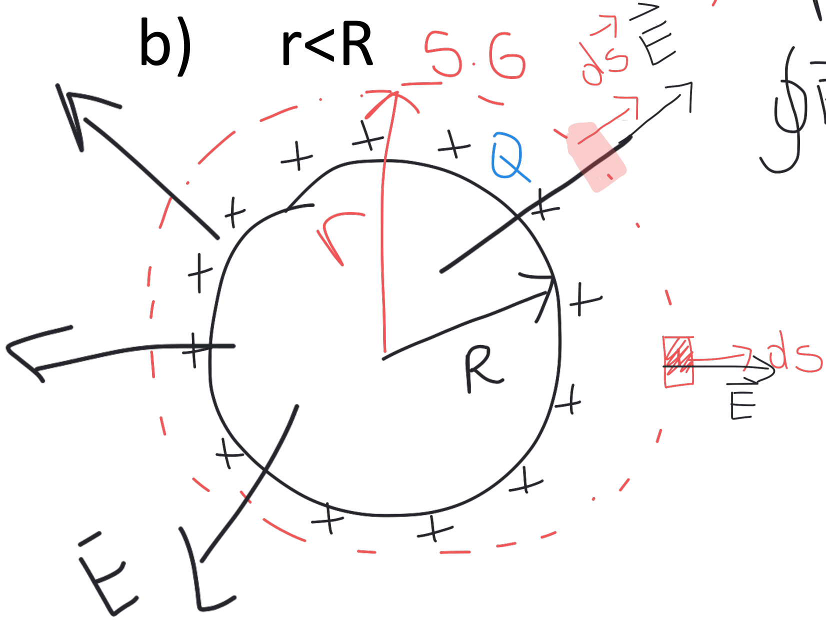
Aplicaciones de la Ley de Gauss

I. Campo eléctrico de una esfera

i. Se tiene una **esfera conductora** de radio R y carga Q. Determine el campo eléctrico para

a) r>R
b) r<R



a) Sup. Gaussiana con $r > R$

$$\oint \vec{E}\cdot\vec{ds} = \frac{Q_{enc}}{\varepsilon_0} \quad [Q_{enc} = Q]$$

$$\oint E\cdot ds\cos 0 = \frac{Q_{enc}}{\varepsilon_0}$$

$$E\int ds = \frac{Q}{\varepsilon_0}$$

$$E\cdot S = \frac{Q}{\varepsilon_0}$$

Sup. Gauss

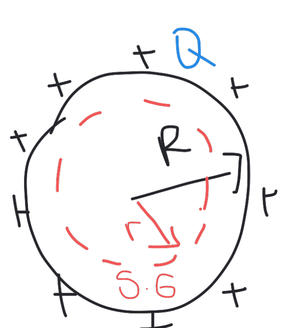
$$S_{esf} = 4\pi r^2$$

$$V_{esf} = \frac{4}{3}\pi r^3$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi r^2 \varepsilon_0}\ (N/C)\quad [r > R]}$$

b) Sup Gauss con $r < R$



$$Q_{enc} = 0$$

$$E = 0\ (N/C)$$

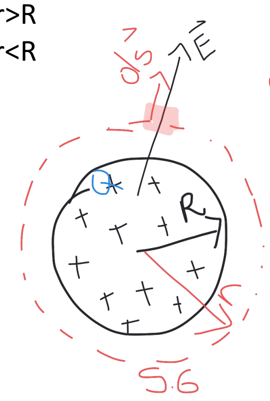
I. Campo eléctrico de una esfera

ii. Se tiene una **esfera aislante** (no conductora) de radio R y carga Q distribuida en todo su volumen de manera **uniforme**, determine el campo eléctrico para:

a) r>R

b) r<R

$\rho = cte$



a) Sup. Gauss $r > R$

$Q_{enc} = Q$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint E\, ds \cos 0 = \frac{Q}{\varepsilon_0}$$

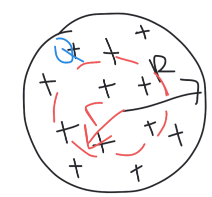
$$E \oint ds = \frac{Q}{\varepsilon_0}$$

$$E \cdot S = \frac{Q}{\varepsilon_0}$$

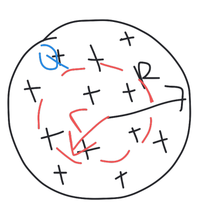
$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E_{ext} = \frac{Q}{4\pi r^2 \varepsilon_0} \ (N/C)}$$

b) Sup. Gauss con $r < R$



b) Sup. Gauss con $r < R$



Q_{enc} : $\rho = cte$

$$\rho_T = \frac{Q_T}{V_T}$$

$$\rho_T = \rho_{enc}$$

$$\frac{Q_T}{V_T} = \frac{Q_{enc}}{V_{enc}} \rightarrow Q_{enc} = \frac{Q_T \cdot V_{enc}}{V_T}$$

$$Q_{enc} = \frac{Q \cdot \cancel{\tfrac{4}{3}\pi} r^3}{\cancel{\tfrac{4}{3}\pi} R^3}$$

$$Q_{enc} = \frac{Q r^3}{R^3}$$

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q r^3}{R^3 \varepsilon_0} \rightarrow \boxed{E_{int} = \frac{Q r}{4\pi R^3 \varepsilon_0} \ (N/C)}$$

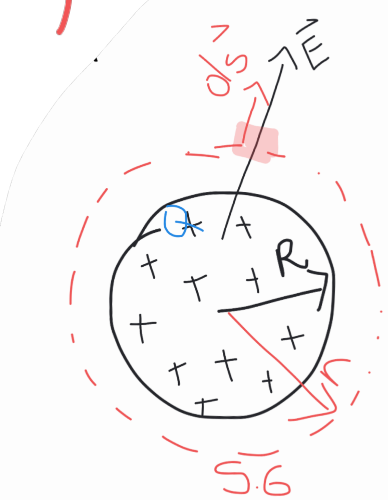
Campo eléctrico de una esfera

iii. Se tiene una **esfera aislante** de radio R, con densidad de carga $\rho=Ar$, (A es una constante y r es la distancia radial), determine el campo eléctrico para: (con carga$_T$ Q)

a) r>R

b) r<R

a)



a) Sup. Gauss $r>R$

$Q_{enc}=Q$

$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$

$\oint E\,ds\cos 0 = \frac{Q}{\varepsilon_0}$

$E\oint ds = \frac{Q}{\varepsilon_0}$

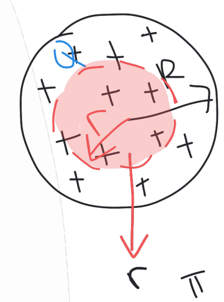
$E\cdot S = \frac{Q}{\varepsilon_0}$

$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$

$$\boxed{E_{ext} = \frac{Q}{4\pi r^2 \varepsilon_0}\ (N/C)}$$

b)

b) Sup. Gauss con $r<R$



Q_{enc}:

$\rho \neq cte$

$Q = \int \rho \cdot dv$

$\rho = cte \rightarrow \rho = \frac{Q}{V}$

$\rho = cte \rightarrow Q = \int \rho \cdot dv$

$Q = \int_0^r\int_0^\pi\int_0^{2\pi} Ar' \cdot r'^2 \operatorname{sen}\theta\, d\phi\, d\theta\, dr'$

$= A \cdot 2\pi \cdot 2 \int_0^r r'^3 dr'$

$= \cancel{4}\pi A \cdot \frac{r^4}{\cancel{4}}$

$Q_{enc} = \pi A r^4$

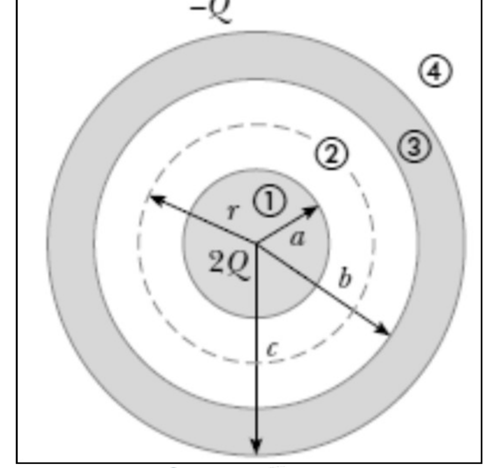
$\underbrace{\oint \vec{E}\cdot d\vec{s}} = \frac{Q_{enc}}{\varepsilon_0}$

$E(4\cancel{\pi} \cancel{r^2}) = \frac{\cancel{\pi} A r^{\cancel{4}\,2}}{\varepsilon_0}$

$$\boxed{E_{int} = \frac{A r^2}{4\varepsilon_0}\ (N/C)}$$

Campo electico de una esfera dentro de un cascaron esférico.

Una esfera aislante sólida de radio "a" tiene una carga positiva neta 2Q. Un cascarón conductor esférico de radio interno "b" y radio externo "c" es concéntrico con la esfera sólida y tiene una carga neta –Q. Utilizando la Ley de Gauss, determine el campo eléctrico en las regiones identificadas como 1, 2, 3 y 4 de la figura.



Región 1 ; Sup. Gauss con $r<a$

$\rho = cte$

$\rho_T = \rho_{enc}$

$$\frac{Q_T}{V_T} = \frac{Q_{enc}}{V_{enc}}$$

$$Q_{enc} = \frac{Q_T \cdot V_{enc}}{V_T} = 2Q \cdot \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3}$$

$$Q_{enc} = \frac{2Qr^3}{a^3}$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \cdot (4\pi r^2) = \frac{2Qr^3}{a^3 \varepsilon_0}$$

$$\boxed{E_1 = \frac{Qr}{2\pi a^3 \varepsilon_0} \ (N/C)} \quad (r<a)$$

Región 2: Sup. Gauss con $a<r<b$

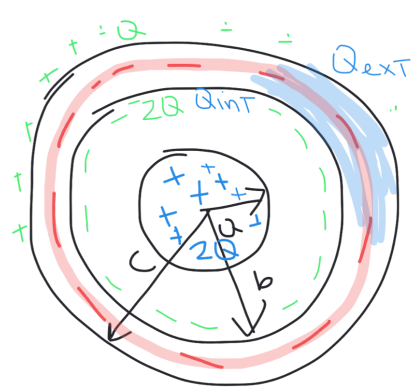
$Q_{enc} = 2Q$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{2Q}{\varepsilon_0}$$

$$(a<r<b) \quad \boxed{E_2 = \frac{Q}{2\pi r^2 \varepsilon_0} \ (N/C)}$$

Región 3: Sup. Gauss con $b<r<c$



$Q_{inT} + Q_{exT} = -Q$

$-2Q + Q_{exT} = -Q$

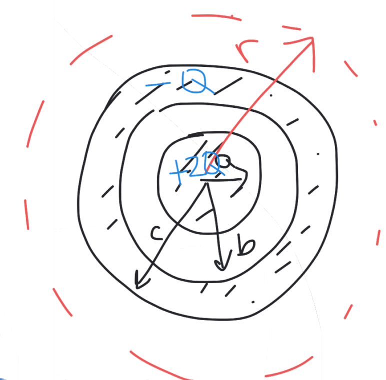
$Q_{exT} = Q$

$Q_{enc} = \underbrace{+2Q}_{\text{carga esf}} \underbrace{-2Q}_{Q_{inT}\ \text{cond}}$

$Q_{enc} = 0$

$E_3 = 0 \ (N/C)$

Región 4: Sup Gauss $r>c$



$Q_{enc} = +2Q - Q$

$Q_{enc} = Q$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

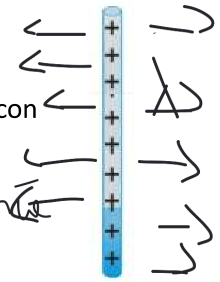
$$\boxed{E_4 = \frac{Q}{4\pi r^2 \varepsilon_0} \ (N/C)}$$

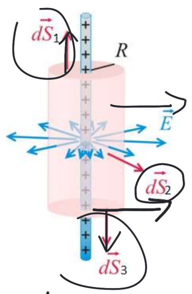
$r>c$

Aplicaciones de la Ley de Gauss

III. Campo electico de un hilo conductor infimito

Determinar el campo eléctrico de una hilo infinito cargado con carga +q

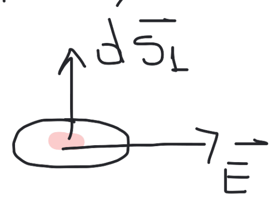




Sup. Gauss con simetría cilíndrica con R y L

$$\phi_E = \cancel{\phi_1}^{\,0} + \phi_2 + \cancel{\phi_3}^{\,0}$$

$$\phi_1 = \oint \vec{E} \cdot d\vec{S}_1$$



$$\oint E\, ds\, \cancel{\cos 90}^{\,0}$$

$$\phi_1 = 0 = \phi_3$$

$$\phi_2 = \oint \vec{E} \cdot d\vec{S}_2 = \oint E \cdot ds_2 \cdot \cos 0$$

$$= E \oint ds_2$$

$$\phi_2 = E \cdot S$$

$$\left.\begin{array}{l} \phi = \oint \vec{E} \cdot d\vec{s} \\ \phi = \dfrac{q}{\varepsilon_0} \end{array}\right\} \begin{array}{l} \phi_T = E \cdot S \\ \text{Gauss} \quad ES = \dfrac{q}{\varepsilon_0} \end{array}$$

→ sup. Gauss

$$E(2\pi RL) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\pi RL\varepsilon_0}$$

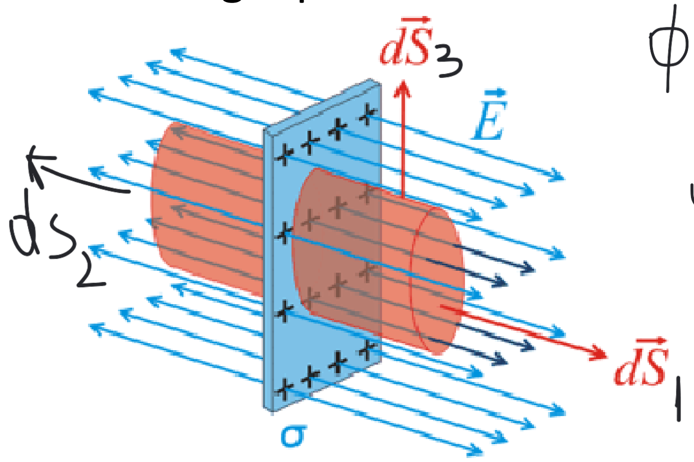
$$\lambda = \frac{q}{L}$$

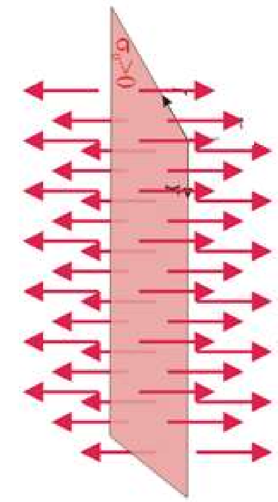
$$\boxed{E = \frac{\lambda}{2\pi R\varepsilon_0} \quad (N/C)}$$

Aplicaciones de la Ley de Gauss

IV. Campo electico de un plano infinito

Determinar el campo eléctrico debido a un plano infinito con carga q





$$\phi_T = \phi_{S_1} + \phi_{S_2} + \phi_{S_3}$$

$$\phi_{S_1} = \oint \vec{E}\cdot d\vec{S}_1 \quad (\text{tapa}_1)$$

$$= \oint E\, dS \cos 0$$

$$\phi_1 = ES$$

$$\phi_{S_2} = \oint \vec{E}\cdot d\vec{S}_2 \quad (\text{tapa}_2)$$

$$= \oint E\, dS_2 \cos 0$$

$$\phi_2 = ES$$

$$\phi_{S_3} = \oint \vec{E}\cdot d\vec{S}_3 \quad (\text{manto})$$

$$= \oint E\, dS_3 \cancel{\cos 90}^{\,0}$$

$$\phi_3 = 0$$

$$\phi_T = 2ES$$

aplicando Gauss

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

$$2ES = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2S\varepsilon_0} \;;\; \sigma = \frac{q}{S}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0} \ (N/C)}$$