Problemas de los viernes

1

$$T = \frac{1}{2} m(\dot{x}_1^2 + \dot{x}_2^2) \qquad V = \frac{1}{2}k(x_1)^2 + \frac{1}{2}3k(x_2 - x_1)^2 + \frac{1}{2}k(x_2)^2$$

$$L = \frac{1}{2} m(\dot{x}_1^2 + \dot{x}_2^2) - k(2x_1^2 - 3x_2x_1 + 2x_2^2)$$

$$\frac{\partial L}{\partial x_1} = -k(4x_1 - 3x_2) \qquad \frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m\ddot{x}_1$$

$$\frac{\partial L}{\partial x_2} = -k(4x_2 - 3x_1) \qquad \frac{\partial L}{\partial \dot{x}_2} = m\dot{x}_2 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m\ddot{x}_2$$

Por tanto las ecuaciones de movimiento

Para x_1

$$m\ddot{x}_1 + k(4x_1 - 3x_2) = 0$$

Para x_2

$$m\ddot{x}_2 + k(4x_2 - 3x_1) = 0$$

$$T_{ij} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \qquad V_{ij} = \begin{bmatrix} 4k & -3k \\ -3k & 4k \end{bmatrix}$$

Entonces

$$\begin{vmatrix} -4k - w^2 m & -3k \\ -3k & 4k - w^2 m \end{vmatrix} = 0 \Longrightarrow \begin{array}{l} (4k - w^2 m)^2 - 9k^2 = 0 \\ 4k - w^2 m = \pm 3k^2 \\ -w^2 m = -k \quad ó \quad -w^2 m = -7k \end{array}$$

$$w_1 = \sqrt{\frac{-k}{m}} \qquad w_2 = \sqrt{\frac{7k}{m}}$$

Las amplitudes

Para ω_1

$$\begin{bmatrix} 4K - m\frac{k}{m} & -3K \\ -3K & 4K - m\frac{k}{m} \end{bmatrix} = \begin{bmatrix} 3k & -3k \\ -3k & 3k \end{bmatrix} \quad \text{Det} = 0$$

$$\left.\begin{aligned} 3Ka_1 - 3Ka_2 &= 0 \\ -3Ka_1 + 3Ka_2 &= 0 \end{aligned}\right\} \quad a_1 = a_2 \quad \text{Amplitudes iguales}$$

Para ω_2

$$\begin{pmatrix} 4K - m\frac{7K}{m} & -3K \\ -3K & 4K - m\frac{7K}{m} \end{pmatrix} = \begin{pmatrix} -3K & -3K \\ -3K & -3K \end{pmatrix}$$

$$\begin{aligned} -3Ka_1 - 3Ka_2 &= 0 \\ -3Ka_1 - 3Ka_2 &= 0 \end{aligned} \quad a_1 = -a_2 \quad \text{Amplitudes opuestas}$$

2) Para el segundo caso

las desplazamientos $\sqrt{l^2 + y^2} - l$

$$T = \frac{1}{2} m (\dot{y}_1^2) + \frac{1}{2} (\dot{y}_2)^2 m \qquad V = \frac{1}{2} K \left(\sqrt{l^2 + y_1^2} - l\right)^2 + \frac{1}{2} K \left(\sqrt{l^2 + y_2^2} - l\right)^2 + \frac{1}{2} K \left(\sqrt{l^2 + (y_2 - y_1)^2} - l\right)^2$$

Si decimos

$$V = 3l^2 K - K\left(-\sqrt{l^2 + y_1^2} - \sqrt{l^2 + y_2^2} - \sqrt{l^2 + (y_1 - y_2)^2} - y_1 y_2 + y_1^2 + y_2^2\right.$$

Si $l \gg y$

$$V = 3l^2 K - 3Kl + \left(y_1^2 + y_2^2 - y_1 y_2\right)\left(K - \frac{K}{l}\right)$$

$$V = \left(y_1^2 + y_2^2 - y_1 y_2\right)\left(K - \frac{K}{l}\right)$$

$$T = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \qquad V_{ij} = \begin{bmatrix} K - \frac{K}{l} & \frac{K}{l} - K \\ \frac{K}{l} - K & K - \frac{K}{l} \end{bmatrix}$$

$$\begin{bmatrix} \left(2K-\frac{2K}{l}\right)-\omega^2 m & \frac{K}{l}-K \\ \left(\frac{K}{l}-K\right) & 2K-\frac{2K}{l}-\omega_2 m \end{bmatrix} \qquad \left(\left(2K-\frac{2K}{l}\right)-\omega^2 m\right)^2-\left(\frac{K}{l}-K\right)^2=0$$

$$2\left(K-\frac{K}{l}\right)-\omega^2 m = \pm\frac{K}{l}-K$$

$$m\omega_1^2=\frac{K}{l}-K-2K+\frac{2K}{l}$$

$$\omega_1^2=\frac{3\left(K-\frac{K}{l}\right)}{m} \qquad \omega_2^2=\frac{K-K/l}{m}$$

Amplitudes ω_1

$$\begin{bmatrix} \frac{K}{l}-K & \frac{K}{l}-K \\ \frac{K}{l}-K & \frac{K}{l}-K \end{bmatrix} \qquad \left(\frac{K}{l}-K\right)a_1+\left(\frac{K}{l}-K\right)a_2=0$$

$$-a_1=a_2$$

$$\left(\frac{K}{l}-K\right)a_2+\left(\frac{K}{l}-K\right)a_1=0$$

Amplitudes $\omega_2 \Longrightarrow a_1=a_2$

3)

$$T=\frac{1}{2}m(\dot{x}_1)^2+\frac{1}{2}m(\dot{x}_2)^2 \qquad V=\frac{1}{2}Kx_1^2+\frac{1}{2}Kx_2^2+\frac{1}{2}K(x_2-x_1)^2+\frac{Kq_1q_2}{(l+(x_2-x_1))^2}$$

$$L=\frac{1}{2}m\dot{x}_1^2+\frac{1}{2}m\dot{x}_2^2-K(x_1^2-x_2x_1+x_2^2)-\frac{Kq^2}{l+(x_2-x_1)}$$

Lw E.M

$$m\ddot{x}_1+2x_1K-Kx_2+\frac{Kq^2}{(l+(x_2-x_1))^2}=0$$

$$m\ddot{x}_2+2Kx_2-\frac{K_eq^2}{l+(x_2-x_1)}-Kx_1=0$$

Expandiendo por Taylor y tomando el termino relevante para pequeñas oscilaciones

$$m\ddot{x}_1+2Kx_1-Kx_2+\frac{2K_eq^2}{l^3}(x_2-x_1)$$

$$m\ddot{x}_2+2Kx_2-Kx_1+\frac{2K_eq^2}{l^3}(x_2-x_1)$$

$$T = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \qquad V = \begin{bmatrix} 2K + \frac{2K_e q^2}{l^3} & -K - \frac{2K_e q^2}{l^3} \\ -K - \frac{2Kq^2}{l^3} & 2K + \frac{2K_e q^2}{l^3} \end{bmatrix}$$

$$\begin{vmatrix} 2\left(K + \frac{K_e q^2}{l^3}\right) - w^2 m & -K - \frac{2K_e q^2}{l^3} \\ -K - \frac{2K_e q^2}{l^3} & 2\left(K + \frac{K_e q^2}{l^3}\right) - w^2 m \end{vmatrix} = 0 \qquad \text{Si } \frac{K_e q^2}{l^3} = N$$

$$\left(2(K+N) - w^2 m\right)^2 - (-K-2N)^2 = 0$$

$$w_1 = \sqrt{\frac{3K + \frac{4K_e q^2}{l^3}}{m}} \qquad w_2 = \sqrt{\frac{K}{m}}$$

Estos modos $a_1 = a_2$ y $a_1 = -a_1$