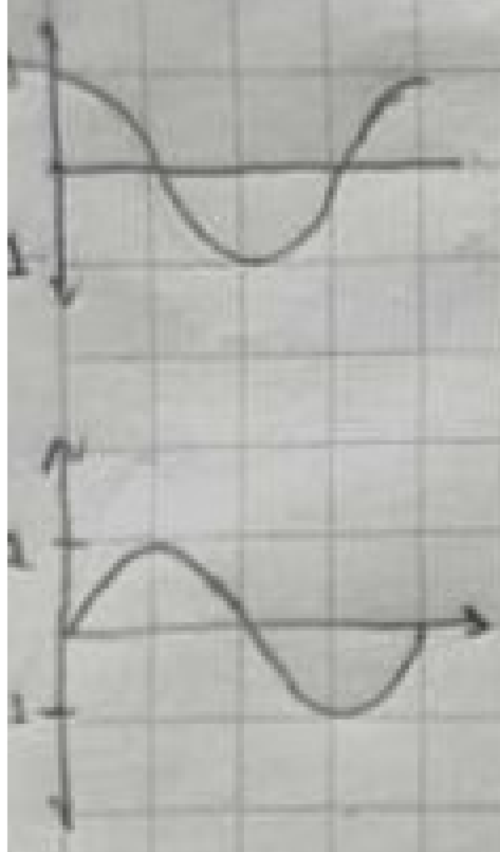
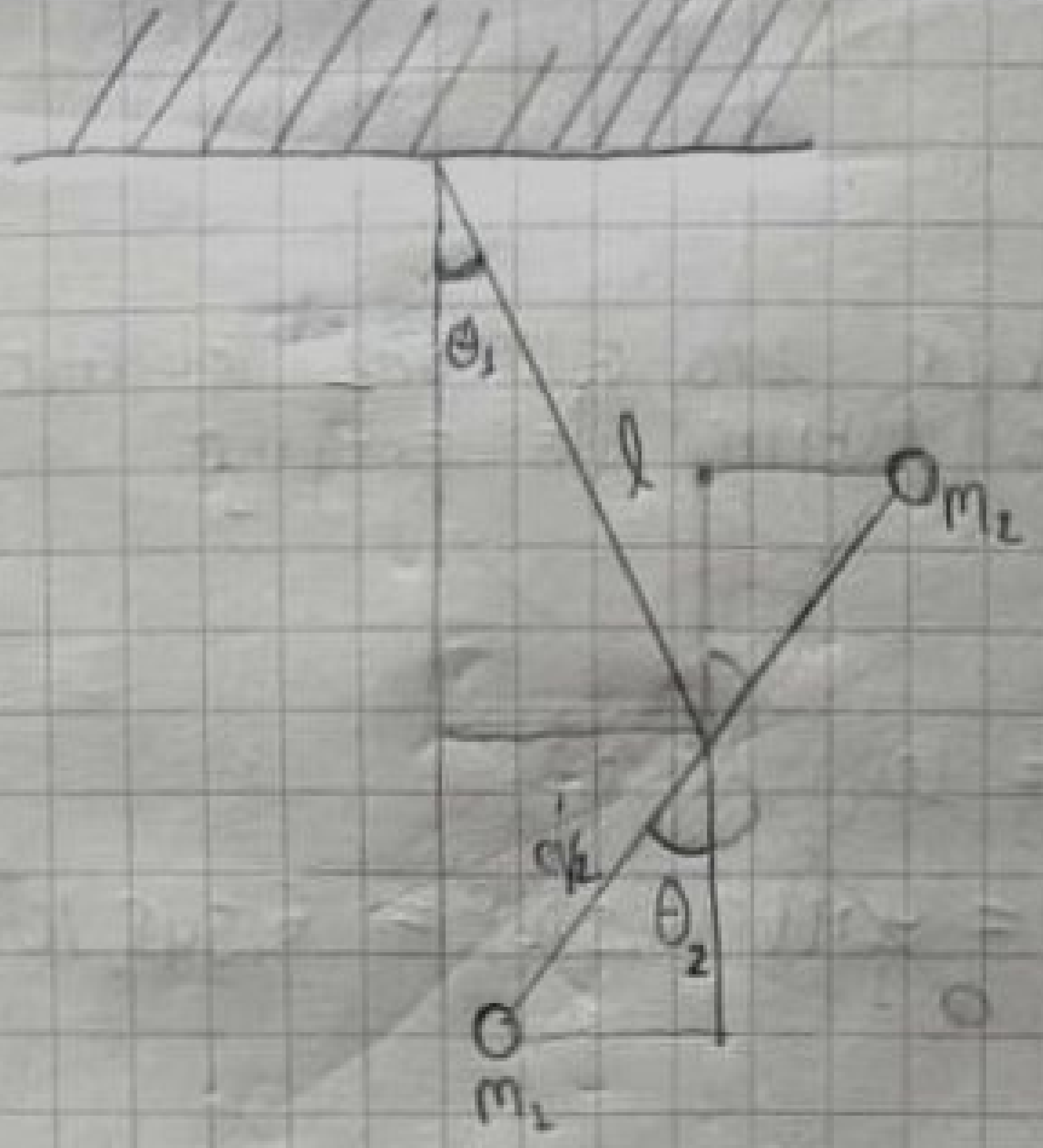
Punto 1





$x_1 = l\,sen\,\theta_1 + a/2\,sen\,\theta_2$

$y_1 = l\cos\theta_1 + a/2\cos\theta_2$

$x_2 = l\,sen\,\theta_1 - a/2\,sen\,\theta_2$

$y_2 = l\cos\theta_1 - a/2\cos\theta_2$

$\dot{x}_1 = l\cos\theta_1\dot{\theta}_1 + a/2\cos\theta_2\dot{\theta}_2$

$\dot{y}_1 = -l\,sen\theta_1\dot{\theta}_1 - a/2\,sen\theta_2\dot{\theta}_2$

$\dot{x}_2 = l\cos\theta_1\dot{\theta}_1 - a/2\cos\theta_2\dot{\theta}_2$

$\dot{y}_2 = -l\,sen\theta_1\dot{\theta}_1 + a/2\,sen\theta_2\dot{\theta}_2$

$V = m_1 g(l\cos\theta_1 + a/2\cos\theta_2) + m_2 g(l\cos\theta_1 - a/2\cos\theta_2)$

$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2 = (l\cos\theta_1\dot{\theta}_1 + a/2\cos\theta_2\dot{\theta}_2)^2 + (-l\,sen\theta_1\dot{\theta}_1 - a/2\,sen\theta_2\dot{\theta}_2)^2$

$= l^2\cos^2\theta_1\dot{\theta}_1^2 + (a/2)^2\cos^2\theta_2\dot{\theta}_2^2 + la\cos\theta_1\cos\theta_2\dot{\theta}_1\dot{\theta}_2 + l^2 sen^2\theta_1\dot{\theta}_1^2 + (a/2)^2 sen^2\theta_2\dot{\theta}_2^2 + la\,sen\theta_1 sen\theta_2\dot{\theta}_1\dot{\theta}_2$

$v_1^2 = l^2\dot{\theta}_1^2 + (a/2)^2\dot{\theta}_2^2 + la\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + sen\theta_1 sen\theta_2)$

$v_2^2 = (l\cos\theta_1\dot{\theta}_1 - a/2\cos\theta_2\dot{\theta}_2)^2 + (-l\,sen\theta_1\dot{\theta}_1 + a/2\,sen\theta_2\dot{\theta}_2)^2$

$= l^2\cos^2\theta_1\dot{\theta}_1^2 - la\cos\theta_1\cos\theta_2\dot{\theta}_1\dot{\theta}_2 + (a/2)^2\cos^2\theta_2\dot{\theta}_2^2 + l^2 sen^2\theta_1\dot{\theta}_1^2 - la\,sen\theta_1 sen\theta_2\dot{\theta}_1\dot{\theta}_2 + (a/2)^2 sen^2\theta_2\dot{\theta}_2^2$

$v_2^2 = l^2\dot{\theta}_1^2 + (a/2)^2\dot{\theta}_2^2 - la\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + sen\theta_1 sen\theta_2)$

$T = \frac{1}{2}\left((m_1+m_2)(l^2\dot{\theta}_1^2 + (a/2)^2\dot{\theta}_2^2) + la\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + sen\theta_1 sen\theta_2)(m_1-m_2)\right)$

$L = \frac{1}{2}\left((m_1+m_2)(l^2\dot{\theta}_1^2 + (a/2)^2\dot{\theta}_2^2) + la\dot{\theta}_1\dot{\theta}_2(\cos\theta_1\cos\theta_2 + sen\theta_1 sen\theta_2)(m_1-m_2)\right) -$

$-\left(m_1 g(l\cos\theta_1 + a/2\cos\theta_2) + m_2 g(l\cos\theta_1 - a/2\cos\theta_2)\right)$

$\frac{\partial L}{\partial \theta_1} = \frac{1}{2}(m_1-m_2)la\dot{\theta}_1\dot{\theta}_2(-sen\theta_1\cos\theta_2 + \cos\theta_1 sen\theta_2) + g l\,sen\theta_1(m_1+m_2)$

$\frac{\partial L}{\partial \dot{\theta}_1} = \frac{1}{2}(m_1+m_2)(2l^2\dot{\theta}_1) + la\dot{\theta}_2(\cos\theta_1\cos\theta_2 + sen\theta_1 sen\theta_2)(m_1-m_2))$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \frac{1}{2}\left((m_1+m_2)(2l^2\ddot{\theta}_1) + (m_1-m_2)la(\ddot{\theta}_2\cos(\theta_1+\theta_2) - \dot{\theta}_2 sen(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2))\right)$

Entonces para θ_1

$$l^2\ddot{\theta}_1(m_1+m_2) + \frac{1}{2}(m_1-m_2)la\left(\ddot{\theta}_2\cos(\theta_1+\theta_2) - \dot{\theta}_2\operatorname{sen}(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)\right) - \frac{1}{2}(m_2-m_1)l\dot{\theta}_1\dot{\theta}_2\operatorname{sen}(\theta_2-\theta_1)$$

$$-gl\operatorname{sen}\theta_1(m_1+m_2) = 0$$

$$(m_1+m_2)(l^2\ddot{\theta}_1 - gl\operatorname{sen}\theta_1) + \frac{1}{2}(m_1-m_2)\left(\ddot{\theta}_2\cos(\theta_1+\theta_2) - \dot{\theta}_2\operatorname{sen}(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) - al\dot{\theta}_1\dot{\theta}_2\operatorname{sen}(\theta_2-\theta_1)\right) = 0$$

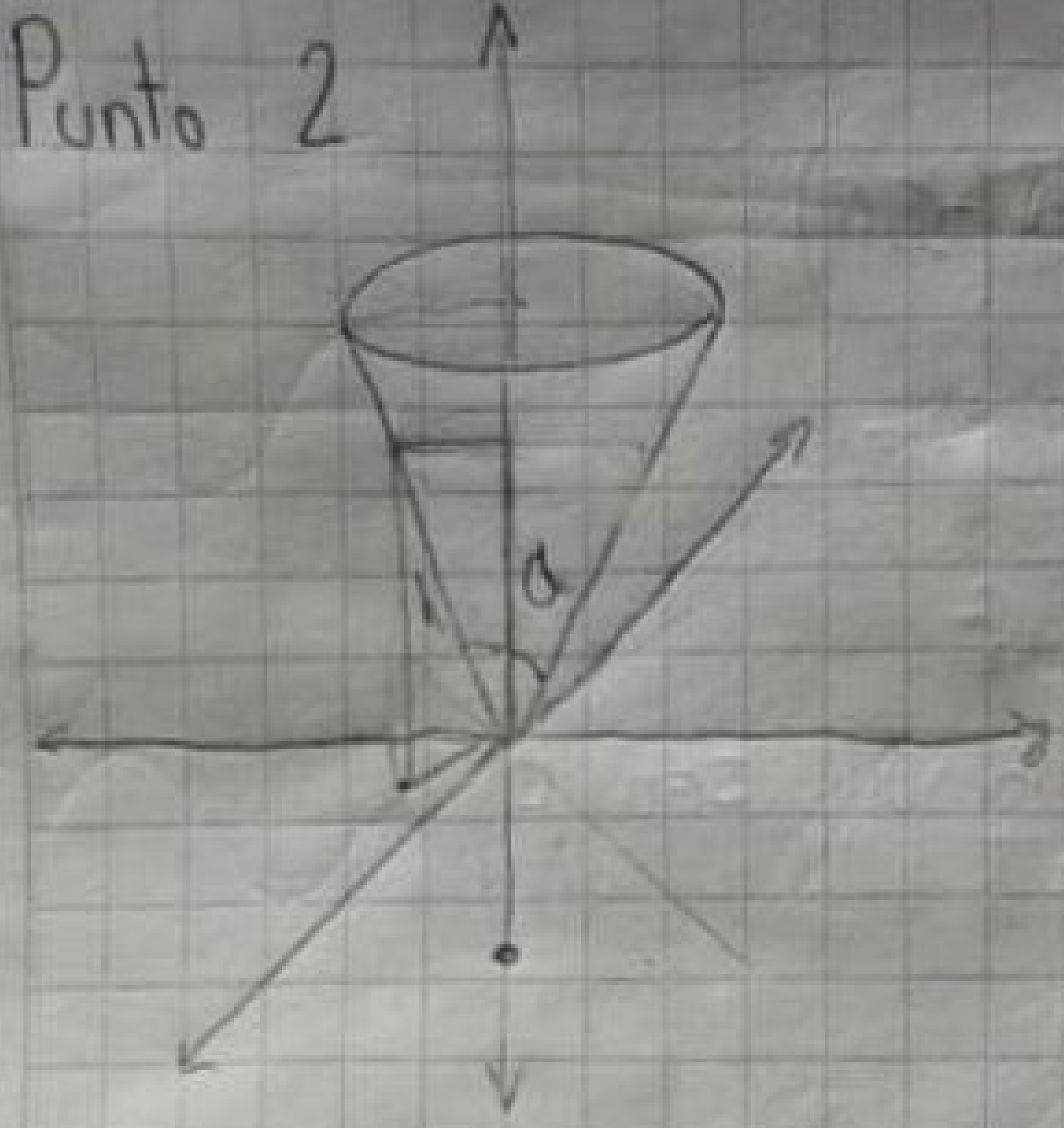
$$\frac{\partial \mathcal{L}}{\partial \theta_2} = \frac{1}{2}(m_1-m_2)la\dot{\theta}_1\dot{\theta}_2\operatorname{sen}(\theta_1-\theta_2) + a/2\, g\operatorname{sen}\theta_2(m_1-m_2)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = \frac{1}{2}\left((m_1+m_2)(2(a/2)^2\dot{\theta}_2) + la\dot{\theta}_1\cos(\theta_1+\theta_2)(m_1-m_2)\right)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) = \frac{1}{2}\left((m_1+m_2)2(a/2)^2\ddot{\theta}_2 + (m_1-m_2)la\left(\ddot{\theta}_1\cos(\theta_1+\theta_2) - \dot{\theta}_2\operatorname{sen}(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)\right)\right)$$

$$(m_1+m_2)(a/2)^2\ddot{\theta}_2 + (m_1-m_2)\left(la\left(\ddot{\theta}_1\cos(\theta_1+\theta_2) - \dot{\theta}_2(\dot{\theta}_1+\dot{\theta}_2)\operatorname{sen}(\theta_1+\theta_2)\right)\right) - \frac{1}{2}la\dot{\theta}_1\dot{\theta}_2\operatorname{sen}(\theta_1-\theta_2) - a/2\, g\operatorname{sen}\theta_2 = 0$$

Punto 2



$x_2 = 0$

$y_2 = 0$

$z_2 = l - \frac{z}{\cos a}$

$\dot{z}_2 = -\frac{\dot{z}}{\cos(a)}$

$x_1 = r\cos\theta \qquad r = z\tan a$

$y_1 = r\,\text{sen}\,\theta$

$z_1 = z$

$x_1 = z\tan a\cos\theta$

$y_1 = z\tan a\,\text{sen}\,\theta$

$\dot{z}_1 = \dot{z}$

$\dot{x}_1 = \dot{z}_1\tan a\cos\theta - z_1\tan a\,\text{sen}\,\theta\,\dot{\theta}$

$\dot{x}_1 = \dot{z}_1\tan a\,\text{sen}\,\theta + z_1\tan a\cos\theta\,\dot{\theta}$

$$V = m_1 g z_1 + m_2 g\left(l - \frac{z}{\cos a}\right)$$

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2 = (\dot{z}\tan a\cos\theta - z\tan a\,\text{sen}\,\theta\,\dot{\theta})^2 + (\dot{z}\tan a\,\text{sen}\,\theta + z\tan a\cos\theta\,\dot{\theta})^2 + \dot{z}^2$$

$$= \dot{z}^2\tan^2 a - 2\dot{z}z\tan^2 a\cos\theta\,\text{sen}\,\theta\,\dot{\theta} + 2\dot{z}z\tan^2 a\cos\theta\,\text{sen}\,\theta\,\dot{\theta} + z^2\tan^2 a\,\dot{\theta}^2(\text{sen}^2\theta + \cos^2\theta) + \dot{z}^2$$

$$= \dot{z}^2(\tan^2 a + 1) + z^2\tan^2 a\,\dot{\theta}^2(1 - 2\,\text{sen}^2\theta)$$

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2 = 0 + 0 + \left(\frac{\dot{z}}{\cos a}\right)^2 = \frac{\dot{z}^2}{\cos^2 a}$$

$$T = \frac{1}{2}m_1\left(\dot{z}^2(\sec^2 a) + z^2\tan^2 a\,\dot{\theta}^2(1 - 2\,\text{sen}^2\theta)\right) + \frac{1}{2}m_2\frac{\dot{z}^2}{\cos^2 a}$$

$$L = \frac{1}{2}m_1\left(\dot{z}^2\sec^2 a + z^2\tan^2 a\,\dot{\theta}^2(1 - 2\,\text{sen}^2\theta)\right) + \frac{1}{2}m_2\frac{\dot{z}^2}{\cos^2 a} - m_1 g z_1 - m_2 g\left(l - \frac{z}{\cos a}\right)$$

$$\frac{\partial L}{\partial\theta} = \frac{1}{2}m_1(-z^2\tan^2 a\,\dot{\theta}^2\,4\,\text{sen}\,\theta\cos\theta)$$

$$\frac{\partial L}{\partial\dot{\theta}} = \frac{1}{2}m_1\left(z^2\tan^2 a\,2\dot{\theta}(1 - 2\,\text{sen}^2\theta)\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) = \frac{1}{2}m_1\tan^2 a\left(2z\dot{z}\dot{\theta} + z^2 2\ddot{\theta} - (2z\dot{z}\dot{\theta}\,2\,\text{sen}^2\theta) - z^2 2\ddot{\theta}\,2\,\text{sen}^2\theta - z^2 2\dot{\theta}^2\cdot 4\,\text{sen}\,\theta\cos\theta\right.$$

$$\frac{1}{2}m_1\tan^2 a\left((2z\dot{z}\dot{\theta} + 2z^2\ddot{\theta})(1 - 2\,\text{sen}^2\theta) - 8z^2\dot{\theta}^2\,\text{sen}\,\theta\cos\theta + z^2\dot{\theta}^2\,4\cos\theta\,\text{sen}\,\theta\right) = 0$$

$$\frac{1}{2}m_1\tan^2 a\left((2z\dot{z}\dot{\theta} + 2z^2\ddot{\theta})(1 - 2\,\text{sen}^2\theta) - 4z^2\dot{\theta}^2\,\text{sen}\,\theta\cos\theta\right) = 0$$

Ahora Para z

$$\frac{\partial L}{\partial z} = m_1 z \tan^2 a \dot{\theta}^2 (1 - 2\,sen^2\theta) - m_1 g + \frac{m_2 g}{\cos a}$$

$$\frac{\partial L}{\partial \dot{z}} = m_1 \dot{z} \sec^2 a + \frac{m_2 \dot{z}}{\cos^2 a} = \dot{z} \sec^2 a (m_1 + m_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = \ddot{z} \sec^2 a (m_1 + m_2)$$

$$\ddot{z} \sec^2 a (m_1 + m_2) - m_1 z \tan^2 a \dot{\theta}^2 (1 - 2\,sen^2\theta) + m_1 g - \frac{m_2 g}{\cos a} = 0$$

Las fuerzas involucradas

Para m_1

$$T\,sen\,a = m_1 \frac{v^2}{r}$$

Para m_2

$$T\cos a = m_2 g$$

Dividiendo

$$\frac{sen\,a}{\cos a} = \frac{m_1 \frac{v^2}{r}}{m_2 g}$$

$$\tan a = \frac{m_1 \frac{v^2}{r}}{m_2 g}$$

Despejando r y reemplazando $v = wr$

$$r = \frac{m_2 g \tan a}{m_1 w^2}$$

El lagrangiano de un sistema de particulas se define como la diferencia entre su energía cinética y potencial

$$\mathcal{L} = T - V$$

Para este sistema la energía cinética proviene del cilindro pequeño el cual tiene dos componentes : traslación y rotación

Traslación $\Rightarrow$ El cilindro A que se mueve sobre la circunferencia del cilindro B.

$$T_{traslacional} = \frac{1}{2} m v^2$$

donde la velocidad tiene componentes en x y y . Por lo tanto

$$T_{traslacional} = \frac{1}{2} m_A (v_x^2 + v_y^2)$$ donde v_x, v_y dependen de θ y su derivada temporal

Rotación $\Rightarrow$ Al rededor de su propio eje

$$T_{rotacional} = \frac{1}{2} I \phi^2$$

donde I es el momento de inercia de un cilindro , por lo tanto

$$T_{rotacional} = \frac{1}{2}\left(\frac{1}{2} m_A a^2\right) \phi^2$$

Para la energía potencial tenemos que

$V = mgy$ donde $y = b\cos(\theta) + a$, por lo tanto

$$V = m_A g (a\cos(\theta) + b)$$

Como A rueda sin deslizarse sobre B existe una relación geométrica entre las distancias recorridas

$$a\,d\phi = b\,d\theta$$

Esto implica que la relación entre las velocidades angulares es

$$a\phi = b\theta$$

Entonces

$$T_{trasl} = \frac{1}{2} m_A (b\dot{\theta})^2$$

$$T_{rot} = \frac{1}{4} m_A a^2 \phi^2 \Rightarrow \text{usando la relación de } a\phi = b\theta$$

$$\hookrightarrow T_{rot} = \frac{1}{4} m_A a^2 \left(\frac{b}{a}\dot{\theta}\right)^2 = \frac{1}{4} m_A b^2 \dot{\theta}^2$$

Para hallar la energía cinética total

$$T = T_{trasl} + T_{rot}$$

$$T = \frac{1}{2} m_A (b\dot{\theta})^2 + \frac{1}{4} m_A b^2 \dot{\theta}^2$$

$$T = \frac{3}{4} m_A b^2 \dot{\theta}^2$$

$$\mathcal{L} = T - V$$

$$\mathcal{L} = \frac{3}{4} m_A b^2 \dot{\theta}^2 - m_A g (b + a\cos\theta)$$

Ecuación de lagrange $\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \Rightarrow \frac{\partial}{\partial \dot{\theta}}\left(\frac{3}{4} m_A b^2 \dot{\theta}^2 - m_A g (b + a\cos\theta)\right)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = 2 \cdot \frac{3}{4} m_A b^2 \dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{3}{2} m_A b^2 \dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} \Rightarrow \frac{\partial}{\partial \theta}\left(\frac{3}{4} m_A b^2 \dot{\theta}^2 - m_A g (b + a\cos\theta)\right)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -(-m_A g\, a \operatorname{sen}\theta)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = m_A g\, a \operatorname{sen}\theta$$

Entonces:

$$\frac{d}{dt}\left(\frac{3}{2} m_A b^2 \dot{\theta}\right) - m_A g\, a \operatorname{sen}\theta$$

La ecuación de movimiento es:

La separación de los cilindros ocurre cuando la F_N se anula es decir:

$$\cancel{m_A} b\dot{\theta}^2 = \cancel{m_A} g\cos\theta$$

$$\dot{\theta}^2 = \frac{g\cos\theta}{b}$$

La ecuación de la fuerza radial es:

$$N + m_A g\cos\theta = m_A b\dot{\theta}^2$$

sustituyendo:

$$N + m_A g\cos\theta = m_A \cancel{b} \frac{g\cos\theta}{\cancel{b}}$$

$$N + m_A g\cos\theta = m_A g\cos\theta$$

Despejando N tenemos que

$$N = m_A g\cos\theta - m_A g\cos\theta = 0$$

La F de fricción estática F_F es la que garantiza que A ruede sin deslizarse sobre B, es la fuerza de fricción es la que genera la aceleración angular

$$F_F = m_A b\ddot{\theta} \Rightarrow$$ ecuación de mov tangencial

$$\ddot{\theta} = \frac{-ga}{3b^2}\operatorname{sen}\theta$$ por lo tanto

$$F_F = m_A \cancel{b}\left(-\frac{ga}{3b^2}\operatorname{sen}\theta\right) \Rightarrow -\frac{m_A g a \operatorname{sen}\theta}{3b}$$

Punto 4

Consideramos una cuerda de longitud L y masa M

si $N = \frac{M}{L}$ y dividimos el problema en la parte de la mesa y la parte que cuelga

$$T_{mesa} = \frac{1}{2}\int_0^{L-x} N\dot{x}^2 dx = \frac{1}{2}N\dot{x}^2(L-x)$$

$$T_{cuelga} = \frac{1}{2}\int_0^{x} N\dot{x}^2 dx = \frac{1}{2}N\dot{x}^2 x$$

$$T = \frac{1}{2}N\dot{x}^2(L-x) + \frac{1}{2}N\dot{x}x = \frac{1}{2}\frac{M}{L}\dot{x}^2 L = \frac{1}{2}M\dot{x}^2$$

$$V = Ng\int_0^{x} -y\,dy = -Ng\frac{x^2}{2} = -\frac{Mgx^2}{2L}$$

$$L = \frac{1}{2}M\dot{x}^2 + \frac{Mgx^2}{2L}$$

$$\frac{\partial L}{\partial \dot{x}} = M\dot{x} \qquad \frac{\partial L}{\partial x} = \frac{Mgx}{L}$$

$$\boxed{M\ddot{x} - \frac{Mgx}{L} = 0}$$

La solución de esta ecuación

$$x(t) = A\cos\left(\sqrt{\frac{g}{L}}t\right) + B\sin\left(\sqrt{\frac{g}{L}}t\right)$$