Problema 1.

$y = |x|$; $y = x^2 - 2$

$|x| = x^2 - 2$

para $x \geq 0$

$x = x^2 - 2 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

para $x < 0$

$-x = x^2 - 2 \Rightarrow x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\Rightarrow x = -2 \quad y \quad x = 2$

por lo tanto

$f(x) = |x|$

$g(x) = x^2 - 2$

$A = \int_{-2}^{2} (|x| - (x^2 - 2)) \, dx$

$= \int_{-2}^{0} -x - x^2 + 2 \, dx + \int_{0}^{2} x - x^2 + 2 \, dx$

$= \left[-\frac{x^2}{2} - \frac{x^3}{3} + 2x \right]\Big|_{-2}^{0} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x \right]\Big|_{0}^{2}$

$= \left[\frac{(-2)^2}{2} - \frac{(-2)^3}{3} + 2(-2) \right] + \left[\frac{2^2}{2} - \frac{2^3}{3} + 2(2) \right]$

$= \frac{10}{3} + \frac{10}{3} = \boxed{\frac{20}{3} \, u^2}$

Problema 2

$y = x^2$; $y = 4x - x^2$, $x = 4$

$x = 4$

$x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$2(x^2 - 2x) = 0$

$2x(x - 2) = 0$

$\Rightarrow x = 0$ y $x = 2$

Para Resolver el problema se usará el método de Cascarones cilíndricos o arandelas

$2\pi \int_0^2 (4-x)(4x - x^2 - x^2) \, dx$

$= 2\pi \int_0^2 (4-x)(4x - 2x^2) \, dx = 2\pi \int_0^2 16x - 8x^2 - 4x^2 + 2x^3 \, dx$

$= 2\pi \left[8x^2 - \dfrac{8}{3}x^3 - \dfrac{4}{3}x^3 + \dfrac{x^4}{2} \right] \Big|_0^2$

$= 2\pi \left[8x^2 - 4x^3 + \dfrac{x^4}{2} \right] \Big|_0^2 = 2\pi \left(\left[8(2)^2 - 4(2)^3 + \dfrac{(2)^4}{2} \right] - [0] \right)$

$= 2\pi [32 - 32 + 8]$

$= \boxed{16\pi}$