Problema 1

a) $B = \{-7 + 4x,\ 2 - 3x\}$, $P = 17 - 6x$

$\alpha_1, \alpha_2 \in \mathbb{R}$

entonces

$\alpha_1 B_1 + \alpha_2 B_2 = P$

$\alpha_1(-7+4x) + \alpha_2(2-3x) = 17 - 6x$ $\qquad -\frac{68}{7} \quad -\frac{42}{7}$

$-7\alpha_1 + 4x\alpha_1 + 2\alpha_2 - 3x\alpha_2 = 17 - 6x$

$$\Rightarrow \begin{cases} -7\alpha_1 + 2\alpha_2 = 17 \\ 4x\alpha_1 - 3x\alpha_2 = -6x \end{cases}$$

Resolviendo el sistema

$$\left(\begin{array}{cc|c} -7 & 2 & 17 \\ 4 & -3 & -6 \end{array}\right)\left(-\tfrac{1}{7}\right) = \left(\begin{array}{cc|c} 1 & -\frac{2}{7} & \frac{17}{7} \\ 4 & -3 & -6 \end{array}\right)(-4) = \left(\begin{array}{cc|c} 1 & -\frac{2}{7} & \frac{17}{7} \\ 0 & -\frac{13}{7} & \frac{26}{7} \end{array}\right)\left(-\tfrac{7}{13}\right)$$

$$\left(\begin{array}{cc|c} 1 & -\frac{2}{7} & \frac{17}{7} \\ 0 & 1 & -2 \end{array}\right)\left(\tfrac{2}{7}\right) = \left(\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & -2 \end{array}\right)$$

$$[P]_B = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

Problema 1

B) $B = \{1 + 2x + 2x^2, 2x - x^2, -1 - 2x\}$

$P = -1 + 6x - 8x^2 \qquad \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$

$B_1\alpha_1 + B_2\alpha_2 + B_3\alpha_3 = P$

$(1 + 2x + 2x^2)\alpha_1 + (2x - x^2)\alpha_2 + (-1 - 2x)\alpha_3 = -1 + 6x - 8x^2$

$\alpha_1 + 2x\alpha_1 + 2x^2\alpha_1 + 2x\alpha_2 - x^2\alpha_2 - \alpha_3 - 2x\alpha_3 = -1 + 6x - 8x^2$

$$\Rightarrow \begin{cases} 2x^2\alpha_1 - x^2\alpha_2 = -8x^2 \\ 2x\alpha_1 + 2x\alpha_2 - 2x\alpha_3 = 6x \\ \alpha_1 - \alpha_3 = -1 \end{cases}$$

Resolviendo el sistema

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 2 & 2 & -2 & 6 \\ 2 & -1 & 0 & -8 \end{array}\right)(-2)(-2) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 2 & 0 & 8 \\ 0 & -1 & 2 & -6 \end{array}\right)\left(\tfrac{1}{2}\right) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & -1 & 2 & -6 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 2 & -2 \end{array}\right)\left(\tfrac{1}{2}\right) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)(1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$[P]_B = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix}$$

Problema 2

a) $B = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right\}$

$[M]_B = \begin{pmatrix} 4 \\ -3 \\ 8 \\ 10 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \quad \alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}$

Si

$\alpha_1 B_1 + \alpha_2 B_2 + \alpha_3 B_3 + \alpha_4 B_4 = M$

entonces

$(4)\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + (-3)\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} + (8)\begin{pmatrix} 2 & 0 \\ -1 & 0 \end{pmatrix} + (10)\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = M$

$\begin{pmatrix} 4 & 4 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 16 & 0 \\ -8 & 0 \end{pmatrix} + \begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix} = M$

$\begin{pmatrix} 33 & 24 \\ 22 & 40 \end{pmatrix} = M$

b) $M = \begin{pmatrix} 4 & -1 \\ -4 & -4 \end{pmatrix}$

$\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}$

$\alpha_1 B_1 + \alpha_2 B_2 + \alpha_3 B_3 + \alpha_4 B_4 = M$

$$\alpha_1 \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 2 & 0 \\ -1 & 0 \end{pmatrix} + \alpha_4 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 & \alpha_1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} -\alpha_2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 2\alpha_3 & 0 \\ -\alpha_3 & 0 \end{pmatrix} + \begin{pmatrix} \alpha_4 & 2\alpha_4 \\ 3\alpha_4 & 4\alpha_4 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 - \alpha_2 + 2\alpha_3 + \alpha_4 & \alpha_1 + 2\alpha_4 \\ 3\alpha_4 - \alpha_3 & 4\alpha_4 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -4 & -4 \end{pmatrix}$$

$$\Rightarrow \left\{ \begin{array}{l} \alpha_1 - \alpha_2 + 2\alpha_3 + \alpha_4 = 4 \\ \alpha_1 + 0 + 0 + 2\alpha_4 = -1 \\ 0 + 0 - \alpha_3 + 3\alpha_4 = -4 \\ 4\alpha_4 = -4 \end{array} \right\}$$

$\Rightarrow \alpha_4 = -1$

$\alpha_3 = 4 + 3\alpha_4 = (4 + 3(-1)) = 1$

$\alpha_3 = 1$

$\alpha_1 = -1 - 2\alpha_4 = -1 - 2(-1) = 1$

$\alpha_1 = 1$

$\alpha_2 = \alpha_1 + 2\alpha_3 + \alpha_4 - 4 = 1 + 2(1) + (-1) - 4 = 2$

$\alpha_2 = 2$

$$\Rightarrow [M]_B = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \end{pmatrix}$$

Problema 3

$S = \{v_1, v_2\}$ y $T = \{W_1, W_2\}$

$v_1 = (1,1)$, $v_2 = (1,2)$, $w_1 = (1,3)$ y $w_2 = (1,4)$

$Q_{S \to T}$

$$v_1 = a_{11} W_1 + a_{21} W_2$$
$$v_2 = a_{12} W_1 + a_{22} W_2$$

Para $[V_1]_T$

$$(1,1) = a_{11}(1,3) + a_{21}(1,4)$$
$$= (a_{11}, 3a_{11}) + (a_{21}, 4a_{21})$$
$$= (a_{11} + a_{21}, 3a_{11} + 4a_{21})$$

$a_{11} + a_{21} = 1$
$3a_{11} + 4a_{21} = 1$ $\Rightarrow \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 3 & 4 & 1 \end{array}\right)^{(-3)} = \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -2 \end{array}\right)(-1) = \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right)$

como $[V_1]_T = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

ahora para

$[V_2]_T = \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix}$

$$(1,2) = a_{12}(1,3) + a_{22}(1,4)$$
$$= (a_{12}, 3a_{12}) + (a_{22}, 4a_{22})$$
$$= (a_{12} + a_{22}, 3a_{12} + 4a_{22})$$

$$a_{12} + a_{22} = 1$$
$$3a_{12} + 4a_{22} = 2$$
$$\Rightarrow \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 3 & 4 & 2 \end{array}\right)(-3) = \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \end{array}\right)(-1)$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right)$$

$$\Rightarrow [V_2]_T = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

como

$$Q_{S\to T} = \left([V_1]_T \; [V_2]_T \right) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

en tonces

$$Q_{S\to T} = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$$

Problema 4

$S = \{(-1,2,1), (0,1,1), (-2,2,1)\}$ y $T = \{(-1,1,0), (0,1,0)$
$(0,1,1)\}$

$[V]_S = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

a)

Para $T_{T \to S}$

$T_1 = a_{11} S_1 + a_{21} S_2 + a_{31} S_3$
$T_2 = a_{12} S_1 + a_{22} S_2 + a_{32} S_3$
$T_3 = a_{13} S_1 + a_{32} S_2 + a_{33} S_3$

Para

$[T_1]_S = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$(-1,1,0) = a_{11}(-1,2,1) + a_{21}(0,1,1) + a_{31}(-2,2,1)$
$= (-a_{11}, 2a_{11}, a_{11}) + (0, a_{21}, a_{21}) + (-2a_{31}, 2a_{31}, a_{31})$
$= (-a_{11} - 2a_{31}, 2a_{11} + a_{21} + 2a_{31}, a_{11} + a_{21} + a_{31})$

$-a_{11} - 2a_{31} = -1$
$2a_{11} + a_{21} + 2a_{31} = 1$
$a_{11} + a_{21} + a_{31} = 0$

$= \left(\begin{array}{ccc|c} -1 & 0 & -2 & -1 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right)^{(-1)} = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right)^{(-2)(-1)}$

$= \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -1 & -1 \end{array}\right)^{(-1)} = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)_{(-2)(2)} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$[T_1]_S = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

para $[T_2]_S = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$

$$(0,1,0) = a_{12}(-1,2,1) + a_{22}(0,1,1) + a_{32}(-2,2,1)$$
$$= (-a_{12}, 2a_{12}, a_{12}) + (0, a_{22}, a_{22}) + (-2a_{32}, 2a_{32}, a_{32})$$
$$= (-a_{12} - 2a_{32}, 2a_{12} + a_{22} + 2a_{32}, a_{12} + a_{22} + a_{32})$$

$-a_{12} - 2a_{32} = 0$
$2a_{12} + a_{22} + a_{32} = 1$
$a_{12} + a_{22} + a_{32} = 0$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \end{array}\right) (-2)(1) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right) (-1)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right) (-1) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 1 \end{array}\right) \left(-\frac{1}{2}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right) (-1)(-1)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1/2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \end{array}\right) (-1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 \end{array}\right)$$

$$[T_2]_S = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$$

para $[T_3]_S = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$

$$(0,1,1) = a_{13}(-1,2,1) + a_{23}(0,1,1) + a_{33}(-2,2,1)$$

$$(0,1,1) = (-a_{13} - 2a_{33},\ 2a_{13} + a_{23} + 2a_{33},\ a_{13} + a_{23} + a_{33})$$

$$-a_{13} - 2a_{33} = 0$$
$$2a_{13} + a_{23} + 2a_{33} = 1$$
$$a_{13} + a_{23} + a_{33} = 1$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ -1 & 0 & -2 & 1 \end{array}\right) (-2)(1) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right) (-1)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 1 \end{array}\right) (-1) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 2 \end{array}\right) (-\tfrac{1}{2}) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right) (-1)(-1)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) (-1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$[T_3]_S = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$T_{T \to S} = \left([T_1]_S\ [T_2]_S\ [T_3]_S \right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$T_{T \to S} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & -1 \end{pmatrix}$$

Para

$T_{S \to T}$

$S_1 = a_{11} T_1 + a_{21} T_2 + a_{31} T_3$
$S_2 = a_{12} T_1 + a_{22} T_2 + a_{32} T_3$
$S_3 = a_{13} T_1 + a_{23} T_2 + a_{33}$

como $[S_1]_T = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$(-1, 2, 1) = a_{11}(-1, 1, 0) + a_{21}(0, 1, 0) + a_{31}(0, 1, 1)$
$= (-a_{11}, a_{11}, 0) + (0, a_{21}, 0) + (0, a_{31}, a_{31})$
$= (-a_{11}, a_{11} + a_{21} + a_{31}, a_{31})$

$\Rightarrow -a_{11} = -1 \Rightarrow a_{11} = 1$
$a_{11} + a_{21} + a_{31} = 2 \Rightarrow a_{21} = 0$
$a_{31} = 1$

entonces $[S_1]_T = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

ahora para $[S_2]_T = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$

$$(0,1,1) = a_{12}(-1,1,0) + a_{22}(0,1,0) + a_{32}(0,1,1)$$
$$= (-a_{12},\ a_{12}+a_{22}+a_{32},\ a_{32})$$

$\Rightarrow$ $a_{12} = 0$ $\qquad$ $a_{12}+a_{22}+a_{32} = 1$

$a_{32} = 1$ $\qquad$ $0 + a_{22} + 1 = 1$

$a_{22} = 0$

$$\Rightarrow [S_2]_T = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

para $[S_3]_T = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$

$$(-2,2,1) = a_{13}(-1,1,0) + a_{23}(0,1,0) + a_{33}(0,1,1)$$
$$= (-a_{13},\ a_{13}+a_{23}+a_{33},\ a_{33})$$

$\Rightarrow -a_{13} = -2 \Rightarrow a_{13} = 2$

$a_{33} = 1$

$a_{13} + a_{23} + a_{33} = 2$

$a_{23} = -1$

$$\Rightarrow [S_3]_T = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

como

$$T_{S\to T} = \left[[S_1]_T \; [S_2]_T \; [S_3]_T \right] = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

entonces

$$T_{S\to T} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

b) como

$$V = \alpha_1 S_1 + \alpha_2 S_2 + \alpha_3 S_3 \qquad \text{pero } [V]_S = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$\Rightarrow$

$$V = (2)(-1,2,1) + (0)\cancel{(0,1,1)} + (1)(-2,2,1)$$
$$= (-2,4,2) + (-2,2,1)$$
$$= (-4,6,3)$$

ahora puesto que $[V]_T =$

$$V = \beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3$$

entonces

$$(-4,6,3) = \beta_1(-1,1,0) + \beta_2(0,1,0) + \beta_3(0,1,1)$$
$$= (-\beta_1, \beta_1, 0) + (0, \beta_2, 0) + (0, \beta_3, \beta_3)$$

$(-4, 6, 3) = (-\beta_1, \beta_1 + \beta_2 + \beta_3, \beta_3)$

$\Rightarrow -\beta_1 = -4 \Rightarrow \beta_1 = 4$

$\beta_3 = 3$

$\beta_1 + \beta_2 + \beta_3 = 6$

$4 + \beta_2 + 3 = 6$

$\beta_2 = -1$

$$[V]_T = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

c) $$(T_{T\to S})(T_{S\to T}) = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$a_{11} = 1 + 1 = 2$

$a_{12} = 1$

$a_{13} = 2 - 1 + 1 = 2$

$a_{21} = -1$

$a_{22} = 0$

$a_{23} = -2 + \frac{1}{2} = -\frac{3}{2}$

$a_{31} = -1$

$a_{32} = -1$

$a_{33} = \frac{1}{2} - 1 = -\frac{1}{2}$

$$= [A] = \begin{pmatrix} 2 & 1 & 2 \\ -1 & 0 & -\frac{3}{2} \\ -1 & -1 & -\frac{1}{2} \end{pmatrix}$$

Problema 5

$S = \{v_1, v_2\}$ y $T = \{t, t+1\}$

$\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} = [T_{S \to T}]$

$T_{S \to T} = \left([V_1]_T \quad [V_2]_T \right) = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$

$\Rightarrow [V_1]_T = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$

$\Rightarrow [V_2]_T = \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$

como

$V_1 = a_{11} T_1 + a_{21} T_2$

$V_2 = a_{12} T_1 + a_{22} T_2$

$V_1 = (2)(t) + (-1)(t+1)$

$V_1 = 2t - t - 1 \Rightarrow V_1 = t - 1$

$V_2 = (3)(t) + (2)(t+1)$

$V_2 = 3t + 2t + 2$

$V_2 = 5t + 2$

$S = \{V_1, V_2\} = \{(t-1), (5t+2)\}$

Problema 6

a) $V = \text{gen}\left\{ \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \right\}$

$v_1 = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ y $v_2 = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

$w_1 = v_1 = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

$$w_2 = v_2 - \frac{w_1 \cdot v_2}{w_1 \cdot w_1} w_1 = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - \frac{\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}}{\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}} \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$w_2 = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - \frac{1}{7}\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 4/7 \\ 2/7 \\ -1/7 \end{pmatrix} = \begin{pmatrix} 3/7 \\ 12/7 \\ 34/7 \end{pmatrix}$$

ahora normalizamos

$$U_1 = \frac{w_1}{|w_1|} = \frac{1}{\sqrt{21}}\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4/\sqrt{21} \\ 2/\sqrt{21} \\ -1/\sqrt{21} \end{pmatrix}$$

$$U_2 = \frac{w_2}{|w_2|} = \frac{1}{\sqrt{\frac{187}{7}}}\begin{pmatrix} 3/7 \\ 12/7 \\ 34/7 \end{pmatrix} = \begin{pmatrix} \frac{3}{7}\left(\sqrt{\frac{187}{7}}\right) \\ \frac{12}{7}\left(\sqrt{\frac{187}{7}}\right) \\ \frac{34}{7}\left(\sqrt{\frac{187}{7}}\right) \end{pmatrix}$$

$\therefore$ El conjunto de vectores $\{U_1, U_2\}$ es una base ortonormal

Problema 7

$$v_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad y \quad v_2 = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$$

$$w_1 = v_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$w_2 = v_2 - \frac{w_1 \cdot v_2}{w_1 \cdot w_1} w_1 = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} - \frac{-9}{6} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$w_2 = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} + -\frac{3}{2} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -\frac{3}{2} \\ 3 \\ -3/2 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix}$$

el conjunto $\{w_1, w_2\}$ es ortogonal,

$$u_1 = \frac{w_1}{|w_1|} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix}$$

$$u_2 = \frac{w_2}{|w_2|} = \sqrt{2} \begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix} = \begin{pmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$$

$\therefore$ el conjunto $\{u_1, u_2\}$ es una base ortonormal

Problema 8

$S=\{v_1, v_2, v_3\}$ y $T=\{w_1, w_2, w_3\}$

$w_1=(0,1,1)$, $w_2=(1,0,0)$ y $w_3=(1,1,0)$

$$P_{T\to S}=\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

como

$$P_{T\to S}=([w_1]_S\ [w_2]_S\ [w_3]_S)$$

además

$$[w_1]_S=\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \quad [w_2]_S=\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} \quad [w_3]_S=\begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

entonces

$$w_1 = a_{11}v_1 + a_{21}v_2 + a_{31}v_3$$

$$(0,1,1) = 0\cdot v_1 + \left(\tfrac{1}{2}\right)\cdot v_2 + \left(-\tfrac{1}{2}\right)v_3$$

$$w_2 = a_{12}v_1 + a_{22}v_2 + a_{32}v_3$$

$$(1,0,0) = v_1 + \left(-\tfrac{1}{2}\right)v_2 + \tfrac{1}{2}v_3$$

w.

$w_3 = a_{13} v_1 + a_{23} v_2 + a_{33} v_3$

$(1,1,0) = 0v_1 + 0v_2 + v_3$

$\Rightarrow \boxed{(1,1,0) = v_3}$ pero

$(0,1,1) = \frac{1}{2} v_2 + \left(-\frac{1}{2}\right) v_3 = \frac{1}{2} v_2 + \left(-\frac{1}{2}\right)(1,1,0)$

$= \frac{1}{2} v_2 + \left(-\frac{1}{2}, -\frac{1}{2}, 0\right)$

$(0,1,1) - \left(-\frac{1}{2}, -\frac{1}{2}, 0\right) = \frac{1}{2} v_2$

$2\left(\frac{1}{2}, \frac{1}{2}, 1\right) = v_2$

$\boxed{(1,1,2) = v_2}$

a demas

$(1,0,0) = v_1 + \left(-\frac{1}{2}\right) v_2 + \frac{1}{2} v_3$

$(1,0,0) - \left(-\frac{1}{2}\right) v_2 - \frac{1}{2} v_3 = v_1$

$(1,0,0) - \left(-\frac{1}{2}, -\frac{1}{2}, -1\right) - \left(\frac{1}{2}, \frac{1}{2}, 0\right) = v_1$

$\boxed{(1,0,1) = v_1}$

Problema 9

Si $S = \{ t^2 + t + 1,\ t^2 + 2t + 3,\ t^2 + 1 \}$

$T = \{ t+1,\ t^2,\ t^2+1 \}$

para $S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

$P_{T \to S}$ para $T = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

$T_1 = a_{11} S_1 + a_{21} S_2 + a_{31} S_3$

$T_2 = a_{12} S_1 + a_{22} S_2 + a_{32} S_3$

$T_3 = a_{13} S_1 + a_{23} S_2 + a_{33} S_3$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = a_{11} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + a_{21} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + a_{31} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{11} \\ a_{11} \end{pmatrix} + \begin{pmatrix} a_{21} \\ 2a_{21} \\ 3a_{21} \end{pmatrix} + \begin{pmatrix} a_{31} \\ 0 \\ a_{31} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} + a_{21} + a_{31} \\ a_{11} + 2a_{21} \\ a_{11} + 3a_{21} + a_{31} \end{pmatrix} \Rightarrow \begin{matrix} a_{11} + a_{21} + a_{31} = 0 \\ a_{11} + 2a_{21} = 1 \\ a_{11} + 3a_{21} + a_{31} = 1 \end{matrix} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 3 & 1 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & 0 & 1 \end{array} \right) (-2) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & -1 \end{array} \right) \tfrac{1}{2} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1/2 \end{array} \right) (1)(-1)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1/2 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array} \right) (-1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -1/2 \end{array} \right) \Rightarrow [T_1]_S = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \\ -1/2 \end{pmatrix}$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} = a_{12}\begin{pmatrix}1\\1\\1\end{pmatrix} + a_{22}\begin{pmatrix}1\\2\\3\end{pmatrix} + a_{32}\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}a_{12} + a_{22} + a_{32}\\ a_{12} + 2a_{22}\\ a_{12} + 3a_{22} + a_{32}\end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c}1&1&1&1\\1&2&0&0\\1&3&1&0\end{array}\right)(-1)(-1)$$

$$= \left(\begin{array}{ccc|c}1&1&1&1\\0&1&-1&-1\\0&2&0&-1\end{array}\right)(-2) = \left(\begin{array}{ccc|c}1&1&1&1\\0&1&-1&-1\\0&0&2&1\end{array}\right)\left(\tfrac{1}{2}\right) = \left(\begin{array}{ccc|c}1&1&1&1\\0&1&-1&-1\\0&0&1&\frac{1}{2}\end{array}\right)(1)(-1)$$

$$= \left(\begin{array}{ccc|c}1&1&0&\frac{1}{2}\\0&1&0&-1/2\\0&0&1&1/2\end{array}\right)(-1) = \left(\begin{array}{ccc|c}1&0&0&1\\0&1&0&-1/2\\0&0&1&1/2\end{array}\right)$$

$$[T_2]_S = \begin{pmatrix}a_{12}\\a_{22}\\a_{32}\end{pmatrix} = \begin{pmatrix}1\\-1/2\\1/2\end{pmatrix}$$

para $[T_3]_S = \begin{pmatrix}a_{13}\\a_{23}\\a_{33}\end{pmatrix}$

$$\begin{pmatrix}1\\0\\1\end{pmatrix} = a_{13}\begin{pmatrix}1\\1\\1\end{pmatrix} + a_{23}\begin{pmatrix}1\\2\\3\end{pmatrix} + a_{33}\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}a_{13} + a_{23} + a_{33}\\ a_{13} + 2a_{23}\\ a_{13} + 3a_{23} + a_{33}\end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c}1&1&1&1\\1&2&0&0\\1&3&1&1\end{array}\right)(-1)(-1)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & 0 & 0 \end{array}\right)(-2) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 2 \end{array}\right)\left(\tfrac{1}{2}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)(1)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)(-1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$$[T_3]_S = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$P_{T \to S} = \left([T_1]_S \; [T_2]_S \; [T_3]_S \right) = \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & -1/2 & 0 \\ -1/2 & 1/2 & 1 \end{pmatrix}$$

para $Q_{S \to T}$

$S_1 = a_{11} T_1 + a_{21} T_2 + a_{31} T_3$
$S_2 = a_{12} T_1 + a_{22} T_2 + a_{32} T_3$
$S_3 = a_{13} T_1 + a_{23} T_2 + a_{33} T_3$

para $[S_1]_T = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = a_{11} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + a_{21} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + a_{31} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ a_{11} \\ a_{11} \end{pmatrix} + \begin{pmatrix} a_{21} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} a_{31} \\ 0 \\ a_{31} \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{21} + a_{31} \\ a_{11} \\ a_{11} + a_{31} \end{pmatrix} \Rightarrow \begin{matrix} a_{21} + a_{31} = 1 \\ a_{11} = 1 \\ a_{11} + a_{31} = 1 \end{matrix} \Rightarrow a_{31} = 0 \Rightarrow a_{21} = 1$

$\therefore [S_1]_T = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

para $[S_2]_T = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = a_{12} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + a_{22} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + a_{32} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{matrix} a_{21} + a_{31} = 1 \\ a_{12} = 2 \\ a_{12} + a_{32} = 3 \end{matrix} \Rightarrow a_{32} = 1 \Rightarrow a_{21} = 0$

$$[S_2]_T = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

para $[S_3]_T = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = a_{13}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + a_{23}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + a_{33}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{array}{l} a_{23} + a_{33} = 1 \\ a_{13} = 0 \quad \Rightarrow a_{33} = 1 \Rightarrow a_{23} = 0 \\ a_{13} + a_{33} = 1 \end{array}$$

$$[S_3]_T = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$Q_{S\to T} = ([S_1]_T \ [S_2]_T \ [S_3]_T) = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Problema 10

$S = \{(0,-2,3), (0,1,1), (1,1,0)\}$

$T = \{(0,-1,1), (0,3,0), (1,-1,1)\}$

$[v]_T = (2,1,3)$ y $[v]_S = (-1,4,1)$

a) $P_{T \to S}$

$T_1 = a_{11} S_1 + a_{21} S_2 + a_{31} S_3$

$T_2 = a_{12} S_1 + a_{22} S_2 + a_{32} S_3$

$T_3 = a_{13} S_1 + a_{23} S_2 + a_{33} S_3$

Para $[T_1]_S = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$(0,-1,1) = a_{11}(0,-2,3) + a_{21}(0,1,1) + a_{31}(1,1,0)$

$(0,-1,1) = (0,-2a_{11},3a_{11}) + (0,a_{21},a_{21}) + (a_{31},a_{31},0)$

$(0,-1,1) = (a_{31}, -2a_{11} + a_{21} + a_{31}, 3a_{11} + a_{21})$

$\Rightarrow \quad a_{31} = 0$

$-2a_{11} + a_{21} + a_{31} = -1 \quad \Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 0 & 1 \\ -2 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)(\tfrac{1}{3}) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 1/3 \\ -2 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$3a_{11} + a_{21} = 1$

$= \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 1/3 \\ 0 & 5/3 & 1 & -1/3 \\ 0 & 0 & 1 & 0 \end{array}\right)(\tfrac{3}{5}) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 1/3 \\ 0 & 1 & 3/5 & -\tfrac{1}{5} \\ 0 & 0 & 1 & 0 \end{array}\right)\begin{matrix}(-\tfrac{1}{3}) \\ (-\tfrac{3}{5})\end{matrix} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2/5 \\ 0 & 1 & 0 & -1/5 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$$[T_1]_S = \begin{pmatrix} 2/5 \\ -1/5 \\ 0 \end{pmatrix}$$

Para $[T_2]_S = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$

$$(0,3,0) = a_{12}(0,-2,3) + a_{22}(0,1,1) + a_{32}(1,1,0)$$

$$(0,3,0) = (a_{32}, -2a_{12} + a_{22} + a_{32}, 3a_{12} + a_{22})$$

$$a_{32} = 0$$

$$-2a_{12} + a_{22} + a_{32} = 3 \Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ -2 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right)^{(\frac{1}{3})} = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ -2 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$3a_{12} + a_{22} = 0$$

$$= \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ 0 & 5/3 & 1 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right) (\tfrac{3}{5}) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ 0 & 1 & 3/5 & 9/5 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{matrix} (-\frac{1}{3}) \\ (-\frac{3}{5}) \end{matrix} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3/5 \\ 0 & 1 & 0 & 9/5 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$[T_2]_S = \begin{pmatrix} -3/5 \\ 9/5 \\ 0 \end{pmatrix}$$

Para $[T_3]_S = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$

$(0,-1,1) = a_{13}(0,-2,3) + a_{23}(0,1,1) + a_{33}(1,1,0)$

$(0,-1,1) = (a_{33}, -2a_{13} + a_{23} + a_{33}, 3a_{13} + a_{23})$

$a_{33} = 0$

$-2a_{13} + a_{23} + a_{33} = -1 \Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 0 & 1 \\ -2 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right) \left(\frac{1}{3}\right) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 1/3 \\ -2 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$3a_{13} + a_{23} = 1$

$= \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 1/3 \\ 0 & 5/3 & 1 & -1/3 \\ 0 & 0 & 1 & 0 \end{array}\right) \left(\frac{3}{5}\right) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 1/3 \\ 0 & 1 & 3/5 & -1/5 \\ 0 & 0 & 1 & 0 \end{array}\right) \begin{array}{c} \left(-\frac{1}{3}\right) \\ \left(-\frac{3}{5}\right) \end{array} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2/5 \\ 0 & 1 & 0 & -1/5 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$[T_3]_S = \begin{pmatrix} 2/5 \\ -1/5 \\ 0 \end{pmatrix}$

$P_{T \to S} = ([T_1]_S [T_2]_S [T_3]_S) = \begin{pmatrix} 2/5 & -3/5 & 2/5 \\ -1/5 & 9/5 & -1/5 \\ 0 & 0 & 0 \end{pmatrix}$

b) ¿$[v]_S$?

$[v]_T = (2,1,3)$

$$[v]_T = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \Rightarrow v = \alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3$$

$$\Rightarrow v = (2)(0,-1,1) + (1)(0,3,0) + (3)(1,-1,1)$$

$$v = (0,-2,2) + (0,3,0) + (3,-3,3)$$

$$\boxed{v = (3,-2,5)}$$

$$\Rightarrow v = \beta_1 S_1 + \beta_2 S_2 + \beta_3 S_3 \,/\, [v]_S = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

$$(3,-2,5) = \beta_1(0,-2,3) + \beta_2(0,1,1) + \beta_3(1,1,0)$$

$$(3,-2,5) = (0,-2\beta_1,3\beta_1) + (0,\beta_2,\beta_2) + (\beta_3,\beta_3,0)$$

$$(3,-2,5) = (\beta_3, -2\beta_1 + \beta_2 + \beta_3, 3\beta_1 + \beta_2)$$

$\beta_3 = 3$

$-2\beta_1 + \beta_2 + \beta_3 = -2$

$3\beta_1 + \beta_2 = 5$

$$\Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 0 & 5 \\ -2 & 1 & 1 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \left(\tfrac{1}{3}\right) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 5/3 \\ -2 & 1 & 1 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 5/3 \\ 0 & \frac{5}{3} & 1 & 4/3 \\ 0 & 0 & 1 & 3 \end{array}\right) \left(\tfrac{3}{5}\right) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & 5/3 \\ 0 & 1 & \frac{3}{5} & \frac{4}{5} \\ 0 & 0 & 1 & 3 \end{array}\right) \left(-\tfrac{3}{5}\right) = \left(\begin{array}{ccc|c} 1 & 1/3 & 0 & \frac{5}{3} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

$$\Rightarrow [v]_S = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

c) $a_{S \to T}$

$$S_1 = a_{11} T_1 + a_{21} T_2 + a_{31} T_3$$
$$S_2 = a_{12} T_1 + a_{22} T_2 + a_{32} T_3$$
$$S_3 = a_{13} T_1 + a_{23} T_2 + a_{33} T_3$$

Para $[S_1]_T = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$$(0,-2,3) = a_{11}(0,-1,1) + a_{21}(0,3,0) + a_{31}(1,-1,1)$$
$$(0,-2,3) = (0,-a_{11},a_{11}) + (0,3a_{21},0) + (a_{31},-a_{31},a_{31})$$
$$(0,-2,3) = (a_{31}, -a_{11}+3a_{21}-a_{31}, a_{11}+a_{31})$$

$$a_{31} = 0$$
$$-a_{11} + 3a_{21} - a_{31} = -2$$
$$a_{11} + a_{31} = 3$$

$\Rightarrow a_{11} = 3$

$\Rightarrow a_{21} = \frac{1}{3}$

$$\therefore [S_1]_T = \begin{pmatrix} 3 \\ 1/3 \\ 0 \end{pmatrix}$$

$$[S_2]_T = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$$

$$(0,1,1) = a_{12}(0,-1,1) + a_{22}(0,3,0) + a_{32}(1,-1,1)$$

$$(0,1,1) = (a_{32}, -a_{12} + 3a_{22} - a_{32}, a_{12} + a_{32})$$

$a_{32} = 0$

$-a_{12} + 3a_{22} - a_{32} = 1$

$a_{12} + a_{32} = 1$

como $a_{32} = 0$, entonces

$a_{12} = 1$

ademas

$a_{22} = \frac{2}{3}$

$$[S_2]_T = \begin{pmatrix} 1 \\ 2/3 \\ 0 \end{pmatrix}$$

$$[S_3]_T = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

$$(1,1,0) = a_{13}(0,-1,1) + a_{23}(0,3,0) + a_{33}(1,-1,1)$$

$$(1,1,0) = (a_{33}, -a_{13} + 3a_{23} - a_{32}, a_{13} + a_{33})$$

$a_{33} = 1$

$-a_{13} + 3a_{23} - a_{32} = 1$

$a_{13} + a_{33} = 0$

como $a_{33} = 1$, entonces

$a_{13} = -1$

ademas

$a_{23} = \frac{1}{3}$

$$\Rightarrow [S_3]_T = \begin{pmatrix} -1 \\ 1/3 \\ 1 \end{pmatrix}$$

como

$$a_{S\to T} = ([S_1]_T\ [S_2]_T\ [S_3]_T) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{S\to T} = \begin{pmatrix} 3 & 1 & -1 \\ 1/3 & 2/3 & 1/3 \\ 0 & 0 & 1 \end{pmatrix}$$

d) ¿ $[V]_T$?

si $[V]_S = (-1, 4, 1)$, entonces

$$[V]_S = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$$

ademas

$$V = \alpha_1 S_1 + \alpha_2 S_2 + \alpha_3 S_3$$

$$V = (-1)(0,-2,3) + (4)(0,1,1) + (1)(1,1,0)$$

$$V = (0, 2, -3) + (0, 4, 4) + (1, 1, 0)$$

$$V = (1, 7, 1)$$

ahora si $V = \beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3$

ademas $[V]_T = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$

$(1,7,1) = \beta_1(0,-1,1) + \beta_2(0,3,0) + \beta_3(1,-1,1)$

$(1,7,1) = (0,-\beta_1,\beta_1) + (0,3\beta_2,0) + (\beta_3,-\beta_3,\beta_3)$

$(1,7,1) = (\beta_3, -\beta_1 + 3\beta_2 - \beta_3, \beta_1 + \beta_3)$

$\Rightarrow \beta_3 = 1$

$-\beta_1 + 3\beta_2 - \beta_3 = 7$

$\beta_1 + \beta_3 = 1$

como $\beta_3 = 1$, entonces

$\beta_1 = 0$

además

$\beta_2 = \frac{8}{3}$

$\therefore [V]_T = \begin{pmatrix} 0 \\ 8/3 \\ 1 \end{pmatrix}$

Problema 11

$S = \{v_1, v_2, v_3\}$ y $T = \{w_1, w_2, w_3\}$

$v_1 = (1, 0, 1)$, $v_2 = (-1, 0, 0)$ y $v_3 = (0, 1, 2)$

$$P_{T \to S} = \begin{pmatrix} -2 & -5 & -2 \\ -1 & -6 & -2 \\ 1 & 2 & 1 \end{pmatrix}$$

$$P_{T \to S} = \left([W_1]_S \ [W_2]_S \ [W_3]_S \right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

ademas

$$[W_1]_S = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \quad [W_2]_S = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} \quad [W_3]_S = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

$$W_1 = a_{11} V_1 + a_{21} V_2 + a_{31} V_3$$
$$W_2 = a_{12} V_1 + a_{22} V_2 + a_{32} V_3$$
$$W_3 = a_{13} V_1 + a_{23} V_2 + a_{33} V_3$$

$$W_1 = (-2)(1, 0, 1) + (-1)(-1, 0, 0) + (1)(0, 1, 2)$$
$$W_1 = (-2, 0, -2) + (1, 0, 0) + (0, 1, 2)$$
$$\boxed{W_1 = (-1, 1, 0)}$$

$w_2 = (-5)(1,0,1) + (-6)(-1,0,0) + (2)(0,1,2)$

$w_2 = (-5,0,-5) + (6,0,0) + (0,2,4)$

$\boxed{w_2 = (1,2,-1)}$

$w_3 = (-2)(1,0,1) + (-2)(-1,0,0) + (1)(0,1,2)$

$w_3 = (-2,0,-2) + (2,0,0) + (0,1,2)$

$\boxed{w_3 = (0,1,0)}$

Problema 12

$E = \{e_r, e_\theta, e_\varphi\}$ vectores base en coordenadas esféricas

$C = \{e_\rho, e_\varphi, e_z\}$ vectores base en coordenadas cilíndricas

$e_r = (\operatorname{sen}\theta\cos\varphi, \operatorname{sen}\theta\operatorname{sen}\varphi, \cos\theta)$

$e_\theta = (\cos\theta\cos\varphi, \cos\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta)$

$e_\varphi = (-\operatorname{sen}\varphi, \cos\varphi, 0)$

$e_\rho = (\cos\varphi, \operatorname{sen}\varphi, 0)$

$e_\varphi = (-\operatorname{sen}\varphi, \cos\varphi, 0)$

$e_z = (0, 0, 1)$

$T_{E \to C}$

$e_r = a_{11} e_\rho + a_{21} e_\varphi + a_{31} e_z$

$e_\theta = a_{12} e_\rho + a_{22} e_\varphi + a_{32} e_z$

$e_\varphi = a_{13} e_\rho + a_{23} e_\varphi + a_{33} e_z$

$[e_r]_C = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$(\operatorname{sen}\theta \cos\varphi, \operatorname{sen}\theta \operatorname{sen}\varphi, \cos\theta) = a_{11}(\cos\varphi, \operatorname{sen}\varphi, 0) + a_{21}(-\operatorname{sen}\varphi, \cos\varphi, 0) + a_{31}(0, 0, 1)$

$$\Rightarrow \operatorname{sen}\theta \cos\varphi = a_{11} \cos\varphi - \operatorname{sen}\varphi\, a_{21}$$
$$\operatorname{sen}\theta \operatorname{sen}\varphi = a_{11} \operatorname{sen}\varphi + \cos\varphi\, a_{21}$$
$$\cos\theta = a_{31}$$

$$\Rightarrow a_{11} = \frac{\operatorname{sen}\theta \operatorname{sen}\varphi - \cos\varphi\, a_{21}}{\operatorname{sen}\varphi}$$

$$\boxed{a_{11} = \operatorname{sen}\theta - \cot\varphi\, a_{21}}$$

$$\operatorname{sen}\theta \cos\varphi = (\operatorname{sen}\theta - \cot\varphi\, a_{21})\cos\varphi - \operatorname{sen}\varphi\, a_{21}$$
$$\operatorname{sen}\theta \cos\varphi = \operatorname{sen}\theta \cos\varphi - \cot\varphi \cos\varphi\, a_{21} - \operatorname{sen}\varphi\, a_{21}$$
$$0 = a_{21}(\cot\varphi \cos\varphi + \operatorname{sen}\varphi) \Rightarrow \boxed{a_{21} = 0}$$

$$[e_r]_c = \begin{pmatrix} \operatorname{sen}\theta \\ 0 \\ \cos\theta \end{pmatrix}$$

$$[e_\theta]_c = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$$

$$(\cos\theta\cos\varphi, \cos\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta) = a_{12}(\cos\varphi, \operatorname{sen}\varphi, 0)$$
$$+ a_{22}(-\operatorname{sen}\varphi, \cos\varphi, 0) + a_{32}(0, 0, 1)$$

$$\Rightarrow \cos\theta\cos\varphi = \cos\varphi\, a_{12} - \operatorname{sen}\varphi\, a_{22}$$
$$\cos\theta\operatorname{sen}\varphi = \operatorname{sen}\varphi\, a_{12} + \cos\varphi\, a_{22}$$
$$-\operatorname{sen}\theta = a_{32}$$

$$\Rightarrow a_{22} = \frac{\cos\theta\operatorname{sen}\varphi - \operatorname{sen}\varphi\, a_{12}}{\cos\varphi}$$
$$\boxed{a_{22} = \cos\theta\tan\varphi - \tan\varphi\, a_{12}}$$

$$\cos\theta\cos\varphi = \cos\varphi\, a_{12} - \operatorname{sen}\varphi(\cos\theta\tan\varphi - \tan\varphi\, a_{12})$$
$$\cos\theta\cos\varphi + \operatorname{sen}\varphi\cos\theta\tan\varphi = \cos\varphi\, a_{12} + \tan\operatorname{sen}\varphi\, a_{12}$$
$$\cos\theta(\cos\varphi + \operatorname{sen}\varphi\tan\varphi) = a_{12}(\cos\varphi + \operatorname{sen}\varphi\tan\varphi)$$

$$\cos\theta = a_{12}$$

$$a_{22} = \tan\varphi(\cos\theta - \cos\theta) = 0$$

$$[e_\theta]_c = \begin{pmatrix} \cos\theta \\ 0 \\ -\operatorname{sen}\theta \end{pmatrix}$$

$$[e_\varphi]_c = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

$$(-\operatorname{sen}\varphi, \cos\varphi, 0) = a_{13}(\cos\varphi, \operatorname{sen}\varphi, 0) + a_{23}(-\operatorname{sen}\varphi, \cos\varphi, 0)$$
$$+ a_{33}(0, 0, 1)$$

$$-\operatorname{sen}\varphi = \cos\varphi\, a_{13} - \operatorname{sen}\varphi\, a_{23}$$
$$\cos\varphi = \operatorname{sen}\varphi\, a_{13} + \cos\varphi\, a_{23}$$
$$0 = a_{33}$$

$$\Rightarrow a_{13} = \frac{\cos\varphi - \cos\varphi\, a_{23}}{\operatorname{sen}\varphi} = \cot\varphi\,(1 - a_{23})$$

$$-\operatorname{sen}\varphi = \cos\varphi\,(\cot\varphi - \cot\varphi\, a_{23}) - \operatorname{sen}\varphi\, a_{23}$$

$$-\operatorname{sen}\varphi - \cos\varphi\cot\varphi = -\cos\varphi\cot\varphi\, a_{23} - \operatorname{sen}\varphi\, a_{23}$$

$$-\operatorname{sen}\varphi - \cos\varphi\cot\varphi = a_{23}(-\cos\varphi\cot\varphi - \operatorname{sen}\varphi)$$

$$\boxed{1 = a_{23}}$$

$$\Rightarrow a_{13} = 0$$

$$[e_\varphi]_c = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow T_{E\to C} = \left([e_r]_C \; [e_\theta]_C \; [e_\varphi]_C\right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$T_{E\to C} = \begin{pmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}$$

$Q \; c \to E$

$e_\rho = a_{11} e_r + a_{21} e_\theta + a_{31} e_\varphi$

$e_\varphi = a_{12} e_r + a_{22} e_\theta + a_{32} e_\varphi$

$e_z = a_{13} e_r + a_{23} e_\theta + a_{33} e_\varphi$

$[e_\rho]_E = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$

$(\cos\varphi, \operatorname{sen}\varphi, 0) = a_{11}(\operatorname{sen}\theta\cos\varphi, \operatorname{sen}\theta\operatorname{sen}\varphi, \cos\theta) +$
$a_{21}(\cos\theta\cos\varphi, \cos\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta) +$
$a_{31}(-\operatorname{sen}\varphi, \cos\varphi, 0)$

$\Rightarrow \operatorname{sen}\theta\cos\varphi\, a_{11} + \cos\theta\cos\varphi\, a_{21} - \operatorname{sen}\varphi\, a_{31} = \cos\varphi$
$\operatorname{sen}\theta\operatorname{sen}\varphi\, a_{11} + \cos\theta\operatorname{sen}\varphi\, a_{21} + \cos\varphi\, a_{31} = \operatorname{sen}\varphi$
$\cos\theta\, a_{11} - \operatorname{sen}\theta\, a_{21} = 0$

$$\Delta = \begin{vmatrix} \operatorname{sen}\theta\cos\varphi & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta\operatorname{sen}\varphi & \cos\varphi \\ \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta\cos\varphi & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta\operatorname{sen}\varphi & \cos\varphi \end{vmatrix}$$

$\to \cos^2\theta\operatorname{sen}^2\varphi$
$\to \operatorname{sen}^2\theta\cos^2\varphi$
$\to 0$
$\to 0$
$\to \operatorname{sen}^2\theta\operatorname{sen}^2\varphi$
$\to \cos^2\theta\cos^2\varphi$

$\cos^2\theta\operatorname{sen}^2\varphi + \operatorname{sen}^2\theta\cos^2\varphi + \operatorname{sen}^2\theta\operatorname{sen}^2\varphi + \cos^2\theta\cos^2\varphi$

$= \cos^2\theta(\operatorname{sen}^2\varphi + \cos^2\varphi) + \operatorname{sen}^2\theta(\cos^2\varphi + \operatorname{sen}^2\varphi)$

$= 1$

$\Rightarrow \Delta = 1$

$$\Delta_1 = \left| \begin{array}{ccc} \cos\psi & \cos\theta\cos\psi & -\operatorname{sen}\psi \\ \operatorname{sen}\psi & \cos\theta\operatorname{sen}\psi & \cos\psi \\ 0 & -\operatorname{sen}\theta & 0 \\ \cos\psi & \cos\theta\cos\psi & -\operatorname{sen}\psi \\ \operatorname{sen}\psi & \cos\theta\operatorname{sen}\psi & \cos\psi \end{array} \right| \begin{array}{l} \to 0 \\ \to \operatorname{sen}\theta\cos^2\psi \\ \to 0 \\ \to 0 \\ \to \operatorname{sen}^2\psi\operatorname{sen}\theta \\ \to 0 \end{array}$$

$$= \operatorname{sen}\theta\cos^2\psi + \operatorname{sen}^2\psi\operatorname{sen}\theta$$

$$= \operatorname{sen}\theta(\cos^2\psi + \operatorname{sen}^2\psi)$$

$$\boxed{\Delta_1 = \operatorname{sen}\theta}$$

$$\Delta_2 = \left| \begin{array}{ccc} \operatorname{sen}\theta\cos\psi & \cos\psi & -\operatorname{sen}\psi \\ \operatorname{sen}\theta\operatorname{sen}\psi & \operatorname{sen}\psi & \cos\psi \\ \cos\theta & 0 & 0 \\ \operatorname{sen}\theta\cos\psi & \cos\psi & -\operatorname{sen}\psi \\ \operatorname{sen}\theta\operatorname{sen}\psi & \operatorname{sen}\psi & \cos\psi \end{array} \right| \begin{array}{l} \to \cos\theta\operatorname{sen}^2\psi \\ \to 0 \\ \to 0 \\ \to 0 \\ \to 0 \\ \to \cos\theta\cos^2\psi \end{array}$$

$$= \cos\theta\operatorname{sen}^2\psi + \cos\theta\cos^2\psi$$

$$= \cos\theta(\operatorname{sen}^2\psi + \cos^2\psi)$$

$$\boxed{\Delta_2 = \cos\theta}$$

$$\Delta_3 = \left| \begin{array}{ccc} \operatorname{sen}\theta\cos\psi & \cos\theta\cos\psi & \cos\psi \\ \operatorname{sen}\theta\operatorname{sen}\psi & \cos\theta\operatorname{sen}\psi & \operatorname{sen}\psi \\ \cos\theta & \operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta\cos\psi & \cos\theta\cos\psi & \cos\psi \\ \operatorname{sen}\theta\operatorname{sen}\psi & \cos\theta\operatorname{sen}\psi & \operatorname{sen}\psi \end{array} \right| \begin{array}{l} \to -\cos^2\theta\operatorname{sen}\psi\cos\psi \\ \to -\operatorname{sen}^2\theta\operatorname{sen}\psi\cos\psi \\ \to 0 \\ \to 0 \\ \to \operatorname{sen}^2\theta\operatorname{sen}\psi\cos\psi \\ \to \cos^2\theta\operatorname{sen}\psi\cos\psi \end{array}$$

$$= \operatorname{sen}^2\theta\operatorname{sen}\psi\cos\psi + \cos^2\theta\operatorname{sen}\psi\cos\psi - \cos^2\theta\operatorname{sen}\psi\cos\psi - \operatorname{sen}^2\theta\operatorname{sen}\psi\cos$$

$$\Delta_3 = 0$$

$$\Rightarrow a_{11} = \frac{\Delta_1}{\Delta} = \frac{\operatorname{sen}\theta}{1} = \operatorname{sen}\theta$$

$$a_{21} = \frac{\Delta_2}{\Delta} = \frac{\cos\theta}{1} = \cos\theta$$

$$a_{31} = \frac{\Delta_3}{\Delta} = \frac{0}{1} = 0$$

$$\Rightarrow [e_\rho]_E = \begin{pmatrix} \operatorname{sen}\theta \\ \cos\theta \\ 0 \end{pmatrix}$$

para $\quad [e_\varphi]_E = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$

$$(-\operatorname{sen}\varphi, \cos\varphi, 0) = a_{12}(\operatorname{sen}\theta\cos\varphi, \operatorname{sen}\theta\operatorname{sen}\varphi, \cos\theta)$$
$$+ a_{22}(\cos\theta\cos\varphi, \cos\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta)$$
$$+ a_{32}(-\operatorname{sen}\varphi, \cos\varphi, 0)$$

$$\Rightarrow \begin{aligned} \operatorname{sen}\theta\cos\varphi\, a_{12} + \cos\theta\cos\varphi\, a_{22} + (-\operatorname{sen}\varphi\, a_{32}) &= -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi\, a_{12} + \cos\theta\operatorname{sen}\varphi\, a_{22} + \cos\varphi\, a_{32} &= \cos\varphi \\ \cos\theta\, a_{12} \quad + (-\operatorname{sen}\theta)a_{22} &= 0 \end{aligned}$$

$$\Delta_1 = \left|\begin{array}{ccc} -\operatorname{sen}\varphi & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ \cos\varphi & \cos\theta\operatorname{sen}\varphi & \cos\varphi \\ 0 & -\operatorname{sen}\theta & 0 \\ -\operatorname{sen}\varphi & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ \cos\varphi & \cos\theta\operatorname{sen}\varphi & \cos\varphi \end{array}\right| \begin{array}{l} \to 0 \\ \to -\operatorname{sen}\theta\operatorname{sen}\varphi\cos\varphi \\ \to 0 \\ \to 0 \\ \to \operatorname{sen}\theta\operatorname{sen}\varphi\cos\varphi \\ \to 0 \end{array}$$

$$= \operatorname{sen}\theta\operatorname{sen}\varphi\cos\varphi - \operatorname{sen}\theta\operatorname{sen}\varphi\cos\varphi$$

$$\Delta_1 = 0$$

$$\Delta_2 = \begin{vmatrix} \operatorname{sen}\theta\cos\varphi & -\operatorname{sen}\varphi & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\varphi & \cos\varphi \\ \cos\theta & 0 & 0 \\ \operatorname{sen}\theta\cos\varphi & -\operatorname{sen}\varphi & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\varphi & \cos\varphi \end{vmatrix}$$

dado que la columna 2 y 3 son iguales

$\Delta_2 = 0$

$$\Delta_3 = \begin{vmatrix} \operatorname{sen}\theta\cos\varphi & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta\operatorname{sen}\varphi & \cos\varphi \\ \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta\cos\varphi & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta\operatorname{sen}\varphi & \cos\varphi \end{vmatrix}$$

$\to \cos^2\theta\cdot\operatorname{sen}^2\varphi$
$\to \operatorname{sen}^2\theta\cos^2\varphi$
$\to 0$
$\to 0$
$\to \operatorname{sen}^2\theta\operatorname{sen}^2\varphi$
$\to \cos^2\theta\cos^2\varphi$

$$= \cos^2\theta\operatorname{sen}^2\varphi + \operatorname{sen}^2\theta\cos^2\varphi + \operatorname{sen}^2\theta\operatorname{sen}^2\varphi + \cos^2\theta\cos^2\varphi$$
$$= \cos^2\theta(\operatorname{sen}^2\varphi + \cos^2\varphi) + \operatorname{sen}^2\theta(\cos^2\varphi + \operatorname{sen}^2\varphi)$$
$$= \cos^2\theta + \operatorname{sen}^2\theta$$
$$= 1$$

$\Delta_3 = 1$

$$a_{12} = \frac{\Delta_1}{\Delta} = \frac{0}{1} = 0$$
$$a_{22} = \frac{\Delta_2}{\Delta} = \frac{0}{1} = 0$$
$$a_{32} = \frac{\Delta_3}{\Delta} = \frac{1}{1} = 1$$

$$\Rightarrow [e_\varphi]_E = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

para $[e_z] = \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$

$(0,0,1) = a_{13}(\operatorname{sen}\theta\cos\varphi, \operatorname{sen}\theta \operatorname{sen}\varphi, \cos\theta) +$
$a_{23}(\cos\theta\cos\varphi, \cos\theta\operatorname{sen}\varphi, -\operatorname{sen}\theta) +$
$a_{33}(-\operatorname{sen}\varphi, \cos\varphi, 0)$

$\Rightarrow$

$\operatorname{sen}\theta\cos\varphi\, a_{13} + \cos\theta\cos\varphi\, a_{23} - \operatorname{sen}\varphi\, a_{33} = 0$

$\operatorname{sen}\theta\operatorname{sen}\varphi\, a_{13} + \cos\theta\operatorname{sen}\varphi\, a_{23} + \cos\varphi\, a_{33} = 0$

$\cos\theta\, a_{13} - \operatorname{sen}\theta\, a_{23} = 1$

$$\Delta_1 = \left| \begin{array}{ccc} 0 & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ 0 & \cos\theta\operatorname{sen}\varphi & \cos\varphi \\ 1 & -\operatorname{sen}\theta & 0 \\ 0 & \cos\theta\cos\varphi & -\operatorname{sen}\varphi \\ 0 & \cos\theta\operatorname{sen}\varphi & \cos\varphi \end{array} \right| \begin{array}{l} \to \operatorname{sen}^2\varphi\cos\theta \\ \to 0 \\ \to 0 \\ \to 0 \\ \to 0 \\ \to \cos^2\varphi\cos\theta \end{array}$$

$= \operatorname{sen}^2\varphi\cos\theta + \cos^2\varphi\cos\theta$

$= \cos\theta(\operatorname{sen}^2\varphi + \cos^2\varphi)$

$\Delta_1 = \cos\theta$

$$\Delta_2 = \left| \begin{array}{ccc} \operatorname{sen}\theta\cos\varphi & 0 & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & 0 & \cos\varphi \\ \cos\theta & 1 & 0 \\ \operatorname{sen}\theta\cos\varphi & 0 & -\operatorname{sen}\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & 0 & \cos\varphi \end{array} \right| \begin{array}{l} \to 0 \\ \to -\cos^2\varphi\operatorname{sen}\theta \\ \to 0 \\ \to 0 \\ \to -\operatorname{sen}^2\varphi\operatorname{sen}\theta \\ \to 0 \end{array}$$

$= -\cos^2\varphi\operatorname{sen}\theta - \operatorname{sen}^2\varphi\operatorname{sen}\theta$

$= -\operatorname{sen}\theta(\cos^2\varphi + \operatorname{sen}^2\varphi)$

$\Delta_2 = -\operatorname{sen}\theta$

$$\Delta_3 = \left| \begin{array}{ccc} \operatorname{sen}\theta \cos\psi & \cos\theta\cos\psi & 0 \\ \operatorname{sen}\theta \operatorname{sen}\psi & \cos\theta \operatorname{sen}\psi & 0 \\ \cos\theta & -\operatorname{sen}\theta & 1 \\ \operatorname{sen}\theta\cos\psi & \cos\theta\cos\psi & 0 \\ \operatorname{sen}\theta\operatorname{sen}\psi & \cos\theta\operatorname{sen}\psi & 0 \end{array} \right. \quad \begin{array}{l} \to 0 \\ \to 0 \\ \to -\operatorname{sen}\theta\operatorname{sen}\psi\cos\theta\cos\psi \\ \to \cos\theta\operatorname{sen}\psi\operatorname{sen}\theta\cos\psi \\ \to 0 \\ \to 0 \end{array}$$

$$= \operatorname{sen}\psi\operatorname{sen}\theta\cos\theta\cos\psi - \operatorname{sen}\theta\operatorname{sen}\psi\cos\theta\cos\psi$$

$\Delta_3 = 0$

$$\Rightarrow \quad a_{13} = \frac{\Delta_1}{\Delta} = \frac{\cos\theta}{1} = \cos\theta$$

$$a_{23} = \frac{\Delta_2}{\Delta} = \frac{-\operatorname{sen}\theta}{1} = -\operatorname{sen}\theta$$

$$a_{33} = \frac{\Delta_3}{\Delta} = \frac{0}{1} = 0$$

$$\Rightarrow [e_z]_E = \begin{pmatrix} \cos\theta \\ -\operatorname{sen}\theta \\ 0 \end{pmatrix}$$

$$Q_{C\to E} = ([e_\rho]_E \; [e_\psi]_E \; [e_z]_E) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$Q_{C\to E} = \begin{pmatrix} \operatorname{sen}\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\operatorname{sen}\theta \\ 0 & 1 & 0 \end{pmatrix}$$

Problema 13

$$x_1 + x_2 + 2x_4 = 0$$
$$-2x_1 - 2x_2 + x_3 - 5x_4 = 0$$
$$x_1 + x_2 - x_3 + 3x_4 = 0$$
$$4x_1 + 4x_2 - x_3 + 9x_4 = 0$$

$$= \left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ -2 & -2 & 1 & -5 & 0 \\ 1 & 1 & -1 & 3 & 0 \\ 4 & 4 & -1 & 9 & 0 \end{array}\right) (2)(-1)(-4)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right) (1)(1) = \left(\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 - x_4 = 0 \Rightarrow x_3 = x_4 = \alpha \in \mathbb{R}$

$x_1 + x_2 + 2x_4 = 0 \quad x = -x_2 - 2x_4 = -\beta - 2\alpha, \ \beta \in \mathbb{R}$

$x = -\beta - 2\alpha, \ y = \beta, \ z = \alpha$ y $w = \alpha$

$$X = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -\beta - 2\alpha \\ \beta \\ \alpha \\ \alpha \end{pmatrix} = \beta \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$V = \{v_1, v_2\} \ / \ v_1 = (-1, 1, 0, 0), \ v_2 = (-2, 0, 1, 1)$

$w_1 = v_1 = (-1, 1, 0, 0)$

$$w_2 = w_2 - \frac{w_1 \cdot v_2}{w_1 \cdot w_1} w_1 = (-2, 0, 1, 1) - \left(\frac{2}{2}\right)(-1, 1, 0, 0)$$

$$w_2 = (-2, 0, 1, 1) - (-1, 1, 0, 0)$$

$$w_2 = (-1, -1, 1, 1)$$

tenemos el conjunto $\{w_1, w_2\}$ es ortogonal por lo que normalizamos

$$v_1 = \frac{w_1}{|w_1|} = \frac{1}{\sqrt{2}}(-1, 1, 0, 0) = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right)$$

$$v_2 = \frac{w_2}{|w_2|} = \frac{1}{2}(-1, -1, 1, 1) = \left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

Por lo tanto el conjunto de vectores $v = \{v_1, v_2\}$ es una base ortonormal para el espacio solución

Problema 14

$S = \{ (1,-2,3), (-2,2,2), (5/7, 4/7, 1/7) \}$

$v = (12,-6,6) \qquad \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$

$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$

$(12,-6,6) = \alpha_1(1,-2,3) + \alpha_2(-2,2,2) + \alpha_3\left(\frac{5}{7}, \frac{4}{7}, \frac{1}{7}\right)$

$= (\alpha_1, -2\alpha_1, 3\alpha_1) + (-2\alpha_2, 2\alpha_2, 2\alpha_2) + \left(\frac{5}{7}\alpha_3, \frac{4\alpha_3}{7}, \frac{\alpha_3}{7}\right)$

$= \left(\alpha_1 - 2\alpha_2 + \frac{5}{7}\alpha_3, -2\alpha_1 + 2\alpha_2 + \frac{4\alpha_3}{7}, 3\alpha_1 + 2\alpha_2 + \frac{\alpha_3}{7}\right)$

$$\Rightarrow \begin{cases} \alpha_1 - 2\alpha_2 + \frac{5}{7}\alpha_3 = 12 \\ -2\alpha_1 + 2\alpha_2 + \frac{4}{7}\alpha_3 = -6 \\ 3\alpha_1 + 2\alpha_2 + \frac{\alpha_3}{7} = 6 \end{cases} = \left(\begin{array}{ccc|c} 1 & -2 & \frac{5}{7} & 12 \\ -2 & 2 & \frac{4}{7} & -6 \\ 3 & 2 & \frac{1}{7} & 6 \end{array}\right)(2)(-$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 5/7 & 12 \\ 0 & -2 & 2 & 18 \\ 0 & 8 & -2 & -30 \end{array}\right)\left(-\frac{1}{2}\right) = \left(\begin{array}{ccc|c} 1 & -2 & 5/7 & 12 \\ 0 & 1 & -1 & -9 \\ 0 & 8 & -2 & -30 \end{array}\right)(-8)$$

$$= \left(\begin{array}{ccc|c} 1 & -2 & 5/7 & 12 \\ 0 & 1 & -1 & -9 \\ 0 & 0 & 6 & 42 \end{array}\right)\left(\frac{1}{6}\right) = \left(\begin{array}{ccc|c} 1 & -2 & 5/7 & 12 \\ 0 & 1 & -1 & -9 \\ 0 & 0 & 1 & 7 \end{array}\right)(1)\left(-\frac{5}{7}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7 \end{array}\right) (2) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7 \end{array}\right)$$

$$\Rightarrow [v]_S = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$$

Problema 15

$S = \{ v_1, v_2, v_3 \}$

$v_1 = (1,-1,1)$, $v_2 = (-2,3,-1)$ y $v_3 = (1,2,-4)$

$w_1 = v_1 = (1,-1,1)$

$$w_2 = v_2 - \frac{w_1 \cdot v_2}{w_1 \cdot w_1} w_1 = (-2,3,-1) - \frac{-6}{3}(1,-1,1)$$
$$= (-2,3,-1) - (-2,2,-2)$$
$$= (0,1,1)$$

$$w_3 = v_3 - \frac{w_1 \cdot v_3}{w_1 \cdot w_1} w_1 - \frac{w_2 \cdot v_3}{w_2 \cdot w_2} w_2$$
$$= (1,2,-4) - \frac{(-5)}{3}(1,-1,1) - \frac{(-2)}{2}(0,1,1)$$
$$= (1,2,-4) - \left(-\frac{5}{3}, \frac{5}{3}, -\frac{5}{3}\right) - (0,-1,-1)$$
$$= (1,3,3) + \left(\frac{5}{3}, -\frac{5}{3}, \frac{5}{3}\right)$$
$$= \left(\frac{8}{3}, \frac{4}{3}, \frac{14}{3}\right)$$

$\{ w_1, w_2, w_3 \}$ son ortogonales

$$v_1 = \frac{w_1}{|w_1|} = \frac{1}{\sqrt{3}}(1,-1,1) = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$v_2 = \frac{w_2}{|w_2|} = \frac{1}{\sqrt{2}}(0,1,1) = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$v_3 = \frac{w_3}{|w_3|} = \frac{\sqrt{69}}{46}\left(\frac{8}{3}, \frac{4}{3}, \frac{14}{3}\right) = \left(\frac{4\sqrt{69}}{69}, \frac{2\sqrt{69}}{69}, \frac{7\sqrt{69}}{69}\right)$$

$\therefore$ el conjunto $\{v_1, v_2, v_3\}$
es una base ortonormal para el
espacio vectorial $\mathbb{R}^3$