$$f(t) = \begin{cases} \cos wt & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

sol

para obtener la transformada de f(t) utilizamos un cambio de variable

Sea $t = KT$, entonces

$$f(KT) = \cos w KT$$

Aplicamos la definición de la transformada Z

$$F(z) = \sum_{K=0}^{\infty} z^{-K} \cos wKT = 1 + z^{-1}\cos wT + z^{-2}\cos 2wT + \cdots$$

Utilizamos la identidad de euler

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

Sustituyendo en la serie y aplicando la propiedad de linealidad

$$F(z) = \sum_{K=0}^{\infty} \left(\frac{e^{jwKT} + e^{-wKT}}{2} \right) z^{-K}$$

$$= \sum_{K=0}^{\infty} \frac{e^{jwT}}{2} z^{-K} + \sum_{K=0}^{\infty} \frac{e^{-jwKT}}{2} z^{-K}$$

Pero esto es la transformada para el exp que ya obtuvimos, por lo tanto, sustituimos

$$F(z) = \frac{1}{2} \cdot \frac{z}{z - e^{j\omega T}} + \frac{1}{2} \cdot \frac{z}{z - e^{-j\omega T}}$$

$$F(z) = \frac{1}{2}\left[\frac{z}{z - e^{j\omega T}} + \frac{z}{z - e^{-j\omega T}} \right]$$

$$F(z) = \frac{1}{2}\left(\frac{z(z - e^{-j\omega T}) + z(z - e^{j\omega T})}{(z - e^{j\omega T})(z - e^{-j\omega T})} \right)$$

$$F(z) = \frac{1}{2}\left(\frac{z(2z - (e^{-j\omega T} + e^{j\omega T}))}{z^2 - z e^{-j\omega T} - z e^{j\omega T} + 1} \right)$$

$$F(z) = \frac{1}{2}\left(\frac{z\left(2z - 2\,\frac{e^{-j\omega T} + e^{j\omega T}}{2}\right)}{z^2 - 2z\left(\frac{e^{-j\omega T} + e^{j\omega T}}{2}\right) + 1} \right)$$

$$F(z) = \cancel{\frac{1}{2}}\left(\frac{\cancel{2} z(z - \cos\omega T)}{z^2 - 2z\cos\omega T + 1} \right)$$

$$F(z) = \frac{z(z - \cos\omega T)}{z^2 - 2z\cos\omega T + 1}$$

$$F(z) = \begin{cases} t^2 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

Sol:

$$F(z) = \sum_{k=0}^{\infty} t^2 z^{-k}$$

$$F(z) = \sum_{k=0}^{\infty} t \cdot t\, z^{-k}$$

por el teorema de multiplicación por una rampa

$$F(z) = -zT \frac{d\, t z^{-k}}{dt} = -zT \frac{d\, kT z^{-k}}{dt}$$

$$= -zT \frac{d}{dt}\left[0 + Tz^{-1} + 2Tz^{-2} + 3Tz^{-3}\right]$$

factorizamos T y sustituimos la serie

$$= -zT^2 \frac{d}{dt}\left(\frac{z}{(z-1)^2}\right)$$

derivamos

$$= -zT^2\left(\frac{(1)(z-1)^2 - (2(z-1))(z)}{(z-1)^4}\right)$$

Factorizamos $(z-1)$ del numerador

$$F(z) = -zT \frac{(z-1)(z-1-2z)}{(z-1)^4}$$

$$= -zT\left(\frac{-z-1}{(z-1)^3}\right)$$

$$= zT\left(\frac{z+1}{(z-1)^3}\right)$$

$$F(z) = T\,\frac{z(z+1)}{(z-1)^3}$$

$$G(s) = \frac{10}{s(s+2)(s+4)}, \quad T = 0.1$$

$$\frac{10}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$\frac{10}{s(s+2)(s+4)} = \frac{A(s+2)(s+4) + B\,s(s+4) + C\,s(s+2)}{s(s+2)(s+4)}$$

$$10 = A(s^2+6s+8) + B(s^2+4s) + C(s^2+2s)$$

$$A+B+C=0 \Rightarrow B = -C-A$$

$$6A+4B+2C=0 \Rightarrow 2 = -6A-4B \Rightarrow C = -3A-2B$$

$$C = -\frac{15}{4} - 2B \Rightarrow C = -\frac{15}{4} + \frac{10}{2} = \frac{5}{4}$$

$$8A = 10 \Rightarrow \boxed{A = \frac{5}{4}}$$

$$B = +\frac{15}{4} - 2B - \frac{5}{4} \Rightarrow \boxed{B = -\frac{10}{4} = -\frac{5}{2}}$$

$$G(s) = \frac{5}{4}\cdot\frac{1}{s} - \frac{5}{2}\left(\frac{1}{s+2}\right) + \frac{5}{4}\left(\frac{1}{s+4}\right)$$

$$G(z) = \frac{5}{4}\left(\frac{z}{z-1}\right) - \frac{5}{2}\left(\frac{z}{z-e^{-2T}}\right) + \frac{5}{4}\left(\frac{z}{z-e^{-4T}}\right)$$

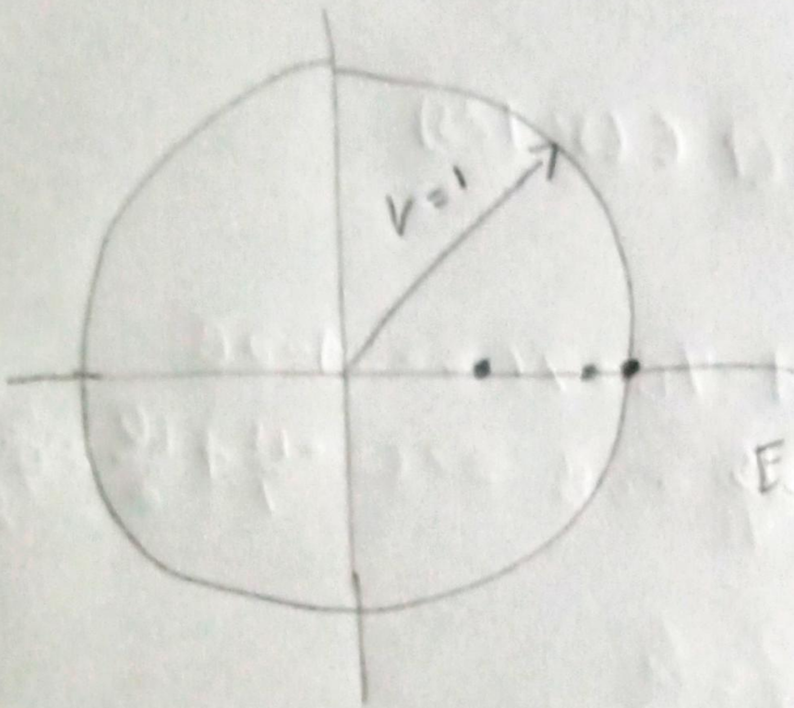
$$G(z) = \frac{5}{4}\left(\frac{z}{z-1}\right) - \frac{5}{2}\left(\frac{z}{z-0.81}\right) + \frac{5}{4}\left(\frac{z}{z-0.67}\right)$$

$$G(z) = \frac{5z}{4z-4} - \frac{2.5z}{z-0.81} + \frac{1.25z}{z-0.67}$$

$$G(z) = \frac{0.25z^2 + 0.06353z}{4(z-1)(z-0.81)(z-0.67)}$$

Por lo tanto

Tenemos que $z_1 = 1$, $z_2 = 0.81$, $z_3 = 0.67$



Estable.