Problema 4.81

$\int_c \phi \, dr$ desde $(1,1,0)$ hasta $(2,4,0)$

para $\phi = x^3y + 2y$

(a)

$y = x^2, z = 0$

(b)

c $(0,0,0)$ y $(1,1,0)$

para el inciso (a)

$x = t \Rightarrow y = t^2, z = 0$

$r = x\hat{i} + y\hat{j} + z\hat{k}$

$r = t\hat{i} + t^2\hat{j} + 0\hat{k}$

$dr = (\hat{i} + 2t\hat{j})dt$

$\phi = t^5 + 2t^2$

$$\int_c \phi \, dr = \int_c (t^5 + 2t^2)(\hat{i} + 2t\hat{j}) \, dt = \hat{i}\int_1^2 (t^5 + 2t^2) dt + \hat{j}\int_1^4 (2t^6 + 4t^3) dt$$

$$= \hat{i}\left[\frac{t^6}{6} + \frac{2}{3}t^3\right]\Big|_1^2 + \hat{j}\left[\left(\frac{2}{7}t^7 + t^4\right]\Big|_1^4$$

$$= \left[\frac{91}{6}\right]\hat{i} + \left[\frac{33015}{7}\right]\hat{j}$$

$$= \left(\frac{91}{6}, \frac{33015}{7}, 0\right)$$

para el inciso (B) $A=(1,1,0)$ $B=(2,4,0)$

$x=(1-t)A_1+tB_1 \Rightarrow x=1+t$

$y=(1-t)A_2+tB_2 \Rightarrow y=1+3t$

$z=(1-t)A_3+tB_3 \Rightarrow z=0$

$r=(1+t)\hat{\imath}+(1+3t)\hat{\jmath}$

$dr=\hat{\imath}+3\hat{\jmath}$

$\phi=(1+t)^3(1+3t)+(1+3t)$

$\phi=3t^4+10t^3+12t^2+9t+2$

$$\int_c (3t^4+10t^3+12t^2+9t+2)(\hat{\imath}+3\hat{\jmath})\,dt$$

$$=\hat{\imath}\int_1^2 (3t^4+10t^3+12t^2+9t+2)\,dt$$

$$+\hat{\jmath}\int_1^4 (9t^4+30t^3+36t^2+27t+6)\,dt$$

$$=\hat{\imath}\left[\frac{3}{5}t^5+\frac{10t^4}{4}+\frac{12}{3}t^3+\frac{9t^2}{2}+2t\right]\Big|_1^2+\hat{\jmath}\left[\frac{9}{5}t^5+\frac{30}{4}t^4+\frac{36}{3}t^2+\frac{27}{2}t^2+6t\right]\Big|_1^4$$

$=\hat{\imath}[160]+\hat{\jmath}[4806]$

$=[160,4806,0]$

problema 4.82

$\int_C f \cdot dr$ para $f = y\hat{i} + (x+z)^2\hat{j} + (x-z)^2\hat{k}$

desde $(0,0,0)$ hasta $(2,4,0)$

(a) $y = x^2$, $z = 0$

(b) $y = 2x$

para el inciso (a)

$x = t \quad y = t^2 \quad z = 0$

$r = x\hat{i} + y\hat{j} + 0\hat{k}$

$r = t\hat{i} + t^2\hat{j} + 0\hat{k}$

$dr = \hat{i} + 2t\hat{j} + \hat{k}$

$f = t^2\hat{i} + t^2\hat{j} + t^2\hat{k}$

$f \cdot dr = t^2 + 2t^3 +$

$\int_0^2 f \cdot dr = \frac{t^3}{3} + \frac{2t^4}{4} = \frac{t^3}{3} + \frac{t^4}{2}\Big|_0^2$

$= \frac{8}{3} + 8 = \frac{24+8}{3} = \boxed{\frac{32}{3}}$

Para el inciso (b)

$x = t \quad y = 2x \quad z = 0 \Rightarrow y = 2t$

$r = t\hat{i} + 2t\hat{j} + 0\hat{k}$

$dr = \hat{i} + 2\hat{j} + \hat{k}$

$F = 2t\hat{i} + t^2\hat{j}$

$F \cdot dr = 2t + 2t^2$

$\int_0^2 F \cdot dr = t^2 + \frac{2}{3}t^3 = 4 + \frac{16}{3} = \frac{12+16}{3} = \boxed{\frac{28}{3}}$

Problema 4.83 $\int_C F \cdot dr$

$F = 2xy^2\hat{\imath} + 2(x^2y + y)\hat{\jmath}$ desde $(0,0,0)$ hasta $(2,4,0)$

(a)

$y = x^2, z = 0$

(b) $y = 2x$

para el inciso (a)

$x = t, \quad y = x^2, \quad z = 0 \Rightarrow y = t^2$

$r = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$r = t\hat{\imath} + t^2\hat{\jmath} + 0\hat{k}$

$dr = \hat{\imath} + 2t\hat{\jmath}$

$F = 2t^5\hat{\imath} + (2t^4 + 2t^2)\hat{\jmath}$

$F \cdot dr = 2t^5 + 4t^5 + 4t^3$

$= 6t^5 + 4t^3$

$\int_0^2 6t^5 + 4t^3\, dt = [t^6 + t^4]\Big|_0^2 = \boxed{80}$

para el inciso (b)

$x = t \quad y = 2x \quad z = 0$

$\Rightarrow \quad y = 2t$

$r = t\hat{\imath} + 2t\hat{\jmath} + t\hat{k}$

$dr = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

$F = 8t^3\hat{\imath} + (4t^3 + 4t)\hat{\jmath}$

$F \cdot dr = 8t^3 + 8t^3 + 8t = 16t^3 + 8t$

$\int_0^2 F \cdot dr = \left[\frac{16t^4}{4} + \frac{8t^2}{2}\right]\Big|_0^2 = 4t^4 + 4t^2 = \boxed{80}$

Problema 4.84

$\int_c F \cdot dr \quad F = 3x\hat{i} + (2xz-y)\hat{j} + zK$

desde $(0,0,0)$ hasta $(2,1,3)$ a lo largo

de

(a)

$x = 2t^2, y = t, z = 4t^2 - t$

(b)

la recta desde $(0,0,0)$ hasta $(2,1,3)$

Para el inciso (a)

$r = x\hat{i} + y\hat{j} + z\hat{k}$

$r = 2t^2\hat{i} + t\hat{j} + (4t^2-t)\hat{k}$

$dr = 4t\hat{i} + \hat{j} + (8t-1)\hat{k}$

$F = 3(2t^2)\hat{i} + (4(2t^2)(4t^2-t))\hat{j} + (4t^2-t)\hat{k}$

$F = 6t^2\hat{i} + (16t^4 - 4t^3)\hat{j} + (4t^2-t)\hat{k}$

$F \cdot dr = 24t^3 + 16t^4 - 4t^3 + 32t^3 - 12t^2 + t$

$\int_0^1 F \cdot dr = \frac{16t^5}{5} + \frac{52t^4}{4} - \frac{12t^3}{3} + \frac{t^2}{2} \Big|_0^1$

$= \left[\frac{16}{5}t^5 + 13t^4 - 4t^3 + \frac{t^2}{2} \right] \Big|_0^1$

$= \frac{16}{5} + 13 - 4 + \frac{1}{2} = \frac{16}{5} + \frac{19}{2} = \frac{32+95}{10}$

$= \boxed{\frac{127}{10}}$

Para el inciso (b)

$x = 2t \quad y = t \quad z = 3t$

$r = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$r = 2t\hat{\imath} + t\hat{\jmath} + 3t\hat{k}$

$dr = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$F = 6t\hat{\imath} + 11t\hat{\jmath} + 3t\,k$

$F \cdot dr = 12t + 11t + 9t = 32t$

$\int_0^1 F \cdot dr = \frac{32t^2}{2}\Big|_0^1 = \boxed{16}$

Problema 4.85

$\int_C F \times dr \quad F = y\hat{\imath} + x\hat{\jmath}$ desde $(0,0,0)$ hasta $(3,9,0)$

$y = \frac{x^3}{3}, \quad z = 0$

$$\int_C F \times dr = \hat{\imath}\int_C (F_2\, dz - F_3\, dy) + \hat{\jmath}\int_C (F_3\, dx - F_1\, dz)$$

$$+ \hat{k}\int_C (F_1\, dy - F_2\, dx)$$

$$\Rightarrow \hat{\imath}\int_{(0,0)}^{(9,0)} x\, dz - 0\, dy + \hat{\jmath}\int_{(0,0)}^{(3,0)} 0\, dx - y\, dz$$

$$+ \hat{k}\int_{(0,0)}^{(3,9)} y\, dy - x\, dx$$

$$= \hat{\imath}\left[xz\right]\Big|_{(0,0)}^{(9,0)} + \hat{\jmath}\left[-yz\right]\Big|_{(0,0)}^{(3,0)} + \hat{k}\left[\frac{y^2}{2} - \frac{x^2}{2}\right]\Big|_{(0,0)}^{(3,9)}$$

$$= \hat{\imath}[0] + \hat{\jmath}[0] + 36\hat{k}$$

$$= [0, 0, 36]$$

problema 4.86 ¿ $\oint a \cdot dr = 0$?

dem:

si a es un vector cte

$\Rightarrow$ la ecuación paramétrica de una cte es una cte

$r =$ cte

$\Rightarrow dr = 0$

entonces

$a \cdot dr = 0$

$\Rightarrow \oint a \cdot dr = 0$

qed

$$¿\oint a \times dr = 0?$$

dem:

si a es un vector cte

las ecuaciones paramétricas son

$x=0$, $y=0$, $z=0$

$r=0$

$$\Rightarrow dr = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$a \times dr = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$a \times dr = 0$$

$$\Rightarrow \oint a \times dr = 0$$

qed