Actividad 3 correspondiente a la Unidad 3

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Instrucciones: Resolver las integrales, indicar si son convergentes o son divergentes

**5.** $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\, dx$

u=x-2 -> du=dx

$$\lim_{b\to\infty} \int_1^b \frac{1}{u^{\frac{3}{2}}} du$$

$$= \lim_{b\to\infty} -\frac{2}{\sqrt{u}} \Big|_1^b$$

$$= \lim_{b\to\infty} -\frac{2}{\sqrt{b}} + \frac{2}{\sqrt{1}}$$

$$= \lim_{b\to\infty} -\frac{1}{\sqrt{b}} + 2$$

$$= 2$$

La integral converge y converge a 2

**7.** $\int_{-\infty}^{0} \frac{1}{3-4x}\, dx$

u = 3-4x -> du=-4dx

$$\lim_{a\to\infty} \frac{-1}{4} \int_a^3 \frac{1}{u} du$$

$$= \lim_{a\to\infty} \frac{-1}{4} [\ln u] \Big|_a^3$$

$$= \lim_{a\to\infty} \frac{-1}{4} [\ln 3 - ln\ a]$$

$$= \infty$$

La integral diverge

**9.** $\int_{2}^{\infty} e^{-5p}\, dp$

u=-5p ->du=-5dp

$$\lim_{b\to\infty} \frac{-1}{5} \int_{-10}^{\mathrm{b}} e^{u} \mathrm{du}$$

$$= \lim_{b\to\infty} \frac{-1}{5} [e^{u}] \Big|_{-10}^{b}$$

$$= \lim_{b\to\infty} \frac{-1}{5} [e^{b} - e^{-10}]$$

$$= \frac{-1}{5} [0 - e^{-10}]$$

$$= \frac{1}{5} [e^{-10}]$$

La integral converge

**11.** $\int_{0}^{\infty} \frac{x^2}{\sqrt{1 + x^3}}\, dx$

u = 1+x^3 -> du=3x^2 dx

$$\lim_{b\to\infty} \frac{1}{3x^2} \int_{1}^{\mathrm{b}} \frac{x^2}{\mathrm{u}^{\frac{1}{2}}} \mathrm{du}$$

$$= \lim_{b\to\infty} \int_{1}^{\mathrm{b}} \frac{1}{\mathrm{u}^{\frac{1}{2}}} \mathrm{du}$$

$$= \lim_{b\to\infty} 2\mathrm{u}^{\frac{1}{2}} \Big|_{1}^{b}$$

$$= \lim_{b\to\infty} 2[\mathrm{b}^{\frac{1}{2}} - 1^{\frac{1}{2}}]$$

$$= 2[\infty - 1]$$

$$= \infty$$

La integral diverge

**13.** $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

u = -x^2 -> du =-2x dx

$$\lim_{b\to-\infty} \frac{1}{-2x}\int_0^{\mathrm{b}} \mathrm{x}e^{u}\mathrm{du} + \lim_{a\to-\infty} \frac{1}{-2x}\int_a^0 \mathrm{x}e^{u}\mathrm{du}$$

$$= \lim_{b\to-\infty} \frac{1}{-2}e^{u}\Big|_0^b + \lim_{a\to-\infty} \frac{1}{-2}e^{u}\Big|_a^0$$

$$= \frac{1}{-2}(0) + \frac{1}{-2}(0)$$

$$= 0$$

La integral converge y converge a 0

**15.** $\int_0^{\infty} \mathrm{sen}^2\alpha\, d\alpha$

$$sen^2\alpha = \frac{1}{2} - \frac{cos2\alpha}{2}$$

$$\lim_{b\to\infty} \int_0^b \frac{1}{2} - \frac{cos2\alpha}{2}\mathrm{d\alpha}$$

$$= \lim_{b\to\infty} (\frac{\alpha}{2} - \frac{sen2\alpha}{4})\Big|_0^b$$

$$= \lim_{b\to\infty} \left(\frac{\alpha}{2} - \frac{sen2\alpha}{4}\right) - \left(\frac{0}{2} - \frac{sen(2*0)}{4}\right)$$

$$= \left(\frac{\infty}{2} - \frac{sen\infty}{4}\right)$$

La integral es divergente

**17.** $\int_{1}^{\infty} \frac{1}{x^2 + x}\, dx$

$$\lim_{b\to\infty} \int_{1}^{b} \frac{1}{x} - \frac{1}{x+1}\ \mathrm{dx}$$

$$= \lim_{b\to\infty} (lnx - \ln x + 1)\Big|_{1}^{b}$$

$$= \lim_{b\to\infty} (lnb - \ln b) - (ln1 - \ln 2)$$

$$= -(0 - ln2)$$

$$= ln2$$

La converge y converge a ln2

**19.** $\int_{-\infty}^{0} ze^{2z}\, dz$

u = z -> du=dz   dv=e^2z dz -> v=(e^2z)/2

$$\lim_{a\to\infty} \frac{z}{2}e^{2z} - \int_{a}^{0} \frac{e^{2z}}{2}\ \mathrm{dz}$$

$$= \lim_{a\to\infty} [\frac{z}{2}e^{2z} - \frac{e^{2z}}{4}]\Big|_{a}^{0}$$

$$= \lim_{a\to\infty} [\frac{0}{2} * e^{2*0} - \frac{e^{2*0}}{4}] - [\frac{a}{2}e^{2a} - \frac{e^{2a}}{4}]$$

$$= -\frac{1}{4} - [0 - 0]$$

$$= -\frac{1}{4}$$

La integral converge y converge a -1/4

**31.** $\int_{-2}^{3} \frac{1}{x^4}\, dx$

$$\int_0^3 \frac{1}{x^4}\text{dx} + \int_{-2}^{0} \frac{1}{x^4}\text{dx}$$

$$= \left[\frac{-1}{4x^3}\right] |_0^3 + \left[\frac{-1}{4x^3}\right] |_{-2}^0$$

$$= \left[\frac{-1}{4(3)^3}\right] - \left[\frac{-1}{4(0)^3}\right] + \left[\frac{-1}{4(0)^3}\right] - \left[\frac{-1}{4(-2)^3}\right]$$

La integral es divergente

**33.** $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\, dx$

$$\int_1^9 \frac{1}{\sqrt[3]{x-1}}\text{dx} + \int_0^1 \frac{1}{\sqrt[3]{x-1}}\text{dx}$$

u = x-1 -> du = dx

$$\int_0^8 \frac{1}{u^{\frac{1}{3}}}\text{du} + \int_{-1}^0 \frac{1}{u^{\frac{1}{3}}}\text{du}$$

$$= \frac{3u^{\frac{2}{3}}}{2} |_0^8 + \frac{3u^{\frac{2}{3}}}{2} |_{-1}^0$$

$$= [\frac{3(8)^{\frac{2}{3}}}{2} - \frac{3(0)^{\frac{2}{3}}}{2}] + [\frac{3(0)^{\frac{2}{3}}}{2} - \frac{3(-1)^{\frac{2}{3}}}{2}]$$

$$= \frac{3(8)^{\frac{2}{3}}}{2} - \frac{3(-1)^{\frac{2}{3}}}{2}$$

$$= 6 - \frac{3}{2} = \frac{9}{2}$$

La integral converge y converge a 9/2

**35.** $\int_0^3 \frac{dx}{x^2 - 6x + 5}$

$$\int_1^3 \frac{1}{x^2 - 6x + 5}\,\mathrm{dx} + \int_0^1 \frac{1}{x^2 - 6x + 5}\,\mathrm{dx}$$

$$[\frac{-1}{x} - \ln 6x + \frac{x}{5}]\Big|_1^3 + [\frac{-1}{x} - \ln 6x + \frac{x}{5}]\Big|_0^1$$

$$[(\frac{-1}{3} - \ln 18 + \frac{3}{5}) - ([\frac{-1}{1} - \ln 6 + \frac{1}{5}])] + [(\frac{-1}{1} - \ln 6 + \frac{1}{5}) - ([\frac{-1}{0} - \ln 0 + \frac{0}{5}])]$$

La integral es divergente

**37.** $\int_{-1}^0 \frac{e^{1/x}}{x^3}\,dx$

$$u = \frac{1}{x}$$

$$du = -\frac{1}{x^2}dx$$

$$dv = \frac{e^{\frac{1}{x}}dx}{x^2}$$

$$v = -e^{\frac{1}{x}}$$

$$\frac{-e^{\frac{1}{x}}}{x} - \int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^2}dx$$

$$[\frac{-e^{\frac{1}{x}}}{x} - e^{\frac{1}{x}}]\Big|_{-1}^0$$

$$[\frac{-e^{\frac{1}{x}} - xe^{\frac{1}{x}}}{x}]\Big|_{-1}^0$$

$$= \frac{-e^{\frac{1}{0}} - (0)e^{\frac{1}{0}}}{0} - \frac{-e^{\frac{1}{0}} - (0)e^{\frac{1}{0}}}{0}$$

La integral diverge