

Actividad 1 correspondiente a la Unidad 3

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Instrucciones: Calcula cada uno de los siguientes límites: Recuerda que sólo puedes usar la regla de L'Hopital cuando tienes una indeterminación de cociente.

5. $\lim_{x \rightarrow -2} \frac{x^2 - 1}{x^2 - x}$

$$\begin{aligned}\lim_{x \rightarrow -2} \frac{(-2)^2 - 1}{(-2)^2 - (-2)} \\&= \frac{(-2)^2 - 1}{(-2)^2 - (-2)} \\&= \frac{3}{6} \\&= \frac{1}{2}\end{aligned}$$

7. $\lim_{x \rightarrow 1} \frac{x^9 - 1}{x^5 - 1}$

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{9x^8}{5x^4} \\&\lim_{x \rightarrow 1} \frac{9x^4}{5} \\&= \frac{9(1)^4}{5} \\&= \frac{9}{5}\end{aligned}$$

$$9. \lim_{x \rightarrow (\pi/2)^+} \frac{\cos x}{1 - \sin x}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{-\cos x}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan x$$

$$= \tan \frac{\pi}{2}$$

$$= -\infty$$

$$11. \lim_{t \rightarrow 0} \frac{e^t - 1}{t^3}$$

$$\lim_{t \rightarrow 0} \frac{e^t}{3t^2}$$

$$\lim_{t \rightarrow 0} e^t * \lim_{t \rightarrow 0} \frac{1}{3t^2}$$

$$= 1 * \infty$$

$$= \infty$$

$$13. \lim_{x \rightarrow 0} \frac{\tan px}{\tan qx}$$

$$\lim_{x \rightarrow 0} \frac{p \sec^2(px)}{q \sec^2(qx)}$$

$$\frac{p}{q} \lim_{x \rightarrow 0} \frac{\sec^2(px)}{\sec^2(qx)}$$

$$= \frac{p \sec^2(p(0))}{q \sec^2(q(0))}$$

$$= \frac{p}{q}(1)$$

$$= \frac{p}{q}$$

$$15. \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$$

$$\lim_{x \rightarrow \infty} \frac{2}{\sqrt{x}}$$

$$= \frac{2}{\sqrt{\infty}}$$

$$= \frac{2}{\sqrt{\infty}}$$

$$= 0$$

$$17. \lim_{x \rightarrow 0^+} \frac{\ln x}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{x}}{1}$$

$$\lim_{x \rightarrow 0} \frac{1}{x}$$

$$= -\infty$$

$$19. \lim_{x \rightarrow \infty} \frac{e^x}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{3x^2}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{3x^2}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{6x}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{6}$$

$$= \infty$$

$$\text{21. } \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{2x}$$

$$\lim_{x \rightarrow 0} \frac{e^x}{2}$$

$$= \frac{1}{2}$$

$$\text{23. } \lim_{x \rightarrow 0} \frac{\tanh x}{\tan x}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{sech}^2 x}{\sec^2 x}$$

$$= \frac{\operatorname{sech}^2 0}{\sec^2 0}$$

$$= \frac{1}{1}$$

$$= 1$$

$$\text{25. } \lim_{t \rightarrow 0} \frac{5^t - 3^t}{t}$$

$$\lim_{t \rightarrow 0} \frac{5^t \ln(5) - 3^t \ln(3)}{1}$$

$$= 5^0 \ln(5) - 3^0 \ln(3)$$

$$= \ln(5) - \ln(3)$$

$$= \ln\left(\frac{5}{3}\right)$$

$$27. \lim_{x \rightarrow 0} \frac{\operatorname{sen}^{-1} x}{x}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} \\ &= \frac{1}{\sqrt{1-(0)^2}} \\ &= \frac{1}{\sqrt{1}} \\ &= 1 \end{aligned}$$

$$29. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{2x} \\ \lim_{x \rightarrow 0} \frac{\cos x}{2} \\ &= \frac{\cos(0)}{2} \\ &= \frac{1}{2} \end{aligned}$$

$$31. \lim_{x \rightarrow 0} \frac{x + \operatorname{sen} x}{x + \cos x}$$

$$\begin{aligned} &= \frac{0 + \operatorname{sen}(0)}{0 + \cos(0)} \\ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

$$33. \lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\frac{-x + 1}{x}}{-\pi \operatorname{sen}(\pi x)} \\ & \lim_{x \rightarrow 0} \frac{(-\pi \operatorname{sen}(\pi x))(1 - x)}{x} \\ & = \frac{(-\pi \operatorname{sen}(\pi))(1 - 1)}{1} \\ & = 0 \end{aligned}$$

$$35. \lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2}$$

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{ax - a}{2(x - 1)} \\ & \lim_{x \rightarrow 1} \frac{a(x - 1)}{2(x - 1)} \\ & \lim_{x \rightarrow 1} \frac{a}{2} \\ & = \frac{a}{2} \end{aligned}$$

$$37. \lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{x - \operatorname{sen} x}{4x^3} \\ & \lim_{x \rightarrow 0} \frac{1 - \cos x}{12x^2} \\ & \lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{24x} \\ & \lim_{x \rightarrow 0} \frac{\cos x}{24} \end{aligned}$$

$$= \frac{\cos(0)}{24}$$

$$= \frac{1}{24}$$

39. $\lim_{x \rightarrow \infty} x \sin(\pi/x)$

$$\lim_{x \rightarrow \infty} \frac{\sin \frac{\pi}{x}}{\frac{1}{x}}$$

$$\lim_{x \rightarrow \infty} \frac{-\frac{\pi}{x^2} \cos \frac{\pi}{x}}{-\frac{1}{x^2}}$$

$$\lim_{x \rightarrow \infty} \pi \cos \frac{\pi}{x}$$

$$= \pi(1)$$

$$= \pi$$

41. $\lim_{x \rightarrow 0} \cot 2x \sin 6x$

$$\lim_{x \rightarrow 0} \frac{\sin 6x}{\tan 2x}$$

$$\lim_{x \rightarrow 0} \frac{6 \cos 6x}{2 \sec^2 2x}$$

$$\lim_{x \rightarrow 0} \frac{3 \cos 6x}{\sec^2 2x}$$

$$= \frac{3 \cos 6(0)}{\sec^2 2(0)}$$

$$= 3$$

$$\boxed{43.} \quad \lim_{x \rightarrow \infty} x^3 e^{-x^2}$$

$$\lim_{x \rightarrow \infty} \frac{x^3}{\frac{1}{e^{-x^2}}}$$

$$\lim_{x \rightarrow \infty} \frac{3x^2}{e^{x^2} 2x}$$

$$\lim_{x \rightarrow \infty} \frac{3x}{e^{x^2} 2}$$

$$\lim_{x \rightarrow \infty} \frac{3}{4xe^{x^2}}$$

$$\frac{3}{4} \lim_{x \rightarrow \infty} \frac{1}{xe^{x^2}}$$

$$= \frac{3}{4} * \frac{1}{\infty}$$

$$= 0$$