Problema 1

$$\int_c \vec{F} \times d\vec{r}, \quad \vec{F} = y\hat{\imath} + x\hat{\jmath} \quad (0,0,0) \quad (3,9,0)$$

$$y = \frac{x^3}{3}, \quad z = 0$$

Sea $x = t$

$$\Rightarrow \quad y = \frac{t^3}{3}$$

$$r = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad r = t\hat{\imath} + \frac{t^3}{3}\hat{\jmath} + 0\hat{k}$$

$$dr = (\hat{\imath} + t^2\hat{\jmath})\,dt$$

$$F = \frac{t^3}{3}\hat{\imath} + t\hat{\jmath}$$

$$\Rightarrow \vec{F} \times d\vec{r} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{t^3}{3} & t & 0 \\ 1 & t^2 & 0 \end{vmatrix} = 0\hat{\imath} + 0\hat{\jmath} + \left(\frac{t^5}{3} - t\right)\hat{k}$$

$$\hat{k}\int_0^3 \left(\frac{t^5}{3} - t\right) dt = \hat{k}\left[\frac{t^6}{18} - \frac{t^2}{2}\right]\Bigg|_0^3$$

$$= \boxed{[0, 0, 36]}$$

Problema 4 $\oint_C P\,dx + Q\,dy = \iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

como

$F = P\hat{\imath} + Q\hat{\jmath}$ y $ds = k\,dx\,dy$

$$\nabla \times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & 0 \end{vmatrix} = -\frac{\partial Q}{\partial z}\hat{\imath} + \frac{\partial P}{\partial z}\hat{\jmath} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

$F \cdot dr = P\,dx + Q\,dy$

$\nabla \times F \cdot ds = \nabla \times F \cdot k\,dx\,dy = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

$$\boxed{\therefore \oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy \quad \text{qed}}$$

calcular $\oint_C (3x + 4y)\,dx + (2x - 3y)\,dy$

donde C es el círculo $x^2 + y^2 = 4$

$x = 2\cos t$, $y = 2\,\mathrm{sen}\,t$ $\quad 0 \leq t \leq 2\pi$

$$\int_{t=0}^{2\pi} (3(\cos t) + 4(\mathrm{sen}\,t\ 2))(-2\,\mathrm{sen}\,t) + (2(2\cos t) - 3(2\,\mathrm{sen}\,t))(2\cos t)\,dt$$

$$= \int_{t=0}^{2\pi} \left(\cos^2 t - 2\,\mathrm{sen}^2 t - 3\frac{\mathrm{sen}(2t)}{2}\right) dt$$

$$= \boxed{-8\pi}$$

Problema 5

$$\vec{F}(\rho,\phi) = f_\rho(\rho,\phi)\,\hat{e}_\rho + f_\phi(\rho,\phi)\,\hat{e}_\phi$$

$\nabla \times \bar{F}$

$$\bar{\nabla}\times\bar{F} = \begin{vmatrix} \hat{e}_\rho & \hat{e}_\phi & \hat{e}_z \\ \frac{\partial}{d\rho} & \frac{\partial}{d\phi} & \frac{\partial}{dz} \\ f & f & 0 \end{vmatrix} = \frac{\partial f_\phi}{dz}\hat{e}_\rho + \frac{\partial f_\rho}{dz}\hat{e}_\phi + \left(\frac{\partial f_\phi}{d\rho} - \frac{\partial f_\rho}{d\phi}\right)\hat{e}_z$$

Como
$f_\rho(\rho,\phi)$ y $f_\phi(\rho,\phi)$ no depende de z

$$\Rightarrow \frac{\partial f_z}{dz} = \frac{\partial(f(\rho,\phi))}{dz} = 0$$

además

$$\frac{\partial f_1}{dz} = \frac{\partial(f_\phi(\rho,\phi))}{dz} = 0$$

$$\therefore \nabla\times\bar{F} = \left(\frac{\partial(f_\phi(\rho,\phi))}{d\rho} - \frac{\partial(f_\rho(\rho,\phi))}{d\phi}\right)\hat{e}_z$$

y por tanto solo tiene componente en z qed