$$\frac{1}{C_0}\int du + \frac{1}{C_0}\int \frac{du}{u} = x + c_1$$

$$\frac{u}{C_0} + \frac{1}{C_0}\ln u = x + c_1$$

$$\frac{C_0 y - 1}{C_0} + \frac{1}{C_0}\ln|C_0 y - 1| = x + c_1$$

$$\Rightarrow \boxed{\frac{y}{C_0} + \frac{1}{C_0}\ln|C_0 y - 1| = x + C_3} \quad \text{con } C_3 = C_1 + \frac{1}{C_0}$$

c) Resolver la sig. ecuación diferencial

$$(y+1)y'' = (y')^2$$

Sol. $(y+1)y'' = (y')^2 \Rightarrow \dfrac{y''}{(y')^2} = \dfrac{1}{1+y}$

que es equivalente a $\dfrac{\cancel{y'}\,y''}{\cancel{y'}\,y'} = \dfrac{y'}{1+y}$

Sea $u = y' \Rightarrow \dfrac{u'}{u} = \dfrac{y'}{1+y}$

es decir $\dfrac{d}{dx}\ln u = \dfrac{d}{dx}\ln|1+y|$

integrando tendremos que $\ln u = \ln|1+y| + c_1$

Aplicando $e^{(\;)}$ en ambos lados

$$y' = u = e^{\cancel{\ln u}} = e^{(\ln(1+y) + c_1)}$$

$$y' = C_0(1+y), \quad C_1 = \ln C_0$$

Separando variables e integrando

$$\int \frac{dy}{1+y} = \int C_0\,dx \Rightarrow \ln|1+y| = C_0 x + C_2$$

$$\therefore \boxed{y(x) = e^{C_2 + C_0 x} - 1}$$