

Problema 1

$$y = 12 - x^2$$

$$y = x^2 - 6$$

$$12 - x^2 = x^2 - 6$$

$$0 = 2x^2 - 18$$

$$0 = 2(x^2 - 9)$$

$$0 = 2(x-3)(x+3)$$

$$\Rightarrow x = \pm 3$$

$$\Rightarrow f(x) = 12 - x^2$$

$$g(x) = x^2 - 6$$

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_{-3}^3 [12 - x^2 - (x^2 - 6)] dx$$

$$= \int_{-3}^3 [18 - 2x^2] dx$$

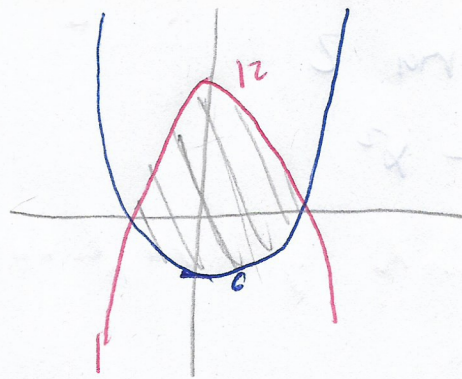
$$= 18x - \frac{2}{3}x^3 \Big|_{-3}^3$$

$$= \left[18(3) - \frac{2}{3}(3)^3 \right] - \left[18(-3) - \frac{2}{3}(-3)^3 \right]$$

$$= [54 - 18] - [-54 + 18]$$

$$= 36 + 36$$

$$= \boxed{72}$$



Problem 2

$$y = x - x^2$$

$$x = 2 \text{ c/c}$$

$$y = 0$$

$$x - x^2 = 0$$

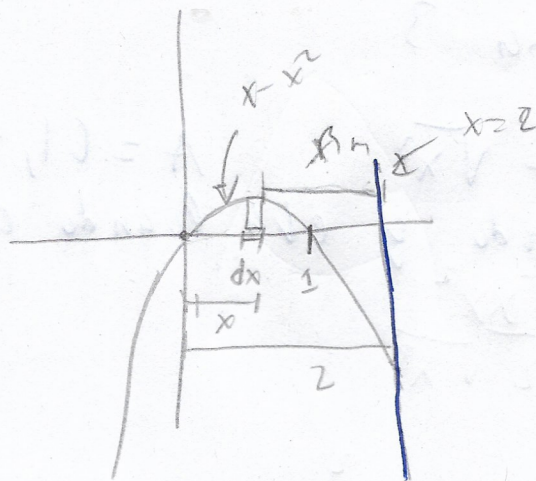
$$x(1-x) = 0$$

$$x_1 = 0$$

$$x_2 = 1$$

$$f(x) = x - x^2$$

$$g(x) = 0$$



$$x + R_m = 2$$

$$R_m = 2 - x$$

$$V = 2\pi \int_0^1 (2-x)(x-x^2) dx$$

$$= 2\pi \int_0^1 (2x^2 - 2x^3 - x^2 + x^3) dx$$

$$= 2\pi \left[x^2 - x^3 + \frac{x^4}{4} \right]_0^1$$

$$= 2\pi \left[\frac{1}{4} \right]$$

$$= \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

Problema 3

$$y = 5 - \sqrt{x^3} \quad A = (1, 1) \quad B = (4, -3)$$

derivando y evaluando en el intervalo

$$y' = -\frac{3x^2}{2\sqrt{x^3}}$$

$$y' = -\frac{3(1)^2}{2\sqrt{(1)^3}} = -\frac{3}{2\sqrt{8}}$$

$$y' = -\frac{3(4)^2}{2\sqrt{(4)^3}} = -\frac{48}{16}$$

Integrando

$$L = \int_1^4 \sqrt{1 + \left(-\frac{3x^2}{2\sqrt{x^3}}\right)^2} dx$$

$$= \int_1^4 \sqrt{1 + \frac{9}{4}x} dx \Rightarrow u = 1 + \frac{9}{4}x$$

$$du = \frac{9}{4} dx$$

$$= \int \frac{4}{9} \sqrt{u} du$$

$$\frac{4}{9} du = dx$$

$$= \left[\frac{4}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} \right]_1^4 = \frac{8}{9} \left[\sqrt{u} \right]_1^4 = \frac{8}{9} \left[\sqrt{1 + \frac{9}{4}x} \right]_1^4$$

$$= \frac{1}{27} (9x + 4)^{\frac{3}{2}} \Big|_1^4$$

regresando el cambio de variable y evaluando

$$= \left[\frac{1}{27} (40)^{\frac{3}{2}} \right] - \left[\frac{1}{27} (13)^{\frac{3}{2}} \right]$$

$$= \frac{1}{27} \left[\sqrt{(40)^3} - \sqrt{(13)^3} \right]$$

$$= \frac{1}{27} [252.9 - 46.8]$$

$$= \frac{1}{27} [206.18]$$

$$= \boxed{7.6}$$