

$$3. \sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$$

comparación  $\left| \frac{a_{n+1}}{a_n} \right| \leq R < 1$  (radio)

$$\left| \frac{\frac{x^{n+1}}{\sqrt{n+1}}}{\frac{x^n}{\sqrt{n}}} \right| = \left| \frac{\sqrt{n}x^n \cdot x}{x^n \sqrt{n+1}} \right| = \left| x \frac{\sqrt{n}}{\sqrt{n+1}} \right| = \left| x \sqrt{\frac{n}{n+1}} \right|$$

$$\lim_{n \rightarrow \infty} \sqrt{\frac{n}{n+1}} \cdot |x| = \lim_{n \rightarrow \infty} \sqrt{\frac{1}{1+\frac{1}{n}}} |x| = |x| = L < 1$$

$$\Rightarrow |x| < 1 = \boxed{-1 < x < 1}$$

intervalo de convergencia

$$\boxed{R=1}$$

radio de convergencia

para  $x = 1$

$$\sum_{n=1}^{\infty} \frac{1^n}{\sqrt{n}} = \text{diverge}$$

para  $x = -1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = \text{converge}$$

$$\Rightarrow \boxed{-1 \leq x < 1}$$

intervalo de convergencia

$$5. \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$$

$$\left| \frac{(-1)^n x^{n+1}}{(n+1)^3} \right| = \left| \frac{n^3 (-1)^n x^n x}{(-1)^{n-1} x^n (n+1)^3} \right| = \frac{|x|^3}{(n+1)^3}$$

$$\lim_{n \rightarrow \infty} \frac{n^3}{(n+1)^3} |x| = \left( \lim_{n \rightarrow \infty} \frac{n}{n+1} \right)^3 |x| = \left( \lim_{n \rightarrow \infty} \frac{\frac{n}{n+h}}{1 + \frac{h}{n+h}} \right)^3 |x|$$

$$\left( \lim_{n \rightarrow \infty} 1 \right)^3 |x| = 1^3 |x| = |x| = L$$

$$|x| < 1 \Rightarrow -1 < x < 1$$

$$R = 1$$

para  $x = 1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 1^n}{n^3} = \text{converge}$$

para  $x = -1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (-1)^n}{n^3} = \text{converge}$$

$$\therefore \boxed{-1 \leq x \leq 1}$$

$$7. \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right| = \left| \frac{n! x^{n+1}}{x^n (n+1)!} \right| = \left| \frac{x}{n+1} \right| = \frac{1}{n+1} |x|$$

$$\lim_{n \rightarrow \infty} \frac{1}{n+1} |x| = 0, |x| < 1$$

$$\Rightarrow |x| \cdot 0 < 1$$

$$-a < x < \infty$$

$$R = \infty$$

$$9. \sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$$

$$\Rightarrow \left| \frac{(-1)^{n+1} \frac{(n+1)^2 x^{n+1}}{2^{n+1}}}{(-1)^n n^2 x^n} \right| = \left| \frac{2^n (-1)^n (-1) (n+1)^2 x^n x}{2^n (-1)^n n^2 x^n} \right| \\ = \left| \frac{(-1) \cdot (n+1)^2 x}{2^n} \right| = \frac{(n+1)^2}{2^n} \frac{1}{2} |x| = \left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \frac{1}{2} |x|$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \frac{1}{2} |x| = \frac{1}{2} |x| = L < 1$$

$$\Rightarrow |x| < 2 \Rightarrow -2 < x < 2 \Rightarrow R = 2$$

param  $x = 2$

$$\sum_{n=1}^{\infty} (-1)^n n^2 (2)^n = \text{diverge}$$

param  $x = -\frac{1}{2}$

$$\sum_{n=1}^{\infty} (-1)^n n^2 (-2)^n = \text{cond. converge}$$

$$\therefore -2 < x < 2$$

$$\text{II. } \sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$$

$$\left| \frac{(-2)^{n+1} x^{n+1}}{\sqrt[4]{n+1}} \right| = \left| \frac{(-x)^n (-2) x^n \sqrt[4]{n}}{(-2)^n x^n \sqrt[4]{n+1}} \right| \\ = \sqrt[4]{\frac{n}{n+1}} |x|$$

$$\lim_{n \rightarrow \infty} \sqrt[4]{\frac{n}{n+1}} |x| = \lim_{n \rightarrow \infty} \sqrt[4]{\frac{\frac{n}{n}}{1 + \frac{1}{n}}} |x| = |x| = L < 1$$

$$\Rightarrow |x| < 1 \Rightarrow -1 < x < 1$$

Param  $x = \frac{1}{2}$

$$\sum_{n=1}^{\infty} (-2)^n \left(\frac{1}{2}\right)^n = \text{converge}$$

Param  $x = -\frac{1}{2}$

$$\sum_{n=1}^{\infty} (-2)^n \left(-\frac{1}{2}\right)^n = \text{converge}$$

$$\therefore -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$13. \sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$$

$$\left| \frac{(-1)^{n+1} x^{n+1}}{4^{n+1} \ln(n+1)} \right| = \left| \frac{4^n \ln n (-1)^n (-1)^{n+1} x^n}{4^n 4 \ln(n+1) (-1)^n x^n} \right| = \frac{\ln n}{\ln(n+1)} \cdot \frac{1}{4} |x|$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{\ln(n+1)} \cdot \frac{1}{4} |x| = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{n} + 1} \cdot \frac{1}{4} |x| = \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \frac{1}{4} |x| =$$

$$= \lim_{n \rightarrow \infty} \frac{1}{4} |x| = \frac{1}{4} |x| = L < 1$$

$$\Rightarrow \frac{1}{4} |x| < 1 \Rightarrow |x| < 4 \Rightarrow -4 \leq x \leq 4$$

Param  $x=4$

$$\sum_{n=2}^{\infty} (-1)^n \frac{4^n}{4^n \ln n} = \text{converge}$$

Param  $x=-4$

$$\sum_{n=2}^{\infty} (-1)^n \frac{(-4)^n}{4^n \ln n} = \text{diverge}$$

$$\boxed{-4 < x \leq 4}$$

$$15. \sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

$$\left| \frac{(x-2)^{n+1}}{(n+1)^2+1} \right| = \frac{(n+1)(x-2)^n(x-2)}{(x-2)^n ((n+1)^2+1)} = \frac{n^2+1}{(n^2+2n+2)} |x-2|$$

$$\lim_{n \rightarrow \infty} \frac{n^2+1}{n^2+2n+2} |x-2| = \lim_{n \rightarrow \infty} \frac{\frac{n^2}{n^2} + \frac{1}{n^2}}{\frac{2n}{n^2} + \frac{2n}{n^2} + \frac{2}{n^2}} |x-2|$$

$$= \lim_{n \rightarrow \infty} 1 |x-2| = |x-2| = L < 1$$

$$\Rightarrow |x-2| < 1 \Rightarrow -1 < x-2 < 1 \Rightarrow 1 < x < 3$$

para  $x=3$

$$\sum_{n=0}^{\infty} \frac{(3-2)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{1^n}{n^2+1} = \text{converge}$$

$\boxed{R=2}$

para  $x=1$

$$\sum_{n=0}^{\infty} \frac{(1-2)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1} = \text{converge}$$

$$\boxed{1 \leq x \leq 3}$$

$$17. \sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$

$$\left| \frac{\frac{3^{n+1} (x+4)^{n+1}}{\sqrt{n+1}}}{\frac{3^n (x+4)^n}{\sqrt{n}}} \right| = \left| \frac{\sqrt{n} 3^n (x+4)^n (x+4)}{3^n (x+4)^n \sqrt{n+1}} \right| = \frac{\sqrt{n}}{\sqrt{n+1}} |x+4|$$

$$\lim_{n \rightarrow \infty} \sqrt{\frac{n}{n+1}} |x+4| = \lim_{n \rightarrow \infty} \sqrt{\frac{n}{n+1}} 3|x+4| = 3|x+4|$$

$$\Rightarrow 3|x+4| < 1 \Rightarrow |x+4| < \frac{1}{3} \Rightarrow -\frac{1}{3} < x+4 < \frac{1}{3}$$

$$\Rightarrow -4 - \frac{1}{3} < x < \frac{1}{3} - 4 \Rightarrow -\frac{13}{3} < x < -\frac{11}{3}$$

param  $x = -\frac{13}{3}$

$$\boxed{R = \frac{1}{3}}$$

$$\sum_{n=1}^{\infty} \frac{3^n (-\frac{13}{3} + 4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n (-\frac{1}{3})^n}{\sqrt{n}} = \text{diverge}$$

param  $x = -\frac{11}{3}$

$$\sum_{n=1}^{\infty} \frac{3^n (-\frac{11}{3} + 4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n (\frac{1}{3})^n}{\sqrt{n}} = \text{diverge}$$

$$\therefore -\frac{13}{3} < x < -\frac{11}{3}$$

$$19. \sum_{n=1}^{\infty} \frac{(x-z)^n}{n^n}$$

$$\left| \frac{\frac{(x-z)^{n+1}}{(n+1)^{n+1}}}{\frac{(x-z)^n}{n^n}} \right| = \left| \frac{n^n (x-z)^n (x-z)}{(x-z)^n (n+1)^{n+1}} \right| = \frac{n^n}{(n+1)^{n+1}} |x-z|$$

$$\lim_{n \rightarrow \infty} \frac{n^n}{(n+1)^{n+1}} |x-z| = 0 \cdot |x-z| = L < 1$$

$$\Rightarrow 0 \cdot |x-z| < 1$$

$$-\infty < x < \infty$$

$$\therefore R = \infty$$

$$21. \sum_{n=1}^{\infty} \frac{n}{b^n} (x-a)^n, b > 0$$

$$\left| \frac{\frac{(n+1)}{b^{(n+1)}} (x-a)^{n+1}}{\frac{n}{b^n} (x-a)^n} \right| = \left| \frac{(n+1)(x-a)^n (x-a) b^n}{n b^n b (x-a)^n} \right| = \left| \frac{(n+1)(x-a)}{n b} \right| = \frac{n+1}{n} \left| \frac{x-a}{b} \right|$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n+1}{n} \cdot \frac{1}{b} |x-a| = \lim_{n \rightarrow \infty} 1 + \frac{1}{n} \frac{1}{b} |x-a| = \frac{1}{b} |x-a| < 1$$

$$\Rightarrow |x-a| < b \Rightarrow -b < x-a < b \Rightarrow a-b < x < b+a$$

$$R = b$$

para  $x = b+a$

$$\sum_{n=1}^{\infty} \frac{n}{b^n} (b+a-a)^n = \sum_{n=1}^{\infty} \frac{n}{b^n} b^n = \text{divergente}$$

para  $x = a-b$

$$\sum_{n=1}^{\infty} \frac{n}{b^n} (a-b-a)^n = \sum_{n=1}^{\infty} \frac{n}{b^n} (-b)^n = \text{divergente}$$

$$\therefore a-b < x < b+a$$