Actividad 3 correspondiente a la Unidad 3

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Instrucciones: Resolver las integrales, indicar si son convergentes o son divergentes

5. $\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$

u=x-2 -> du=dx

$$\lim_{b\to\infty}\int_1^b \frac{1}{u^{\frac{3}{2}}}du$$

$$= \lim_{b\to\infty} -\frac{2}{\sqrt{u}}\ \Big|_1^b$$

$$= \lim_{b\to\infty} -\frac{2}{\sqrt{b}}+\frac{2}{\sqrt{1}}$$

$$= \lim_{b\to\infty} -\frac{1}{\sqrt{b}}+2$$

$$=2$$

La integral converge y converge a 2

7. $\int_{-\infty}^0 \frac{1}{3-4x}\,dx$

u = 3-4x -> du=-4dx

$$\lim_{a\to\infty}\frac{-1}{4}\int_a^3 \frac{1}{u}du$$

$$= \lim_{a\to\infty}\frac{-1}{4}[\ln u]\ \Big|_a^3$$

$$= \lim_{a\to\infty}\frac{-1}{4}[\ln 3 - ln\ a]$$

$$=\infty$$

La integral diverge

9. $\int_{2}^{\infty} e^{-5p}\, dp$

u=-5p ->du=-5dp

$$\lim_{b\to\infty} \frac{-1}{5}\int_{-10}^{\mathrm{b}} e^{u}\mathrm{du}$$

$$= \lim_{b\to\infty} \frac{-1}{5}[e^{u}]\,\Big|_{-10}^{b}$$

$$= \lim_{b\to\infty} \frac{-1}{5}[e^{b} - e^{-10}]$$

$$= \frac{-1}{5}[\infty - e^{-10}]$$

$$= \frac{-1}{5}[\infty]$$

$$= \infty$$

La integral diverge

11. $\int_{0}^{\infty} \frac{x^{2}}{\sqrt{1 + x^{3}}}\, dx$

u = 1+x^3 -> du=3x^2 dx

$$\lim_{b\to\infty} \frac{1}{3x^2}\int_{1}^{\mathrm{b}} \frac{x^2}{\mathrm{u}^{\frac{1}{2}}}\mathrm{du}$$

$$= \lim_{b\to\infty} \int_{1}^{\mathrm{b}} \frac{1}{\mathrm{u}^{\frac{1}{2}}}\mathrm{du}$$

$$= \lim_{b\to\infty} 2\mathrm{u}^{\frac{1}{2}}\,\Big|_{1}^{b}$$

$$= \lim_{b\to\infty} 2[\mathrm{b}^{\frac{1}{2}} - 1^{\frac{1}{2}}]$$

$$= 2[\infty - 1]$$

$$= \infty$$

La integral diverge

13. $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

u = -x^2 -> du =-2x dx

$$\lim_{b\to-\infty}\frac{1}{-2x}\int_0^{\mathrm{b}} \mathrm{x}e^{u}\mathrm{du}+\lim_{a\to-\infty}\frac{1}{-2x}\int_a^0 \mathrm{x}e^{u}\mathrm{du}$$

$$=\lim_{b\to-\infty}\frac{1}{-2}e^{u}\Big|_0^b+\lim_{a\to-\infty}\frac{1}{-2}e^{u}\Big|_a^0$$

$$=\frac{1}{-2}(0)+\frac{1}{-2}(0)$$

$$=0$$

La integral converge y converge a 0

15. $\int_0^{\infty} \mathrm{sen}^2\alpha\,d\alpha$

$$sen^2\alpha=\frac{1}{2}-\frac{cos2\alpha}{2}$$

$$\lim_{b\to\infty}\int_0^b \frac{1}{2}-\frac{cos2\alpha}{2}\mathrm{d\alpha}$$

$$=\lim_{b\to\infty}\left(\frac{\alpha}{2}-\frac{sen2\alpha}{4}\right)\Big|_0^b$$

$$=\lim_{b\to\infty}\left(\frac{\alpha}{2}-\frac{sen2\alpha}{4}\right)-\left(\frac{0}{2}-\frac{sen(2*0)}{4}\right)$$

$$=\left(\frac{\infty}{2}-\frac{sen\infty}{4}\right)$$

La integral es divergente

17. $\int_1^{\infty} \frac{1}{x^2 + x}\, dx$

$$\lim_{b \to \infty} \int_1^b \frac{1}{x^2} + \frac{1}{x}\ \mathrm{dx}$$

$$= \lim_{b \to \infty} \left(\frac{-1}{x} + \ln x\right)\Big|_1^b$$

$$= \lim_{b \to \infty} \left(\frac{-1}{b} + \ln b\right) - (1 + \ln 1)$$

$$= (0 + \infty) - (1 + 0)$$

$$= \infty$$

La integral es divergente

19. $\int_{-\infty}^{0} ze^{2z}\, dz$

u = z -> du=dz   dv=e^2z dz -> v=(e^2z)/2

$$\lim_{a \to \infty} \frac{z}{2} e^{2z} - \int_a^0 \frac{e^{2z}}{2}\ \mathrm{dz}$$

$$= \lim_{a \to \infty} \left[\frac{z}{2} e^{2z} - \frac{e^{2z}}{4}\right]\Big|_a^0$$

$$= \lim_{a \to \infty} \left[\frac{0}{2} * e^{2*0} - \frac{e^{2*0}}{4}\right] - \left[\frac{a}{2} e^{2a} - \frac{e^{2a}}{4}\right]$$

$$= -\frac{1}{4} - [0 - 0]$$

$$= -\frac{1}{4}$$

La integral converge y converge a -1/4

31. $\int_{-2}^{3} \frac{1}{x^4}\, dx$

$$\int_0^3 \frac{1}{x^4}\mathrm{dx} + \int_{-2}^{0} \frac{1}{x^4}\mathrm{dx}$$

$$= \left[\frac{-1}{4x^3}\right] |_0^3 + \left[\frac{-1}{4x^3}\right] |_{-2}^0$$

$$= \left[\frac{-1}{4(3)^3}\right] - \left[\frac{-1}{4(0)^3}\right] + \left[\frac{-1}{4(0)^3}\right] - \left[\frac{-1}{4(-2)^3}\right]$$

Como

$$\frac{-1}{4(0)^3}$$

No está definido, entonces

La integral es divergente

33. $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\, dx$

$$\int_1^9 \frac{1}{\sqrt[3]{x-1}}\mathrm{dx} + \int_0^1 \frac{1}{\sqrt[3]{x-1}}\mathrm{dx}$$

u = x-1 -> du = dx

$$\int_0^8 \frac{1}{u^{\frac{1}{3}}}\mathrm{du} + \int_{-1}^0 \frac{1}{u^{\frac{1}{3}}}\mathrm{du}$$

$$= \frac{3u^{\frac{2}{3}}}{2} |_0^8 + \frac{3u^{\frac{2}{3}}}{2} |_{-1}^0$$

$$= [\frac{3(8)^{\frac{2}{3}}}{2} - \frac{3(0)^{\frac{2}{3}}}{2}] + [\frac{3(0)^{\frac{2}{3}}}{2} - \frac{3(-1)^{\frac{2}{3}}}{2}]$$

$$= \frac{3(8)^{\frac{2}{3}}}{2} - \frac{3(-1)^{\frac{2}{3}}}{2}$$

$$= 6 - \frac{3}{2} = \frac{9}{2}$$

La integral converge y converge a 9/2

35. $\int_0^3 \frac{dx}{x^2 - 6x + 5}$

$$\int_1^3 \frac{1}{x^2 - 6x + 5}\,\mathrm{dx} + \int_0^1 \frac{1}{x^2 - 6x + 5}\,\mathrm{dx}$$

$$[\frac{-1}{x} - \ln 6x + \frac{x}{5}]\Big|_1^3 + [\frac{-1}{x} - \ln 6x + \frac{x}{5}]\Big|_0^1$$

$$[(\frac{-1}{3} - \ln 18 + \frac{3}{5}) - ([\frac{-1}{1} - \ln 6 + \frac{1}{5}])] + [(\frac{-1}{1} - \ln 6 + \frac{1}{5}) - ([\frac{-1}{0} - \ln 0 + \frac{0}{5}])]$$

Como

$$\frac{-1}{0}$$

Además, ln 0 no está definido, entonces

La integral es divergente

37. $\int_{-1}^0 \frac{e^{1/x}}{x^3}\,dx$

$$u = \frac{e^{\frac{1}{x}}}{x^3}$$

$$du = -\frac{e^{\frac{1}{x}}(3x+1)}{x^5}$$

$$dv = dx$$

$$v = x$$

$$\frac{e^{\frac{1}{x}}}{x^2} - \int_{-1}^0 -\frac{e^{\frac{1}{x}}(3x+1)}{x^4}\,dx$$

$$[\frac{e^{\frac{1}{x}}}{x^2} - \frac{e^{\frac{1}{x}}}{x} + e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2}]\Big|_{-1}^0$$

$$[\frac{xe^{\frac{1}{x}} - e^{\frac{1}{x}}}{x}]\Big|_{-1}^0$$

$$= -\frac{2}{e}$$

La integral converge y converge a -2/e