Ejemplo 1: Resolver $x^2y'' - 2xy' - 4y = 0$

Si $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)y^{m-2}$
$x^2m(m-1)x^{m-2} - 2xmx^{m-1} - 4x^m = 0$
$x^m[m(m-1) - 2m - 4] = x^m[m^2 - 3m - 4] = 0$
$\therefore (m+1)(m-4) = 0 \Rightarrow m_1 = -1 \ , \ m_2 = 4$

$$y_h(x) = c_1x^{-1} + c_2x^4$$

Ejemplo 2: Resolver $4x^2y'' + 8xy' + y = 0$

Si $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)y^{m-2}$
$4x^2m(m-1)x^{m-2} + 8xmx^{m-1} + x^m = 0$
$x^m[4m(m-1) + 8m + 1] = 0 \therefore 4m^2 - 4m + 8m + 1 = 4m^2 + 4m + 1 = 0$
$\Rightarrow (2m+1)^2 = 0 \ , \ m_1 = -\dfrac{1}{2} = m_2$

$$y_h(x) = c_1x^{-\frac{1}{2}} + c_2x^{-\frac{1}{2}}\ln(x)$$

Ejercicios: Resolver

a) $x^2y'' - 2y = 0$

b) $x^2y'' - 3xy' - 2y = 0$

c) $3x^2y'' + 6xy' + y = 0$