1. Obtenga el resultado de

$$\frac{i}{1+\frac{i}{1+\frac{1}{i}}} + \frac{1}{1+\frac{1}{1+\frac{1}{i}}}$$

Solución:

$$\frac{i}{1+\frac{i}{1+\frac{1}{i}}} + \frac{1}{1+\frac{1}{1+\frac{1}{i}}} = \frac{i}{1+\frac{i}{1-i}} + \frac{1}{1+\frac{1}{1-i}} = \frac{i}{1+\frac{1}{2}+\frac{1}{2}i} + \frac{1}{1+\frac{1}{2}+\frac{1}{2}i}$$

$$= \frac{i}{\frac{3}{2}+\frac{1}{2}i} + \frac{1}{\frac{3}{2}+\frac{1}{2}i} = \frac{1}{5}+\frac{3}{5}i+\frac{3}{5}-\frac{1}{5}i$$

$$= \underline{\frac{4}{5}+\frac{2}{5}i}$$

2. Encontrar las raíces de $\left[\frac{1-i}{1+i}\right]^{\frac{1}{5}}$ y graficarlas en el plano z

Solución:

$$z = \frac{1-i}{1+i} = -i$$

$$\Rightarrow |z| = \sqrt{(-1)^2} = 1$$

$$\theta = \frac{3\pi}{2}$$

$$z = e^{\frac{3\pi}{2}i + 2k\pi i} = e^{(\frac{3}{2}+2k)\pi i} = e^{(\frac{3+4k}{2})\pi i}$$

$$\Rightarrow \sqrt[5]{z} = \left(e^{(\frac{3+4k}{2})\pi i}\right)^{1/5} = e^{(\frac{3+4k}{10})\pi i}, \quad k = 0, 1, 2, 3, 4$$

para

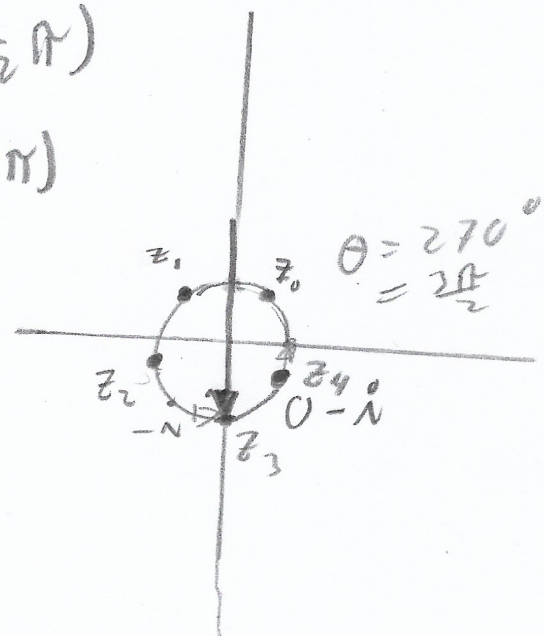
$$k=0 \Rightarrow z_0 = e^{\frac{3}{10}\pi i} = \cos(\tfrac{3}{10}\pi) + i\,\mathrm{sen}(\tfrac{3}{10}\pi)$$

$$k=1 \Rightarrow z_1 = e^{\frac{7}{10}\pi i} = \cos(\tfrac{7}{10}\pi) + i\,\mathrm{sen}(\tfrac{7}{10}\pi)$$

$$k=2 \Rightarrow z_2 = e^{\frac{11}{10}\pi i} = \cos(\tfrac{11}{10}\pi) + i\,\mathrm{sen}(\tfrac{11}{10}\pi)$$

$$k=3 \Rightarrow z_3 = e^{\frac{3}{2}\pi i} = \cos(\tfrac{3}{2}\pi) + i\,\mathrm{sen}(\tfrac{3}{2}\pi)$$

$$k=4 \Rightarrow z_4 = e^{\frac{19}{10}\pi i} = \cos(\tfrac{19}{10}\pi) + i\,\mathrm{sen}(\tfrac{19}{10}\pi)$$



3. Resolver la ecuación $\cos z = 1$

solución:

$\cos z = 1$

Como

$$\cos z = \frac{e^{iz}+e^{-iz}}{2} = \frac{e^{iz}+\frac{1}{e^{iz}}}{2} = \frac{e^{2iz}+1}{2e^{iz}} = i$$

$$\Rightarrow e^{2iz}+1 = 2i\,e^{iz}$$

$$\Rightarrow e^{2iz}-2i\,e^{iz}+1=0$$

tenemos que

$$e^{iz} = \frac{2i\pm\sqrt{-4-4}}{2} = \frac{2i+\sqrt{-8}}{2} = \frac{2i\pm 2i\sqrt{2}}{2}$$

$$= i \pm i\sqrt{2}$$

Entonces

$$e^{iz} = i \pm i\sqrt{2} = (i\pm\sqrt{2})i$$

aplicando ln

$$\ln e^{iz} = \ln((i\pm\sqrt{2})i)$$

$$\Rightarrow iz + 2k\pi i = \ln[(i\pm\sqrt{2})i]$$

despejando iz

$$iz = \ln[(1\pm\sqrt{2})i] - 2k\pi i$$

por lo tanto

$$z = \frac{\ln[(1\pm\sqrt{2})i]-2k\pi i}{i} = -i[\ln[(1\pm\sqrt{2})i]-2k\pi$$

$$z = \begin{cases} -i\left[\ln|1+\sqrt{2}| + i\left(\frac{\pi}{2} + 2c\pi\right)\right] - 2k\pi \\ -i\left[\ln|1-\sqrt{2}| + i\left(\frac{3\pi}{2} + 2a\pi\right)\right] - 2k\pi \end{cases}$$

$$= \begin{cases} -i\left[\ln(1+\sqrt{2})\right] + \left(\frac{\pi}{2} + 2b\pi\right) \\ -i\left[\ln(1-\sqrt{2})\right] + \left(\frac{3\pi}{2} + 2d\pi\right) \end{cases}$$

4. Encuentre $f'(z)$ por definición si;

$$f(z) = \frac{1}{z}$$

solución:

$$f'(z) = \lim_{\Delta z \to 0} \frac{\left(\frac{1}{z+\Delta z}\right) - \frac{1}{z}}{\Delta z} = \lim_{\Delta z \to 0} \frac{\frac{z-(z+\Delta z)}{z(z+\Delta z)}}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{\frac{z - z - \Delta z}{z(z+\Delta z)}}{\Delta z} = \lim_{\Delta z \to 0} \frac{\frac{-\Delta z}{z(z+\Delta z)}}{\frac{\Delta z}{1}}$$

$$= \lim_{\Delta z \to 0} \frac{-\Delta z}{\Delta z\, z(z+\Delta z)} = \lim_{\Delta z \to 0} -\frac{1}{z(z+\Delta z)}$$

Evaluando el limite

$$-\frac{1}{z(z+0)} = -\frac{1}{z \cdot z} = -\frac{1}{z^2}$$

por lo tanto

$$\boxed{f'(z) = -\frac{1}{z^2}}$$