Problema 2

5. $\phi(x,y,z) = xy + yz + zx$, hallar

$$\nabla\phi = \frac{d(xy+yz+zx)}{dx}\hat{\imath} + \frac{d(xy+yz+zx)}{dy}\hat{\jmath} + \frac{d(xy+yz+zx)}{dz}\hat{k}$$

$$= (y+z)\hat{\imath} + (x+z)\hat{\jmath} + (y+x)\hat{k}$$

evaluando en $(1,1,3)$

tenemos que

$$(1+3)\hat{\imath} + (1+3)\hat{\jmath} + (1+1)\hat{k}$$

$$= (4)\hat{\imath} + (4)\hat{\jmath} + (2)\hat{k}$$

$$= \boxed{[4,4,2]}$$

ahora para el segundo punto $\frac{d\phi}{ds} = \nabla\phi \cdot T$

para $T = \dfrac{\hat{\imath}+\hat{\jmath}+\hat{k}}{[[1,1,1]\cdot[1,1,1]]^{\frac{1}{2}}} = \dfrac{\hat{\imath}+\hat{\jmath}+\hat{k}}{[1^2+1^2+1^2]^{\frac{1}{2}}} = \dfrac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$

$$d\phi = \frac{(\hat{\imath}+\hat{\jmath}+\hat{k})}{\sqrt{3}} \cdot (4\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$$

$$= \frac{4}{\sqrt{3}} + \frac{4}{\sqrt{3}} + \frac{2}{\sqrt{3}}$$

$$= \boxed{\frac{10}{\sqrt{3}}}$$

Problema 3

Si $F = 3xyz^2 + 2xy^3\,\hat{j} - x^2yz\,\hat{k}$

$$\nabla \times F = \left[\frac{d(-x^2yz)}{dy} - \frac{d(2xy^3)}{dz}\right]\hat{i} - \left[\frac{d(-x^2yz)}{dx} - \frac{d(3xyz^2)}{dz}\right]\hat{j}$$
$$+ \left[\frac{d(2xy^3)}{dx} - \frac{d(3xyz^2)}{dy}\right]\hat{k}$$

$$= (-x^2z)\,\hat{i} + (8xyz)\,\hat{j} + (2y^3 - 3xz^2)\,\hat{k}$$

ahora evaluando en $(1,-1,1)$

$$[-(1)(1)]\,\hat{i} + [8(1)(-1)(1)]\,\hat{j} + [2(-1)^3 - 3(1)(1)^2]\,\hat{k}$$

$$= -1\,\hat{i} - 8\,\hat{j} - 5\,\hat{k} \qquad [-2-3] = -5$$

$$= \boxed{[-1, -8, -5]}$$

$$\nabla \cdot F = \frac{d(3xyz^2)}{dx} + \frac{d(2xy^3)}{dy} + \frac{d(-x^2yz)}{dz}$$

$$= 3yz^2 + 6xy^2 - x^2y$$

ahora evaluando en $(1,-1,1)$

$$3(-1)(1)^2 + 6(1)(-1)^2 - (1)^2(-1)$$

$$= -3 + 6 + 1$$

$$= \boxed{4}$$

Problema 5

$x^2+y^2-z-1=0$

$(1,1,1)$ $\hat{n}$ $\qquad$ y $R=\{(x,y)\mid x^2+y^2=1\}$

Verificamos si el punto pertenece a la superficie

$(1)^2+(1)^2-1-1=0$

$1+1-2=0$

$0=0$

calculamos el gradiente

$$\frac{d(x^2+y^2-z-1)}{dx}\hat{\imath}+\frac{d(x^2+y^2-z-1)}{dy}\hat{\jmath}+\frac{d(x^2+y^2-z-1)}{dz}\hat{k}$$

$$=2x\hat{\imath}+2y\hat{\jmath}-\hat{k}$$

evaluando en $(1,1,1)$

$$2(1)\hat{\imath}+2(1)\hat{\jmath}-k$$

$$=2\hat{\imath}+2\hat{\jmath}-\hat{k}$$

$$=(2,2,-1)$$

$$\hat{n}=\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{\left[(2,2,-1)\cdot(2,2,-1)\right]^{\frac{1}{2}}}=\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{9}}=\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{3}$$

$$\Rightarrow \boxed{n=\left[\frac{2}{3},\frac{2}{3},-\frac{1}{3}\right]}$$