

Solución

$$V_o(t) = R_F \dot{i}_1(t) + R_V (\dot{i}_1(t) - \dot{i}_2(t)) \dots (1)$$

$$\frac{1}{C} \int \dot{i}_2(t) dt + R_V (\dot{i}_2(t) - \dot{i}_1(t)) = 0 \dots (2)$$

$$V_o(t) = \frac{1}{C} \int \dot{i}_2(t) dt \dots (3)$$

Aplicando la transformada de Laplace

$$V_o(s) = R_F I_1(s) + R_V I_1(s) - R_V I_2(s) \dots (4)$$

$$\frac{1}{sC} I_2(s) + R_V I_2(s) - R_V I_1(s) = 0 \dots (5)$$

$$V_o(s) = \frac{1}{sC} I_2(s) \dots (6)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ V.S & F.T & V.E \end{array}$$

Despejamos las variables dependientes

de (4)

$$I_1(s) (R_F + R_V) = V_o(s) + R_V I_2(s)$$

$$I_1(s) = \frac{1}{R_F + R_V} V_o(s) + \frac{R_V}{R_F + R_V} I_2(s) \dots (7)$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ V.S & F.T & V.E & F.T & V.E \end{array}$$

de (5)

$$I_2(s) \left(\frac{1}{sC} + R_v \right) = R_v I_1(s)$$

$$I_2(s) = \frac{R_v \left(\frac{1}{sC R_v} \right)}{1 + sC R_v} I_1(s)$$

$$I_2(s) = \frac{sC R_v}{1 + sC R_v} I_1(s) \dots (8)$$

↓
V.S

↓
F.T

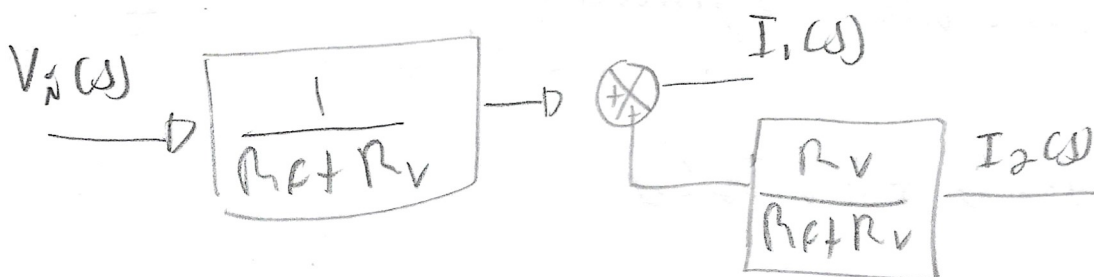
↓
V.E

Generamos un diagrama de bloques particular para cada ecuación.

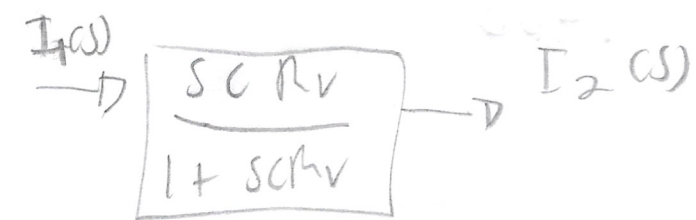
Para la ecuación (6)



Para la ecuación (7)

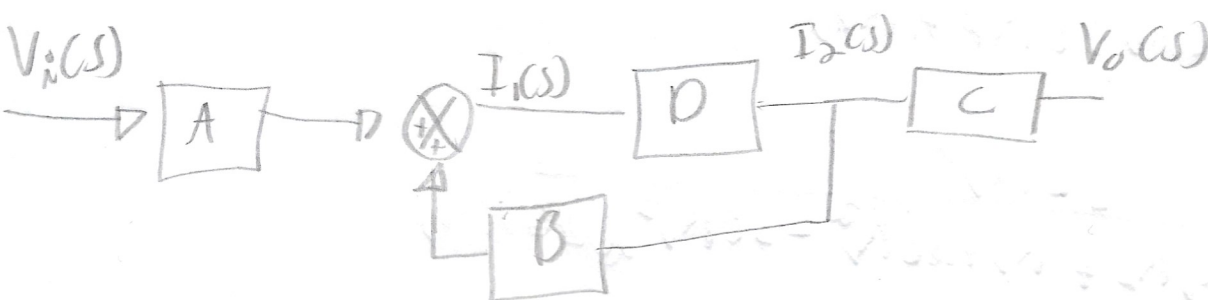


Para la ecuación (8)



Relacionamos los diagramas de bloques

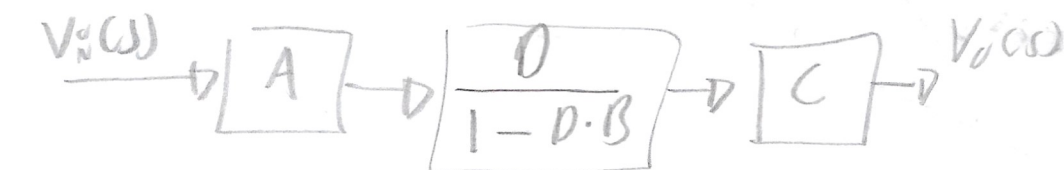
$$A = \frac{1}{R_f + R_v}, \quad B = \frac{R_v}{R_v + R_f}, \quad C = \frac{1}{sC}, \quad D = \frac{sC R_v}{1 + sC R_v}$$



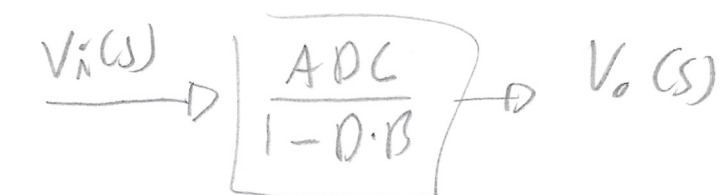
Como

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - I(s) G(s)}$$

Entonces



Aplicamos algebra de bloques



Sustituyendo A, B, C y D en $\frac{A \cdot DC}{1 - DB}$

Tenemos que

$$\frac{ADC}{1 - DB} = \frac{\left(\frac{1}{R_F + R_V}\right) \left(\frac{SCR_V}{1 + SCR_V}\right) \left(\frac{1}{SC}\right)}{1 - \left(\frac{SCR_V}{1 + SCR_V}\right) \left(\frac{R_V}{R_F + R_V}\right)}$$

$$= \frac{\left(\frac{1}{R_F + R_V}\right) \left(\frac{SCR_V}{1 + SCR_V}\right) \left(\frac{1}{SC}\right)}{(1 + SCR_V)(R_F + R_V) - (SCR_V^2)}$$

$$= \frac{SCR_V}{(R_F + SCR_V R_F + R_V + SCR_V^2 - SCR_V^2)(SC)}$$

$$= \frac{R_V}{R_F + SCR_V R_F + R_V}$$

$$= \frac{R_V}{SCR_V R_F + R_F + R_V}$$