Problema $\underline{1}$

$S = \{(0, K, K^2); (1, 1, 0); (0, K, 2K)\}$ sea una base de $\mathbb{R}^3$

$$\begin{vmatrix} 0 & K & K^2 \\ 1 & 1 & 0 \\ 0 & K & 2K \end{vmatrix}$$

$$\begin{matrix} 0 & K & K^2 \\ 1 & 1 & 0 \end{matrix}$$

$\to 0$, $\to 0$, $\to -K^2$; $\to 0$, $\to K^3$, $\to 0$

$K^3 - K^2 \neq$ es el determinante

$\Rightarrow K^3 - K^2 \neq 0$

$K^2(K-1) \neq 0$

$K(K(K-1) \neq 0$

$\Rightarrow \boxed{K \neq 1 \quad K \neq 0}$

$(x, y, z) = \alpha_1(0, K, K^2) + \alpha_2(1, 1, 0) + \alpha_3(0, K, 2K)$

$= (0, \alpha_1 K, \alpha_1 K^2) + (\alpha_2, \alpha_2, 0) + (0, \alpha_3 K, \alpha_3 2K)$

$\Rightarrow \alpha_2 = x$

$\alpha_1 K + \alpha_2 + \alpha_3 K = y$

$\alpha_1 K^2 + 2\alpha_3 K = z$

$\boxed{K \neq 1 \text{ y } K \neq 0, \quad K \in \mathbb{R}}$

# Problema 2

$B = \{(1,1,0); (0,1,1); (1,0,0)\}$

$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$

$\alpha_1(1,1,0) + \alpha_2(0,1,1) + \alpha_3(1,0,0) = (0,0,0)$

$(\alpha_1, \alpha_1, 0) + (0, \alpha_2, \alpha_2) + (\alpha_3, 0, 0) = (0,0,0)$

$(\alpha_1 + \alpha_3, \alpha_1 + \alpha_2, \alpha_2) = (0,0,0)$

$\alpha_1 + \alpha_3 = 0$
$\alpha_1 + \alpha_2 = 0$
$\alpha_2 = 0$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right)^{(-1)} = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)^{(-1)} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

no pueden generarse mas bases debido a que es un sistema libre y es linealmente independiente.

Problema 3

$v_1 = (1,0,2)$, $v_2 = (0,1,1)$

Sea $v_3 = (1,2,-1)$, entonces

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -4$$

$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$

$\alpha_1 (1,0,2) + \alpha_2 (0,1,1) + \alpha_3 (1,2,-1) = (0,0,0)$

$(\alpha_1, 0, 2\alpha_1) + (0, \alpha_2, \alpha_2) + (\alpha_3, 2\alpha_3, -\alpha_3) = (0,0,0)$

$\alpha_1 + \alpha_3 = 0$

$\alpha_2 + 2\alpha_3 = 0$

$2\alpha_1 + \alpha_2 - \alpha_3 = 0$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 1 & -1 & 0 \end{array}\right)(-2) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right)(-1)$$

Si $L = \{v_1, v_2, v_3\}$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -5 & 0 \end{array}\right)\left(\tfrac{1}{-5}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)(-2)(-1)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{array}{l} \alpha_1 = 0 \\ \alpha_2 = 0 \\ \alpha_3 = 0 \end{array}$$

$L$ es sistema libre

$\mathbb{R}^3 \;/ \Rightarrow L \subseteq B$, $B$ base $\mathbb{R}^3 \mid L = B$

$\Rightarrow L$ es base de $\mathbb{R}^3$

Problema 4

a) $W = \{(2,-1,0);(1,0,-1);(0,0,1)\}$

$$\begin{vmatrix} 2 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \to 0 \\ \to 0 \\ \to 1 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 0 \\ 1 & 0 & -1 \end{matrix} \begin{matrix} \to 0 \\ \to 0 \\ \to 0 \end{matrix}$$

determinante = 1

el conjunto W es combinación lineal además es linealmente independiente ya que tiene una solución única

$\therefore$ W es base de $\mathbb{R}^3$.

b) $C = \{(1,0,0);(2,-1,0);(1,0,-1)\}$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} \begin{matrix} \to 0 \\ \to 0 \\ \to 0 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 \\ 2 & -1 & 0 \end{matrix} \begin{matrix} \to +1 \\ \to 0 \\ \to 0 \end{matrix}$$

determinante = 1

el conjunto C es combinación lineal, además es linealmente independiente

$\therefore$ C es base de $\mathbb{R}^3$

c) $B = \{(2,1,0); (1,0,-1); (0,1,0)\}$

$$\begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix}$$

$\begin{matrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{matrix}$

determinante $= 2$

el conjunto B es combinación lineal, además es linealmente independiente

$\boxed{\therefore B \text{ es base de } \mathbb{R}^3}$

Problema 5

a) $v_1 = (1,0,1,0)$  $v_2 = (0,1,-1,2)$  $v_3 = (0,2,2,1)$

$\alpha_1(1,0,1,0) + \alpha_2(0,1,-1,2) + \alpha_3(0,2,2,1) = (0,0,0,0)$

$(\alpha_1, 0, \alpha_1, 0) + (0, \alpha_2, -\alpha_2, 2\alpha_2) + (0, 2\alpha_3, 2\alpha_3, \alpha_3) = (0,0,0$

$\alpha_1 = 0$
$\alpha_2 + 2\alpha_3 = 0$
$\alpha_1 - \alpha_2 + 2\alpha_3 = 0$
$2\alpha_2 + \alpha_3 = 0$

$$= \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & -1 & 2 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) (-1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) (1)(-2)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right) \left(\tfrac{1}{4}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{pmatrix} (3)(-2)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

es comb. lineal y linealmente independiente

$\Rightarrow$ $\boxed{\text{Sí es base de } \mathbb{R}^4}$

b) $v_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $v_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $v_3 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $v_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\alpha_1 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + \alpha_4 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} \alpha_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & \alpha_2 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \alpha_3 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \alpha_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow \alpha_1 = 0$
$\alpha_2 = 0$
$\alpha_3 = 0$
$\alpha_4 = 0$

es comb. lineal y linealmente independiente

$\therefore$ Sí es base de $M_{2\times2}$

c) $V_1 = 1 - x^2 \quad V_2 = x$

$\alpha_1(1-x^2) + \alpha_2(x) = 0$

$\alpha_1 - \alpha_1 x^2 + \alpha_2 x = 0$

$-\alpha_1 x^2 + \alpha_2 x + \alpha_1 = 0$

$-\alpha_1 x^2 = 0$

$\alpha_2 x = 0$

$\alpha_1 = 0$

son linealmente independientes y es comb. lineal

$\Rightarrow$ es una base de $P_2$

d) $v_1 = x^2 - 1 \quad v_2 = x - 2 \quad v_3 = x^2 - 3$

$\alpha_1(x^2-1) + \alpha_2(x-2) + \alpha_3(x^2-3) = 0$

$\alpha_1 x^2 - \alpha_1 + \alpha_2 x - 2\alpha_2 + \alpha_3 x^2 - 3\alpha_3 = 0$

$\alpha_1 x^2 + \alpha_3 x^2 = 0$

$\alpha_2 x = 0$

$-\alpha_1 - 2\alpha_2 - 3\alpha_3 = 0$

como $\alpha_2 = 0$ y $\alpha_1 = -\alpha_3$

$-\alpha_1 - 2\alpha_2 - 3\alpha_3 = 0$

$\alpha_3 - 2(0) - 3\alpha_3 = 0$

$\alpha_3 = 0 \Rightarrow \alpha_1 = 0$

son linealmente independientes y es comb lineal

$\Rightarrow$ es una base de $P_2$

Problema 6

a) $v_1 = (1, 2)$

$\alpha_1 v_1 = 0$

$\alpha_1 (1, 2) = (0, 0)$

$(\alpha_1, 2\alpha_1) = (0, 0)$

$\alpha_1 = 0$
$2\alpha_1 = 0$

$$X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ 2\alpha_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

b) $v_1 = (-1, 0, 2)$, $v_2 = (0, 1, 1)$

$\alpha_1 v_1 + \alpha_2 v_2 = 0$

$\alpha_1 (-1, 0, 2) + \alpha_2 (0, 1, 1) = (0, 0, 0)$

$(-\alpha_1, 0, 2\alpha_1) + (0, \alpha_2, \alpha_2) = (0, 0, 0)$

$-\alpha_1 = 0$
$\alpha_2 = 0$
$\alpha_1 + \alpha_2 = 0$

$\Rightarrow \alpha_1 = 0$
$\alpha_2 = 0$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\alpha_1 \\ \alpha_2 \\ \alpha_1 + \alpha_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Problema 7

$\{(1,2,1),(2,1,3),(3,3,4),(1,2,0)\}$

$$\begin{pmatrix}1&2&1\\2&1&3\\3&3&4\\1&2&0\end{pmatrix}(-2)(-3)(-1) = \begin{pmatrix}1&2&1\\0&-3&1\\0&-3&1\\0&0&-1\end{pmatrix}(-\tfrac{1}{3}) = \begin{pmatrix}1&2&1\\0&1&-\tfrac{1}{3}\\0&-3&1\\0&0&-1\end{pmatrix}(3)$$

$$= \begin{pmatrix}1&2&1\\0&1&-\tfrac{1}{3}\\0&0&0\\0&0&1\end{pmatrix}$$

$$\alpha_1(1,2,1) + \alpha_2(2,1,3) + \alpha_3(1,2,0) = (0,0,0)$$

$$(\alpha_1, 2\alpha_1, \alpha_1) + (2\alpha_2, \alpha_2, 3\alpha_2) + (\alpha_3, 2\alpha_3, 0) = (0,0,0)$$

$$\begin{aligned}\alpha_1 + 2\alpha_2 + \alpha_3 &= 0\\ 2\alpha_1 + \alpha_2 + 2\alpha_3 &= 0\\ \alpha_1 + 3\alpha_2 &= 0\end{aligned} = \left(\begin{array}{ccc|c}1&2&1&0\\2&1&2&0\\1&3&0&0\end{array}\right)(-2)(-1) = \left(\begin{array}{ccc|c}1&2&1&0\\0&-3&0&0\\0&1&-1&0\end{array}\right)(-\tfrac{1}{3})$$

$$= \left(\begin{array}{ccc|c}1&2&1&0\\0&1&0&0\\0&1&1&0\end{array}\right)(-1) = \left(\begin{array}{ccc|c}1&2&1&0\\0&1&0&0\\0&0&1&0\end{array}\right)\begin{matrix}(-2)\\(-1)\end{matrix} = \left(\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right)$$

$$x = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}\alpha_1 + 2\alpha_2 + \alpha_3\\ 2\alpha_1 + \alpha_2 + 2\alpha_3\\ \alpha_1 + 3\alpha_2\end{pmatrix} = \begin{pmatrix}1\\2\\1\end{pmatrix}\begin{pmatrix}2\\1\\3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}$$

Problema 8

$v_1 = (1, 2, -2, 1)$, $v_2 = (-3, 3, -9, 6)$, $v_3 = (2, 1, 1, -1)$

$v_4 = (-3, 0, -4, 5)$ $v_5 = (9, 3, 7, -6)$

$\alpha_1(1, 2, -2, 1) + \alpha_2(-3, 3, -9, 6) + \alpha_3(2, 1, 1, -1) + \alpha_4(-3, 0, -4, 5)$
$+ \alpha_5(9, 3, 7, -6) = (0, 0, 0, 0)$

$(\alpha_1, 2\alpha_1, -2\alpha_1, \alpha_1) + (-3\alpha_2, 3\alpha_2, -9\alpha_2, 6\alpha_2) + (2\alpha_3, \alpha_3, \alpha_3, -\alpha_3)$
$+ (-3\alpha_4, 0, -4\alpha_4, -5\alpha_4) + (9\alpha_5, 3\alpha_5, 7\alpha_5, -6\alpha_5) = (0, 0, 0, 0)$

$\alpha_1 - 3\alpha_2 + 2\alpha_3 - 3\alpha_4 + 9\alpha_5 = 0$

$2\alpha_1 + 3\alpha_2 + \alpha_3 + 3\alpha_5 = 0$

$-2\alpha_1 - 9\alpha_2 + \alpha_3 - 4\alpha_4 + 7\alpha_5 = 0$

$\alpha_1 + 6\alpha_2 - \alpha_3 + 5\alpha_4 - 6\alpha_5 = 0$

$$= \left(\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 4 & 0 \\ 0 & 1 & -\frac{1}{3} & 0 & -\frac{5}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$R_1 \Rightarrow \alpha_1 + \alpha_3 + 4\alpha_5 = 0$

$\alpha_2 - \frac{1}{3}\alpha_3 - \frac{5}{3}\alpha_5 = 0$

$\alpha_4 = 0$

$$X = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -\alpha_3 - 4\alpha_5 \\ \frac{1}{3}\alpha_3 + \frac{5}{3}\alpha_5 \\ \alpha_4 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{3} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -4 \\ \frac{5}{3} \\ 0 \\ 0 \end{pmatrix}$$

Problema 9

a) $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 \\ 6 & 0 & -1 & 5 \end{pmatrix} (-3)(-1)(-6) = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & -6 & 1 & -1 \\ 0 & 0 & 3 & -1 \\ 0 & -12 & -1 & -1 \end{pmatrix} (-\tfrac{1}{6}) = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & -6 & 1 & -1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & -3 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & -6 & 1 & -1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\boxed{\text{Rango} = 3 \quad \text{nulidad} = 1}$

b) $\begin{pmatrix} 1 & -2 & 2 \\ -1 & -1 & 0 \\ 3 & 0 & 1 \end{pmatrix} (1)(-3) = \begin{pmatrix} 1 & -2 & 2 \\ 0 & -3 & 2 \\ 0 & 6 & -5 \end{pmatrix} (2) = \begin{pmatrix} 1 & -2 & 2 \\ 0 & -3 & 2 \\ 0 & 0 & -1 \end{pmatrix}$

$\boxed{\text{Rango} = 3 \quad \text{Nulidad} = 0}$

c) $\begin{pmatrix} 3 & -7 & 3 \\ 2 & -5 & 5 \\ 5 & 1 & 3 \\ -3 & 0 & 4 \end{pmatrix} \tfrac{1}{3} = \begin{pmatrix} 1 & -\tfrac{7}{3} & 1 \\ 2 & -5 & 5 \\ 5 & 1 & 3 \\ -3 & 0 & 4 \end{pmatrix} (-2)(-5)(3) = \begin{pmatrix} 1 & -7/3 & 1 \\ 0 & -1/3 & 3 \\ 0 & 38/3 & -2 \\ 0 & -7 & 7 \end{pmatrix} (-3)$

$= \begin{pmatrix} 1 & -7/3 & 1 \\ 0 & 1 & -9 \\ 0 & 38/3 & -2 \\ 0 & -7 & 7 \end{pmatrix} (-\tfrac{38}{3})(7) = \begin{pmatrix} 1 & -\tfrac{7}{3} & 1 \\ 0 & 1 & -9 \\ 0 & 0 & 112 \\ 0 & 0 & -56 \end{pmatrix} \tfrac{1}{112} = \begin{pmatrix} 1 & -\tfrac{7}{3} & 1 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \\ 0 & 0 & -56 \end{pmatrix} (56)$

$= \begin{pmatrix} 1 & -7/3 & 1 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \boxed{\begin{matrix}\text{Rango} = 3 \\ \text{Nulidad} = 1\end{matrix}}$

d) $\begin{pmatrix} 1 & -4 & 2 & -1 \\ 3 & -12 & 6 & -3 \\ 2 & -1 & 0 & 1 \\ 0 & 1 & 3 & -1 \end{pmatrix}^{(-3)(-2)} = \begin{pmatrix} 1 & -4 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 7 & -4 & 3 \\ 0 & 1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -4 & 2 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 7 & -4 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (−7

$= \begin{pmatrix} 1 & -4 & 2 & -1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & -25 & 10 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\boxed{\begin{array}{l} \text{Nulidad} = 1 \\ \text{Rango} = 3 \end{array}}$

e) $\begin{pmatrix} -2 & 4 & 0 & 1 \\ 1 & 0 & 3 & 2 \\ 2 & 1 & 7 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 & 2 \\ -2 & 4 & 0 & 1 \\ 2 & 1 & 7 & 1 \end{pmatrix}^{(2)(-2)} = \begin{pmatrix} 1 & 0 & 3 & 2 \\ 0 & 4 & 6 & 5 \\ 0 & 1 & 1 & -3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 4 & 6 & 5 \end{pmatrix}^{(-4)} = \begin{pmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 2 & 17 \end{pmatrix}$

$\boxed{\begin{array}{l} \text{Nulidad} = 0 \\ \text{Rango} = 3 \end{array}}$

Problema 10

$v_1 = (1, -1, 1)$, $v_2 = (-2, 3, -1)$ $v_3 = (-3, 5, -1)$

$v_4 = (1, 2, -4)$

$$\begin{pmatrix} 1 & -1 & 1 \\ -2 & 3 & -1 \\ -3 & 5 & -1 \\ 1 & 2 & -4 \end{pmatrix} (-2)(3)(-1) = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & -5 \end{pmatrix} (-2)(-3) = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -8 \end{pmatrix} (-\tfrac{1}{8})$$

$$= \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{S = \{(1, -1, 1), (0, 1, 1), (0, 0, 1)\}}$$