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Tarea # 1

Corrección Parcial # 1

(1) Representar en espacio de estados y hallar la función de transferencia.

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2F(t) \quad (1)$$

Variables de estado

$$q_1 = x \qquad F(t) = u$$
$$q_2 = \dot{q}_1 = \dot{x}$$
$$q_3 = \dot{q}_2 = \ddot{x}$$
$$\dot{q}_3 = \dddot{x}$$

Reemplazando en (1)

$$\dot{q}_3 + q_3 + 2q_2 + q_1 = 2u$$
$$\dot{q}_3 = 2u - q_1 - 2q_2 - q_3$$

Representando en espacio de estados

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u]$$

$$[x] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

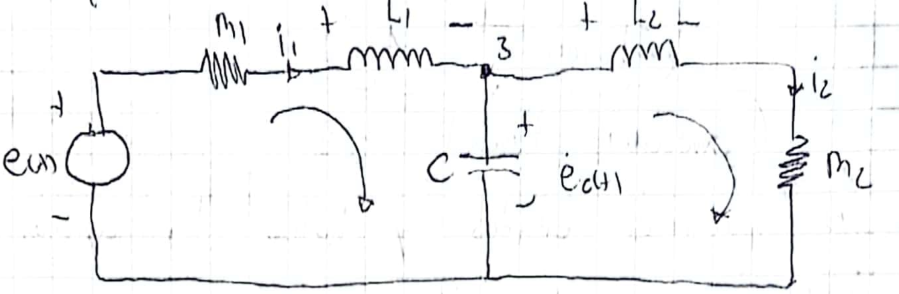
función de transferencia

Aplicando transformada de laplace a la ecuación (1)

$$s^3 \mathcal{L}[x] + s^2 \mathcal{L}[x] + 2s\mathcal{L}[x] + \mathcal{L}[x] = 2\mathcal{L}[F(t)]$$
$$\mathcal{L}[x](s^3 + s^2 + 2s + 1) = 2\mathcal{L}[F(t)]$$
$$\mathcal{L}[x] = X_s \qquad \mathcal{L}[F(t)] = F_s$$
$$X_s(s^3 + s^2 + 2s + 1) = 2F_s$$

$$\frac{X_s}{F_s} = \frac{2}{s^3 + s^2 + 2s + 1} \quad \text{función de transferencia}$$

② Encontrar una expresión en espacio de estados válida para el sistema outputs V_{m2}



Variables de estado i_{L1} i_{L2} V_c

malla 1 $\quad -e(t) + V_{m1} + V_{L1} + V_C = 0$

$$-e(t) + i_{L1} m_1 + V_{L1} + V_C = 0 \quad (1)$$

malla 2 $\quad V_{L2} + V_{m2} - V_C = 0 \quad (2)$

Nodo 3 $\quad i_{L1} - i_C - i_{L2} = 0 \quad (3)$

de (1) $\quad V_{L1} = e(t) - i_{L1} m_1 - V_C$

de (2) $\quad V_{L1} = V_C - V_{m2}$

de (3) $\quad i_C = i_{L2} - i_{L1}$

tomando $\quad V_L = L\dot{i}_L \quad$ y $\quad i_C = C\dot{V}_C$

$$\dot{i}_{L1} = \frac{e(t)}{L_1} - \frac{i_{L1} m_1}{L_1} - \frac{V_C}{L_1}$$

$$\dot{i}_{L2} = \frac{V_C}{L_2} - \frac{V_{m1}}{L_2}$$

$$\dot{V}_C = \frac{i_{L2}}{C} - \frac{i_{L1}}{C}$$

Representando en espacio de estados

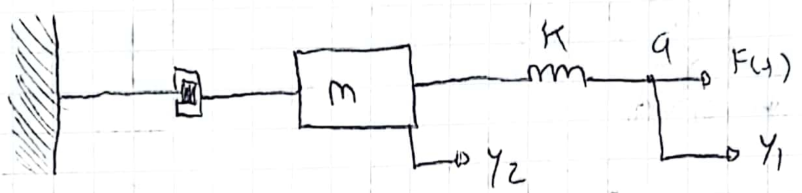
$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} \frac{m_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & \frac{1}{L_2} \\ -\frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} \left[e(t) \right]$$

Para la salida

$V_{m_2} = i_{L_2} m_2$

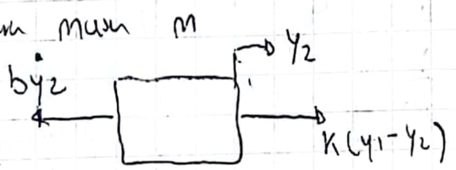
$$[V_{m2}] = \begin{bmatrix} 0 & m_2 & 0 \end{bmatrix} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ V_C \end{bmatrix}$$ } representamos todo en espacio de estados,

③ Hallar una representación en espacio de estados valida para el sistema outputs: y_1 y y_2



Diagramas de cuerpo libre

Para masa m



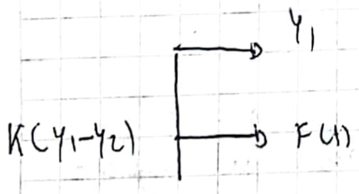
$$\Sigma F = ma$$

$$-b\dot{y}_2 + K(y_1 - y_2) = m\ddot{y}_2$$

$$-b\dot{y}_2 + Ky_1 - Ky_2 = m\ddot{y}_2$$

$$\ddot{y}_2 = \frac{K}{m}y_1 - \frac{K}{m}y_2 - \frac{b}{m}\dot{y}_2 \quad (1)$$

Para punto a



$$\Sigma F = 0$$

$$-K(y_1 - y_2) + F = 0$$

$$-Ky_1 + Ky_2 + F = 0 \quad (2)$$

Variables de estado

$$q_1 = y_1$$
$$q_2 = y_2$$
$$q_3 = \dot{q}_2 = \dot{y}_2$$
$$\dot{q}_3 = \ddot{y}_2$$

Reemplazamos en Ecuaciones 1 y 2

$$\dot{q}_3 = \frac{K}{m}q_1 - \frac{K}{m}q_2 - \frac{b}{m}q_3 \quad ③$$

$$-Kq_1 + Kq_2 + F = 0 \rightarrow q_1 = q_2 + \frac{F}{K}$$

Reemplazando en (3)

$$\dot q_3 = \frac{k}{m}\left(q_2 + \frac{F}{k}\right) - \frac{k}{m} q_2 - \frac{b}{m} q_3$$

$$\cancel{\frac{k}{m}} q_2 + \frac{F}{m} - \cancel{\frac{k}{m}} q_2 - \frac{b}{m} q_3$$

$$\dot q_3 = \frac{F}{m} - \frac{b}{m} q_3$$

Representamos en variables de estado

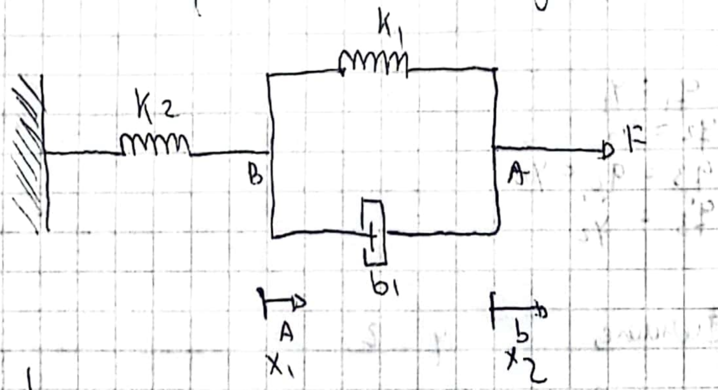
$$\begin{bmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m} \end{bmatrix} [F]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

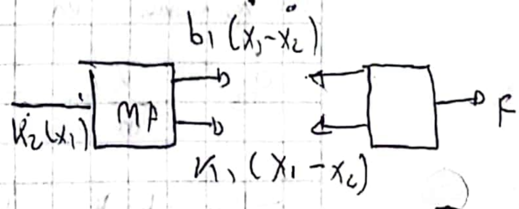
representación de espacio de estados

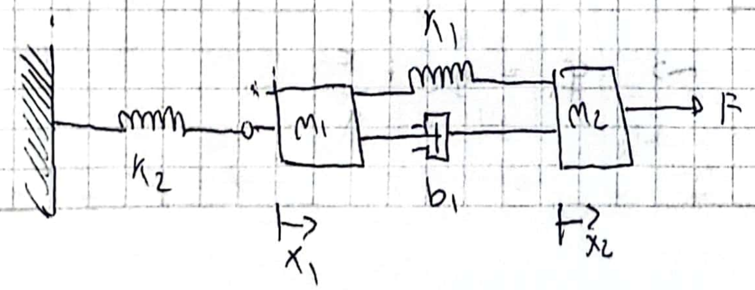
Tarea #2

· Cambia a masa puntual en el diagrama de fuerzas del siguiente ejemplo:



adicionando las masas





Ecuaciones: $+ \rightarrow x \quad \Sigma F = m \cdot a$

$$-K_2 x_1 + b_1(\dot{x}_1 - \dot{x}_2) + K_1(x_1 - x_2) = m_A \ddot{x}_1$$

$$\ddot{x}_1 = -\frac{K_2 x_1}{m_A} + \frac{b_1 \dot{x}_1}{m_A} - \frac{b_1 \dot{x}_2}{m_A} + \frac{K_1 x_1}{m_A} - \frac{K_1 x_2}{m_A} \quad (1)$$

$x_1 = q_1$
$q_2 = \dot{q}_1 = \dot{x}_1$
$q_3 = x_2$
$q_4 = \dot{q}_3 = \dot{x}_2$

$$-b_1(\dot{x}_1 - \dot{x}_2) - K_1(x_1 - x_2) + F = m_B \ddot{x}_2$$

$$\ddot{x}_2 = -\frac{b_1 \dot{x}_1}{m_B} + \frac{b_1 \dot{x}_2}{m_B} - \frac{K_1 x_1}{m_B} + \frac{K_1 x_2}{m_B} + \frac{F}{m_B} \quad (2)$$

Reemplazamos

$$\dot{q}_2 = -\frac{K_2 q_1}{m_A} + \frac{b_1 q_2}{m_A} - \frac{b_1 q_4}{m_A} + \frac{K_1 q_1}{m_A} - \frac{K_1 q_3}{m_A} \quad (1)$$

$$\dot{q}_4 = -\frac{b_1 q_1}{m_B} + \frac{b_1 q_4}{m_B} - \frac{K_1 q_1}{m_B} + \frac{K_1 q_3}{m_B} + \frac{F}{m_B} \quad (2)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_2+K_1}{m_A} & \frac{b_1}{m_A} & -\frac{K_1}{m_A} & -\frac{b_1}{m_A} \\ 0 & 0 & 0 & 1 \\ \frac{-b_1-K_1}{m_B} & 0 & \frac{K_1}{m_B} & \frac{b_1}{m_B} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_B} \end{bmatrix} F$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F$$

Tarea #3 (Espacio de estados - Sistema rotacional)

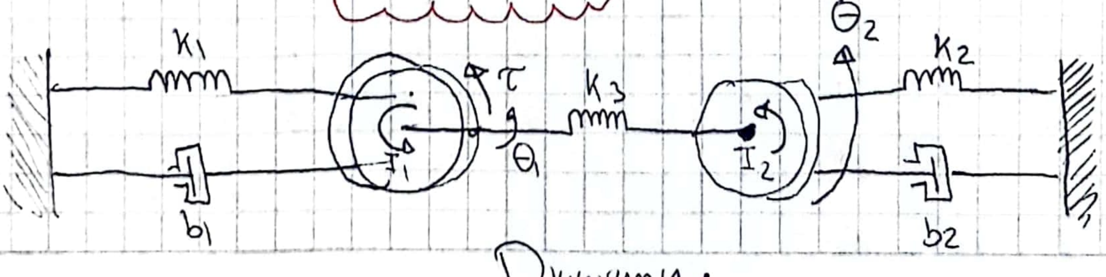
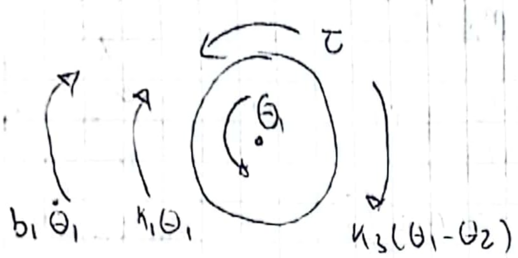


Diagrama:

Encontrar el espacio de estados del siguiente sistema considerando $\theta_1 >> \theta_2$

* Para el disco 1

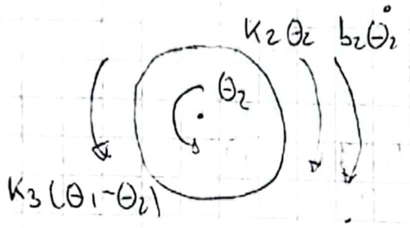


$\cdot \sum F = Ia = I_1\ddot{\theta}_1$

$$\tau - k_1\theta_1 - b_1\dot{\theta}_1 - k_3(\theta_1-\theta_2) = I_1\ddot{\theta}_1$$

$$\hookrightarrow \ddot{\theta}_1 = \frac{\tau}{I_1} + \left(\frac{-k_1-k_3}{I_1}\right)\theta_1 - \frac{b_1}{I_1}\dot{\theta}_1 + \frac{k_3\theta_2}{I_1}$$

* Para el disco 2.



$\cdot \sum F = Ia = I_2\ddot{\theta}_2$

$$k_3(\theta_1-\theta_2) - k_2\theta_2 - b_2\dot{\theta}_2 = I_2\ddot{\theta}_2$$

$$\hookrightarrow \ddot{\theta}_2 = \frac{k_3}{I_2}\theta_1 + \left(\frac{-k_3-k_2}{I_2}\right)\theta_2 - \frac{b_2}{I_2}\dot{\theta}_2$$

Variables de estado:

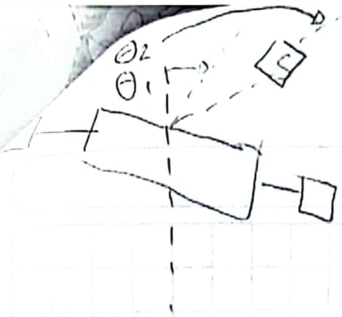
$q_1 = \theta_1$ $\quad$ $q_3 = \theta_2$

$q_2 = \dot{q}_1 = \dot{\theta}_1$ $\quad$ $q_4 = \dot{q}_3 = \dot{\theta}_2$

$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}_1$ $\quad$ $\dot{q}_4 = \ddot{q}_3 = \ddot{\theta}_2$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(k_1+k_3)}{I_1} & -\frac{b_1}{I_1} & \frac{k_3}{I_1} & 0 \\ 0 & 0 & 0 & 1 \\ k_3/I_2 & 0 & -\frac{(k_3+k_2)}{I_2} & -b_2/I_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I_1 \\ 0 \\ 0 \end{bmatrix} \tau$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tau$$

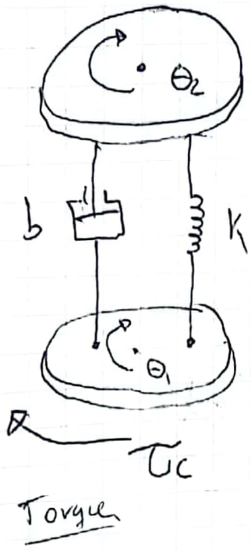


$\Theta_2 \Rightarrow$ Sensor

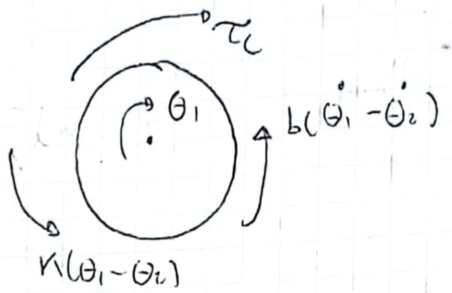
$\Theta_1 \Rightarrow$ body

Tarea #4 (Sistema rotacional del satelite)

Encontrar la representación en espacio de estados del siguiente sistema
Considerar $\Theta_1 >> \Theta_2$



* Para el disco 1



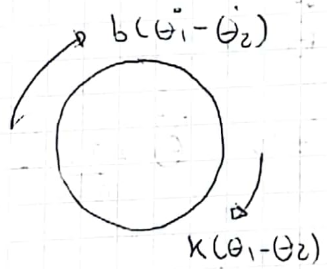
$$\Sigma F = I_1 \ddot{\Theta}_1$$

$$\tau_c - K(\Theta_1 - \Theta_2) - b(\dot{\Theta}_1 - \dot{\Theta}_2) = I_1 \ddot{\Theta}_1$$

$$\Downarrow$$

$$\ddot{\Theta}_1 = \frac{\tau_c}{I_1} - \frac{K\Theta_1}{I_1} + \frac{K\Theta_2}{I_1} - \frac{b\dot{\Theta}_1}{I_1} + \frac{b\dot{\Theta}_2}{I_1}$$

* Para el disco 2



$$\Sigma F = I_2 \ddot{\Theta}_2$$

$$K(\Theta_1 - \Theta_2) + b(\dot{\Theta}_1 - \dot{\Theta}_2) = I_2 \ddot{\Theta}_2$$

$$\Downarrow$$

$$\ddot{\Theta}_2 = \frac{K\Theta_1}{I_2} - \frac{K\Theta_2}{I_2} + \frac{b\dot{\Theta}_1}{I_2} - \frac{b\dot{\Theta}_2}{I_2}$$

* Variables de estado

$$q_1 = \Theta_1 \qquad q_3 = \Theta_2$$

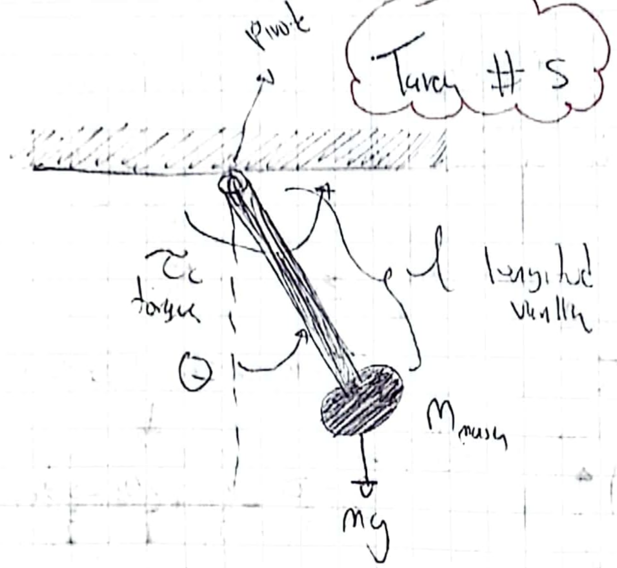
$$q_2 = \dot{q}_1 = \dot{\Theta}_1 \qquad q_A = \dot{q}_3 = \dot{\Theta}_2$$

$$\dot{q}_2 = \ddot{q}_1 = \ddot{\Theta}_1 \qquad \dot{q}_4 = \ddot{q}_3 = \ddot{\Theta}_2$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_A \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/I_1 & -b/I_1 & K/I_1 & b/I_1 \\ 0 & 0 & 0 & 1 \\ K/I_2 & b/I_2 & -K/I_2 & -b/I_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_A \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I_1 \\ 0 \\ 0 \end{bmatrix} \tau_c$$

$$\begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_A \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tau_c.$$

Tarea # 5 (Pendulo)



$I = ml^2$

Ecuación

$\tau_c - mg \cdot l\, sen\,\theta = I\ddot{\theta} \longrightarrow$ reemplazando $I = ml^2$

$T_c - mgl\, sen\,\theta = (ml^2)\ddot{\theta}$

$\ddot{\theta} = \frac{T_c}{ml^2} - \frac{\cancel{m}g\cancel{l}\, sen\,\theta}{\cancel{m}l^{\cancel{2}}} \Rightarrow \ddot{\theta} = \frac{T_c}{ml^2} - \frac{g}{l} sen\,\theta$

$\theta = q_1$
$q_2 = \dot{\theta} = \dot{q}_1$
$\dot{q}_2 = \ddot{\theta} = \ddot{q}_1$

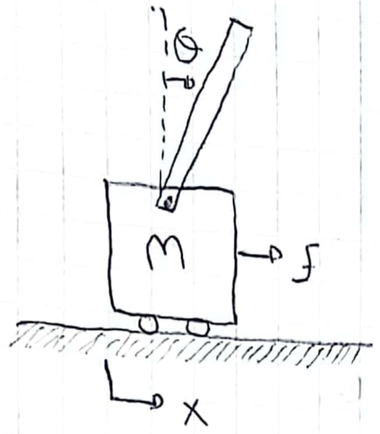
$\ddot{\theta} = \frac{T_c}{ml^2} - \frac{g}{l}\theta \rightarrow$ aproximando $sen\,\theta \rightarrow \theta$

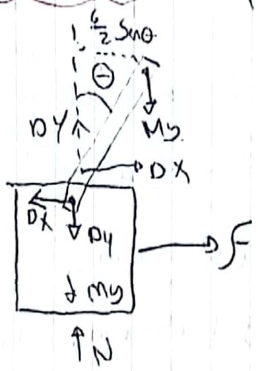
$\dot{q}_2 = \frac{T_c}{ml^2} - \frac{g}{l} q_1$

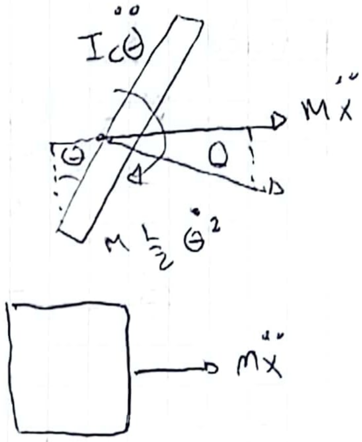
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} T_c$$

$$y = \theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} T_c$$

Tarea #6 (pendulo invertido.







$$+\rightarrow x: \sum F_x = \sum_{i=1}^{2} m_i (a_{ci})_x$$

$$F = m\ddot{x} + M\ddot{x} - M\frac{L}{2}\dot{\theta}^2 \operatorname{Sen}\theta + M\frac{L}{2}\ddot{\theta}\operatorname{Cos}\theta$$

$$+\circlearrowright : \sum M_p = I\alpha + M_{c.f.}\ (mal)$$

$$Mg\cdot\frac{L}{2}\operatorname{Sin}\theta = \frac{1}{12}ML^2\ddot{\theta} + M\ddot{x}\frac{L}{2}\cos\theta + M\frac{L}{2}\ddot{\theta}\cdot\frac{L}{2}$$

Por lo tanto las ecuaciones:

$$(m+M)\ddot{x} + \frac{1}{2}ML\ddot{\theta}\cos\theta - \frac{1}{2}ML\dot{\theta}^2\operatorname{Sen}\theta = F$$

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{x}\cos\theta - \frac{1}{2}MgL\operatorname{Sen}\theta = 0.$$

Para pequeños movimientos angulares $\cos\theta \approx 1$, $\operatorname{Sen}\theta \approx \theta$, $\dot{\theta}^2\theta = 0$

$$(m+M)\ddot{x} + \frac{1}{2}ML\ddot{\theta} = F$$

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{x} - \frac{1}{2}MgL\theta = 0$$

Los estados, la entrada y las salidas especificas son:

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{Bmatrix} \quad u = F \quad y \quad y = \begin{Bmatrix} x \\ \theta \end{Bmatrix}$$

$$\dot{x}_1 = \dot{x} = x_3$$
$$\dot{x}_2 = \dot{\theta} = x_4.$$

teniendo: · Resolviendo por "Cramer"

$$\ddot{x} = \frac{(1/3)\, ML^2 F - (1/4)\, M^2 L^2 g \theta}{(1/12)\, ML^2\, (M + 4m)}$$

$$\ddot{\theta} = \frac{-(1/2)\, MLF + (M+m)\, (1/2)\, MgL\theta}{(1/12)\, ML^2\, (M+4m)}$$

reemplazando:

$$\dot{x}_3 = \frac{4}{M+4m} u - \frac{3Mg}{M+4m} x_2$$

$$\dot{x}_4 = \frac{-6}{L(M+4m)} u + \frac{6(M+m)g}{L(M+4m)} x_2$$

Espacio de estados:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{3Mg}{M+4m} & 0 & 0 \\ 0 & \frac{6(M+m)g}{L(M+4m)} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{4}{M+4m} \\ -\frac{6}{L(M+4m)} \end{bmatrix} u$$

$$y = \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$