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Fracciones Parciales

Ejemplo 2

$$X(s) = \frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)}$$

En fracciones parciales

$$= \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{A}{s+2+j2} + \frac{A^*}{s+2-j2}$$

$$K_1 = s X(s) \Big|_{s=0}$$

$$K_1 = \left. \left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)} \right) \right|_{s=0}$$

$$K_1 = 1$$

$$K_2 = (s+1) X(s) \Big|_{s=-1}$$

$$K_2 = (s+1) \left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)} \right) \Big|_{s=-1}$$

$$K_2 = \frac{-2 + 3 - 4 + 8}{-1(1 - 4 + 8)} = -\frac{10}{5} = -2$$

$$K_2 = -2$$

$$A = (s+2+j2) X(s) \Big|_{s=-2-j2}$$

$$A = (s+2+j2) \left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)} \right) \Big|_{s=-2-j2}$$

$$A = (s + 2 + j2) \left(\frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s+2+j2)(s+2-j2)} \right) \Big|_{s=-2-j2}$$

$$A = \frac{2(-2-j2)^3 + 8[-2-j2]^2 + 4(-2-j2) + 8}{(-2-j2)[(-2-j2)+1][(-2-j2)^2 + 4(-2-j2) + 8]}$$

Resolviendo por partes

$$2s^3 = 2[-2-j2]^3$$

$$= 2 \left[(-2)^3 + 3(-2)^2(-j2) + 3(-2)(-j2)^2 + (-j2)^3 \right]$$

$$\begin{aligned} (-j2)^3 &= (-1)^3 j^3 2^3 \\ &= -1 \cdot j^2 j 8 \\ &= -8(-1)j \\ &= j8 \end{aligned}$$

$$2s^3 = 2[-8 - j24 + 24 + j8]$$

$$2s^3 = 2[16 - j16]$$

$$\boxed{2s^3 = 32 - j32}$$

$$8s^2 = 8[-2+j2]^2$$

$$8s^2 = 8[4 + 2(4j) - 4]$$

$$8s^2 = 8[8j]$$

$$\boxed{8s^2 = j64}$$

$$\text{Numerador: } 32 - j32 + j64 + 4(-2-j2) + 8$$

$$+ 8 - j8 + 8$$

$$\boxed{32 + j24}$$

$$\text{Denominador: } (-2-j2)(-2-j2+1)(-1-j2+2-j2)$$

$$(-2-j2)(-2-j2+1)(-j4)$$

$$(j8-8)(-1-j2)$$

$$-j8 + 16 + 8 + j16$$

$$\boxed{24 + j8}$$

$$A = \frac{32+j24}{24+j8} = \frac{8(4+j3)}{8(3+j)} = \frac{4+j3}{3+j} \cdot \frac{3-j}{3-j} = \frac{12-4j+9j+3}{9-j^2+3+1}$$

$$A = \frac{15+5j}{10} = \frac{3+j}{2} = 1,5 + j0,5$$

$$A = \boxed{1,5 + j0,5}$$

Finalmente se tiene:

$$X(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1,5 + j0,5}{s+2+j2} + \frac{1,5 - j0,5}{s+2-j2}$$