Video 1

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Sistema de control por realimentación de estados

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \begin{cases} OS\% & 9.5\% \\ t_s & 0{,}74 \text{ sg} \end{cases}$$

$$u(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^2+5s^2+4s}$$

$$(s^3+5s^2+4s)\, X_1(s) = U(s)$$

$$\downarrow$$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

Variables de estado:

$$x_1 = x_1$$
$$x_2 = \dot{x}_1$$
$$x_3 = \dot{x}_2 = \ddot{x}_1$$
$$\dot{x}_3 = \dddot{x}_1$$

Reemplazamos

$$\dot{x}_3 + 5x_3 + 4x_2 = u \rightarrow \boxed{\dot{x}_3 = -5x_3 - 4x_2 + u} \quad (1)$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)\, X_1(s)$$
$$= (0s^2 + 20s + 100)\, X_1(s) \rightarrow (20s+100)\, X_1(s)$$
$$= 20\dot{x}_1 + 100x_1 \rightarrow \text{reemplazando en variables de estado}$$

$$\boxed{y = 20x^2 + 100x_1} \quad (2)$$

Reemplazando en variables de estados tenemos finalmente:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\% OS = e^{-(\zeta\pi / \sqrt{1-\zeta^2})} = 100$$

$$0{,}095 = = e^{-(\zeta\pi / \sqrt{1-\zeta^2})} \cdot 100 \rightarrow \ln(0{,}095) = \ln(e^{-(\zeta\pi / \sqrt{1-\zeta^2})})$$

$$-2{,}3539 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \rightarrow \cancel{-}2{,}3539(\sqrt{1-\zeta^2}) = \cancel{-}\zeta\pi$$

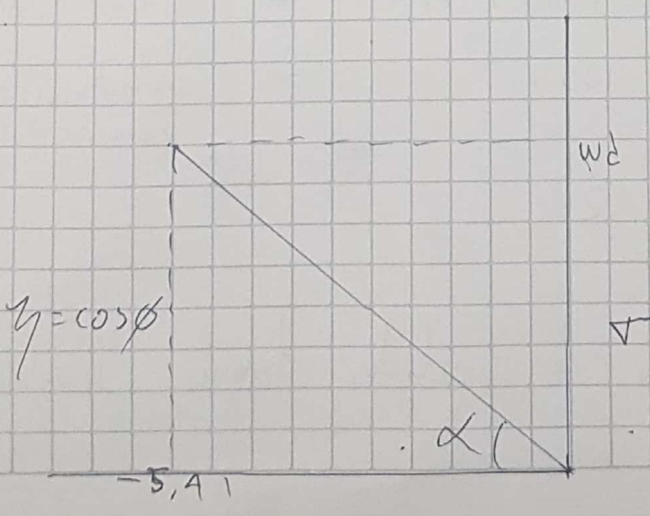
elevando al cuadrado $5.5407(1-\zeta^2) = \zeta^2\pi^2$

$$5{,}5407 - 5{,}5407\zeta^2 - \zeta^2\pi^2 \rightarrow 5{,}5407 = \zeta^2\pi^2 + 5{,}5407\zeta^2$$

$$5{,}5407 = \zeta^2(\pi^2 + 5{,}5407) \rightarrow \zeta^2 = \frac{5{,}5407}{\pi^2 + 5{,}5407}$$

$$\zeta = \sqrt{\frac{5{,}5407}{\pi^2 + 5{,}5407}} = \boxed{\zeta = 0{,}5996}$$

Plano S



$s = \sigma + j\omega d \rightarrow \sigma = \zeta\omega n$

$\phi = \cos^{-1}(0{,}5996) = 53{,}16°$

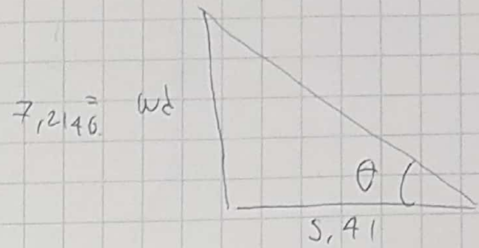
$t_s = \frac{4}{\sigma} \rightarrow 0{,}74 = \frac{4}{\sigma}$

$\sigma = \frac{4}{0{,}74} = 5{,}405$

$\sigma = \zeta\omega_n \rightarrow 5{,}405 = 0{,}5996\,\omega_n \rightarrow \boxed{\omega_n = 9{,}02\ rad/s}$

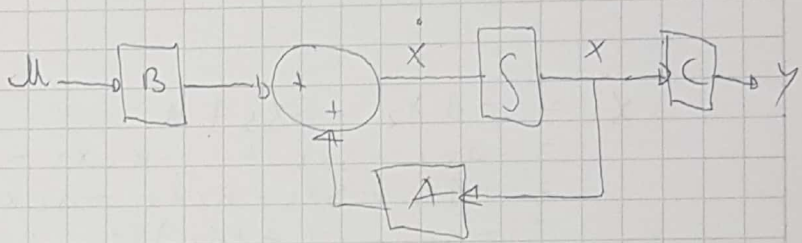
$\tan\theta = \dfrac{wd}{5{,}41} \qquad \tan^{-1}(53{,}16)(5{,}41) = wd$

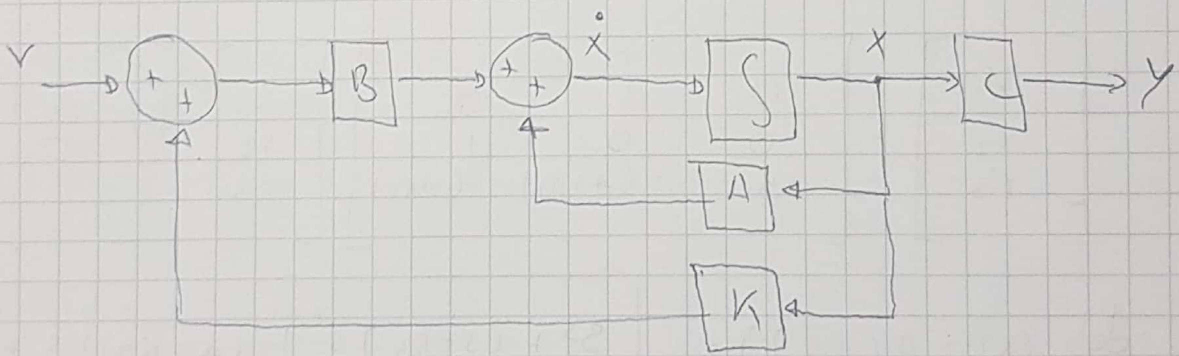
$$\boxed{wd = 7{,}2146}$$



- Realimentación en espacio de estados

$$\left.\begin{aligned} \dot{x} &= Ax + Bx \\ y &= Cx \end{aligned}\right\} \text{matrices}$$





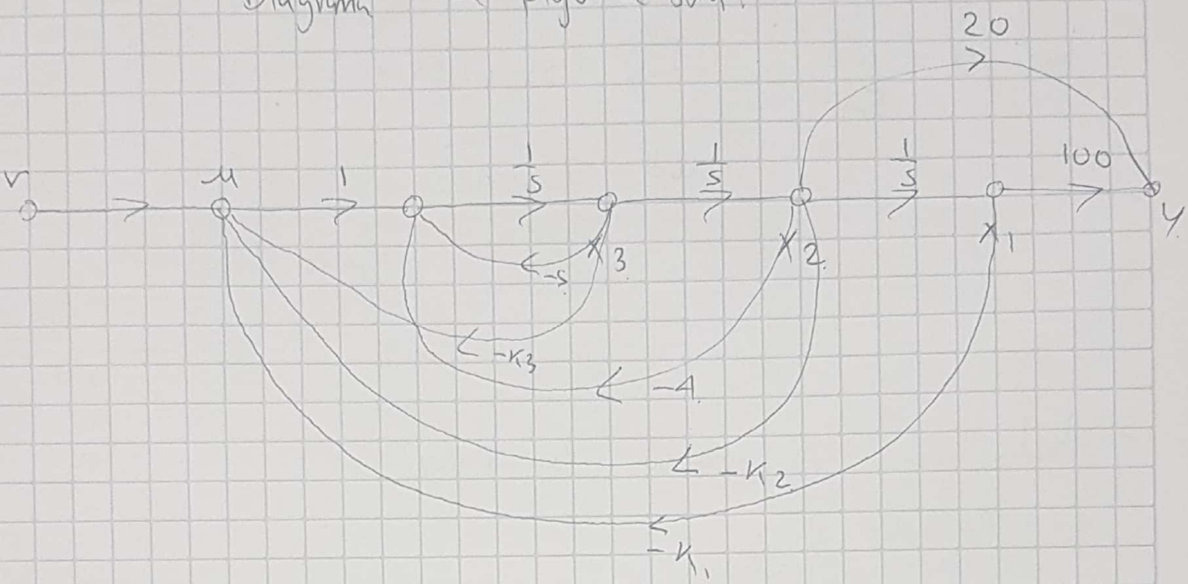
$$\begin{aligned} \dot{x} &= Ax + Bu \\ &= Ax + B(-Kx + r) \\ &= Ax - BKx + Br \end{aligned}$$

$$\boxed{\dot{x} = (A - BK)x + Br}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Diagrama de flujo de señal



$$\dot{x}_3 = -4x_2 - 5x_3 + u$$
$$= -4x_2 - 5x_3 + [-K_3x_3 - K_2x_2 - K_1x_1] + r$$
$$= -4x_2 - 5x_3 - K_3x_3 - K_2x_2 - K_1x_1 + r$$
$$= -K_1x_1 - (4+K_2)x_2 - (5+K_3)x_3 + r$$

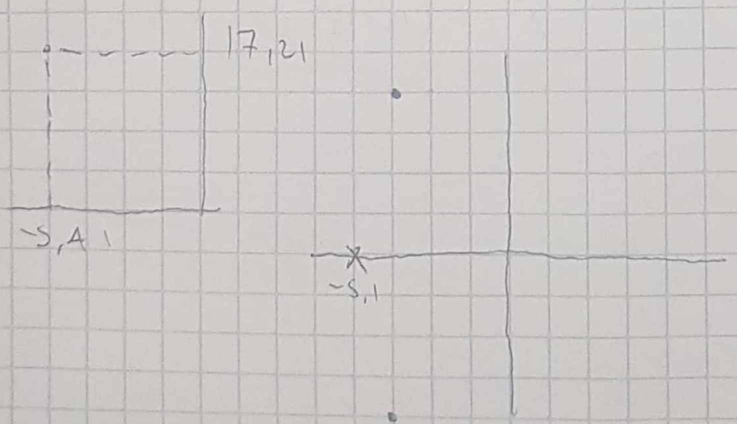
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & -(4+K_2) & -(5+K_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\det(sI - A(BK)) = \boxed{s^3 + (5+K_3)s^2 + (4+K_2)s + K_1 = 0} \quad (A)$$

Ecuación característica del sistema

$$(s+5,4-j7,2)(s+5,4+j7,1)(s+5,1) = T$$

$$T \rightarrow s^3 + 15,9s^2 + 156,22s + 415,83 = 0$$



S.

$$s^3 + (5+k_3)s^2 + (4+k_2)s + k_2 = s^3 + 15,9\,s^2 + 136,22\,s + 413,83$$

$$(5+k_3)\cancel{s^2} = 15,95\,\cancel{s^2} \qquad (4+k_2)s = 136,27s$$

$$5 + k_3 = 15,9 \qquad 4 + k_2 = 136,21$$

$k_1 = 413,83$ $k_3 = 10,9$ $k_2 = 132.22$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413,8 & -136,12 & -15,9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$