Saint Joseph's Institution

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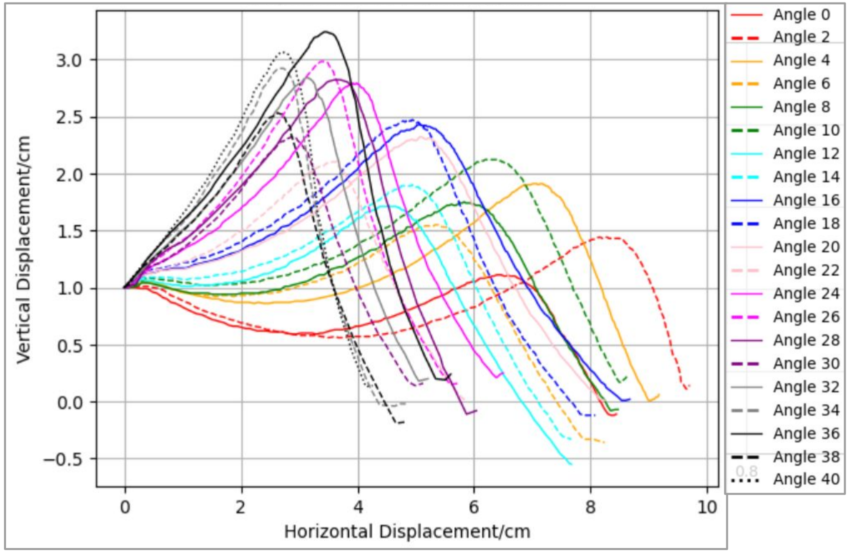
Project Code: US21

Introduction

*"Investigating and predicting the **Trajectory** of a Paper Airplane with different **initial pitch angles**"*

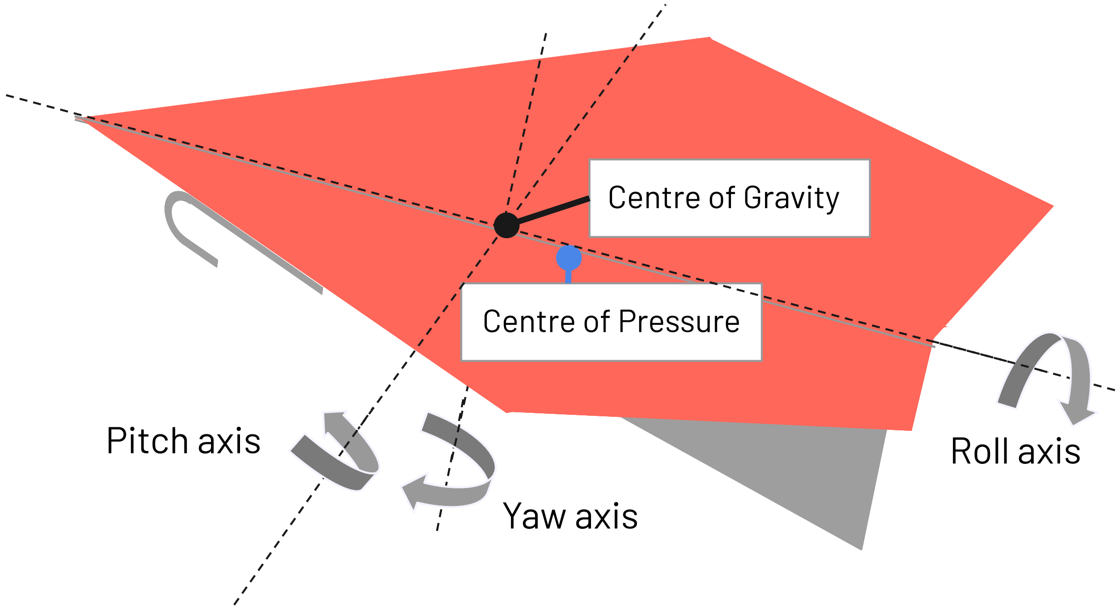


Abstract:
We investigated the effect of the launch angle on the **trajectory** of the paper Airplane.

Rationale:
Paper Airplanes are a cheap and convenient **modelling tool** for the design of MAV (Micro Air Vehicles) as they both have low Reynolds numbers. Investigating the properties of paper planes can help us to better design MAVs.

Literature Review:
Existing literature focuses primarily on the horizontal displacement of the plane with regards to the launch angle. These sources reveal a non-linear relationship with peak horizontal displacement at 5-6° or 20°. There is also a gap in the literature regarding the vertical displacement of the plane, although plane speed is partially investigated.

Introduction: Existing Aerodynamic Concepts



The Suzanne Model - Diagonal View

Three axis of rotation:
1. Pitch - Lateral axis
2. Yaw - Vertical axis
3. Roll - Longitudinal axis

Centre of Gravity:
A point from which weight may be considered to act

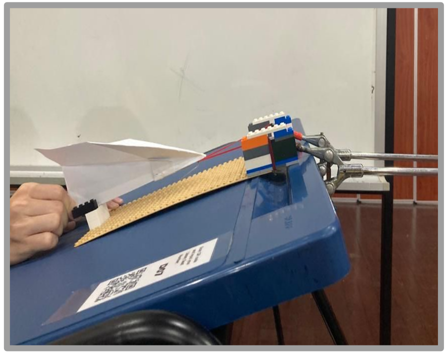
Centre of Pressure:
A point at which resultant aerodynamic forces (lift and drag) may be considered to act

Stability is achieved when the distribution of lift and weight are balanced

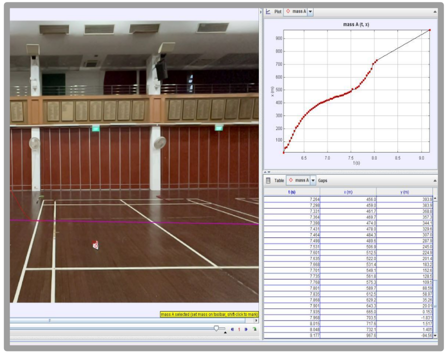
Methodology

Experimental Procedure:

1. Fold one stable Suzanne Model Paper Airplane with copy paper.
2. Measure elastic force of launching device with newton spring meter.
3. Change **initial pitch angle** using retort stands and wooden blocks from 0^o to 40^o with intervals of 2^o, controlling the height at 1.00m.
4. Attach paper airplane to launching device using the paperclip attached.
5. Release paper airplane and record it with a high speed camera.
6. Measure the final distance with a tape measure, tracking the **trajectory** digitally.
7. Repeat steps 4-7 four more times for each angle.



Side view of Launching Device

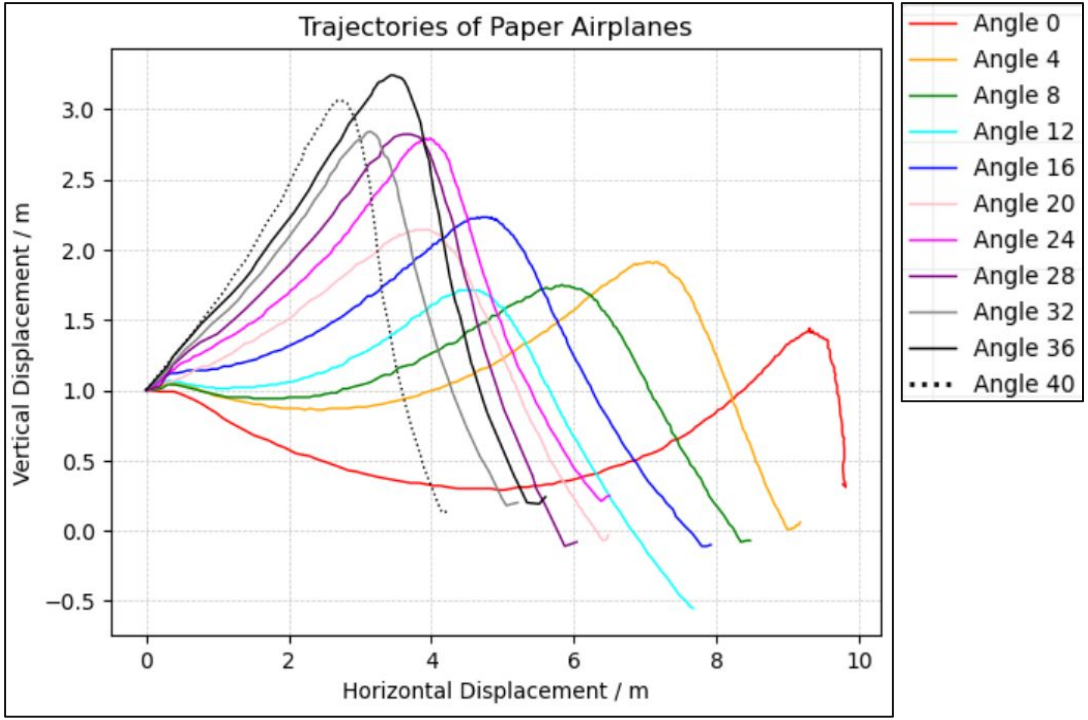


Tracking motion of plane in Tracker.

Independent Variable: Initial Pitch Angle
Dependent Variable: Trajectory (Horizontal Displacement as a function of Vertical Displacement)
Control Variables:

1. Elastic force of launching device
2. Flight Characteristics
3. Wind Speed

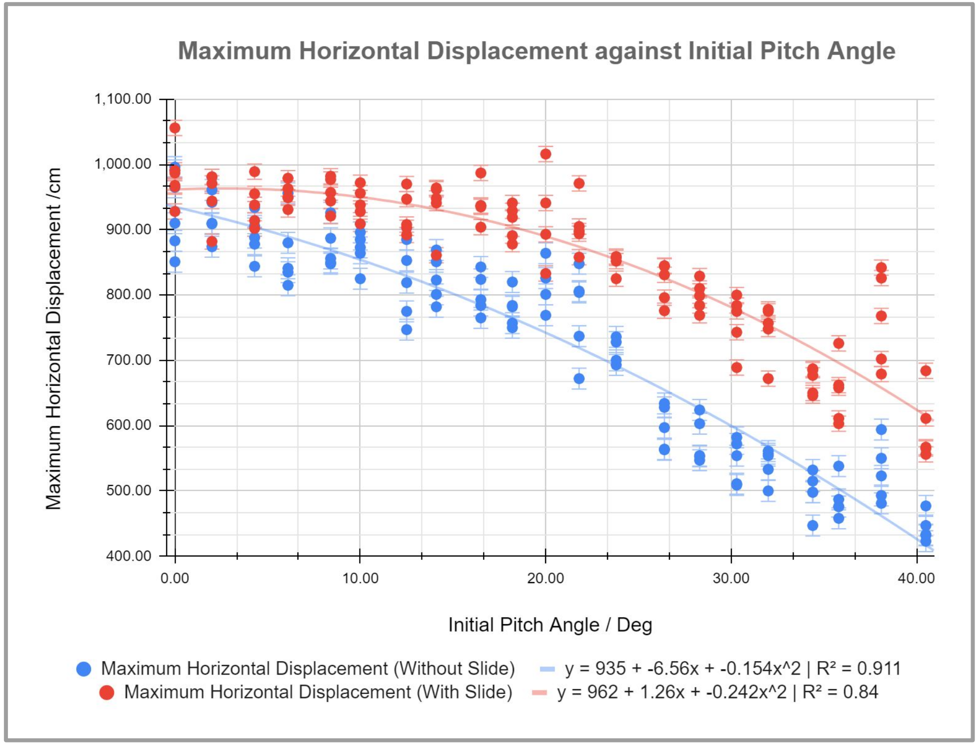
Results



Data Trends:

1. Higher angles have shorter horizontal displacements. *($r=-0.94$, $p<0.001$)*
2. Higher angles reach higher maximum heights. *($r=0.84$, $p<0.001$)*
3. Higher angles have shorter time of flights. *($r=-0.84$, $p<0.001$)*

Statistical Analysis: Horizontal Displacement



It was discovered that as the initial pitch angle increases, the distance the paper airplane slid increased *($r=0.77$, $p<0.001$).*

2 polynomial regression (2-degree) models were subsequently plotted to observe the effect this would have on the data.

With slide:

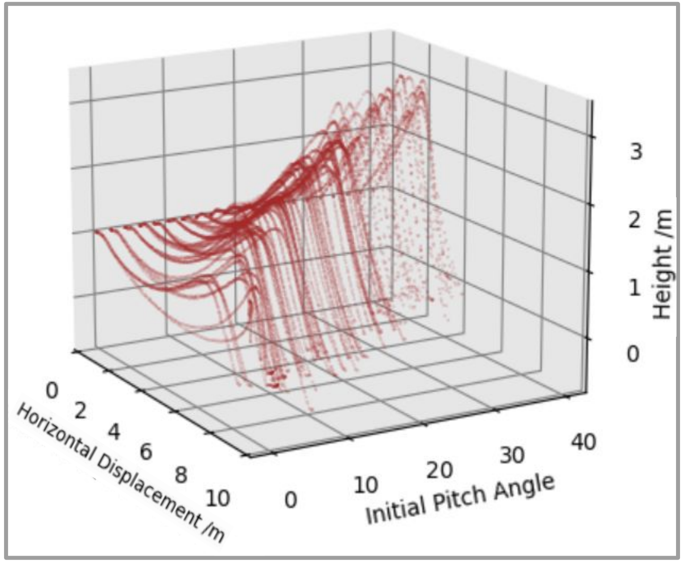
1. Exponential Shape
2. R^2 of 0.84

Without slide:

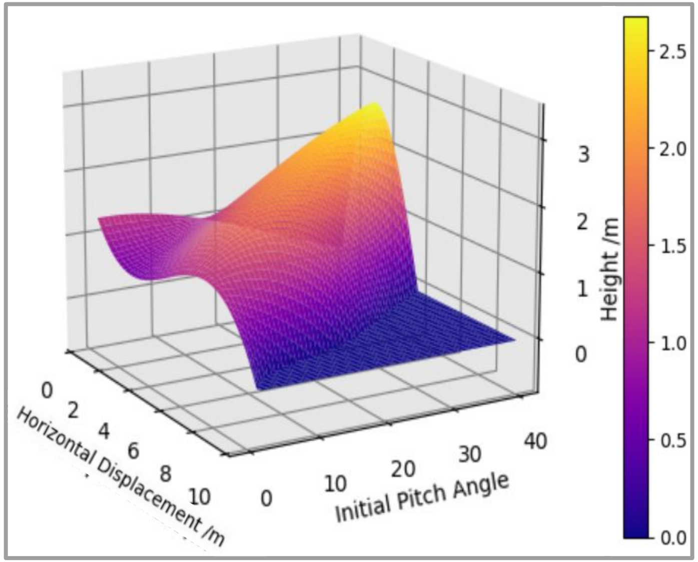
1. Linear Shape
2. Higher R^2 of 0.911

Statistical Analysis: Modelling the Trajectory

3D Scatter Plot of Trajectory Data



3D Predictive Model of Trajectories

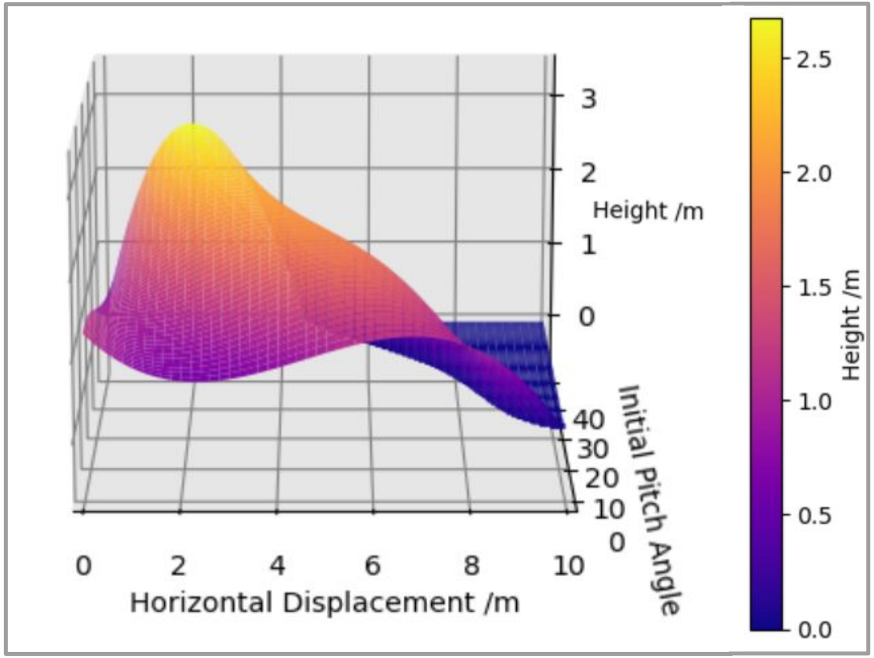


Predictive Model:
Multivariate polynomial spline regression

Spline:
Regresses different segments of the data and pieces them together

Multivariate:
Regresses the *height* (y) as a function of the *horizontal displacement* (x) and *initial pitch angle* (z)

Statistical Analysis: Modelling the Trajectory



Front View of Predictive Model of Trajectories

Evaluation of Predictive Model:
$R^2 = 0.64034$
Standard Error of Estimation: 0.419m

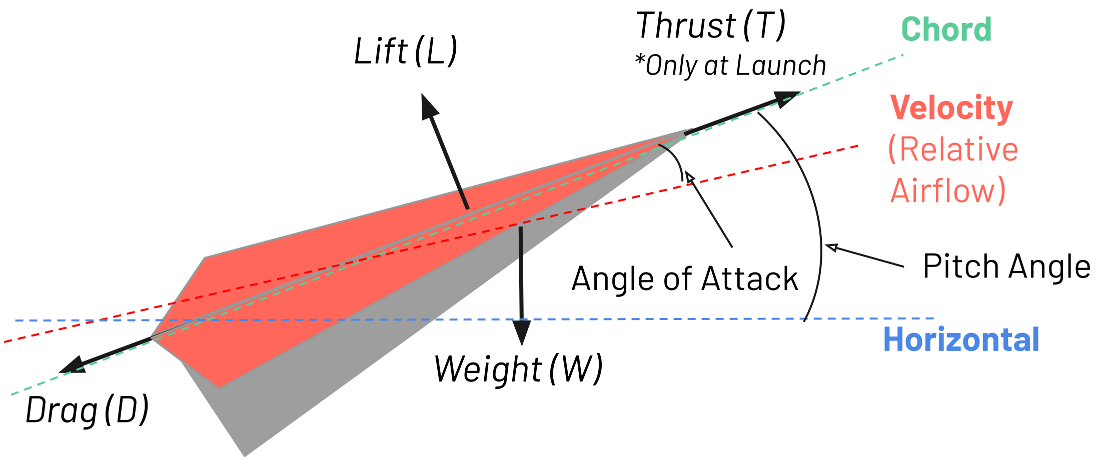
Strengths:

1. Able to accurately model the trajectories of the paper airplane from 0°-40°.
2. Initial height can be taken into consideration by changing the intercept.

Weaknesses:

1. Launching force is fixed at 7.6N.
2. Accuracy of predictions decreases as the variables exceed experimental ranges.

Discussion: Newtonian Model



Angle of Attack: Angle between direction of velocity and chord

Chord: Line between leading and trailing edge of wing

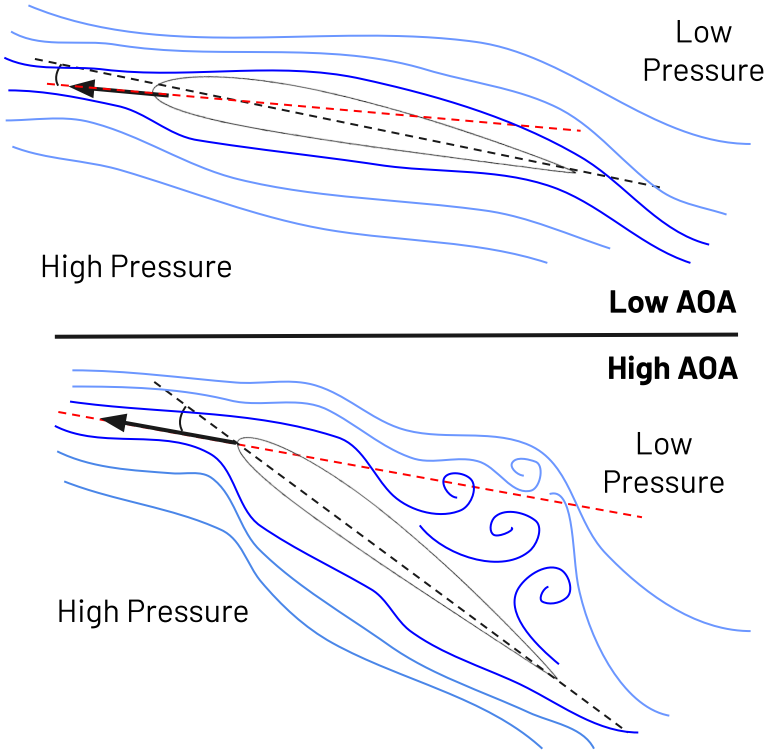
Pitch Angle: Angle between chord and horizontal

Weight: 0.0682N (6.82g)

Thrust from Launcher: 7.6N

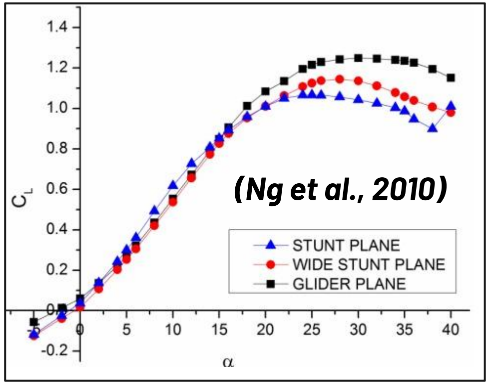
Lift $= \frac{1}{2} C_L \rho v^2 A$ **Drag** $= \frac{1}{2} C_D \rho v^2 A$

Discussion: Mechanism of Lift



Lift Effect of Angle of Attack:

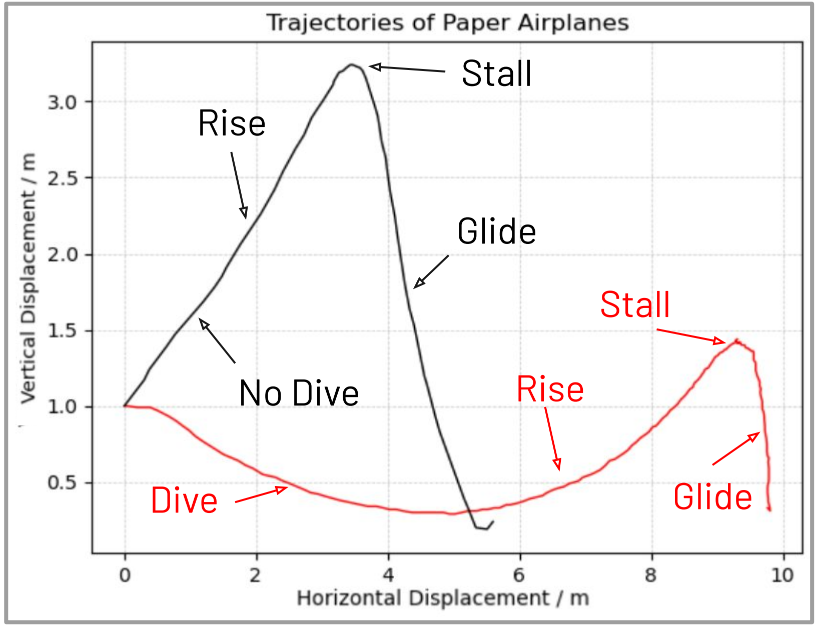
1. There is a greater surface area of the wing in contact with the airflow
2. More air collides with the underside of the wing and compresses, resulting in a higher pressure on the bottom than the top.
3. An increase in relative airflow will increase lift.



Stall Effect:

1. High AOA causes the boundary layer to become turbulent.
2. The detached airflow along the upper surface of the wing reduces lift.

Discussion: Phases of Trajectory



Red - Pitch Angle: 0° Black - Pitch Angle: 40°

Effect of different Pitch Angles:

1. High Pitch angles cause plane to have greater vertical momentum than horizontal.
2. Low Pitch angles cause plane to have greater horizontal momentum than vertical.

Displacement (x)

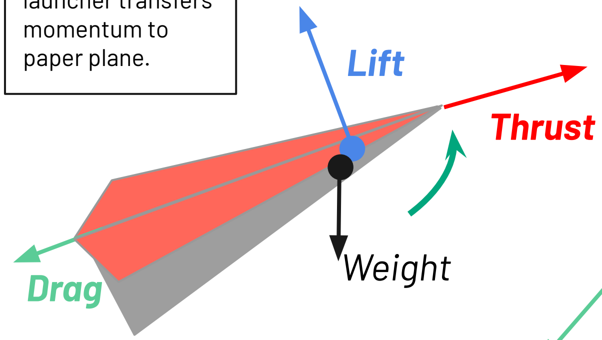
1. Plane with low pitch angle has greater horizontal momentum.
2. Allows plane to maintain forward motion longer.

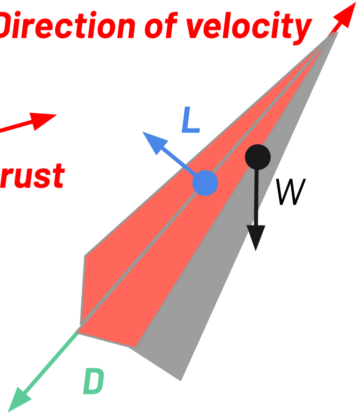
Dive Phase

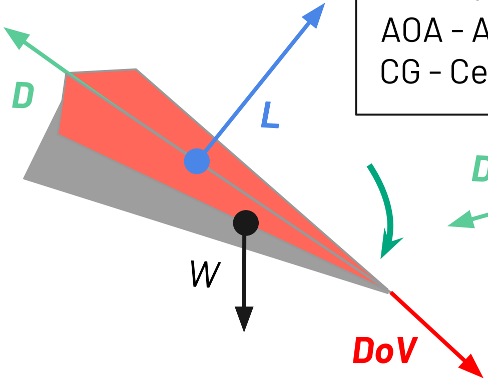
1. Plane with low pitch angle has lesser. vertical momentum.
2. Insufficient velocity to generate sufficient lift resulting in dive phase.
3. Plane has stronger dive as a result.

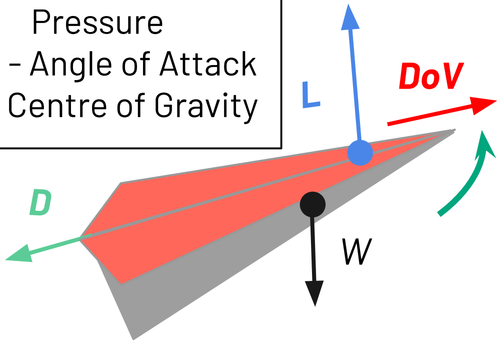
Discussion: Phases of Trajectory

Thrust force from launcher transfers momentum to paper plane.









Legend
CP - Centre of Pressure
AOA - Angle of Attack
CG - Centre of Gravity

<u>Rise Phase:</u>
Velocity (y) increases, causing lift to increase.

CP moves forwards, lift force results in pitch to increase.

AOA increases and lift increases even more.

<u>Stall Phase:</u>
AOA increases until critical stalling angle.

Lift and velocity decreases, CP Shifts backwards.

Drag increases.

<u>Dive/Fall Phase:</u>
As plane falls, drag acts on CP causing tail of plane to move up, resulting in a nose-down pitching moment.

<u>Glide Phase:</u>
Velocity increases as plane accelerates downwards. Lift increases.

CP moves in front of CG, lift force results in nose-up pitching moment.

Conclusion and References

Scientific Conclusion:
Due to changing magnitudes in the aerodynamic forces along with the shifting of the Centre of Pressure relative to the Centre of Gravity, the paper aeroplane experiences different phases in flight throughout its trajectory.

Contradictions with Existing Literature:
Existing literature suggests a polynomial or nonlinear relationship between launch angle and maximum horizontal displacement. However, our findings dispute that it is instead a linear trend when accounting for the distance slid.

Additionally, the trend between initial pitch angle and horizontal displacement appears to be monotonic and does not peak around 20° or 5°-6°. Further analysis suggests that 0° is the optimal launch angle for obtaining maximum horizontal displacement due to the nature of lower angles having a more prevalent dive and rise phase.

Bibliography:

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2. Ismail, N. I., Ali, Z. M., Ishak, I. S., Noor, R. M., & Rabilah, R. (2021). Aerodynamic Performances of Paper Planes. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 77(1), 124–131.
3. Ng, B. F., Kng, Q. M., Pey, Y. Y., & Schlüter, J. U. (2010). On the Aerodynamics of Paper Airplanes. *27th AIAA Applied Aerodynamics Conference.*