AP Calculus AB
AP Classroom: 4.3 – 4.4

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All final solutions are $\boxed{\textit{boxed}}$

List of Questions

Question 1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.
If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}$ at $x = 2$.

Solution: Given that $g(1) = 2$, the point of tangency on the graph of $y = g^{-1}$ is the point $(2, 1)$. (since g^{-1} is an inverse of g, $g(2) = 1$.)
We now want to find the derivative at the point of tangency:
To find $\left[g^{-1}\right]'(2)$, we must use the rule for derivatives of inverses:

$$\begin{aligned}\left[g^{-1}\right]'(x) &= \frac{1}{g'\left(g^{-1}(x)\right)}\\ \left[g^{-1}\right]'(2) &= \frac{1}{g'\left(g^{-1}(2)\right)}\\ &= \frac{1}{g'(1)}\\ &= \frac{1}{5}\end{aligned}$$

Now, we just need to use the point $(2, 1)$ and the derivative at that point to write the tangent line:

$$\boxed{y - 1 = 1/5\,(x - 2)}$$

Question 2

A student attempted to confirm that the function f defined by $f(x) = \frac{x^2+x-6}{x^2-7x+10}$ is continuous at $x = 2$ In which step, if any, does an error first appear?

Step 1:

$$f(x) = \frac{x^2 + x - 6}{x^2 - 7x + 10} = \frac{(x-2)(x+3)}{(x-2)(x-5)}$$

Step 2:

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x+3}{x-5} = \frac{2+3}{2-5} = -\frac{5}{3}$$

Step 3:

$$f(2) = \frac{2+3}{2-5} = -\frac{5}{3}$$

Step 4:

$$\lim_{x \to 2} f(x) = f(2), \text{ so } f \text{ is continuous at } x = 2.$$

Solution: During Step 3, we made a little bit of a mistake. While in the limit in Step 2, we can simplify the rational definition of $f(x)$, when finding the actual value (or lack thereof) of $f(2)$, we ***cannot*** do this.
Even though f is actually undefined at $x = 2$ (removable discontinuity / hole), we incorrectly said it was equal to $\frac{-5}{3}$, and therefore incorrectly stated f as continuous at $x = 2$.

Question 3

Two particles move along the x-axis with velocities given by $v_1(t) = -3t^2 + 8t + 5$ and $v_2(t) = \sin t$ for time $t \geq 0$. At what time t do the two particles have equal acceleration?

Solution:

To find acceleration from give velocity equations, we must take the derivative of the velocity functions to find two new acceleration equations

$$\begin{aligned} a_1(t) &= \frac{d}{dx}\left(v_1(t)\right) \\ &= \frac{d}{dx}\left(-3t^2 + 8t + 5\right) \\ &= -6t + 8 \end{aligned}$$

$$\begin{aligned} a_2(t) &= \frac{d}{dx}\left(v_2(t)\right) \\ &= \frac{d}{dx}\left(\sin t\right) \\ &= \cos t \end{aligned}$$

Now, we need to find the time t where $a_1(t) = a_2(t)$

$$\begin{aligned} a_1(t) &= a_2(t) \\ -6t + 8 &= \cos t \end{aligned}$$

Since this is a calculator required question, just use your calculator to find the intersection of the line and the cosine function. (**Make sure your calculator is in radian mode.**)

$$\boxed{t \approx 1.2866}$$

Question 4

Question 5

Question 6

Question 7

Question 8

Question 9

Question 10

Question 11

Question 12

Question 13

Question 14