

AP Calculus AB
AP Classroom: 4.3 – 4.4

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All final solutions are *boxed*

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Question 1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}$ at $x = 2$.

Solution: Given that $g(1) = 2$, the point of tangency on the graph of $y = g^{-1}$ is the point $(2, 1)$. (since g^{-1} is an inverse of g , $g(2) = 1$.)

We now want to find the derivative at the point of tangency:

To find $[g^{-1}]'(2)$, we must use the rule for derivatives of inverses:

$$\begin{aligned}
 [g^{-1}]'(x) &= \frac{1}{g'(g^{-1}(x))} \\
 [g^{-1}]'(2) &= \frac{1}{g'(g^{-1}(2))} \\
 &= \frac{1}{g'(1)} \\
 &= \frac{1}{5}
 \end{aligned}$$

Now, we just need to use the point $(2, 1)$ and the derivative at that point to write the tangent line:

$$y - 1 = \frac{1}{5}(x - 2)$$

Question 2

A student attempted to confirm that the function f defined by $f(x) = \frac{x^2+x-6}{x^2-7x+10}$ is continuous at $x = 2$. In which step, if any, does an error first appear?

Step 1:

$$f(x) = \frac{x^2 + x - 6}{x^2 - 7x + 10} = \frac{(x - 2)(x + 3)}{(x - 2)(x - 5)}$$

Step 2:

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x + 3}{x - 5} = \frac{2 + 3}{2 - 5} = -\frac{5}{3}$$

Step 3:

$$f(2) = \frac{2+3}{2-5} = -\frac{5}{3}$$

Step 4:

$$\lim_{x \rightarrow 2} f(x) = f(2), \text{ so } f \text{ is continuous at } x = 2.$$

Solution: During Step 3, we made a little bit of a mistake. While in the limit in Step 2, we can simplify the rational definition of $f(x)$, when finding the actual value (or lack thereof) of $f(2)$, we **cannot** do this.

Even though f is actually undefined at $x = 2$ (removable discontinuity / hole), we incorrectly said it was equal to $-\frac{5}{3}$, and therefore incorrectly stated f as continuous at $x = 2$.

Question 3

Two particles move along the x -axis with velocities given by $v_1(t) = -3t^2 + 8t + 5$ and $v_2(t) = \sin t$ for time $t \geq 0$. At what time t do the two particles have equal acceleration?

Solution:

To find acceleration from give velocity equations, we must take the derivative of the velocity functions to find two new acceleration equations

$$\begin{aligned} a_1(t) &= \frac{d}{dx} (v_1(t)) \\ &= \frac{d}{dx} (-3t^2 + 8t + 5) \\ &= -6t + 8 \end{aligned}$$

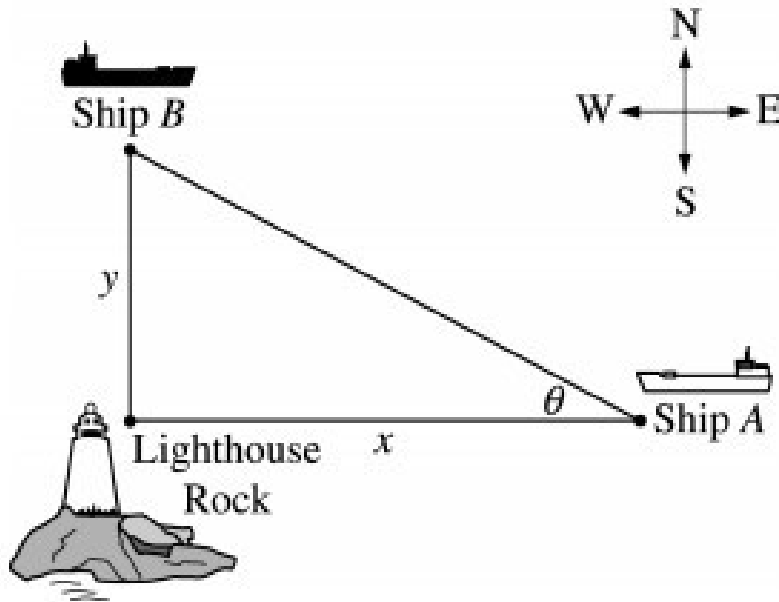
$$\begin{aligned} a_2(t) &= \frac{d}{dx} (v_2(t)) \\ &= \frac{d}{dx} (\sin t) \\ &= \cos t \end{aligned}$$

Now, we need to find the time t where $a_1(t) = a_2(t)$

$$\begin{aligned} a_1(t) &= a_2(t) \\ -6t + 8 &= \cos t \end{aligned}$$

Since this is a calculator required question, just use your calculator to find the intersection of the line and the cosine function. (**Make sure your calculator is in radian mode.**)

$$t \approx 1.2866$$

Question 4

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t , and let y be the distance between Ship B and Lighthouse Rock at time t , as shown in the figure above.

Solution:

Equation (Pythagorean Theorem) of situation: $c^2 = y^2 + x^2$ Find: $\frac{dc}{dt}$

When: $x = 4$ km and $y = 3$ km Other Rates: $\frac{dx}{dt} = -15 \frac{\text{km}}{\text{hr}}$ and $\frac{dy}{dt} = 10 \frac{\text{km}}{\text{hr}}$

c is the hypotenuse / distance between Ship B and Ship A .

Using Pythagorean Theorem: $c = \sqrt{4^2 + 3^2} = 5$

Find derivative of equation:

$$\frac{dc}{dt} \cdot 2c = \frac{dy}{dt} \cdot 2y + \frac{dx}{dt} \cdot 2x$$

Solve for $\frac{dc}{dt}$:

$$\frac{dc}{dt} = \frac{\frac{dy}{dt} \cdot 2y + \frac{dx}{dt} \cdot 2x}{2c}$$

Plug in knowns and solve:

$$\frac{dc}{dt} = \frac{10 \cdot 2(3) - 15 \cdot 2(4)}{2(5)} = \frac{60 - 120}{10} = -\frac{60}{10} = \boxed{-6 \frac{\text{km}}{\text{hr}}}$$

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