Exercise 3

for the lecture

Computational Geometry

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Exercise 1 (Doubly-Connected Edge List)

a) Which of the following equations are always true?.

$$\text{Twin}(\text{Twin}(\vec{e})) = \vec{e} \tag{1}$$

$$\text{Next}(\text{Prev}(\vec{e})) = \vec{e} \tag{2}$$

$$\text{Twin}(\text{Prev}(\text{Twin}(\vec{e}))) = \text{Next}(\vec{e}) \tag{3}$$

$$\text{IncidentFace}(\vec{e}) = \text{IncidentFace}(\text{Next}(\vec{e})) \tag{4}$$

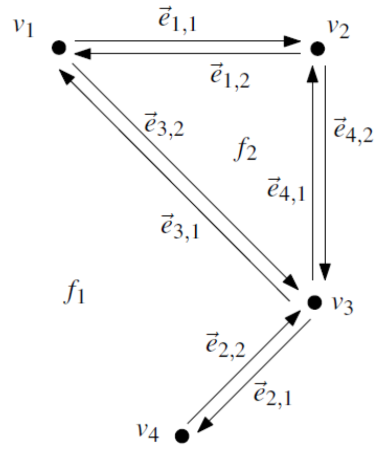
(1) True. See definition of Twin.

(2) True. See definitions of Prev and Next.

(3) False. Consider figure 1 as counterexample, with $\vec{e} := \vec{e}_{2,2}$.

$$\begin{aligned}
&\text{Twin}(\text{Prev}(\text{Twin}(\vec{e}))) \\
=&\text{Twin}(\text{Prev}(\text{Twin}(\vec{e}_{2,2}))) \\
=&\text{Twin}(\text{Prev}(\vec{e}_{2,1})) \\
=&\text{Twin}(\vec{e}_{4,2}) \\
=&\vec{e}_{4,1} \\
\neq&\vec{e}_{3,1} \\
=&\text{Next}(\vec{e}_{2,2})
\end{aligned}$$

(4) True. See defintion of IncidentFace.



Vertex	Coordinates	IncidentEdge
v_1	$(0,4)$	$\vec{e}_{1,1}$
v_2	$(2,4)$	$\vec{e}_{4,2}$
v_3	$(2,2)$	$\vec{e}_{2,1}$
v_4	$(1,1)$	$\vec{e}_{2,2}$

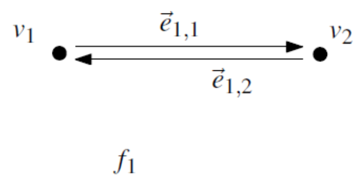
Face	OuterComponent	InnerComponents
f_1	**nil**	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	**nil**

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

Figure 1: An example of a doubly-connected edge list for a simple subdivision. Figure is taken from [1].

b) Give an example of a doubly-connected edge list where for an edge $\vec{e}$ the faces IncidentFace($\vec{e}$) and IncidentFace(Twin($\vec{e}$)) are the same.

Consider this example:



Vertex	Coordinates	IncidentEdge
v_1	$(0,4)$	$\vec{e}_{1,1}$
v_2	$(2,4)$	$\vec{e}_{1,2}$

Face	OuterComponent	InnerComponents
f_1	**nil**	**nil**

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{1,2}$	$\vec{e}_{1,2}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{1,1}$	$\vec{e}_{1,1}$

Figure 2: A very simple example of a doubly-connected edge list.

This is a doubly-connected edge list where $f_1 = \text{IncidentFace}(\vec{e}_{1,1}) = \text{IncidentFace}(\text{Twin}(\vec{e}_{1,1})) = \text{IncidentFace}(\text{Twin}(\vec{e}_{1,2})) = \text{IncidentFace}(\vec{e}_{1,2}) = f_1$ holds.

c) Given a doubly-connected edge list representation for a subdivision where

$$\text{Twin}(\vec{e}) = \text{Next}(\vec{e})$$

holds for every half-edge $\vec{e}$. How many faces can the subdivision have at most?

In this case we have exactly 1 unbounded face. There are only doubly-connected edges like in figure 2 because this is the only case where $\text{Twin}(\vec{e}) = \text{Next}(\vec{e})$ holds. (Of course we might have many of these edges and some of them might be sharing some nodes). By definition each half-edge bounds only one face. We can easily apply the prove from **b)** here and use it to show that this is always the same face. This shows that there can only be exactly 1 unbounded face.

Exercise 2 (Planar Subdivision and Point Set)

References

[1] Mark de Berg, Otfried Cheong, Marc van Kreveld, and Mark Overmars. Computational geometry: algorithms and applications. Springer-Verlag TELOS, 2008.