

Exercise 3

for the lecture

Computational Geometry

Dominik Bendle, Stefan Fritsch, Marcel Rogge and Matthias Tschöpe

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Exercise 1 (Doubly-Connected Edge List)

a) Which of the following equations are always true?.

$$\text{Twin}(\text{Twin}(\vec{e})) = \vec{e} \quad (1)$$

$$\text{Next}(\text{Prev}(\vec{e})) = \vec{e} \quad (2)$$

$$\text{Twin}(\text{Prev}(\text{Twin}(\vec{e}))) = \text{Next}(\vec{e}) \quad (3)$$

$$\text{IncidentFace}(\vec{e}) = \text{IncidentFace}(\text{Next}(\vec{e})) \quad (4)$$

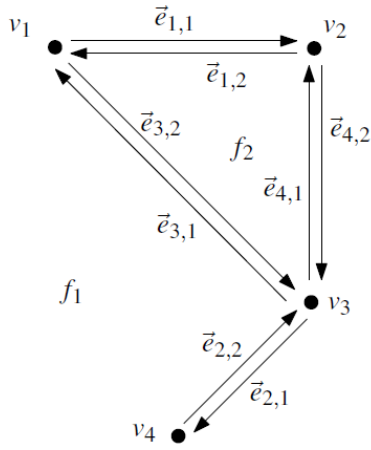
(1) True. See definition of Twin.

(2) True. See definitions of Prev and Next.

(3) False. Consider figure 1 as counterexample, with $\vec{e} := \vec{e}_{2,2}$.

$$\begin{aligned} & \text{Twin}(\text{Prev}(\text{Twin}(\vec{e}))) \\ &= \text{Twin}(\text{Prev}(\text{Twin}(\vec{e}_{2,2}))) \\ &= \text{Twin}(\text{Prev}(\vec{e}_{2,1})) \\ &= \text{Twin}(\vec{e}_{4,2}) \\ &= \vec{e}_{4,1} \\ &\neq \vec{e}_{3,1} \\ &= \text{Next}(\vec{e}_{2,2}) \end{aligned}$$

(4) True. See definition of IncidentFace.



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

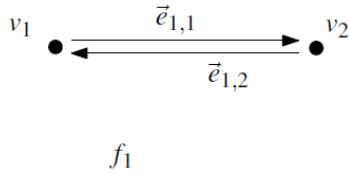
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

Figure 1: An example of a doubly-connected edge list for a simple subdivision. Figure is taken from [1].

b) Give an example of a doubly-connected edge list where for an edge \vec{e} the faces $\text{IncidentFace}(\vec{e})$ and $\text{IncidentFace}(\text{Twin}(\vec{e}))$ are the same.

Consider this example:



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{1,2}$

Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{1,2}$	$\vec{e}_{1,2}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{1,1}$	$\vec{e}_{1,1}$

This is a doubly-connected edge list where $f_1 = \text{IncidentFace}(\vec{e}_{1,1}) = \text{IncidentFace}(\text{Twin}(\vec{e}_{1,1})) = \text{IncidentFace}(\text{Twin}(\vec{e}_{1,2})) = \text{IncidentFace}(\vec{e}_{1,2}) = f_1$ holds.

c) Given a doubly-connected edge list representation for a subdivision where

$$\text{Twin}(\vec{e}) = \text{Next}(\vec{e})$$

holds for every half-edge \vec{e} . How many faces can the subdivision have at most?

Exercise 2 (Planar Subdivision and Point Set)

References

- [1] Mark de Berg, Otfried Cheong, Marc van Kreveld, and Mark Overmars. Computational geometry: algorithms and applications. Springer-Verlag TELOS, 2008.