Exercise 2

for the lecture

Computational Geometry

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Exercise 1 (Point between Line Segments)

Exercise 2 (Line Segment Intersection)

$Q = tree(p_1, p_2, p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4); \; T = tree()$

$p = p_1, U = \{s_1\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s1) \Rightarrow T = tree(s_1)$

$U \cup C = \{s_1\}$

$s' = s_1, s_l = null$

FINDNEWEVENT$(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_1, s_r = null$

FINDNEWEVENT$(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(p_2, p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4)$$

intersection_points $= \emptyset$

$p = p_2, U = \{s_2\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s_2) \Rightarrow T = tree(s_2, s_1)$

$U \cup C = \{s_2\}$

$s' = s_2, s_l = null$

FINDNEWEVENT$(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_2, s_r = s_1$

FINDNEWEVENT$(s'', s_r, p) \Rightarrow s''$ and s_r intersect below the current sweep line in p_{s_1,s_2}.
$$\Rightarrow Q.insert(p_{s_1,s_2})$$
$$\Rightarrow Q = tree(p_{s_1,s_2}, p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4)$$

intersection_points $= \emptyset$

$p = p_{s_1,s_2}, U = \emptyset, L = \emptyset, C = \{s_1, s_2\}$

$|L \cup U \cup C| > 1$ is true $\Rightarrow p = p_{s_1,s_2}$ is intersection of s_1 and s_2

$T.delete(s_1), \; T.delete(s_2), \; T.insert(s_1), \; T.insert(s_2) \Rightarrow T = tree(s_1, s_2)$

$U \cup C = \{s_1, s_2\}$

$s' = s_1, s_l = null$

FINDNEWEVENT$(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_2, s_r = null$

FINDNEWEVENT$(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4)$$

intersection_points $= \{p_{s_1,s_2}\}$

$p = p_3, U = \{s_3\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s_3) \Rightarrow T = tree(s_3, s_1, s_2)$

$U \cup C = \{s_3\}$

$s' = s_3, s_l = null$

FINDNEWEVENT$(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_3, s_r = s_2$

FINDNEWEVENT$(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(q_1, q_3, p_4, q_2, p_5, q_5, q_4)$$

intersection_points $= \{p_{s_1, s_2}\}$

$p = q_1 = q_3, U = \emptyset, L = \{s_1, s_3\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is true $\Rightarrow p = q_1 = q_3$ is intersection of s_1 and s_3

$T.delete(s_1), T.delete(s_3)$, no insertion $\Rightarrow T = tree(s_2)$

$U \cup C = \emptyset$

$s_l = null, s_r = null$

FINDNEWEVENT$(s_l, s_r, p) \Rightarrow s_l$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(p_4, q_2, p_5, q_5, q_4)$$

intersection_points $= \{p_{s_1, s_2}, q_1 = q_3\}$

$p = p_4, U = \{s_4\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s_4) \Rightarrow T = tree(s_4, s_2)$

$U \cup C = \{s_4\}$

$s' = s_4, s_l = null$

FINDNEWEVENT$(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_4, s_r = s_2$

FINDNEWEVENT$(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(q_2, p_5, q_5, q_4)$$

intersection_points $= \{p_{s_1, s_2}, q_1 = q_3\}$

$p = q_2 = p_5, U = \{s_5\}, L = \{s_2\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is true $\Rightarrow p = q_2 = p_5$ is intersection of s_2 and s_5

$T.delete(s_2), T.insert(s_5) \Rightarrow T = tree(s_4, s_5)$

$U \cup C = \{s_5\}$

$s' = s_5, s_l = s_4$

FINDNEWEVENT$(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_5, s_r = null$

FINDNEWEVENT$(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(q_5, q_4)$$

intersection_points $= \{p_{s_1, s_2}, q_1 = q_3, q_2 = p_5\}$

$p = q_5, U = \emptyset, L = \{s_5\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

$T.delete(s_5)$, no insertion $\Rightarrow T = tree(s_4)$

$U \cup C = \emptyset$

$s_l = null, s_r = null$

FINDNEWEVENT$(s_l, s_r, p) \Rightarrow s_l$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree(q_4)$$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3, q_2 = p_5\}$

$p = q_4, U = \emptyset, L = \{s_4\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

$T.delete(s_4)$, no insertion $\Rightarrow T = tree()$

$U \cup C = \emptyset$

$s_l = null, s_r = null$

FINDNEWEVENT$(s_l, s_r, p) \Rightarrow s_l$ and s_r do not intersect below the current sweep line.
$$\Rightarrow Q = tree()$$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3, q_2 = p_5\}$

Exercise 3 (Pyramids Skyline)