Exercise 2

for the lecture

Computational Geometry

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Exercise 1 (Point between Line Segments)

Given a balanced binary tree which lexicographically stores a tuple (x_{hi}, x_{lo}) for each line segment, where x_{hi} and x_{lo} are the x coordinate for the upper and lower end points.
Find the two disjunct line segments that define the region in which the query point $p \in D$ is:

node = tree.root
left_line_segment = null
right_line_segment = null

for i = 0; i < tree.height; i++
    if node.$x_{hi} < p_x$ && node.$x_{lo} < p_x$
        left_line_segment = node
        node = node.right_child
    else if node.$x_{hi} >= p_x$ && node.$x_{lo} >= p_x$
        right_line_segment = node
        node = node.left_child
    else
        position = $(x_{lo} - x_{hi}) * (p_y - 1) - (-1) * (p_x - x_{hi})$
        if position > 0
            left_line_segment = node
            node = node.right_child
        else
            right_line_segment = node
            node = node.left_child

Query point p is in the region between[1] the two lines segments:
$\overline{(left\_line\_segment.x_{hi}, 1) \;\; (left\_line\_segment.x_{lo}, 0)}$
$\overline{(right\_line\_segment.x_{hi}, 1) \;\; (right\_line\_segment.x_{lo}, 0)}$

[1]If a point lies exactly on a line, it is treated as if the point lies in the region to the left of that line.

The algorithm does not work for arbitrary line segments.
Arbitrary line segments can have intersections which causes multiple problems. For example could the two line segments, that define the region in which p lies, be on different branches. This would require to traverse the tree two times. At the same time, could the region, in which the point p lies, be defined by more than two line segments. This would mean that the tree would then have to be traversed an unknown amount of times.

Exercise 2 (Line Segment Intersection)

$Q = tree(p_1, p_2, p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4);\ T = tree()$

$p = p_1, U = \{s_1\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s1) \Rightarrow T = tree(s_1)$

$U \cup C = \{s_1\}$

$s' = s_1, s_l = null$

$\textsc{FindNewEvent}(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_1, s_r = null$

$\textsc{FindNewEvent}(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(p_2, p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4)$

intersection_points $= \emptyset$

$p = p_2, U = \{s_2\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s_2) \Rightarrow T = tree(s_2, s_1)$

$U \cup C = \{s_2\}$

$s' = s_2, s_l = null$

$\textsc{FindNewEvent}(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_2, s_r = s_1$

$\textsc{FindNewEvent}(s'', s_r, p) \Rightarrow s''$ and s_r intersect below the current sweep line in p_{s_1,s_2}.

$\Rightarrow Q.insert(p_{s_1,s_2})$

$\Rightarrow Q = tree(p_{s_1,s_2}, p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4)$

intersection_points $= \emptyset$

$p = p_{s_1,s_2}, U = \emptyset, L = \emptyset, C = \{s_1, s_2\}$

$|L \cup U \cup C| > 1$ is true $\Rightarrow p = p_{s_1,s_2}$ is intersection of s_1 and s_2

$T.delete(s_1),\ T.delete(s_2),\ T.insert(s_1),\ T.insert(s_2)\ \Rightarrow T = tree(s_1, s_2)$

$U \cup C = \{s_1, s_2\}$

$s' = s_1, s_l = null$

$\textsc{FindNewEvent}(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_2, s_r = null$

$\textsc{FindNewEvent}(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(p_3, q_1, q_3, p_4, q_2, p_5, q_5, q_4)$

intersection_points $= \{p_{s_1,s_2}\}$

$p = p_3, U = \{s_3\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s_3) \Rightarrow T = tree(s_3, s_1, s_2)$

$U \cup C = \{s_3\}$

$s' = s_3, s_l = null$

$\textsc{FindNewEvent}(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_3, s_r = s_2$

$\textsc{FindNewEvent}(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(q_1, q_3, p_4, q_2, p_5, q_5, q_4)$

intersection_points $= \{p_{s_1,s_2}\}$

$p = q_1 = q_3, U = \emptyset, L = \{s_1, s_3\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is true $\Rightarrow p = q_1 = q_3$ is intersection of s_1 and s_3

$T.delete(s_1), T.delete(s_3)$, no insertion $\Rightarrow T = tree(s_2)$

$U \cup C = \emptyset$

$s_l = null, s_r = null$

$\textsc{FindNewEvent}(s_l, s_r, p) \Rightarrow s_l$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(p_4, q_2, p_5, q_5, q_4)$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3\}$

$p = p_4, U = \{s_4\}, L = \emptyset, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

no deletion, $T.insert(s_4) \Rightarrow T = tree(s_4, s_2)$

$U \cup C = \{s_4\}$

$s' = s_4, s_l = null$

$\textsc{FindNewEvent}(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_4, s_r = s_2$

$\textsc{FindNewEvent}(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(q_2, p_5, q_5, q_4)$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3\}$

$p = q_2 = p_5, U = \{s_5\}, L = \{s_2\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is true $\Rightarrow p = q_2 = p_5$ is intersection of s_2 and s_5

$T.delete(s_2), T.insert(s_5) \Rightarrow T = tree(s_4, s_5)$

$U \cup C = \{s_5\}$

$s' = s_5, s_l = s_4$

$\textsc{FindNewEvent}(s_l, s', p) \Rightarrow s_l$ and s' do not intersect below the current sweep line.

$s'' = s_5, s_r = null$

$\textsc{FindNewEvent}(s'', s_r, p) \Rightarrow s''$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(q_5, q_4)$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3, q_2 = p_5\}$

$p = q_5, U = \emptyset, L = \{s_5\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

$T.delete(s_5)$, no insertion $\Rightarrow T = tree(s_4)$

$U \cup C = \emptyset$

$s_l = null, s_r = null$

FindNewEvent$(s_l, s_r, p) \Rightarrow s_l$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree(q_4)$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3, q_2 = p_5\}$

$p = q_4, U = \emptyset, L = \{s_4\}, C = \emptyset$

$|L \cup U \cup C| > 1$ is false

$T.delete(s_4)$, no insertion $\Rightarrow T = tree()$

$U \cup C = \emptyset$

$s_l = null, s_r = null$

FindNewEvent$(s_l, s_r, p) \Rightarrow s_l$ and s_r do not intersect below the current sweep line.

$\Rightarrow Q = tree()$

intersection_points $= \{p_{s_1,s_2}, q_1 = q_3, q_2 = p_5\}$

Exercise 3 (Pyramids Skyline)