Diagnosing angular (directional) spectra by Tikhonov regularization (ridge regression, tapered least squares) in numerical studies of wave scattering

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Abstract

WHAT DO YOU WANNA PRESENT? WHICH ONE OF THESE?

- Method
- Usage of method
- Scattering

Right now DDEC is used only for calculation of reflection coefficient and really for nothing in tsunami, but I wanted to use it for identification of scattering energy lobes and maybe filtering them.
Ok, this is "an approach for analysis wave scattering by ddec". Another note: I don't use DDEC for tsunami event, or maybe somehow identify energy lobes? Also you do inverse model.

Realistic numerical simulations of wave propagation usually involve complex patterns of interference that conceals energy transfers. The elementary wave components can be deduced from directional spectra. Here, the energy distribution is found by solving an inverse model. The method is based on plane wave fitting into **coherence** between wave field associated currents and pressure. By fitting analytical model into quad- and co-spectral components the obtained directional distribution of energy provides a way to quantify energy transports after scattering event. This representation is further used in order to study tsunami wave diffraction by an island and reflection of internal tide from a step bathymetry representative of continental shelf. Comparison with known analytical solutions emphasizes ability of the proposed method in delineating energy budgets for complex wave fields.

The method is shown to be useful in understanding of energy transports associated with particular waves of interest **and can found wide spread application to observations and satellite altimetry observations**.
Example, abstract

We present here a new InSAR persistent scatterer (PS) method for analyzing episodic crustal deformation in non-urban environments, with application to volcanic settings. Our method for identifying PS pixels in a series of interferograms is based primarily on phase characteristics and finds low-amplitude pixels with phase stability that are not identified by the existing amplitude-based algorithm. Our method also uses the spatial correlation of the phases rather than a well-defined phase history so that we can observe temporally-variable processes, e.g., volcanic deformation. The algorithm involves removing the residual topographic component of flattened interferogram phase for each PS, then unwrapping the PS phases both spatially and temporally. Our method finds scatterers with stable phase characteristics independent of amplitudes associated with man-made objects, and is applicable to areas where conventional InSAR fails due to complete decorrelation of the majority of scatterers, yet a few stable scatterers are present.

1 Introduction

The oceanic waves interfere and create obscure character (a word, change?) in currents and pressure. The staggered pattern makes difficult to answer the simplest questions about waves: from where are they coming and how strong they are. Hence, there will always be an uncertainty in estimate of energy transfers and budgets. In an example of a standing wave, no energy transfer is taken place, yet two waves are present but oppositely traveling. The problem of diagnosing elementary components inevitably emerges in studies of wave-topography interaction. Energy is transfered into scattered wave field that undoubtedly has large number of components. This might produce spatially (horizontally) tight beams of energy transport. Such phenomena was observed in numerical simulations of tsunami waves (Tang et al. (2012)) and internal waves of tidal frequency (internal tides) (**?**, **?**(???)) (and in observations (Zhao et al. (2016))). To present a method to deduce dynamics of a complex wave field such as aforementioned ones is a chief reason for this note (chapter, paper). And it is application to problems of tsunami wave scattering by submarine island and reflection of low-mode internal tide.

A straightforward method is to obtain spatial frequencies of elementary waves (Barber (1963)) by means of two dimensional Fourier transform. It further can be mapped onto a circle leading to angular spectra. In the field studies this approach was mainly used in relation to surface wave phenomena, e.g. propagation (**?**, reflection (Dickson et al. (1995), Thomson et al. (2005), island diffraction (**?**). And the spectral description was also applied to tsunami trapping (**?**) and internal tide propagation (**?**, **?**). Estimation of spectra is a problem that one encounters that here is dealt by inverse modeling (Long (1986)). As discrete Fourier transform leads to ambiguity in elementary wave direction, additional inferences are necessary. Even though application of the Hilbert transform can resolve the issue (Mercier et al. (2008)) (Will it work with many components? In presence of corrugated surfaces?), it does not utilize all wave field characteristics available. So that constrains of wave dynamics can lessen Gibbs ringing, increase directional resolution and provide a way to super-resolution (**?**) so essential (viable, relevant, crucial) for resolving short-crested waves.

The inverse approach will have to comprise dynamical statements on the wave field. So that angular spectrum will have direct relation to currents and pressure. But such postulate leads to ill-posed system of equations, solution is unstable in relation to small changes in data. The errors in numerical solution might arise to number of reasons with primary one due to numerical and physical friction leading to variable spectrum over sampled patch of numerical ocean. Hence, one should look towards some regularization methods in order to stabilize inverse estimate (**?**).
This note at first will present a method to find angular spectrum using Tikhonov regularization (damped least squares, truncated SVD, ridge regression, tapered least squares). The usual problem with this technique is estimate of regularization parameter that is here dealt by L-curve method (**?**). The developed approach than (Section 3) is illustrated on simple synthetic experiments. And later (sections 4 and 5) is applied to numerical experiments of wave scattering where comparison with the well-known analytical solution is performed. In section 6 concluding remarks are made and drawbacks are outlined.

PIECES: Should I include? Relation to Tikhonov and theoretical studies, L-curve
Subtraction method???
Plane wave?
The simplest way to identify the scattered components is to numerically remove bottom disturbance (Kowalik et al. (2008), Klymak et al. (2016)). Than set new experiment to obtain differences which will provide insights into distribution of energy. This method can provide satisfactory results only if "subtraction" of bathymetry is possible, that is there is no surrounding complexity. And if one is not interested in the energy transfers occurring over the topography which can be a case for a study on trapped wave motions. As well in complex numerical simulations when mesoscale conditions is simulated in the same time as wave phenomena, clearly removement of bathymetry will lead to different background physical conditions such that interaction between waves and background conditions will be erroneous.
In classical observational wave () literature premises of linear wave field are used to obtain directional spectra, $|A(\sigma, \vec{k})|$. Fourier transform can be carried out in order to obtain frequency-wavenumber spectra. This defines distribution of wave energy in frequency-wavenumber space differential intervals. To obtain this different methods might be employed (). Here the trivial approach is considered based on plane-wave fit methods (). The basic approach to find spectral representation is to use underlying wave dynamics and use plane wave fit. This is a traditional approach in analysis of satellite altimetry observations (Ray & Cartwright (2001), Zhao et al. (2016)). In application of regular plane wave fit though there is a drawback that only finite number of components can be resolved. This problem arises as least squares method can infinitely increase amplitude in order to decrease misfit. This will lead to unphysical results. One of the ideas to solve this problem is setting additional restriction on amplitude. So this is why obtaining non-negative wavenumber spectra will bring better performance.
In order to show feasibility of directional wave spectra in numerical experiments we proceed

as follows. In section it is given the basic definition and description of numerical procedure. As well it is studied its characteristics on a synthetic data. With outlined procedure comparison of wavenumber spectra to an analytical solution of SGW diffraction by seamount is carried out (Section 3a) and comparison with reflection of internal tide from step-like continental slope is done with comparison with an appropriate analytical solutions (Section 3b). In section 4 concluding remarks are made.

2 Regularization technique of obtaining directional spectrum

Let pressure (or sea level) in complicated seas to be described by angular spectrum,

$$p(\vec{r},t) = \int_0^{2\pi} d\theta_k S(\theta_k) e^{i\vec{k}(\theta_k)\cdot\vec{r}+\phi(\theta_k)-i\omega t} \tag{1}$$

Here each elementary (monochromatic) sine wave of wavenumber k travels in direction θ with energy $S(\theta)d\theta$ and spatial lag of $\phi(\theta)$. Through out the note temporal dependence is omitted and complex notation is inferred. It is sought to estimate the angular spectrum $S(\theta)$ and phase distribution $\phi(\theta)$ from a finite set of observations. At first, this problem is reformulated in terms of polar Fourier coefficients (?) by Jacobi-Anger expansion (should I cite anything?),

$$p(r,\theta) = e^{i\vec{k}(\theta)\cdot\vec{r}} = \sum_{m=-\infty}^{m=\infty} i^m J_m(kr) e^{im(\theta-\theta_k)} \tag{2}$$

shows that a field at point (r,θ) produced by plane wave can be expanded in series of Bessel functions and angular functions. Than its substitution into (1) and after reorganization leads to

$$p(r,\theta) = \sum_{m=-\infty}^{m=\infty} \Big[\int_0^{2\pi} d\theta_k S(\theta_k) e^{i\phi(\theta_k)} e^{im\theta_k} \Big] i^m J_m(kr) e^{im\theta} \tag{3}$$

Term in brackets (square brackets) represent convolution integrals defining polar Fourier coefficients of order m, $A_m + iB_m$. Thence, series (3) state a model equation to find the unknown coefficients from the known, measured pressure field. Its spatial distribution can be sampled at a set of points (r_i,θ_i) leading to matrix formulation,

$$p_i = M_{im}A_m + iN_{im}A_m + Mm_{im}B_m + iNn_{im}B_m \tag{4}$$

with $M_{im} = (-1)^{m/2} J_m(kr_i)\cos m\theta_i$ for even m and $M_{im} = (-1)^{(m+1)/2} J_m(kr_i)\sin m\theta_i$ for odd m and $Mm_{im} = (-1)^{m/2} J_m(kr_i)\sin m\theta_i$ - even, $Nn_{im} = -(-1)^{m+1/2} J_m(kr_i)\cos m\theta_i$. Now the series is considered over finite number of terms, e.g. m cycles from $-N$ to N. Also note, real and imaginary parts constitute two separate problems allowing "deterministic" definition of the wave field, so that angular spectra and phase distribution can be found as

$$\operatorname{Re} S(\theta_k)e^{i\phi(\theta_k)} = \sum_{m=-N}^{m=N} A_m \cos m\theta_k + B_m \sin m\theta_k \tag{5}$$

$$\operatorname{Im} S(\theta_k)e^{i\phi(\theta_k)} = \sum_{m=-N}^{m=N} A_m \sin m\theta_k - B_m \cos m\theta_k \tag{6}$$

(the assumed model)Further, the model is assumed to consist of plane waves, so kernel function translating spectra into measurable quantities will consist of polarization relation.

The so-chosen basis functions are not orthogonal, this leads to ill-conditioned matrix K, the solution is unstable for small changes in the data. Some sort of regularization is necessary. Here we choose the simplest of most is Tikhonov regularization. The minimization problem to solve is (or to use cost function)

$$\min\{||Kx - y||_2^2 + \alpha||x||_2^2\} \tag{7}$$

Here the first term is part of regular least-squares solution and represents minimization of residual. The second term minimizes variance. One tries to find a balanced solution that has minimum residual and smallest norm trade-off parameter. But success and resultant fit depends on a regularization parameter. The parameter acts as a high pass filter over eigenvectors of K (Here for application the unifrequency component is described. So that the wave field is represented by its wavenumber spectra described by directional propagation of respective linear wave component (Long (1986)),

$$p(\vec{x}, t) = \int_0^{2\pi} b_p(\theta)A(\theta)e^{i\vec{k}(\theta)\cdot\vec{x}-\omega t}d\theta \tag{8}$$

where p are pressure, $A(\theta)$ - differential amplitude of wave traveling at angle θ. $b_{u,v,p}$ is the corresponding transfer function relating spectral representation to physical quantities.
Than cross-spectrum components between observations distanced by $\vec{r}$ is easily found,

$$p_i(\vec{x})p_j(\vec{x}+\vec{r}) = \int_0^{2\pi} b_i(\theta)b_j(\theta)(A(\theta))^2e^{i\vec{k}(\theta)\cdot\vec{r}}d\theta \tag{9}$$

By obtaining the cross-spectrum components at N numerical-grid points between parameters of simulated wave field the directional distribution of amplitude, hence, can be obtained.
The transfer functions for pressure and currents follow dynamical considerations of the waves. Such that for pressure b_p is 1 and for currents corresponding polarization relations can be used. For example, if the cross-spectrum between current and pressure is found, the model (7) will have the form of

$$u_i^\star \cdot p_j = \int \frac{\omega k\cos\theta + ifk\sin\theta}{\rho(\omega^2 - f^2)}e^{-i\vec{k}(\theta)\cdot\vec{x}_i}e^{i\vec{k}(\theta)\cdot\vec{x}_j}S(\theta)d\theta \tag{10}$$

Since in the numerical models, the physical quantities are not random the simple least square with additional condition can be used. Forming matrix equation, the model to minimize will be,

$$C_{ij} = \sum_k \mathbf{b}_{ij}^k S_k \Delta\theta \tag{11a}$$

$$S_k \geq 0 \tag{11b}$$

where C_{ij} represents cross-spectrum between pressure and currents at points i and j in "observational" array, $b_{ij}(\theta)$ corresponding transfer function and $S(\theta_k)$ - directional spectral density function discretized. The circle is discretized at θ_k. The posed problem is classical least square problem for (9a), but additionally the solution is restricted by non-negativity of spectra. This provides constraint on least squares and prevents spurious amplitudes.
This system of equations is formed for each pair of the points. These observational stations arranged as circular pattern antennas. The radius is defined as local wavelength found from the corresponding dispersion relation.
In case of rapidly varying bathymetry a local cutoff to drop stations is based on WKB-alike condition, i.e. variations in wavenumber are much smaller compare to its value.
The formed matrix equations is solved by means of linear programming methods Haskell & Hanson (1981).

2.1 Numerical implementation

For purposes of numerical analysis we will use a circular antenna (Figure 2a). Its directional resolving power (Barber (1963)) is given by Fig. 1b,

$$G(k,l) = 1 + \sum_{i,\ j} 2\cos(2\pi \vec{k}\vec{\Delta_{ij}}) \tag{12}$$

where i, j goes every distance between array points.
To test the developed method it is carried out experiments with synthetic data. The considered data are representative of the problems in following up comparisons with analytical solution. In the first problem 4 waves represent scattering by seamount when one incident wave generates two side lobes and one reflected. Results are given by . It shows that the presented method .....
In the second problem incident wave is interfering with reflected. Here phase is underoging random variations.
WHY I AM DOING THE SYNTHETIC EXPERIMENTS? TO ESTIMATE ERRORS.
(((($b_{ij}(\theta)$ - kernel matrix that transforms directional decomposition into physical observations)))).

2.2 Analytical example, resolution

To investigate properties of the wave spectra let study the analytical expressions for simplified applications.
Consider an array consisting of two points with coordinates $\vec{x}$ and $\vec{x} + \delta\vec{x}$ and two plane waves

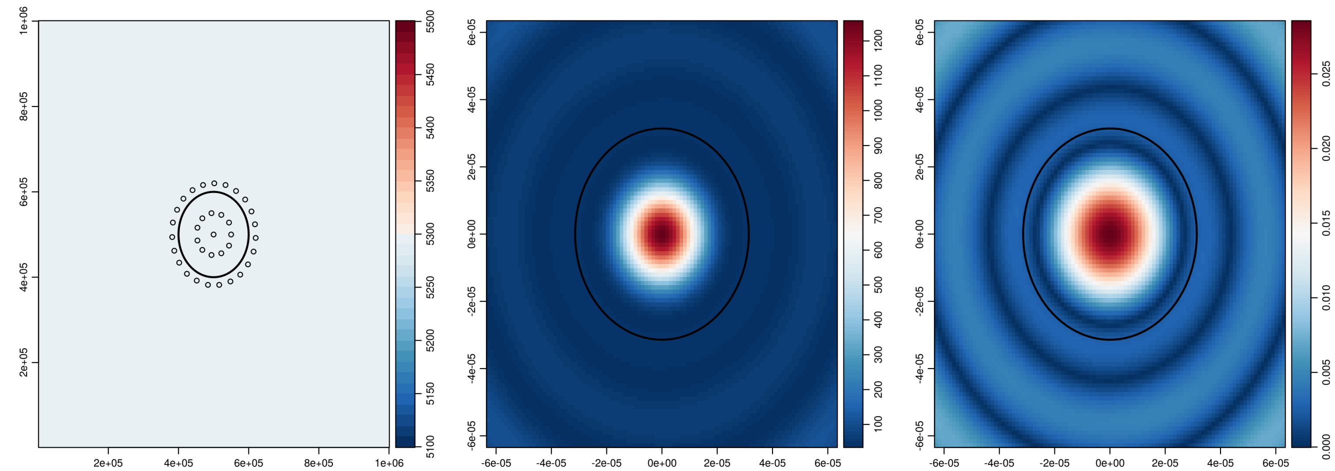

Figure 1: Array organization and its direction resolution (middle) and array response function (right).

$\psi_1 = A_1 e^{i(\vec{k_1}\cdot\vec{x})}$ and $\psi_2 = A_2 e^{i(\vec{k_2}\cdot\vec{x}+\phi)}$. They interfere ($\psi = \psi_1 + \psi_2$) generating the following expressions for cross-spectrum:

$$C_{11} = \psi(\vec{x})\psi(\vec{x})^\star = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\vec{k}\vec{x} - \phi) \quad (13a)$$

$$C_{12} = \psi(\vec{x})\psi(\vec{x}+\delta\vec{x})^\star = A_1^2 e^{-i\vec{k}_1\delta\vec{x}} + A_2^2 e^{-i\vec{k}_2\delta\vec{x}} + 2A_1A_2\cos(\Delta\vec{k}\vec{x} - \phi - \frac{\Delta\vec{k}}{2}\delta\vec{x})e^{-i\frac{\vec{k}_s}{2}\delta\vec{x}} \quad (13b)$$

$$C_{22} = \psi(\vec{x}+\delta\vec{x})\psi(\vec{x}+\delta\vec{x})^\star = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\vec{k}(\vec{x}+\delta\vec{x}) - \phi) \quad (13c)$$

with $\Delta\vec{k} = \vec{k}_1 - \vec{k}_2 = 2|\vec{k}|\sin\frac{\delta\alpha}{2}$. These expressions consist of parts that represent amplitude of the waves and variable signal originating from difference in waves propagation or due to antenna resolution. The proposed method tries to identify magnitudes from C_{11} and C_{22} and direction from C_{12} since autocovariance is subject to fit $\sum_i S_i$ and crosscovariance - to $\sum_i S_i e^{-ik_i\delta\vec{x}}$.
The errors in spectra will arise if variable parts, e.g. $2A_1A_2\cos(\Delta\vec{k}\vec{x} - \phi)$, of cross-spectrum will be large leading to aliasing of energy. In case of aliasing the numerical method cannot distinguish two waves either in their amplitude or direction. The best performance to obtain energy will occur when, (11c), $\cos(\Delta\vec{k}\delta\vec{x}) = 0$, $\Delta\vec{k}\delta\vec{x} = \frac{\pi}{2}(2n+1)$. Oppositely, from (11b) to destroy energy leakage into beat wavenumber $\vec{k}_s$, $\cos(\Delta\frac{1}{2}\vec{k}\delta\vec{x}) = 0$, $\Delta\vec{k}\delta\vec{x} = \pi(2n+1)$. These two conditions lead to uncertainty, that this antenna cannot perform well both for magnitude and direction.

2.3 Synthetic experiments

To test ability of the proposed method and antenna array I carry out synthetic experiments. The aim is to understand how the method can resolve multiple-wave interference. In the scattering events usually it can be found incident (here taken to be 0 degrees), side energy lobe (45 degrees), reflected wave (210 degrees). These three waves are kept constant with amplitude of 1, 0.5, 0.5. Than additionally it is prescribed three wave components with random amplitude, phase and direction. Amplitude is in range from 0 to 0.1. These random wave components represent a noise suggestive of some numerical deficiencies. The goal is to understand what is the uncertainty in defining the three primary components.

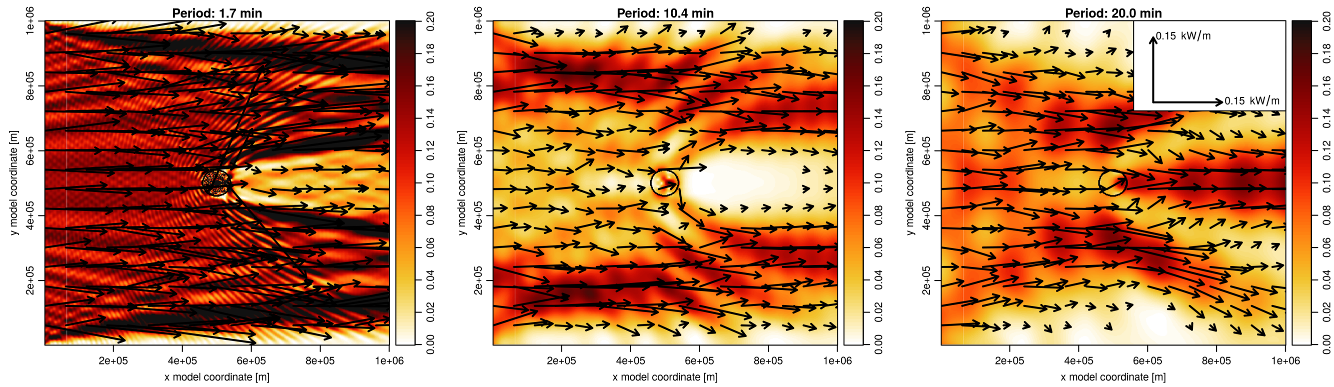

Figure 2: The simulated energy flux maps for SGW diffraction over the shoal.

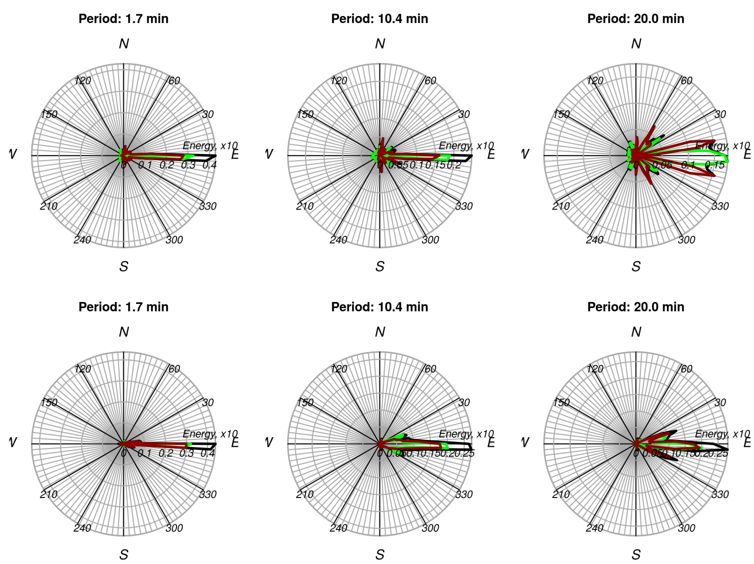

Figure 3: The simulated energy flux maps for SGW diffraction over the shoal.

3 Application

3.1 Tsunami wave diffraction by a round island

Now the method is tested against known analytical problems. The surface gravity wave interaction with submarine seamount. THe solution is known for many years (Longuet-Higgins (1967)).
In the numerical experiment it is used the simplified numerical code developed by (Kowalik et al. (2005)). All nonlinear terms were neglected, while bottom friction is controlled by constant coefficient of $3.3 \cdot 10^{-3}$. The integration is carried out over grid with resolution of 2 km by 2 km. The continuous harmonic wave train is sent towards the shoal of depth 500 m in sea of depth 5300 m from the east side. After the numerical experiment achieved equilibrium the sea level and horizontal currents are sampled. Fourier transform is done and than the presented method here is used to build directional distribution in the wave field. Example map of energy flux is presented on Figure 3.

The directional decomposition shows The analytical solution is shown on Figure 4.

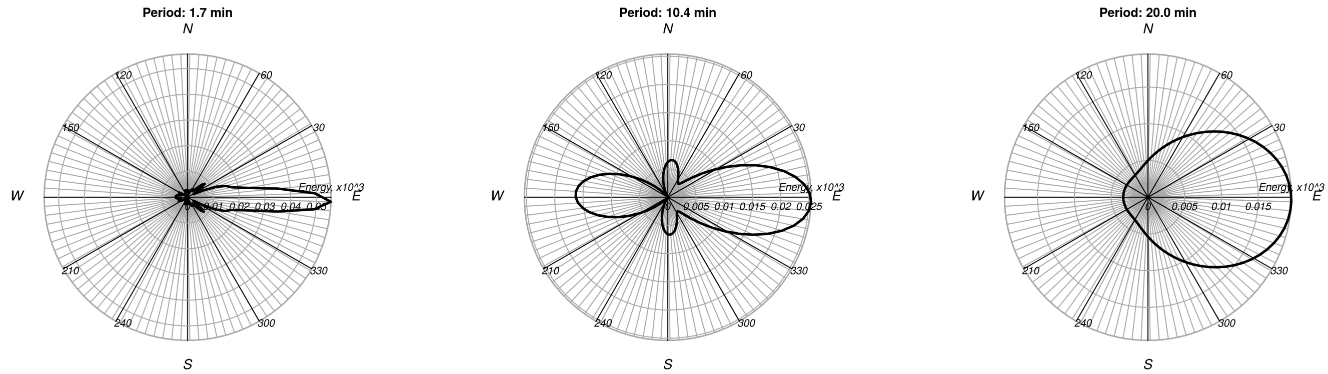

Figure 4: analytical solution

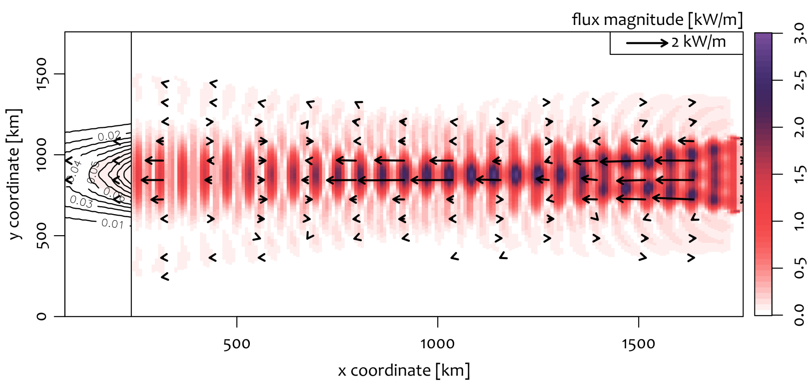

Figure 5: Interference due to oblique reflection.

3.2 Internal tide reflection from step

For internal tide reflection problem it is taken MITgcm numerical model initialized with constant stratification. The continental slope is represented as depth discontinuity arising from flat bottom at 2000 m to variable depth. The numerical results after the equailibrium was achieved are sample, harmonically fitted to M_2 frequency. Than eigenmode decomposition is done. Example of the obtained results is shown by Figure ...

The solution is compared against Chapman & Hendershott (1981) and is shown by

4 Discussion and Conclusions

- Energy preservation
- No phase reconstruction
- Discrete since no a priori model is used
- Useful tool for numerical experiment analysis

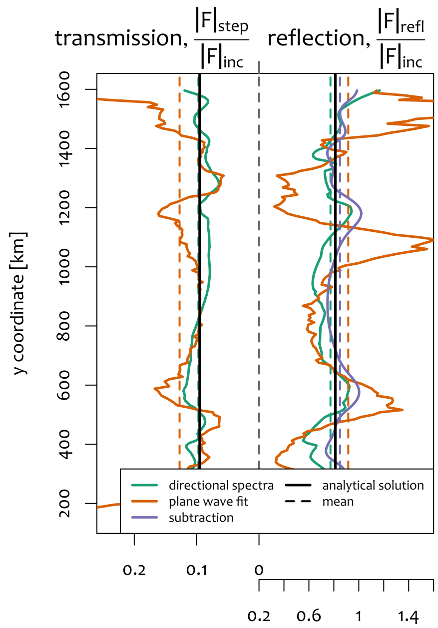

Figure 6: Reflection coefficient

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5 Problems

- I am confusing spectra and amplitude, as well cross-spectrum and covariance

6 TO DO LIST

- Right now my questions are weak. Primarily, it comes from my explanation, why do you need something new? My major answer: a) regular plane wave fit fails for multiple wave components; b) in complex models - it is not that easy to subscribe bathymetry. **That should be my arguments!** Explore them in more detail. Ref: Zhao method, Mercier, Jody's subtraction.
- Monte Carlo and estimate of errors - first thing to do
- Solution for wave scattering by an elliptic seamount
- What is physical meaning of Sf? Ref: Wagner, IT scattering
- Set Chapman experiments
- Set tsunami experiments with Koko Guoyt.

7 pieces

Here the cross-spectral components of observations at points i and j; $S(\theta)$ - directional wave spectra in direction θ and . Here for specifying this relation the regular plane wave representation with corresponding polarization relations is adopted. For example, cross-spectra between velocity at point i and pressure at point j (somewhat related to "energy flux") will be expressed as

with $\vec{k}(\theta) = k(\cos(\theta), \sin(\theta))$ and $\vec{x}_i$, $\vec{x}_j$ will be radius vectors to points i and j. Numerically, the integral representation is disretized and linear model to solve at each computation grid cell