Boolean Algebra: (Switching Algebra)

Three main operators:
- NOT: $()'$, $\overline{()}$, $\sim()$
- AND: $\bullet$
- OR: $+$

Precedence:
- ( )
- NOT
- AND
- OR

Axioms: A set of basic definitions that we assume to be true.

→ use Axioms to develop theorems.

(T3): $X + X = X$

Perfect induction: The theorem needs to be checked to be true for all input combinations (truth Table)

X	$X+X$		
0	$0+0 \xrightarrow{A4D} 0$	$= X$
1	$1+1 \xrightarrow{A3D} 1$	$= X$

(T9): $X + \underline{X \cdot Y} = X$

X	Y	$X \cdot Y$	$X + X \cdot Y$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

For theorems involving more than 3 variables we can use "finite induction" approach to prove them:

→ <u>n variables</u>

① $n=2$, use perfect induction or other approaches to prove the theorem is valid.

② Assume the theorem is valid for "n" variables, then show it is going to be valid for "n+1" variables.

Use Boolean Axiom and Theorems above T9, to prove T9.

$$\boxed{X + X.Y = X}$$

$$\boxed{X.1 + X.Y} \quad \text{used (T1D)}$$

$$= X.(1+Y) \quad \text{used (T8)}$$

$$= X.1 \quad \text{used (T2)}$$

$$= X$$

$$T8: \quad X.Y + X.Z = X.(Y+Z)$$

$$\underset{\uparrow\,1}{Y} \qquad \underset{\uparrow\,Y}{Z} \qquad \underset{\uparrow\,1}{Y} \qquad \underset{\uparrow\,Y}{Z}$$

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use Finite induction to prove DeMorgan's Theorems. (T13)

$$\left\{ \begin{array}{l} (X+Y)' = X'.Y' \\ (X.Y)' = X'+Y' \end{array} \right.$$

$$[A \cdot (B' \cdot C' + B.C)]'$$

$$= A' + (B'.C' + B.C)'$$

$$= A' + (B'.C')' \cdot (B.C)'$$

$$= A' + \underbrace{(B+C) \cdot (B'+C')}_{b}$$

$$\underbrace{B.B'}_{0} + \underbrace{B.C' + C.B'} + \underbrace{C.C'}_{0}$$

$$= A' + B.C' + C.B'$$

$$F(x,y,z) = X.Y + X'.Y' + X'.Z'$$

Express $F(x,y,z)$ as sum of minterms:

$$X.Y = X.Y.\underset{\uparrow \atop (Z+Z')}{1} = X.Y.(Z+Z')$$

$$= X.Y.Z + X.Y.Z'$$

$$X'.Y' = X'.Y'.\underset{\uparrow \atop (Z+Z')}{1} = X'.Y'.(Z+Z')$$

$$= X'.Y'.Z + X'.Y'.Z'$$

$$x'.z' = x'.z'.1 = x'.z'.y + x'.z'.y'$$

$(y+y')$

$$F(x,y,z) = x.y.z + x.y.z' + x'.y'.z + x'y'.z' + x'.z'.y + \cancel{x'.z'.y'}$$

$\boxed{x+x=x}$

$x'.y.z'$

$x+x=x$

$$= \sum_{x,y,z} (m_7, m_6, m_1, m_0, m_2)$$

① Expand the function into sum of products.

② In each product, if there is a literal missing, "AND" the product with "1" and replace "1" with (missing literal + its complement)

$$F(x,y,z) = \overbrace{X \cdot Y + X' \cdot Y'}^{A} + \overset{B}{Y'} \cdot \overset{C}{Z'}$$

Express $F(x,y,z)$ as product of maxterms.

$$\boxed{(A + BC) = (A+B) \cdot (A+C)} \leftarrow \text{Theorem}$$

$$\underbrace{X \cdot Y + X' \cdot Y'}_{A} + \underbrace{Y'}_{B} \cdot \underbrace{Z'}_{C}$$

$$= (\underset{C}{X} \cdot \underset{B}{Y} + \underbrace{X' \cdot Y' + Y'}_{A}) \cdot (\underset{C}{X} \cdot \underset{B}{Y} + \underbrace{X' \cdot Y' + Z'}_{A})$$

$$= \underbrace{(X' \cdot Y' + \underbrace{Y' + Y}_{=1})}_{=1} \cdot (\underset{C}{X'} \cdot \underset{B}{Y'} + \underbrace{Y' + X}_{A}) \cdot (\underset{C}{X'} \cdot \underset{B}{Y'} + \underbrace{Z' + X}_{A}) \cdot (\underset{C}{X'} \cdot \underset{B}{Y'} + \underbrace{Z' + Y}_{A})$$

$$= (X + \underbrace{Y' + Y'}_{Y'}) \cdot (\underbrace{\cancel{X} + Y' + X'}_{=1}) \cdot (\underbrace{X + Z' + X'}_{=1}) \cdot (X + Z' + Y')$$

$$\cdot (Z' + Y + X') \cdot (Z' + \underbrace{Y + Y'}_{=1})$$

$$1 + Z' = 1$$

$$= (X+Y') \cdot (X+\underbrace{Z'+Y'}_{\text{Maxterm}}) \cdot \underbrace{(X'+Y+Z')}_{\text{maxterm}}$$

$$(X+Y') = (X+Y'+\underset{Z \cdot Z'}{\uparrow 0}) = (\underbrace{X+Y'}_{A}+\underset{B}{Z}\cdot\underset{C}{Z'})$$

$$=(X+Y'+Z)\cdot(X+Y'+Z')$$

$$F(x,y,z) = (X+Y'+Z)\cdot(X+Y'+Z')\cdot(X'+Y+Z')$$

$$= \prod_{x,y,z}(M_2, M_3, M_5)$$

$$X \cdot Y + X' \cdot Z + \underline{Y \cdot Z} = \underline{X \cdot Y + X' \cdot Z}$$

$$X \cdot Y + X' \cdot Z + 1 \cdot Y \cdot Z$$

$$X \cdot Y + X' \cdot Z + (X + X') \cdot Y \cdot Z$$

$$= \underline{X \cdot Y} + \underline{X' \cdot Z} + \underline{X \cdot Y \cdot Z} + \underline{X' \cdot Y \cdot Z}$$

$$= X \cdot Y [1 + Z] + X' \cdot Z [1 + Y]$$

$$= X \cdot Y + X' \cdot Z$$