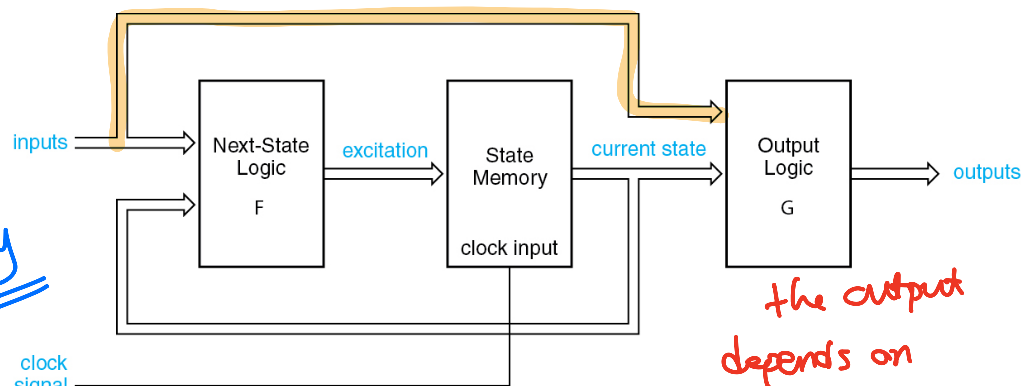
Analysis of Finite State Machines

Goal: Given a logic diagram, predict the behavior of the state machine, i.e., determine the next states and the output of the machine.

Tools:
- State table
- State diagram :
  - Circles : state
  - arcs : transitions between states

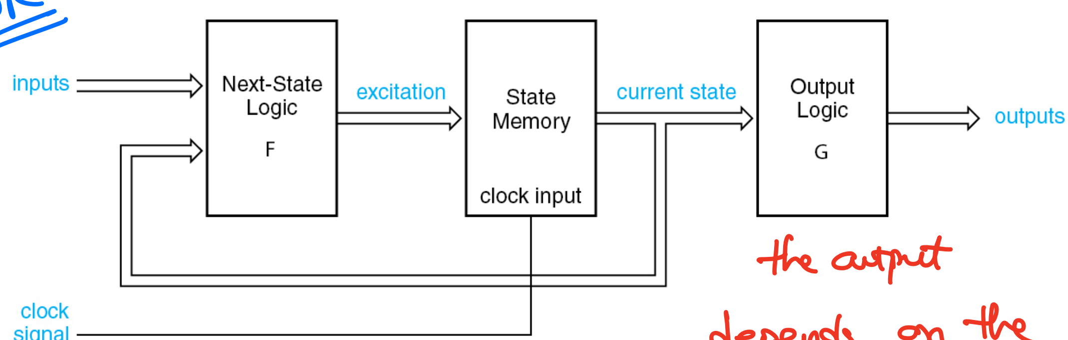
State Machines:

Mealy



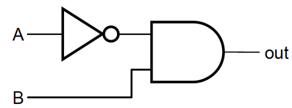
the output depends on the current state and the inputs

Moore



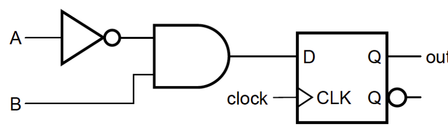
the output depends on the current state

Examples:



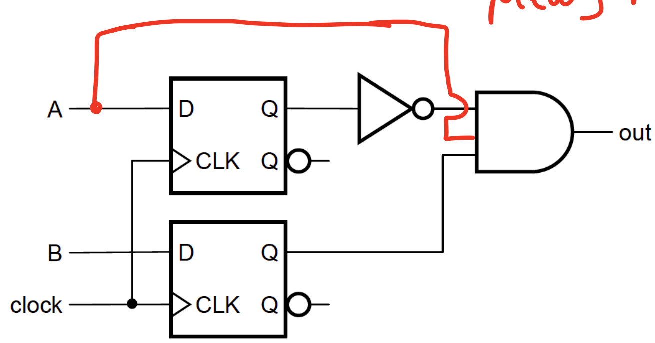
Combinational Circuit
(No clock, No memory)

this is not a sequential circuit.



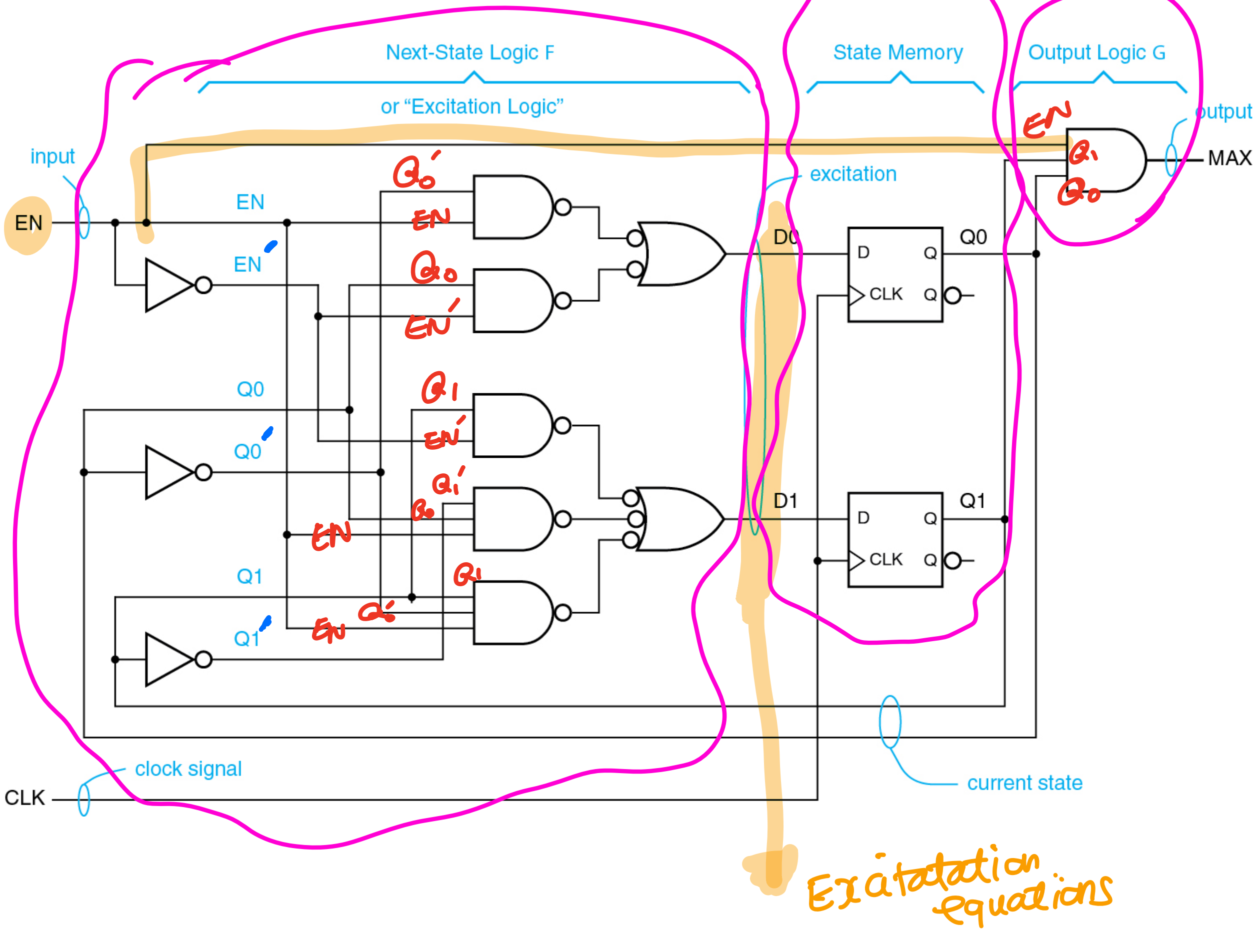
Moore Machine
(No direct feeding of input to the output)

Mealy machine



Moore Machine

Transition and State Tables



- Excitation Equations: equations describing the output of the excitation logic block

$$D_0 = Q_0 \cdot EN' + Q_0' \cdot EN$$

$$D_1 = Q_1 \cdot EN' + Q_0 \cdot Q_1' \cdot EN + Q_0' \cdot Q_1 \cdot EN$$

- Characteristic Equations: equations describing flip-flops' next states

$$Q(t+\varepsilon) = \underset{\text{next state}}{Q^*}$$

$$\begin{cases} Q_0^* = D_0 \\ Q_1^* = D_1 \end{cases}$$

- Transition Equations: equations describing the next values of the state variables as a function of the current states and inputs

$$\begin{cases} Q_0^* = Q_0 \cdot EN' + Q_0' \cdot EN \\ Q_1^* = Q_1 \cdot EN' + Q_0 \cdot Q_1' \cdot EN + Q_0' \cdot Q_1 \cdot EN \end{cases}$$

- Output Equations: equations describing the output

$$MAX = EN \cdot Q_1 \cdot Q_0$$

two state variables: Q_1, Q_0

use the transition equations to find Q_1^*, Q_0^*

Transition and State Tables

	Q_1 Q_0	EN = 0	EN = 1
A	0 0	0 0	0 1
B	0 1	0 1	1 0
C	1 0	1 0	1 1
D	1 1	1 1	0 0

Q_1^*, Q_0^*

(State)

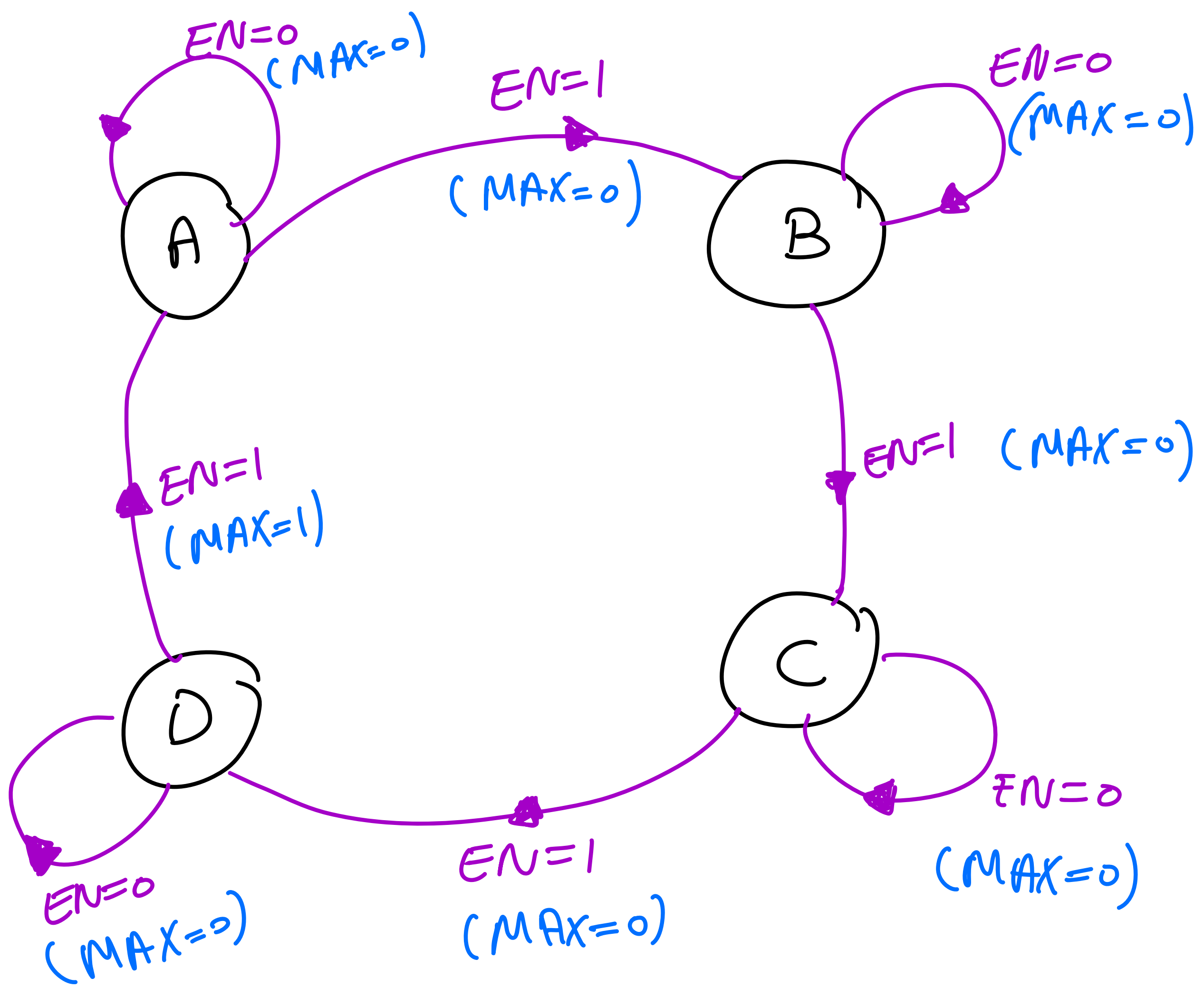
S	EN = 0	EN = 1
A	A	B
B	B	C
C	C	D
D	D	A

S^* (Next state)

	EN = 0	EN = 1
A	A, 0	B, 0
B	B, 0	C, 0
C	C, 0	D, 0
D	D, 0	A, 1

S^*, MAX (Nextstate, output)

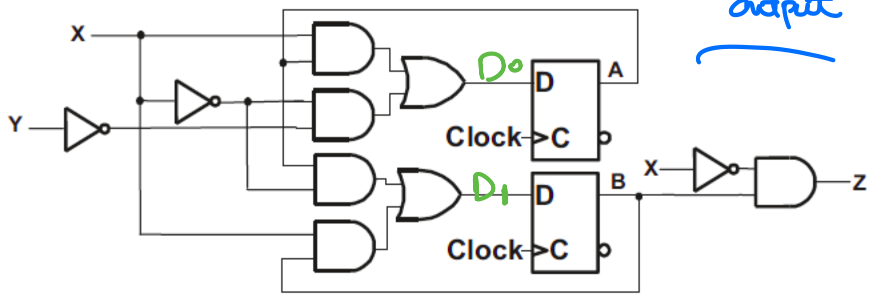
State Diagram



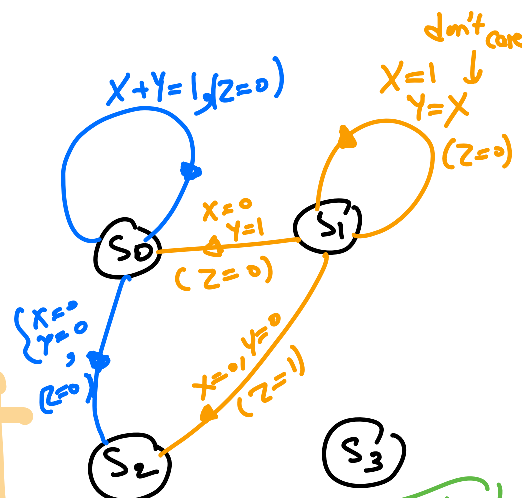
Mealy Machine

Example

Excitation Logic (Next Logic) — Memory — output



Mealy machine, $\begin{cases} 2 \text{ inputs}: x, y \\ 2 \text{ D-FF} \Rightarrow 2 \text{ state variables} \\ 1 \text{ output } (Z) \end{cases}$



Excitation equations: $\begin{cases} D_0 = x \cdot A + x' \cdot y' \\ D_1 = X' \cdot A + X \cdot B \end{cases}$

characteristic equations: $\begin{cases} A^* = D_0 \\ B^* = D_1 \end{cases}$

Transition equations: $\boxed{\begin{cases} A^* = x \cdot A + X' \cdot Y' \\ B^* = X' \cdot A + X \cdot B \end{cases}}$

Complete the state diagram

Output equation: $Z = X' \cdot B$

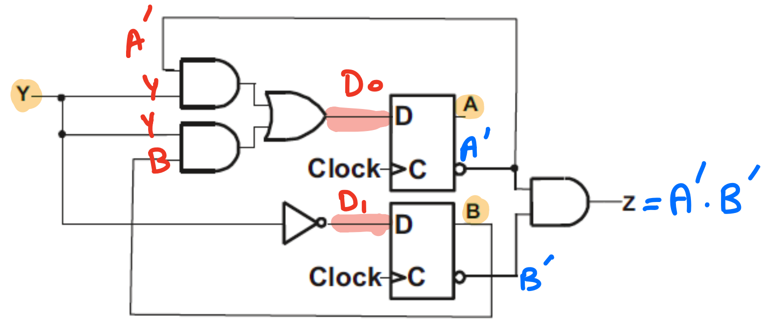
	A	B	XY = 00	01	10	11
S_0	0	0	10	00	00	00
S_1	0	1	10	00	01	01
S_2	1	0	11	01	10	10
S_3	1	1	11	01	11	11

$A^* B^*$

S	XY = 00	01	10	11
S_0	S_2, 0	S_0, 0	S_0, 0	S_0, 0
S_1	S_2, 1	S_0, 1	S_1, 0	S_1, 0
S_2	S_3, 0	S_1, 0	S_2, 0	S_2, 0
S_3	S_3, 1	S_1, 1	S_3, 0	S_3, 0

S^*, Z

Example



→ Moore Machine

→ one input : Y , two FFs → two state variables: A, B
one output: Z

(2) Equations:

Excitation equations: $D_0 = (A' \cdot Y) + (Y \cdot B)$, $D_1 = Y'$

Transition equations: $A^* = (A' \cdot Y) + (Y \cdot B)$, $B^* = Y'$ ✱

output equations: $Z = A' \cdot B'$

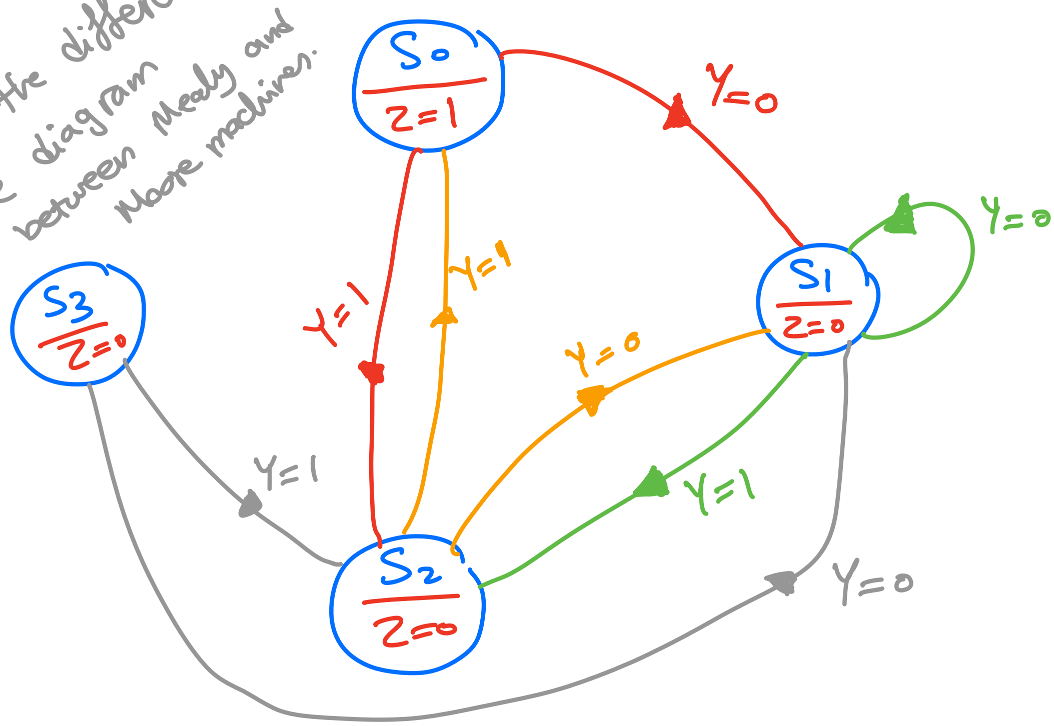
	A B	Y = 0	Y = 1
S_0	0 0	0 1	1 0
S_1	0 1	0 1	1 0
S_2	1 0	0 1	0 0
S_3	1 1	0 1	1 0

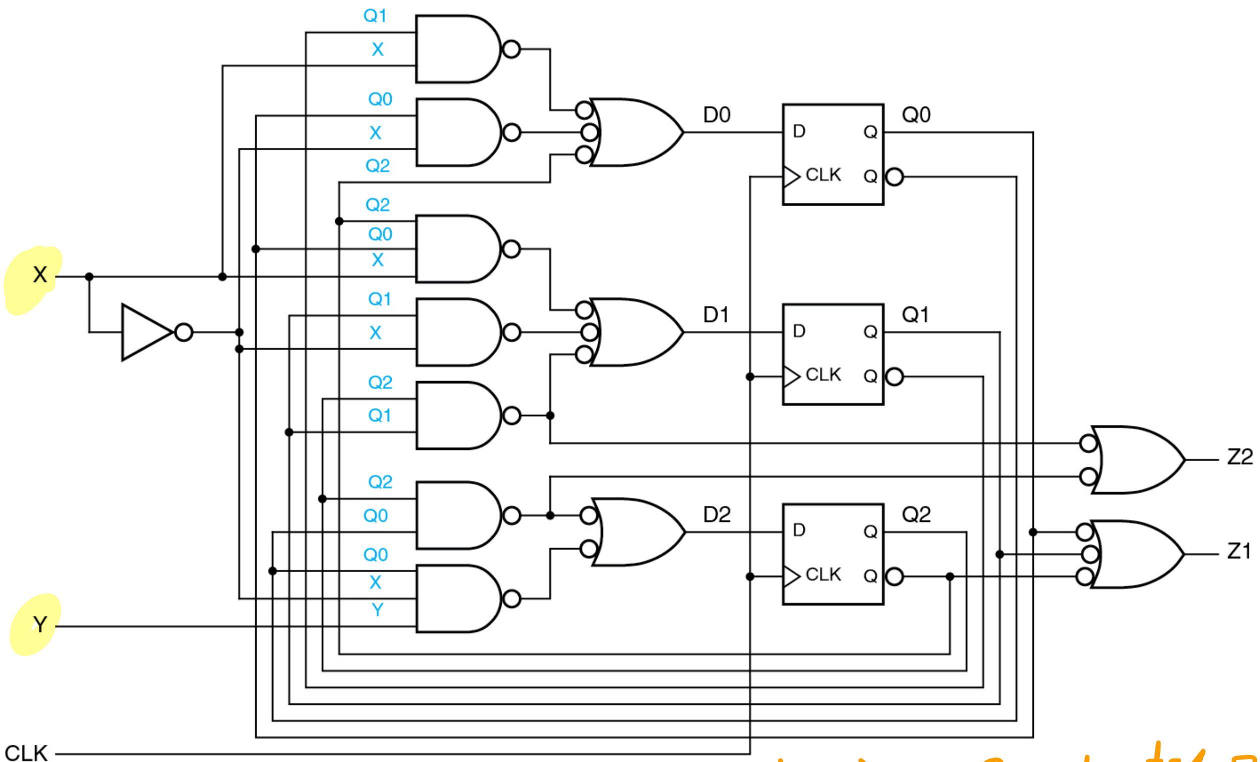
$A^* B^*$

S	Y = 0	Y = 1
S_0	S_1, 1	S_2, 1
S_1	S_1, 0	S_2, 0
S_2	S_1, 0	S_0, 0
S_3	S_1, 0	S_2, 0

S^*, Z

Note the difference in state diagram between Mealy and Moore machines.





two inputs (X, Y), 3 FFs: (3 state variables), 2 outputs (Z_1, Z_2)
Q_0, Q_1, Q_2

(a)

8

	XY				
Q2 Q1 Q0	**00**	**01**	**10**	**11**	**Z1 Z2**
000	000	100	001	001	10
001	001	001	011	011	10
010	010	110	000	000	10
011	011	011	010	010	00
100	101	101	101	101	11
101	001	001	001	001	10
110	111	111	111	111	11
111	011	011	011	011	11
	Q2* Q1* Q0*				

(b)

	XY				
S	**00**	**01**	**10**	**11**	**Z1 Z2**
A	A	E	B	B	10
B	B	B	D	D	10
C	C	G	A	A	10
D	D	D	C	C	00
E	F	F	F	F	11
F	B	B	B	B	10
G	H	H	H	H	11
H	D	D	D	D	11
	S*				