Number Systems :

(Decimal)

base (radix) = 10

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

---

base (radix) = 2 (Binary)

0, 1

---

base (radix) = 8 (octal)

0, 1, 2, 3, 4, 5, 6, 7

---

base (radix) = 16 (Hexadecimal)

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$1734 : 1 \times 10^3 + 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

$10^3 \quad 10^2 \quad 10^1 \quad 10^0$

$d_{p-1} \; d_{p-2} \cdots d_1 \, d_0 \, . \, d_{-1} \; d_{-2} \cdots d_{-n}$

Most significant digit

$2^{(p-1)}$

radix point

Binary System: base (radix) = 2 , (0, 1)

$(11010)_2 \rightarrow 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$

$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$= (26)_{10}$

use "Summation" method to convert binary to decimal

$(110101.11)_2 : 1\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \quad 2^{-1} \quad 2^{-2}$

$+ 1\times2^{-1} + 1\times2^{-2} = (53.75)_{10}$

$0.5 \quad \frac{1}{2} + \frac{1}{4} \quad 0.25$

Decimal to Binary Conversion:

use "Division" method to convert decimal to binary

$(179)_{10}$

	Remainder
$179 \div 2 = 89$	$R = 1$ — LSB (Least Significant Bit)
$89 \div 2 = 44$	$R = 1$
$44 \div 2 = 22$	$R = 0$
$22 \div 2 = 11$	$R = 0$
$11 \div 2 = 5$	$R = 1$
$5 \div 2 = 2$	$R = 1$
$2 \div 2 = 1$	$R = 0$
$1 \div 2 = 0$	$R = 1$ ← MSB (Most Significant Bit)

$$(179)_{10} \equiv (10110011)_2$$

$$(3.703125)_{10}$$

Division by "2" → (11)

$$0.703125 \times 2 = 1.40625$$

$$0.40625 \times 2 = 0.8125$$

$$0.8125 \times 2 = 1.625$$

$$0.625 \times 2 = 1.25$$

$$0.25 \times 2 = 0.5$$

$$0.5 \times 2 = 1 \quad \leftarrow \text{LSB}$$

2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4}

$$(3.703125)_{10} = (11.101101)_2$$

Octal base $(r = 8)$: $(0, 1, 2, 3, 4, 5, 6, 7)$

$\uparrow$ radix

Octal → Decimal : use "Summation" method

$$(\underset{8^2}{1}\,\underset{8^1}{2}\,\underset{8^0}{7}\,.\,\underset{8^{-1}}{4})_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1}$$

$$= (87.5)_{10}$$

Octal → binary : use "Substitution" method

octal	binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

$$(\overset{110}{6}\,\underset{111}{7}\,\overset{011}{3}\,.\,\underset{001}{1}\,\overset{010}{2})_8 \equiv (110\;111\;011\,.\,001\;010)_2$$

binary → octal : use "Substitution" method

$$(\overbrace{100}^{4}\,\overbrace{011}^{3}\,\overbrace{001}^{1}\,\overbrace{110}^{6})_2 \equiv (4316)_8$$

$$(10.1011)_2 \equiv (?)_8$$

$$(\overbrace{010}^{2}.\underbrace{101}_{5}\,\underbrace{100}_{4})_2 \equiv (2.54)_8$$

Decimal to Octal :

$(467)_{10} \equiv (?)_8$ use "Division" method

	$\boxed{R}$
$467 \div 8 = 58$	3 ← LSD
$58 \div 8 = 7$	2
$7 \div 8 = 0$	7 ← MSD

$$(467)_{10} \equiv (723)_8$$

Hexadecimal : $r = 16$

0 → 0000		9	1001
1 → 0001		A	1010
2 → 0010		B	1011
$\boxed{3 \rightarrow 0011}$		C	1100
4 → 0100		D	1101
5 → 0101		E	1110
6 → 0110			
7 → 0111			
8 → 1000		F	1111

Hexadecimal to Binary Conversion:

use "Substitution" method

$$(3A6.C)_{16} \equiv (0011\ 1010\ 0110.1100)_2$$

point

0011 1010 0110 1100

Decimal to Hexadecimal Conversion:

$(3417)_{10} \equiv (?)_{16}$ use "Division" method

	R	
$3417 \div 16 = 213$	9	→ LSD
$213 \div 16 = 13$	5	
$13 \div 16 = 0$	13 → D	→ MSD

$$(3417)_{10} \equiv (D59)_{16}$$

Binary Addition:

Carry out from previous column

$$
\begin{array}{r}
1\ \ \ 1\ 1\ \ \ \ \ \ \\
1\ 1\ 0\ 1\ 1\ 0 \\
+\ \ 0\ 1\ 0\ 1\ 1\ 1 \\
\hline
1\ 0\ 0\ 1\ 1\ 0\ 1
\end{array}
$$

$$
\begin{array}{r}
1\ \ \\
1\,7 \\
+\ \ 8 \\
\hline
2\,5
\end{array}
$$

Decimal

Carry in

Cin	X	Y	Cout	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$
\begin{array}{r}
1\ 1\ 1\ 1\ 1\ 1\ \ \ \\
1\ 1\ 1\ 1\ 1\ 1\ 1 \\
+\ \ \ \ 1\ 1\ 1\ 1\ 1\ 1 \\
\hline
1\ 0\ 1\ 1\ 1\ 1\ 1\ 0
\end{array}
$$

Binary Subtraction:

$$
\begin{array}{r}
\cancel{1}\,0\,0 \\
-\;0\,0\,1 \\
\hline
0\,1\,1
\end{array}
\;\rightarrow\;
\begin{array}{r}
(4)_{10} \\
-\;(1)_{10} \\
\hline
(3)_{10}
\end{array}
$$

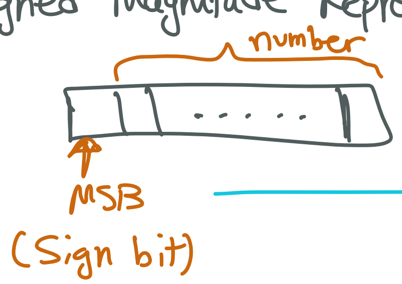
Decimal

$$
\begin{array}{r}
\cancel{1}7 \\
-\;9 \\
\hline
8
\end{array}
$$

$$
\begin{array}{r}
\cancel{1}\,0\,0\,\cancel{1}\,\cancel{1}\,0\,1 \\
-\;1\,0\,1\,1\,1 \\
\hline
0\,1\,1\,0\,1\,1\,0
\end{array}
$$

Negative Numbers
- → Signed Magnitude Representation
- → 2's-Complement Representation

Signed Magnitude Representation:



→ { 0 : Positive Number
1 : Negative Number

Example: 4_bit System:

$0\,100 \equiv (+4)_{10}$

$1\,001 \equiv (-1)_{10}$

8-bit System:

$01010101 \equiv (+85)_{10}$

$11010101 \equiv (-85)_{10}$

For <u>n_bit System</u>

largest positive number: $0\overbrace{111\cdots\cdots1}^{(n-1)\text{ bits}}$

Smallest negative number: $1\underbrace{111\cdots\cdots1}_{(n-1)\text{ bits}}$

$\rightarrow$ "0" $\begin{cases} 0\;0000000 \\ 1\;0000000 \end{cases}$

2's-Complement Representation:

1. Complement the digits
2. Add "1" to LSB
3. Discard $\underbrace{\text{Cout}}_{\text{Carry out}}$ in the MSB

4_bit System : $(0111) \rightarrow (+7)$

① Complement the digits : 1000

② Add "1" to LSB :

$$\begin{array}{r} 1000 \\ +\quad 1 \\ \hline 1001 \end{array} \quad (-7)$$

$2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$(-1) \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= -8 + 1 = -7$

$$\begin{array}{r} \rightarrow 0110 \\ +\quad 1 \\ \hline 0111 \end{array} \rightarrow (+7)$$

+8

$$\begin{array}{r} 01000 \\ 10111 \\ +\quad 1 \\ \hline 11000 \end{array}$$

-8

$$\boxed{\text{2's-Complement}\left[\text{2's-Complement}(A)\right] = A}$$

Range of numbers in a n-bit 2's-Complement System :

$$-2^{n-1} \leq \quad < 2^{n-1} - 1$$

$$\left(\overset{2^6}{1}\,\overset{2^5}{0}\,\overset{2^4}{1}\,\overset{2^3}{1}\,\overset{2^2}{0}\,\overset{2^1}{0}\,\overset{2^0}{1}\right)_2 = (?)_{10}$$

Unsigned binary System: $(89)_{10}$ $\left[1\times2^6+1\times2^4+1\times2^3+1\times2^0 = 89\right]$

Signed-magnitude representation system: $(-25)_{10}$

2's-Complement representation system: $(-39)_{10}$

$$(-1)\times2^6+1\times2^4+1\times2^3+1\times2^0 = -39$$

5-bit

2's Complement System

$$\overset{2^4}{1}\,\overset{2^3}{1}\,\overset{2^2}{0}\,\overset{2^1}{0}\,\overset{2^0}{0}$$

$$\begin{array}{rccccccl}
 & 1 & 1 & 1 & 1 & & & \\
 & 1 & 1 & 1 & 1 & 1 & \rightarrow & 0 \\
+ & & & & & 1 & & \\
\hline
\boxed{\cancel{1}} & 0 & 0 & 0 & 0 & 0 & & \\
\end{array}$$

Addition/Subtraction with 2's-Complement numbers:

Process:
- Similar to unsigned binary addition/Subtraction, but ignore carriers beyond MSB
- Results should be within $(-2^{n-1}) < \cdots < (2^{n-1}-1)$

Overflow detection rules:

Addition:
- $\oplus + \oplus \rightarrow \ominus$ overflow
- $\ominus + \ominus \rightarrow \oplus$ overflow

Subtraction:
- $\oplus - \ominus \rightarrow \ominus$ overflow
- $\ominus - \oplus \rightarrow \oplus$ overflow

2's-Complement system

$n=4$: $\rightarrow$ $\boxed{-8 < \cdots < +7}$

$$\begin{array}{rcccc} & & 0\rightarrow 1 & 1 & \\ & 0 & \cancel{1} & \cancel{0} & 0 \\ - & 0 & 0 & 1 & 1 \\ \hline & 0 & 0 & 0 & 1 \end{array} \qquad \begin{array}{r} +4 \\ - \quad +3 \\ \hline +1 \end{array}$$

$$(+4) - (+3) = (+4) + (-3)$$

$$\begin{array}{rccccl} & 1 & & & & \\ & 0 & 1 & 0 & 0 & \\ + & 1 & 1 & 0 & 0 & \\ + & & & & 1 & (-3) \\ \hline \cancel{1} & 0 & 0 & 0 & 1 & (+1) \end{array}$$

$\boxed{n=5}$ 2'-Complement

$\boxed{-16 < \qquad < +15}$

$$\begin{array}{rccccc} & 0 & 1 & 0 & 0 & 0 \\ + & 0 & 1 & 0 & 0 & 1 \\ \hline & 1 & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{r} \cancel{1}\,\cancel{1}\,0\,0\,0 \\ -\ 0\,1\,0\,0\,1 \\ \hline 0\,1\,1\,1\,1 \end{array} \qquad \text{overflow} \qquad \begin{array}{r} (-8)_{10} \\ -\ (+9)_{10} \\ \hline (-17)_{10} \end{array}$$

$$\begin{array}{r} 1\,1\,0\,0\,0 \\ +\ 1\,0\,1\,1\,0 \\ +\ \qquad 1 \\ \hline \cancel{1}\,0\,1\,1\,1\,1 \end{array}$$

2's-complement of (01001)

(overflow)

Addition/subtraction in Signed-magnitude systems:

- attention to the sign bit
- work with the magnitudes
- then make the decision on the sign

magnitude comparison

			$A > B$	$A < B$
Addition:	$(+A) + (+B)$	$+(A+B)$		
	$(+A) + (-B)$		$+(A-B)$	$-(B-A)$
	$(-A) + (+B)$		$-(A-B)$	$+(B-A)$
	$(-A) + (-B)$	$-(A+B)$		
Subtraction (complete this)	$(+A) - (+B)$		$+(A-B)$	$-(B-A)$
	$(+A) - (-B)$	$+(A+B)$		
	$(-A) - (+B)$	$-(A+B)$		
	$(-A) - (-B)$		$-(A-B)$	$+(B-A)$

__Example:__ $n=4$, Signed-magnitude System

$$\begin{array}{rr} & 0\,0\,1\,1 \\ + & 0\,0\,1\,0 \\ \hline & 0\,1\,0\,1 \end{array} \qquad \begin{array}{rr} & (+3)_{10} \\ + & (+2)_{10} \\ \hline & (+5)_{10} \end{array}$$

$$\begin{array}{lrr} (+A) & & 1\,0\,1\,1 \\ (-B) & + & 0\,0\,1\,0 \\ \hline & & 1\,0\,0\,1 \end{array} \longrightarrow \begin{array}{rr} & 0\,1\,1 \\ - & 0\,1\,0 \\ \hline & 0\,0\,1 \end{array} \qquad \begin{array}{l} A>B \\ (A-B) \end{array}$$

Binary Multiplication :

```
        1 1 0 1 0 1 1 0
  X     0 0 1 0 1 1 0 1
  ---------------------------------
              1 1 0 1 0 1 1 0
  +         0 0 0 0 0 0 0 0 0
  ---------------------------------   <- Sum
  +       1 1 0 1 0 1 1 0 0 0
  ---------------------------------   <- Sum
  +     1 1 0 1 0 1 1 0 0 0 0
  ---------------------------------   <- sum
  +   0 0 0 0 0 0 0 0 0 0 0 0
  + 1 1 0 1 0 1 1 0 0 0 0 0 0         <- sum
  ---------------------------------
0 0 1 0 0 1 0 1 1 0 0 1 1 1 1 0       <- check this
```