CSCI-404 Arti cial Intelligence
Spring 2025, Homework Set 3

Question 1 Arc consistency in constrained satisfaction problems (18 points)

Suppose that we have three variables X , X_2 and X_3, which are defined on the same domain of $\{1, 2, 3\}$. Two binary constraints for these three variables are defined as follows:

$$R_{\ 2} = \{(X\ , X_2), [(2,1), (2,3), (3,2), (3,3)]\}$$
$$R_{\ 3} = \{(X\ , X_3), [(1,2), (2,1), (3,1), (3,3)]\}$$

1. (9 points) Is X arc-consistent with respect to X_2? And is X arc-consistent with respect to X_3? and why? In addition, is X_3 arc-consistent with respect X if the constraints between R and R_3 are undirected (*i.e.*, R_3 is defined as $\{(X_3, X\ ), [(2,1), (1,2), (1,3), (3,3)]\}$ that switches the element order of every two-tuple of $R_{\ 3}$)? and why?
2. (9 points) Suppose that, after some inference, the domain of X is reduced as $\{2, 3\}$ and the constrains in $R_{\ 2}$ and $R_{\ 3}$ for $X\ = 1$ are removed accordingly. To be more specific, $(1, 2)$ is removed from $R_{\ 3}$ due to reducing the domain of X . Now is X still arc-consistent with respect to X_2? And is X arc-consistent with respect to X_3? and why? In addition, is X_3 still arc-consistent with respect X if the constraints between R and R_3 are undirected? and why?

Question 2 Constraint propagation: path consistency (10 points)

1. (5 points) As illustrated in Fig. 1, suppose that three variables and their corresponding domains are defined as follows: $X\ \in \{0,\quad, 4\}, X_m \in \{1,\quad, 5\}, X_j \in \{6,\quad, 10\}$. Are X and X_j path-consistent with respect to X_m for the following binary constraints? **Please justify your response.**

$$\begin{cases} X\ < X_j \\ X\ < X_m \\ X_m < X_j \end{cases} \tag{1}$$

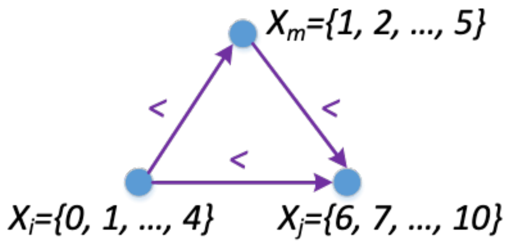

Fig. 1: Path consistency in constrained satisfaction problem.

2. (5 points) As illustrated in Fig. 2, suppose that three variables and their corresponding domains are defined as follows: $X\ \in \{0, \quad ,4\}$, $X_m \in \{1, \quad ,5\}$, $X_j \in \{5, \quad ,10\}$. Are X and X_j path-consistent with respect to X_m for the same constraints in Eq. (1)? If they are not path consistent, what change can we make to the constraint between X and X_j to make them path consistent (*i.e.* filtering via a path constraint) **Please justify your response.**

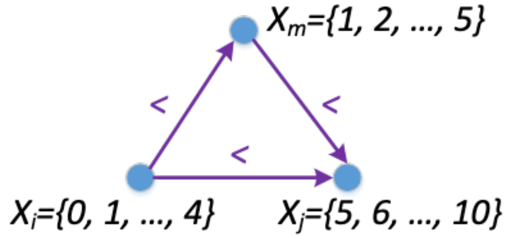

Fig. 2: Path consistency in constrained satisfaction problem.

Question 3 Solving constrained satisfaction problems (24 points)

Suppose that you are in charge of scheduling for computer science classes that meet Mondays, Wednesdays and Fridays. There are 5 classes that meet on these days and 3 professors who will be teaching these classes. You are constrained by the fact that each professor can only teach one class at a time.

The classes are:

- Class 1 - Intro to Programming: meets from 8:00-9:15AM
- Class 2 - Intro to Artificial Intelligence: meets from 8:30-9:45AM
- Class 3 - Natural Language Processing: meets from 9:00-10:15AM
- Class 4 - Computer Vision: meets from 9:30-10:45AM
- Class 5 - Machine Learning: meets from 10:30-11:15AM

The professors are:

- Professor A, who is available to teach Classes 3 and 4.
- Professor B, who is available to teach Classes 2, 3, 4, and 5.
- Professor C, who is available to teach Classes 1, 2, 3, 4, 5.

1. (8 points) Please formulate this problem as a CSP problem in which there is one variable per class, stating the domains, and constraints. Constraints should be specified formally and precisely.
2. (5 points) Please draw the constraint graph (defined on Slide 6, Lecture 14) associated with your CSP. *You only need to draw the labeled nodes, and the edges connecting them. You do not need to illustrate the domains or the speci c constraints.*
3. (3 points) Please show the domains of the variables after running arc-consistency on this initial graph (after having already enforced any unary constraints) which you derived in Question 3.2.
4. (2 points) Please give one solution to this CSP.
5. (3 points) Your CSP should look nearly tree-structured. Briefly explain why we might prefer to solve tree-structures CSPs.
6. (3 points) Please briefly describe the cutset conditioning algorithm for turning these kinds of nearly tree-structured problems into tree-structured ones. State which node(s) should be be pruned to go into the (minimum) cutset.

Question 4 Simulated Annealing (36 points)

WARNING: Please interpret these questions for a version of simulated annealing that **minimizes energy / goes downhill** (be aware that the textbook and slides present di erent interpretations). Please also ignore the choice of direction (**uphill / downhill**) when comparing to Hill Climbing.

1. (15 points; 5 points each) Please indicate if each of the following statement is True or False?
   (a) Simulated Annealing can escape local optima.
   (b) Simulated Annealing with a constant and positive temperature at all times is the same as Hill-Climbing search.
   (c) Simulated Annealing with a linearly decreasing temperature is guaranteed to converge to a global optimal solution after a finite number of iterations.
2. (6 points) What kind of search does Simulated Annealing do (approximately) if the temperature is very large (*i.e.*, close to ∞) at every iteration? Please select your answer from one of the following options and **justify** your answer.
   (a) It will halt immediately and do no search
   (b) Breadth-First search
   (c) A random walk
   (d) Hill-Climbing
3. (6 points) How would Simulated Annealing perform if the temperature T is always very close (but not equal) to 0? You can ignore the termination test. Please pick one option and **justify** your answer.
   (a) A random walk
   (b) Hill-Climbing
4. (9 points) In Simulated Annealing, if the current state's evaluation function value is 9, the selected successor state's value is 12, and the current temperature is $T = 10$, what is the probability of moving to the successor state? (Assume we are minimizing the evaluation function.) Please give your answer as either an expression or a number.

Question 5 Special Cases of Local Search Algorithms (12 points)

1. Give the name of the algorithm that results from each of the following special cases of local beam search:
   (a) (6 points) Local beam search with $k = 1$.
   (b) (6 points) Local beam search with one initial state and no limit on the number of states retained.

Question 6 Binary constraints of constrained satisfaction problems (OPTIONAL, 20 points)

This problem is an OPTIONAL exercise. You are completely free to choose to do it or not. If choose to do it you can get up to 20 bonus points for this homework set.

We can formulate higher-order constraints (*i.e.*, the constraints that involve more than two variables) in a CSP problem using just binary constraints by introducing (possibly several) additional constraints and variables (possibly with new or bigger domains). Please explain how you could do so:

- (5 points) For the constraint $A \quad B = C$
- (5 points) For any ternary constraint (a constraint involving three variables, like above)
- (10 points) For any constraint involving k di erent variables (by iterating the reduction from ternary to binary constraints)