Appendix A
Proof of theorem 1

1) Form (22), solving the following Nash min-max game problem:

$$
\begin{aligned}
J = &\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} \\
&E\{\int_0^{t_f} \bar{\bar e}^T(t)Q\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) - \rho^2\bar v^T(t)\bar v(t)\,dt\} \\
=&\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} \\
&E\{\int_0^{t_f} \bar{\bar e}^T(t)Q\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) - \rho^2\bar v^T(t)\bar v(t) \\
&+ \frac{d}{dt}V(\bar{\bar e}(t),\tilde{\bar\theta}(t))\,dt + V(\bar{\bar e}(0),\tilde{\bar\theta}(0)) - V(\bar{\bar e}(t_f),\tilde{\bar\theta}(t_f))\} \\
=&\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} E\{V(\bar{\bar e}(0),\tilde{\bar\theta}(0)) - V(\bar{\bar e}(t_f),\tilde{\bar\theta}(t_f)) \\
&+ \int_0^{t_f} \frac{1}{2}\bar{\bar e}^T(t)(\bar A^TP + P\bar A + 2Q)\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) \\
&+ \tilde{\bar\theta}^T(t)(\bar u^T\varepsilon(\hat{\bar x}(t))B_1^TP\bar{\bar e}(t) + S\dot{\tilde{\bar\theta}}(t)) + \bar{\bar e}(t)^TPB_2u_\infty(t) \\
&+ \bar{\bar e}^T(t)PB_3\bar v(t) - \rho^2\bar v^T(t)\bar v(t)\,dt\}
\end{aligned} \tag{A1}
$$

By the fact $\dot{\tilde{\bar\theta}}(t) = \dot{\bar\theta}(t)$ in remark 3, we get the adaptive laws in (25). Then, completing squares, we solve the worst-case disturbance $\bar v^*(t)$ in (27) and the optimal H_∞ control input $u_\infty(t)$ in (26) by Nash min-max game as follows

$$
\begin{aligned}
J = &\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} E\{V(\bar{\bar e}(0),\tilde{\bar\theta}(0)) - V(\bar{\bar e}(t_f),\tilde{\bar\theta}(t_f)) \\
&+ \int_0^{t_f} \frac{1}{2}\bar{\bar e}^T(t)(\bar A^TP + P\bar A + 2Q)\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) \\
&+ \bar{\bar e}(t)^TPB_2u_\infty(t) + \bar{\bar e}^T(t)PB_3\bar v(t) - \rho^2\bar v^T(t)\bar v(t)\,dt\} \\
=&\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} E\{V(\bar{\bar e}(0),\tilde{\bar\theta}(0)) - V(\bar{\bar e}(t_f),\tilde{\bar\theta}(t_f)) \\
&+ \int_0^{t_f} \frac{1}{2}\bar{\bar e}^T(t)(\bar A^TP + P\bar A + 2Q - \frac{1}{2R}PB_2B_2^TP \\
&+ \frac{1}{2\rho^2}PB_3B_3^TP)\bar{\bar e}(t) \\
&+ [\frac{1}{2}B_2^TP\bar{\bar e}(t) + Ru_\infty(t)]^T\frac{1}{R}[\frac{1}{2}B_2^TP\bar{\bar e}(t) + Ru_\infty(t)] \\
&- [\frac{1}{2\rho}B_3^TP\bar{\bar e}(t) - \rho\bar v(t)]^T[\frac{1}{2\rho}B_3^TP\bar{\bar e}(t) - \rho\bar v(t)]\,dt\} \\
=&E\{V(\bar{\bar e}(0),\tilde{\bar\theta}(0)) - V(\bar{\bar e}(t_f),\tilde{\bar\theta}(t_f)) + \int_0^{t_f} \frac{1}{2}\bar{\bar e}^T(t)(\bar A^TP \\
&+ P\bar A + 2Q - \frac{1}{2R}PB_2B_2^TP + \frac{1}{2\rho^2}PB_3B_3^TP)\bar{\bar e}(t)\,dt\}
\end{aligned} \tag{A2}
$$

According to Ricatti-like equation in (28) and Lyapunov function in (21), it is obvious that $V(\bar{\bar e}(t_f),\tilde{\bar\theta}(t_f)) \ge 0$, we obtain:

$$
J \le E\{V(\bar{\bar e}(0),\tilde{\bar\theta}(0))\} \tag{A3}
$$

Therefore, the proof of (22) is complete.

2) (A3) is equivalent to

$$
\begin{aligned}
&\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,\infty)} E\{\int_0^\infty \bar{\bar e}^T(t)Q\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t)\,dt\} \\
&\le E\{V(\bar{\bar e}(0),\tilde{\bar\theta}(0)) + \int_0^\infty \rho^2\bar v^T(t)\bar v(t)\,dt\}
\end{aligned} \tag{A4}
$$

Since $\bar v(t) \in L_F^2[0,\infty)$, i.e. $E\{\int_0^\infty \rho^2\bar v^T(t)\bar v(t)\,dt\} < \infty$ for some finite ρ, the r.h.s of (A4) is finite. Therefore, we can conclude that $\bar{\bar e}(t) \to 0$, $u_\infty(t) \to 0$ in the mss as $t \to \infty$.

Appendix B
Proof of theorem 2

1) From (52), solving the following Nash min-max game problem:

$$
\begin{aligned}
J = &\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} \\
&E\{\int_0^{t_f} \bar{\bar e}^T(t)Q\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) - \rho^2\bar v^T(t)\bar v(t)\,dt\} \\
=&\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} \\
&E\{\int_0^{t_f} \bar{\bar e}^T(t)Q\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) - \rho^2\bar v^T(t)\bar v(t) \\
&+ \frac{d}{dt}V(\bar{\bar e}(t),\tilde\theta_{f,g,h}(t))\,dt + V(\bar{\bar e}(0),\tilde\theta_{i,g,h}(0)) \\
&- V(\bar{\bar e}(t_f),\tilde\theta_{f,g,h}(t_f))\} \\
=&\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} \\
&E\{V(\bar{\bar e}(0),\tilde\theta_{f,g,h}(0)) - V(\bar{\bar e}(t_f),\tilde\theta_{f,g,h}(t_f)) \\
&+ \int_0^{t_f} \frac{1}{2}\bar{\bar e}^T(t)(\bar A^TP + P\bar A + 2Q)\bar{\bar e}(t) \\
&+ u_\infty^T(t)Ru_\infty(t) + \bar{\bar e}^T(t)PB_1\Xi^T(\hat{\bar x}(t))\tilde\theta_f(t) \\
&+ \bar{\bar e}^T(t)PB_1\Xi^T(\hat{\bar x}(t))\tilde\theta_g(t)u(t) \\
&+ \bar{\bar e}^T(t)PB_2\Xi^T(\hat{\bar x}(t))\tilde\theta_h(t) + tr(\frac{1}{\gamma_1}\tilde\theta_f(t)\dot{\tilde\theta}_f^T(t)) \\
&+ tr(\frac{1}{\gamma_2}\tilde\theta_g(t)\dot{\tilde\theta}_g^T(t)) + tr(\frac{1}{\gamma_3}\tilde\theta_h(t)\dot{\tilde\theta}_h^T(t)) \\
&+ \bar{\bar e}^T(t)PB_4\bar v(t) + \bar{\bar e}^T(t)PB_3u_\infty(t) - \rho^2\bar v^T(t)\bar v(t)\,dt\}
\end{aligned} \tag{A5}
$$

Substituting (55) to (A5) and completing squares, the worst-case disturbance $\bar v^*(t)$ in (57) and the optimal control input $u_\infty(t)$ in (56) are determined by the Nash min-max game as follows

$$
\begin{aligned}
J = &\min_{u_\infty(t)} \max_{\bar v(t)\in L_F^2[0,t_f)} \\
&E\{V(\bar{\bar e}(0),\tilde\theta_{f,g,h}(0)) - V(\bar{\bar e}(t_f),\tilde\theta_{f,g,h}(t_f)) \\
&+ \int_0^{t_f} \frac{1}{2}\bar{\bar e}^T(t)(\bar A^TP + P\bar A + 2Q)\bar{\bar e}(t) + u_\infty^T(t)Ru_\infty(t) \\
&+ \bar{\bar e}^T(t)PB_3u_\infty(t) + \bar{\bar e}^T(t)PB_4\bar v(t) - \rho^2\bar v^T(t)\bar v(t)\,dt\}
\end{aligned}
$$

$$
\begin{aligned}
&= \min_{u_\infty(t)} \max_{\bar{v}(t)\in L_F^2[0,t_f)} \\
&\quad E\{V(\bar{\bar{e}}(0),\tilde{\theta}_{f,g,h}(0)) - V(\bar{\bar{e}}(t_f),\tilde{\theta}_{f,g,h}(t_f)) \\
&\quad + \int_0^{t_f} \frac{1}{2}\bar{\bar{e}}^T(t)(\bar{A}^T P + P\bar{A} + 2Q - \frac{1}{2}PB_3R^{-1}B_3^TP \\
&\quad + \frac{1}{2\rho^2}PB_4B_4^TP)\bar{\bar{e}}(t) \\
&\quad + [\frac{1}{2}B_3^TP\bar{\bar{e}}(t) + Ru_\infty(t)]^T R^{-1}[\frac{1}{2}B_3^TP\bar{\bar{e}}(t) + Ru_\infty(t)] \\
&\quad - [\frac{1}{2\rho}B_4^TP\bar{\bar{e}}(t) - \rho\bar{v}(t)]^T[\frac{1}{2\rho}B_4^TP\bar{\bar{e}}(t) - \rho\bar{v}(t)]\,dt\} \\
&= E\{V(\bar{\bar{e}}(0),\tilde{\theta}_{f,g,h}(0)) - V(\bar{\bar{e}}(t_f),\tilde{\theta}_{f,g,h}(t_f)) \\
&\quad + \int_0^{t_f} \frac{1}{2}\bar{\bar{e}}^T(t)(\bar{A}^T P + P\bar{A} + 2Q - \frac{1}{2}PB_3R^{-1}B_3^TP \\
&\quad + \frac{1}{2\rho^2}PB_4B_4^TP)\bar{\bar{e}}(t)\,dt\}
\end{aligned}
\tag{A6}
$$

According to (58) and Lyapunov function $V(\bar{\bar{e}}(t_f),\tilde{\theta}_{f,g,h}(t_f)) \geq 0$, we obtain

$$
J \leq E\{V(\bar{\bar{e}}(0),\tilde{\theta}_{f,g,h}(0))\} \tag{A7}
$$

Therefore, the proof of (52) is completed.

2) (A7) is equivalent to

$$
\begin{aligned}
&\min_{u_\infty(t)} \max_{\bar{v}(t)\in L_F^2[0,\infty)} E\{\int_0^\infty \bar{\bar{e}}^T(t)Q\bar{\bar{e}}(t) + u_\infty^T(t)Ru_\infty(t)\,dt\} \\
&\leq E\{V(\bar{\bar{e}}(0),\tilde{\theta}_{f,g,h}(0)) + \int_0^\infty \rho^2\bar{v}^T(t)\bar{v}(t)\,dt\}
\end{aligned}
\tag{A8}
$$

Since $\bar{v}(t) \in L_F^2[0,\infty)$, i.e. $E\{\int_0^\infty \rho^2\bar{v}^T(t)\bar{v}(t)\,dt\} < \infty$ for some finite ρ, the r.h.s of (A8) is finite. Therefore, $\bar{\bar{e}}(t) \to 0$ and $u_\infty(t) \to 0$ in the mss as $t \to \infty$.