Fluïdummechanica

Behoudsvergelijkingen langs stroomlijnen

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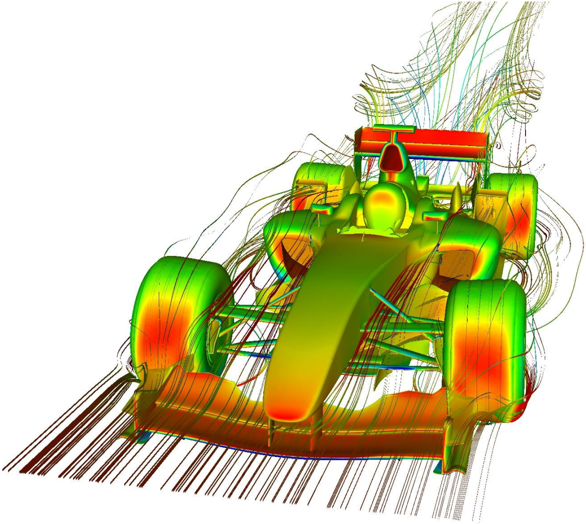
16 september 2015

Inhoud

1. Inleiding
2. Bewegingsvergelijking
3. Bernoulli

Voorbeeld



Bron: http://www.dalco.ch/

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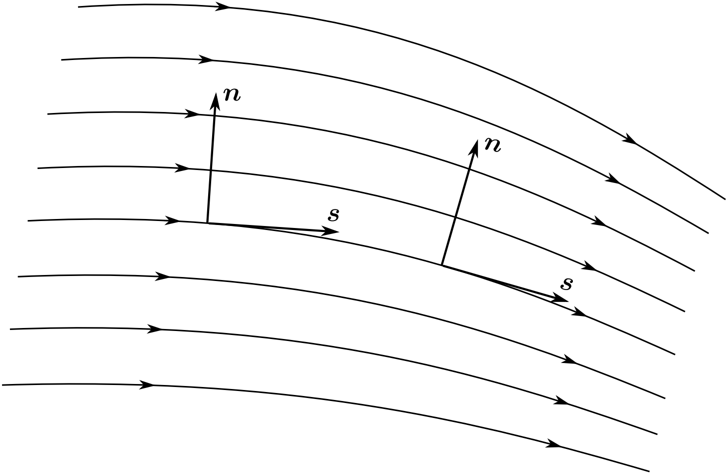
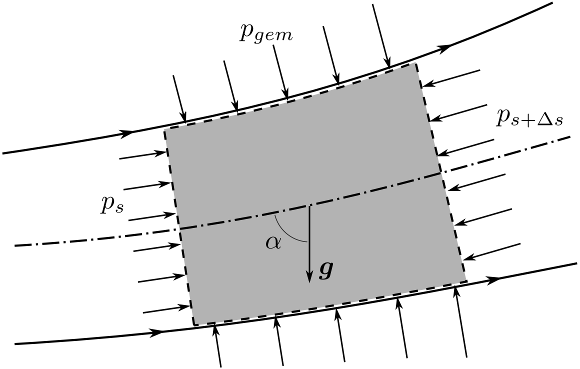
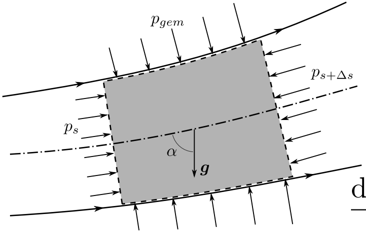
Stroomlijncoordinaten



Bewegingsvergelijking

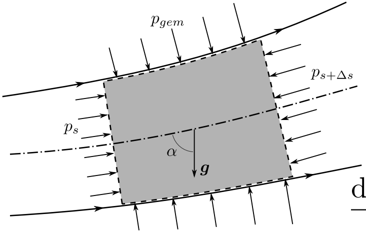


Bewegingsvergelijking



$$\frac{\mathrm{d}\boldsymbol{P}_{CV}}{\mathrm{d}t} + \dot{\boldsymbol{P}}_{\partial CV} = \boldsymbol{F}$$

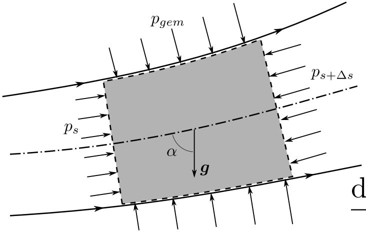
Bewegingsvergelijking



$$\frac{\mathrm{d}\boldsymbol{P}_{CV}}{\mathrm{d}t} + \dot{\boldsymbol{P}}_{\partial CV} = \boldsymbol{F}$$

$$\rho v v_\perp A\big|_{s+\Delta s} - \rho v v_\perp A\big|_{s} = pA\big|_{s} - pA\big|_{s+\Delta s} + p_{gem}\left(A\big|_{s+\Delta s} - A\big|_{s}\right) - \rho g A_{gem} \Delta s \cos\alpha$$

Bewegingsvergelijking

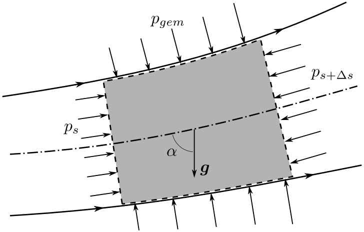


$$\frac{\mathrm{d}\boldsymbol{P}_{CV}}{\mathrm{d}t} + \dot{\boldsymbol{P}}_{\partial CV} = \boldsymbol{F}$$

$$\begin{aligned}\rho v v_\perp A\big|_{s+\Delta s} - \rho v v_\perp A\big|_s &= pA\big|_s - pA\big|_{s+\Delta s} \\ &\quad + p_{gem}\left(A\big|_{s+\Delta s} - A\big|_s\right) - \rho g A_{gem} \Delta s \cos\alpha\end{aligned}$$

$$\begin{aligned}\frac{\rho v v A\big|_{s+\Delta s} - \rho v v A\big|_s}{\Delta s} &= -\frac{pA\big|_{s+\Delta s} - pA\big|_s}{\Delta s} \\ &\quad + p_{gem}\frac{A\big|_{s+\Delta s} - A\big|_s}{\Delta s} - \rho g A_{gem}\frac{z\big|_{s+\Delta s} - z\big|_s}{\Delta s}\end{aligned}$$

Bewegingsvergelijking



$$\frac{\mathrm{d}\rho v v A}{\mathrm{d}s} = -\frac{\mathrm{d}pA}{\mathrm{d}s} + p\frac{\mathrm{d}A}{\mathrm{d}s} - \rho g A\frac{\mathrm{d}z}{\mathrm{d}s}$$

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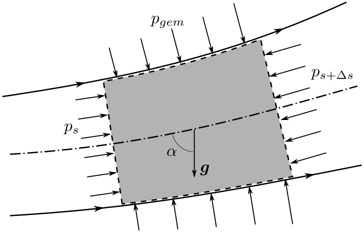

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$$\rho v A \frac{\mathrm{d}v}{\mathrm{d}s} + v\frac{\mathrm{d}\rho v A}{\mathrm{d}s} = -A\frac{\mathrm{d}p}{\mathrm{d}s} - p\frac{\mathrm{d}A}{\mathrm{d}s} + p\frac{\mathrm{d}A}{\mathrm{d}s} - \rho g A \frac{\mathrm{d}z}{\mathrm{d}s}$$

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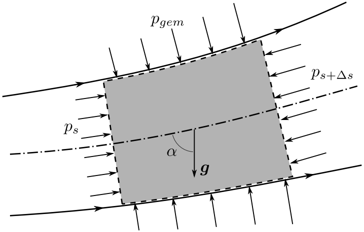

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$$\rho v \frac{\mathrm{d}v}{\mathrm{d}s} + \frac{\mathrm{d}p}{\mathrm{d}s} + \rho g \frac{\mathrm{d}z}{\mathrm{d}s} = 0 \tag{1}$$

Deeltjesversnelling

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \tag{2}$$

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$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\nabla p + \rho \boldsymbol{g} \tag{3}$$

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Integratie van de bewegingsvergelijking

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}s} + \frac{\mathrm{d}p}{\mathrm{d}s} + \rho g \frac{\mathrm{d}z}{\mathrm{d}s} = 0$$

Integratie van de bewegingsvergelijking

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}s} + \frac{\mathrm{d}p}{\mathrm{d}s} + \rho g \frac{\mathrm{d}z}{\mathrm{d}s} = 0$$

$$\int \rho v \frac{\mathrm{d}v}{\mathrm{d}s}\mathrm{d}s + \int \frac{\mathrm{d}p}{\mathrm{d}s}\mathrm{d}s + \int \rho g \frac{\mathrm{d}z}{\mathrm{d}s}\mathrm{d}s = \mathrm{Cst}$$

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$$\Downarrow \quad \rho = \mathrm{Cst}$$

$$\frac{1}{2}\rho v^2 + p + \rho g z = \mathrm{Cst} \tag{4}$$

Bernoulli

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- stationaire stroming

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- langs een stroomlijn

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- constante dichtheid

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Mechanisceh arbeid van een deeltje

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}s} = -\frac{\mathrm{d}p}{\mathrm{d}s} - \rho g \frac{\mathrm{d}z}{\mathrm{d}s}$$

$$W = \int_1^2 F \mathrm{d}s$$

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$$\int_1^2 \rho v \frac{\mathrm{d}v}{\mathrm{d}s} \mathrm{d}s = -\int_1^2 \frac{\mathrm{d}p}{\mathrm{d}s} \mathrm{d}s - \int_1^2 \rho g \frac{\mathrm{d}y}{\mathrm{d}s} \mathrm{d}s$$

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$$\rho \frac{1}{2}(v_2^2 - v_1^2) = -(p_2 - p_1) - \rho g (z_2 - z_1)$$

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$$\rho \frac{1}{2}(v_2^2 - v_1^2) + (p_2 - p_1) + \rho g (z_2 - z_1) = 0 \tag{5}$$

Energiebeschouwingen en irreversibiliteit

$$\dot{m}(u_u + \frac{p_u}{\rho_u} + \frac{1}{2}v_u^2 + gz_u) - \dot{m}(u_i + \frac{p_i}{\rho_i} + \frac{1}{2}v_i^2 + gz_i) = \dot{Q} - \dot{W}_a$$

Energiebeschouwingen en irreversibiliteit

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$$\rho u + p + \frac{1}{2}\rho v^2 + \rho g z = \mathrm{Cst}$$

Energiebeschouwingen en irreversibiliteit

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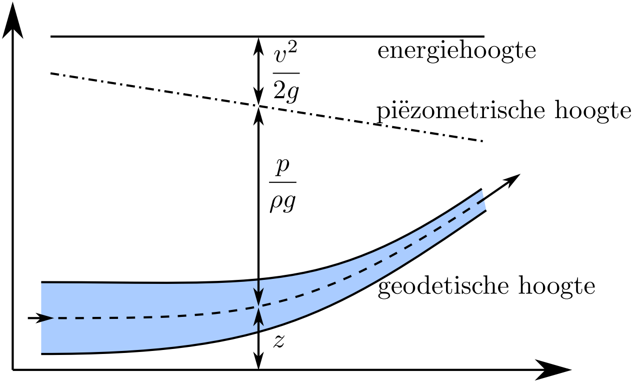
$$\rho u + p + \frac{1}{2}\rho v^2 + \rho g z = \mathrm{Cst}$$

vs

$$p + \frac{1}{2}\rho v^2 + \rho g z = \mathrm{Cst}$$

Grafische voorstelling



Toepassing