Fluïdummechanica

Grenslagen en turbulentie

Brecht Baeten[1]

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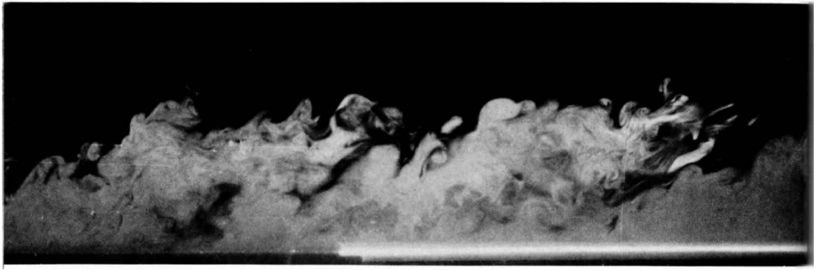
24 november 2016

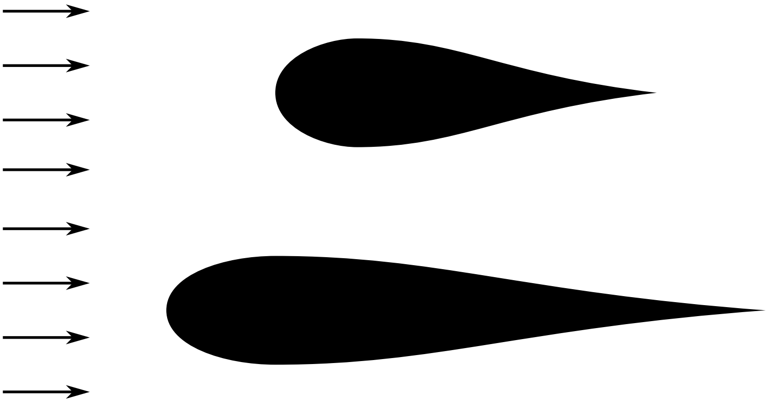
Inhoud

Voorbeeld



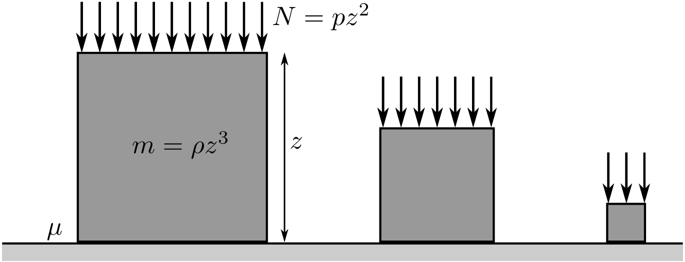
Bron: An Album of Fluid Motion (Van Dyke)

Voorbeeld



Inhoud

No-slip randvoorwaarde



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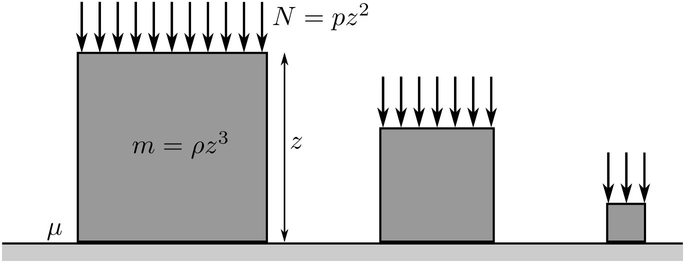

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -\mu N$$

No-slip randvoorwaarde

Bron: https://www.youtube.com/watch?v=cUTkqZeiMow

Het ontstaan van een grenslaag

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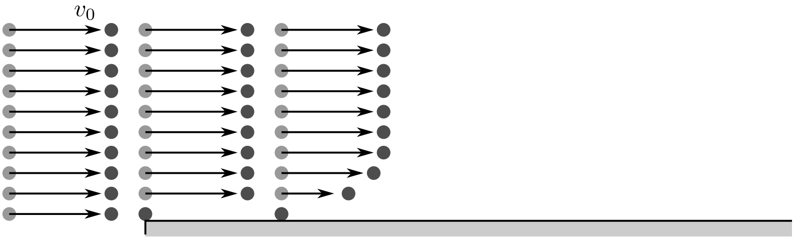

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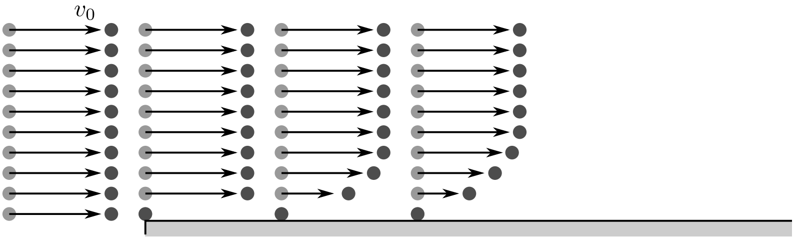

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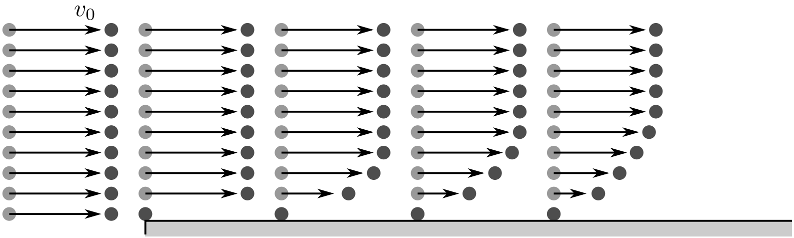

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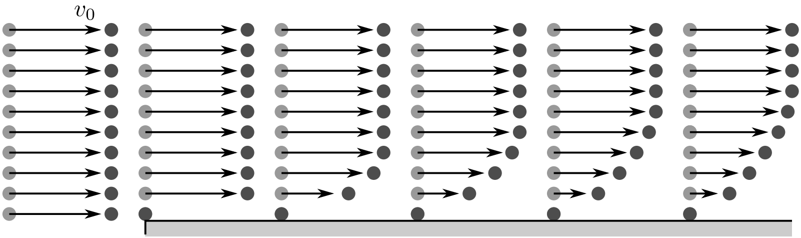

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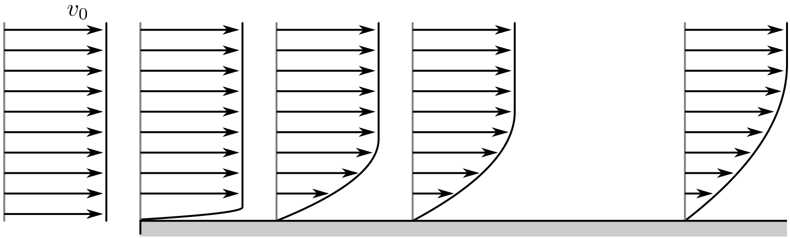

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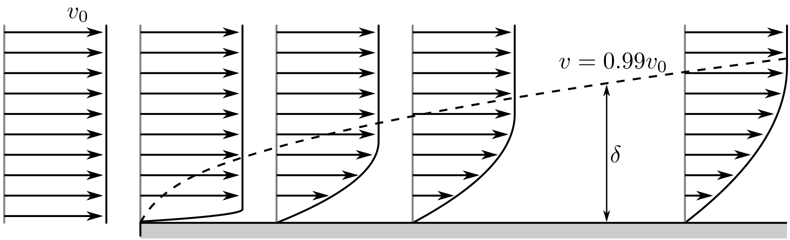

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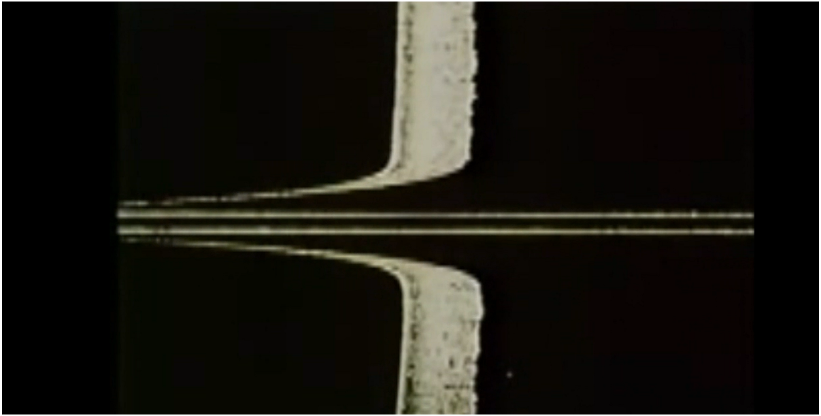

Bron: https://www.youtube.com/watch?v=7SkWxEUXIoM

Snelheidsprofiel

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \left(\frac{\partial^2 v_x}{\partial x^2} \right)$$

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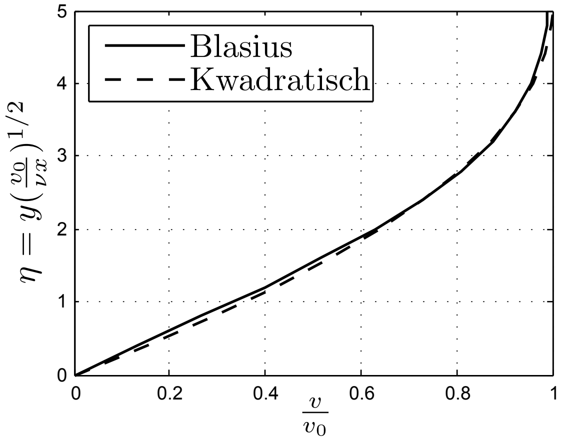
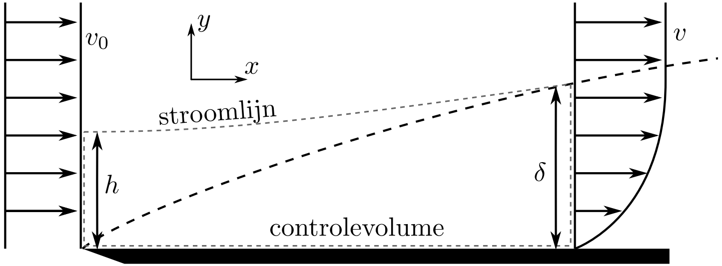


Impulsbalans in een laminaire grenslaag



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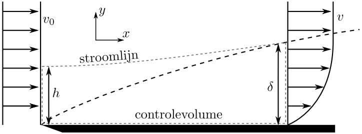

$$-F_d = \int_0^\delta \rho v(y)^2 b\mathrm{d}y - \int_0^h \rho v_0^2 b\mathrm{d}y$$

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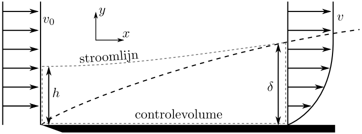

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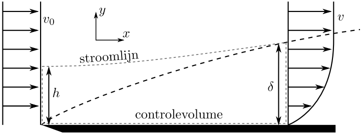

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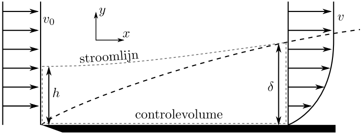

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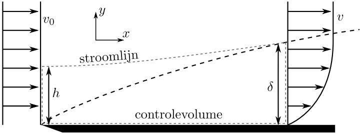

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$$F_d = \rho b \int_0^\delta v(y)(v_0 - v(y))\mathrm{d}y$$

Inhoud

Turbulente grenslaag

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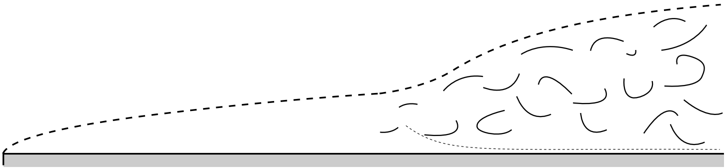

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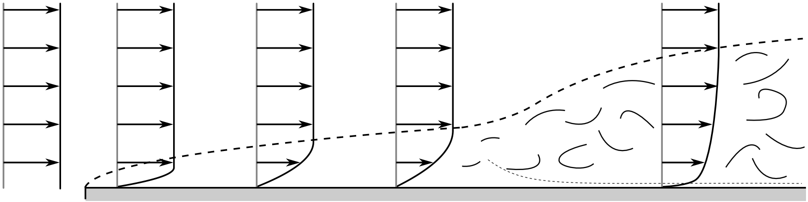

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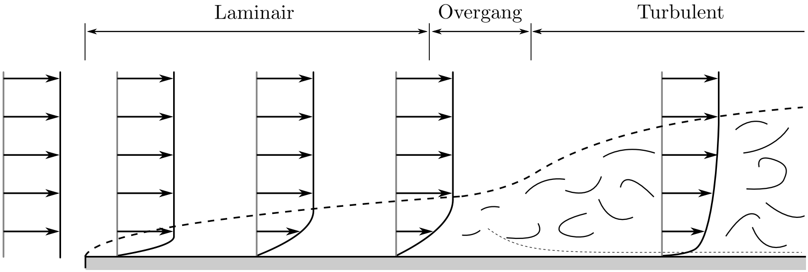
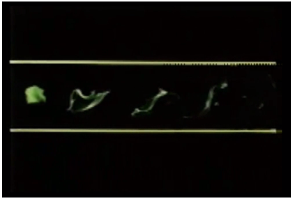
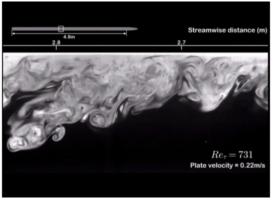
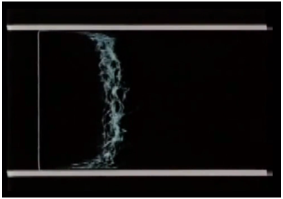
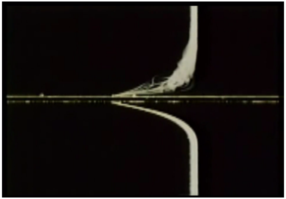


Turbulentie









Bron: https://www.youtube.com/watch?v=1_oyqLOqwnI
Bron: https://www.youtube.com/watch?v=e1TbkLIDWys Bron:
https://www.youtube.com/watch?v=wMxK2GtFFq0

Turbulentie

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- Schijnbaar willekeurig

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- 3 dimensionaal

Laminaire stroming

Bron: https://www.youtube.com/watch?v=p08_KlTKP50

Energiecascade

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = - \frac{\partial p}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right)$$

Energiecascade

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Stel:

$$v_x=\sin\omega x$$

Energiecascade

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$$v_x\frac{\partial v_x}{\partial x}=\omega\sin(\omega x)\cos(\omega x)=\frac{1}{2}\omega\sin(2\omega x)$$

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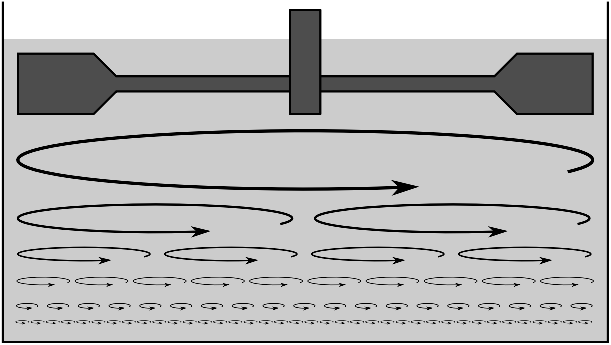

Turbulentie en het Reynoldsgetal

$$\text{Re} = \frac{vx}{\nu} = \frac{\text{traagheidskracht}}{\text{viskeuze krachten}}$$

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$$\text{Re klein} \Longrightarrow \text{Laminair}$$

Turbulentie en het Reynoldsgetal

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Dus

$$\text{Re klein} \implies \text{Laminair}$$

$$\text{Re groot} \implies \text{Turbulent}$$