

Fluïdummechanica

Controle volumes

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16 september 2015

Inhoud

- 1 Inleiding
- 2 Controle massa's
- 3 Controle volumes
- 4 Controle volume met één instroming en één uitstroming

Voorbeeld



Bron: <http://www.nasa.gov/>

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Mechanica en Thermodynamica

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Behoud van massa

$$\frac{dm}{dt} = 0 \quad (1)$$

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$$\frac{d\mathbf{P}}{dt} = \mathbf{F} \quad (2)$$

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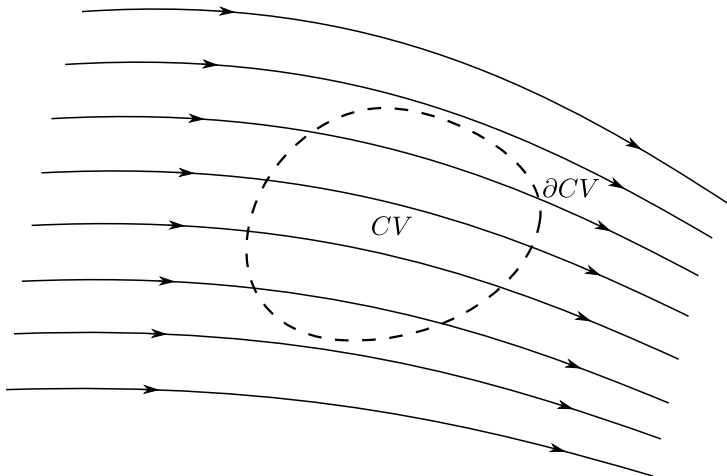
Behoud van energie

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \quad (3)$$

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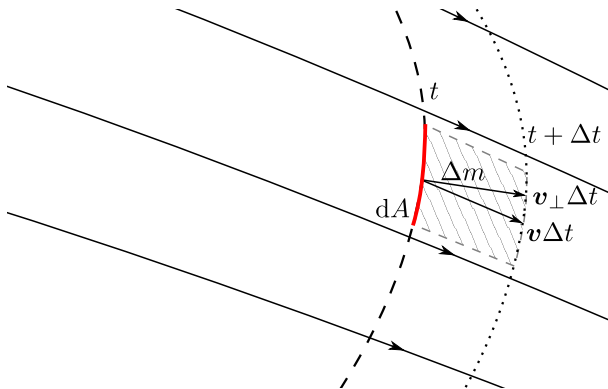
Controlevolume



Behoud van massa

$$\left[\begin{array}{c} \text{De verandering} \\ \text{van massa in} \\ \text{het controlevolume} \end{array} \right] + \left[\begin{array}{c} \text{De netto} \\ \text{massastroom uit} \\ \text{het controlevolume} \end{array} \right] = 0$$

Massastroom



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$$\Delta m = \rho \Delta x_{\perp} dA$$

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$$d\dot{m} = \rho v_{\perp} dA \quad (4)$$

Behoud van massa

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$$\frac{dm_{CV}}{dt} + \dot{m}_{\partial CV} = 0 \quad (5)$$

Behoud van impuls

$$\begin{bmatrix} \text{De verandering} \\ \text{van impuls} \\ \text{in het} \\ \text{controlevolume} \end{bmatrix} + \begin{bmatrix} \text{De netto} \\ \text{impulsstroom} \\ \text{uit het} \\ \text{controlevolume} \end{bmatrix} = \begin{bmatrix} \text{De totale} \\ \text{kracht} \\ \text{op het} \\ \text{controlevolume} \end{bmatrix}$$

Behoud van impuls

$$\left[\begin{array}{c} \text{De verandering} \\ \text{van impuls} \\ \text{in het} \\ \text{controlevolume} \end{array} \right] + \left[\begin{array}{c} \text{De netto} \\ \text{impulsstroom} \\ \text{uit het} \\ \text{controlevolume} \end{array} \right] = \left[\begin{array}{c} \text{De totale} \\ \text{kracht} \\ \text{op het} \\ \text{controlevolume} \end{array} \right]$$

$$\frac{d\mathbf{P}_{CV}}{dt} + \dot{\mathbf{P}}_{\partial CV} = \mathbf{F} \quad (6)$$

Behoud van energie

$$\left[\begin{array}{c} \text{De verandering} \\ \text{van energie} \\ \text{in het} \\ \text{controlevolume} \end{array} \right] + \left[\begin{array}{c} \text{De netto} \\ \text{energiestroom} \\ \text{uit het} \\ \text{controlevolume} \end{array} \right] = \left[\begin{array}{c} \text{De warmtesroom} \\ \text{toegevoegd en} \\ \text{arbeidsstroom} \\ \text{onttrokken aan} \\ \text{het controlevolume} \end{array} \right]$$

Behoud van energie

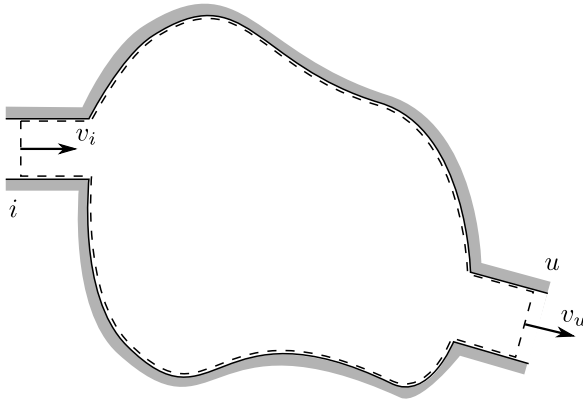
$$\left[\begin{array}{c} \text{De verandering} \\ \text{van energie} \\ \text{in het} \\ \text{controlevolume} \end{array} \right] + \left[\begin{array}{c} \text{De netto} \\ \text{energiestroom} \\ \text{uit het} \\ \text{controlevolume} \end{array} \right] = \left[\begin{array}{c} \text{De warmtestroom} \\ \text{toegevoegd en} \\ \text{arbeidsstroom} \\ \text{onttrokken aan} \\ \text{het controlevolume} \end{array} \right]$$

$$\frac{dE_{CV}}{dt} + \dot{E}_{\partial CV} = \dot{Q} - \dot{W} \quad (7)$$

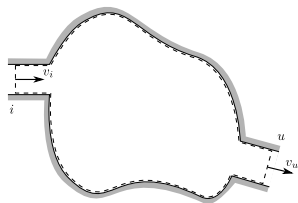
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Controle volume met één instroming en één uitstroming



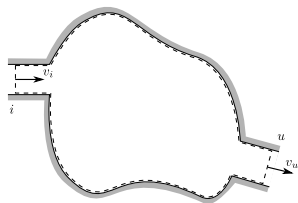
Controle volume met één instroming en één uitstroming



$$\rho_i v_{i,\perp} A_i = \rho_u v_{u,\perp} A_u$$

(8)

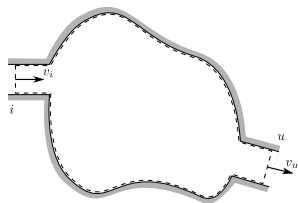
Controle volume met één instroming en één uitstroming



$$\rho_i v_{i,\perp} A_i = \rho_u v_{u,\perp} A_u \quad (8)$$

$$\begin{aligned} F_{x,R} &= p_u A_u n_{x,u} - p_i A_i n_{x,i} + \dot{m}(v_{x,u} - v_{x,i}) \\ F_{y,R} &= p_u A_u n_{y,u} - p_i A_i n_{y,i} + \dot{m}(v_{y,u} - v_{y,i}) \\ F_{z,R} &= p_u A_u n_{z,u} - p_i A_i n_{z,i} + \dot{m}(v_{z,u} - v_{z,i}) \end{aligned} \quad (9)$$

Controle volume met één instroming en één uitstroming

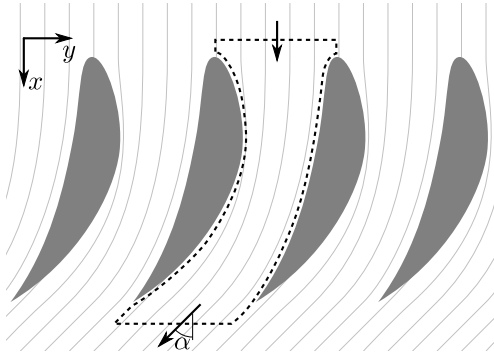


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$$\dot{m}\left(u_u + \frac{p_u}{\rho_u} + \frac{1}{2}v_u^2 + gz_u\right) - \dot{m}\left(u_i + \frac{p_i}{\rho_i} + \frac{1}{2}v_i^2 + gz_i\right) = \dot{Q} - \dot{W}_a \quad (10)$$

Toepassing



Bepaal de horizontale en verticale kracht op één schoep, veronderstel isotherme stroming zonder warmteoverdracht