Fluïdummechanica

Controle volumes

Brecht Baeten[1]

[1]KU Leuven, Technologie campus Diepenbeek,
e-mail: brecht.baeten@kuleuven.be

15 september 2015

Inhoud

Voorbeeld

Bron: http://www.nasa.gov/

Inhoud

Mechanica en Thermodynamica

Mechanica en Thermodynamica

Behoud van massa

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 0 \tag{1}$$

Mechanica en Thermodynamica

Behoud van massa

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 0 \tag{1}$$

Behoud van impuls

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = \mathbf{F} \tag{2}$$

Mechanica en Thermodynamica

Behoud van massa

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 0 \tag{1}$$

Behoud van impuls

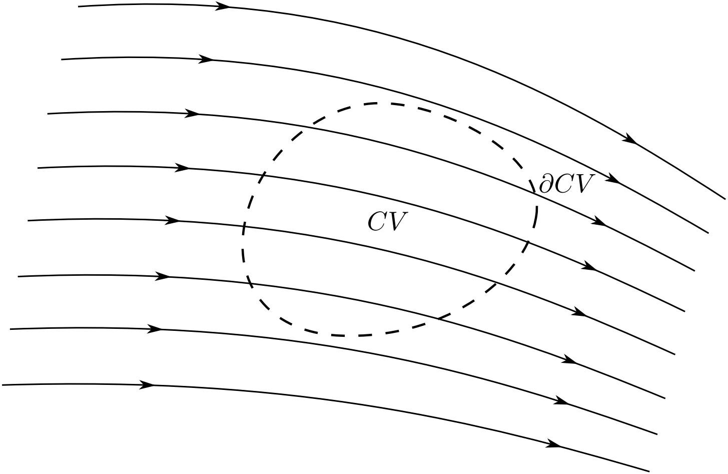
$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = \mathbf{F} \tag{2}$$

Behoud van energie

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \dot{Q} - \dot{W} \tag{3}$$

Inhoud

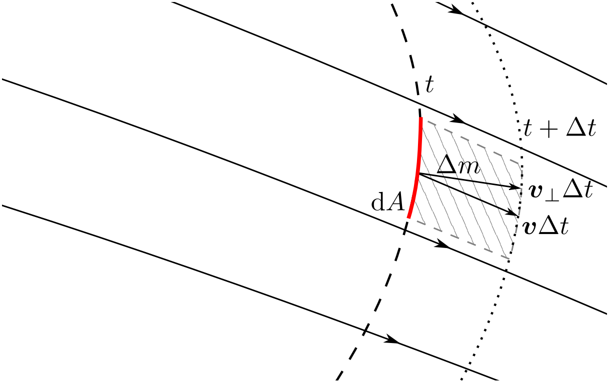
Controlevolume



Behoud van massa

$$\begin{bmatrix} \text{De verandering} \\ \text{van massa in} \\ \text{het controlevolume} \end{bmatrix} + \begin{bmatrix} \text{De netto} \\ \text{massastroom uit} \\ \text{het controlevolume} \end{bmatrix} = 0$$

Massastroom



Massastroom

$$\Delta m = \rho \Delta x_{\perp} \mathrm{d}A$$

Massastroom

$$\Delta m = \rho \Delta x_{\perp} \mathrm{d}A$$

$$\Delta m = \rho v_{\perp} \Delta t \mathrm{d}A$$

Massastroom

$$\Delta m = \rho \Delta x_\perp \mathrm{d}A$$

$$\Delta m = \rho v_\perp \Delta t \mathrm{d}A$$

$$\frac{\Delta m}{\Delta t} = \rho v_\perp \mathrm{d}A$$

Massastroom

$$\Delta m = \rho \Delta x_{\perp} \mathrm{d}A$$

$$\Delta m = \rho v_{\perp} \Delta t \mathrm{d}A$$

$$\frac{\Delta m}{\Delta t} = \rho v_{\perp} \mathrm{d}A$$

$$\mathrm{d}\dot{m} = \rho v_{\perp} \mathrm{d}A \tag{4}$$

Behoud van massa

$$\left[\begin{array}{c}\text{De verandering}\\ \text{van massa in}\\ \text{het controlevolume}\end{array}\right] + \left[\begin{array}{c}\text{De netto}\\ \text{massastroom uit}\\ \text{het controlevolume}\end{array}\right] = 0$$

Behoud van massa

$$\begin{bmatrix} \text{De verandering} \\ \text{van massa in} \\ \text{het controlevolume} \end{bmatrix} + \begin{bmatrix} \text{De netto} \\ \text{massastroom uit} \\ \text{het controlevolume} \end{bmatrix} = 0$$

$$\frac{\mathrm{d}m_{CV}}{\mathrm{d}t} + \dot{m}_{\partial CV} = 0 \tag{5}$$

Behoud van impuls

$$\begin{bmatrix} \text{De verandering} \\ \text{van impuls} \\ \text{in het} \\ \text{controlevolume} \end{bmatrix} + \begin{bmatrix} \text{De netto} \\ \text{impulsstroom} \\ \text{uit het} \\ \text{controlevolume} \end{bmatrix} = \begin{bmatrix} \text{De totale} \\ \text{kracht} \\ \text{op het} \\ \text{controlevolume} \end{bmatrix}$$

Behoud van impuls

$$\begin{bmatrix} \text{De verandering} \\ \text{van impuls} \\ \text{in het} \\ \text{controlevolume} \end{bmatrix} + \begin{bmatrix} \text{De netto} \\ \text{impulsstroom} \\ \text{uit het} \\ \text{controlevolume} \end{bmatrix} = \begin{bmatrix} \text{De totale} \\ \text{kracht} \\ \text{op het} \\ \text{controlevolume} \end{bmatrix}$$

$$\frac{\mathrm{d}\mathbf{P}_{CV}}{\mathrm{d}t} + \dot{\mathbf{P}}_{\partial CV} = \mathbf{F} \tag{6}$$

Behoud van energie

$$\left[\begin{array}{c}\text{De verandering}\\\text{van energie}\\\text{in het}\\\text{controlevolume}\end{array}\right]+\left[\begin{array}{c}\text{De netto}\\\text{energiestroom}\\\text{uit het}\\\text{controlevolume}\end{array}\right]=\left[\begin{array}{c}\text{De warmtesroom}\\\text{toegevoegd en}\\\text{arbeidsstroom}\\\text{onttrokken aan}\\\text{het controlevolume}\end{array}\right]$$

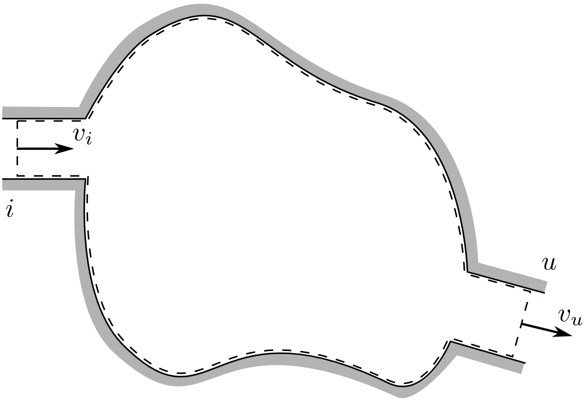
Behoud van energie

$$\begin{bmatrix} \text{De verandering} \\ \text{van energie} \\ \text{in het} \\ \text{controlevolume} \end{bmatrix} + \begin{bmatrix} \text{De netto} \\ \text{energiestroom} \\ \text{uit het} \\ \text{controlevolume} \end{bmatrix} = \begin{bmatrix} \text{De warmtesroom} \\ \text{toegevoegd en} \\ \text{arbeidsstroom} \\ \text{onttrokken aan} \\ \text{het controlevolume} \end{bmatrix}$$

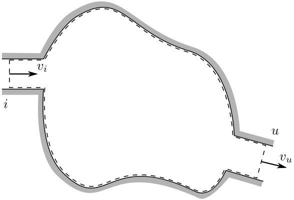
$$\frac{\mathrm{d}E_{CV}}{\mathrm{d}t} + \dot{E}_{\partial CV} = \dot{Q} - \dot{W} \tag{7}$$

Inhoud

Controle volume met één instroming en één uitstroming

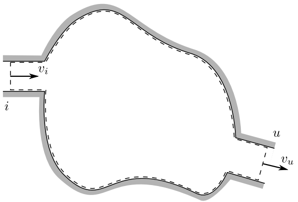


Controle volume met één instroming en één uitstroming



$$\rho_i v_{i,\perp} A_i = \rho_u v_{u,\perp} A_u \tag{8}$$

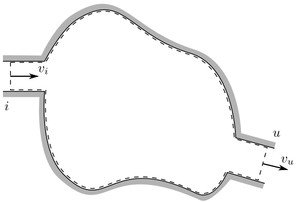
Controle volume met één instroming en één uitstroming



$$\rho_i v_{i,\perp} A_i = \rho_u v_{u,\perp} A_u \tag{8}$$

$$\begin{aligned} F_{x,R} &= p_u A_u n_{x,u} - p_i A_i n_{x,i} + \dot{m}(v_{x,u} - v_{x,i}) \\ F_{y,R} &= p_u A_u n_{y,u} - p_i A_i n_{y,i} + \dot{m}(v_{y,u} - v_{y,i}) \\ F_{z,R} &= p_u A_u n_{z,u} - p_i A_i n_{z,i} + \dot{m}(v_{z,u} - v_{z,i}) \end{aligned} \tag{9}$$

Controle volume met één instroming en één uitstroming

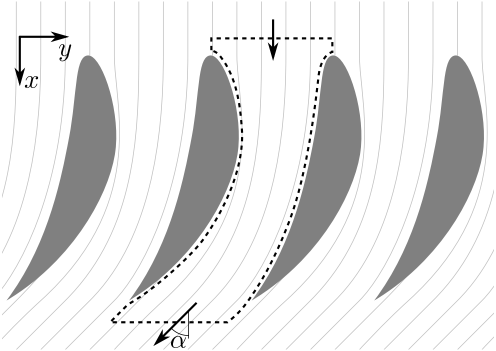


$$\rho_i v_{i,\perp} A_i = \rho_u v_{u,\perp} A_u \tag{8}$$

$$\begin{aligned}
F_{x,R} &= p_u A_u n_{x,u} - p_i A_i n_{x,i} + \dot{m}(v_{x,u} - v_{x,i}) \\
F_{y,R} &= p_u A_u n_{y,u} - p_i A_i n_{y,i} + \dot{m}(v_{y,u} - v_{y,i}) \\
F_{z,R} &= p_u A_u n_{z,u} - p_i A_i n_{z,i} + \dot{m}(v_{z,u} - v_{z,i})
\end{aligned} \tag{9}$$

$$\dot{m}\left(u_u + \frac{p_u}{\rho_u} + \frac{1}{2}v_u^2 + gz_u\right) - \dot{m}\left(u_i + \frac{p_i}{\rho_i} + \frac{1}{2}v_i^2 + gz_i\right) = \dot{Q} - \dot{W}_a \tag{10}$$

Toepassing



Bepaal de horizontale en verticale kracht op één schoep,
veronderstel isotherme stroming zonder warmteoverdracht