Fluïdummechanica

Grenslagen en turbulentie

Brecht Baeten[1]

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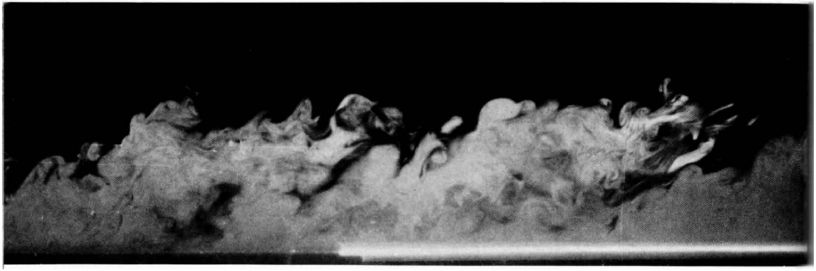
29 november 2016

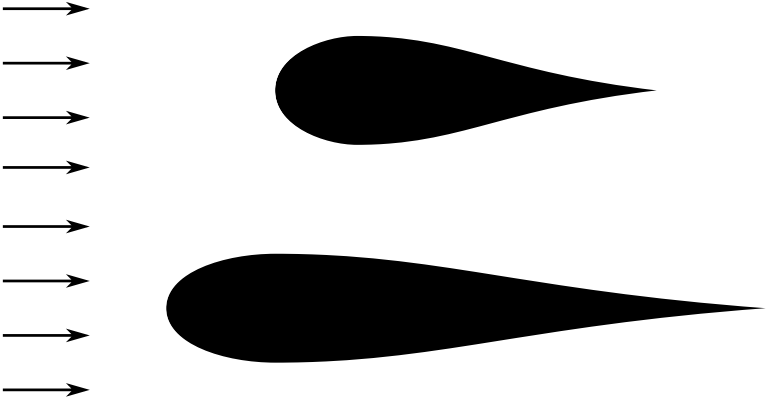
Inhoud

Voorbeeld



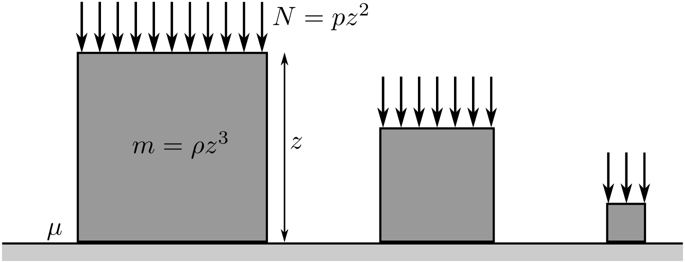
Bron: An Album of Fluid Motion (Van Dyke)

Voorbeeld



Inhoud

No-slip randvoorwaarde



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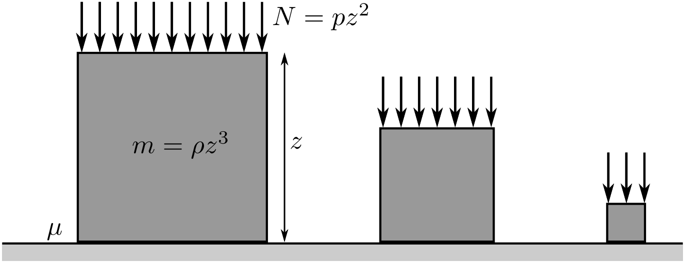

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -\mu N$$

No-slip randvoorwaarde

Bron: https://www.youtube.com/watch?v=cUTkqZeiMow

Het ontstaan van een grenslaag

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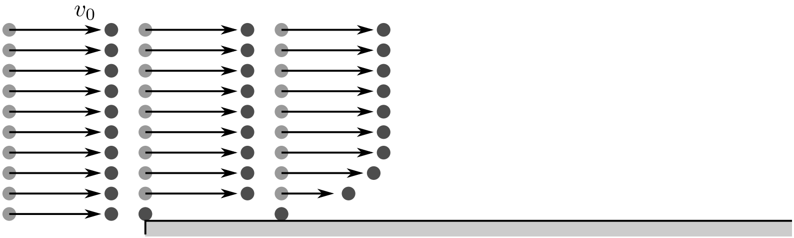

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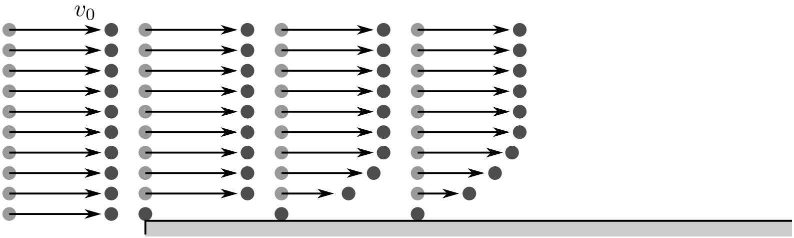

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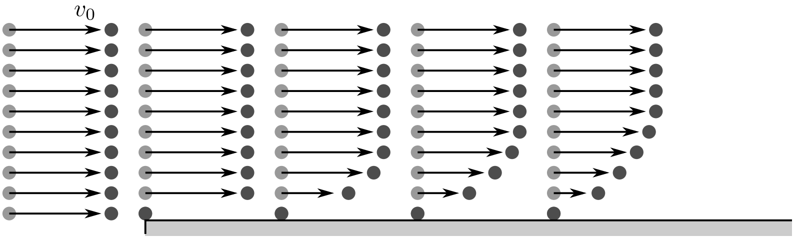

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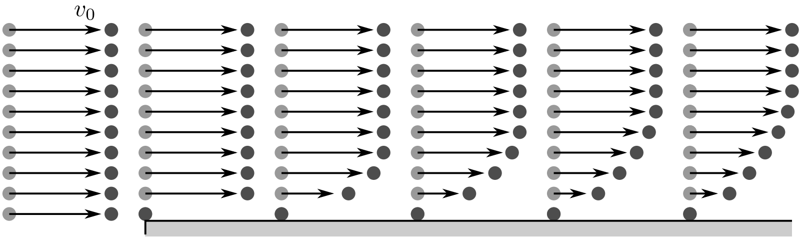

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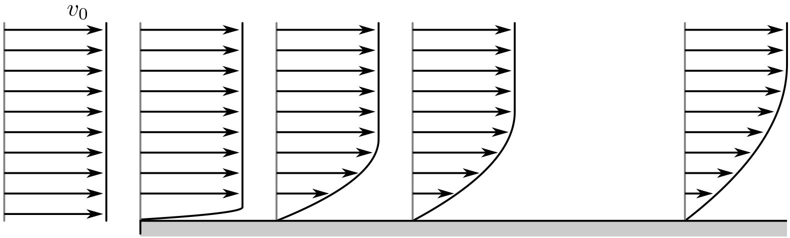

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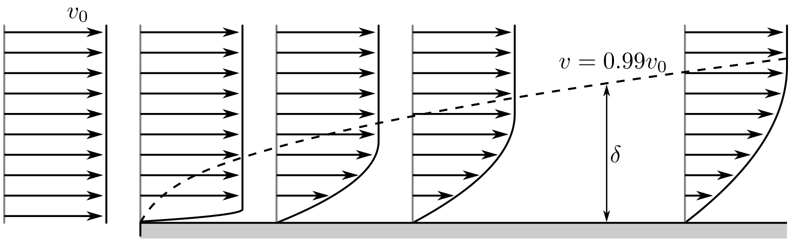

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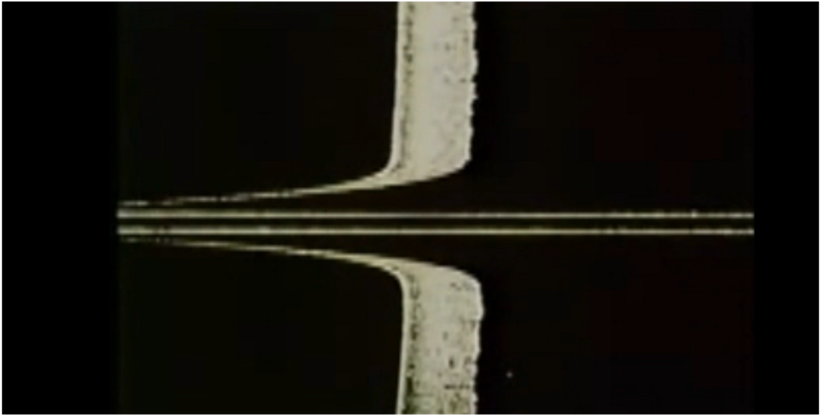

Bron: https://www.youtube.com/watch?v=7SkWxEUXIoM

Snelheidsprofiel

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \left(\frac{\partial^2 v_x}{\partial x^2} \right)$$

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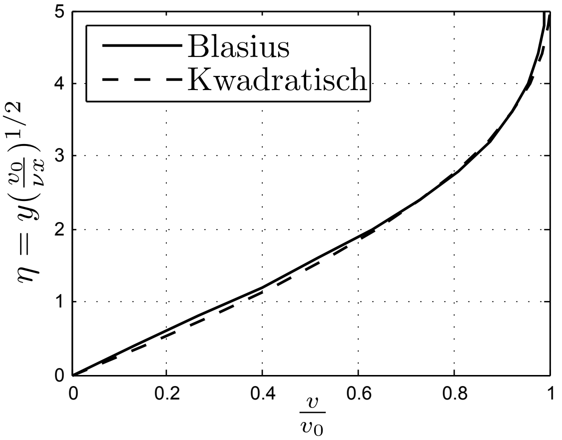
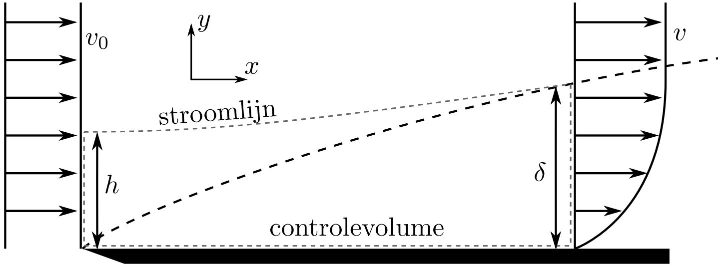


Impulsbalans in een laminaire grenslaag



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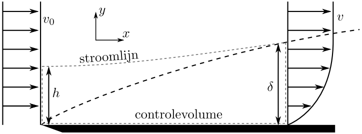

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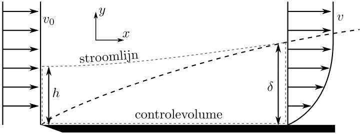

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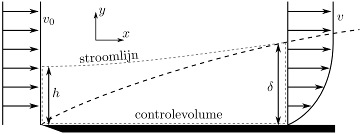

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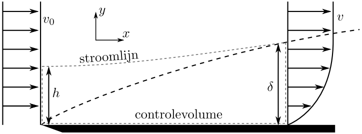

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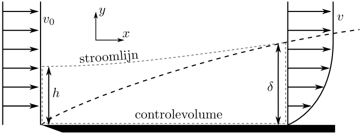

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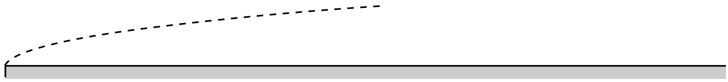
$$F_d = \rho b \int_0^\delta v(y)(v_0 - v(y))\mathrm{d}y$$

Inhoud

Turbulente grenslaag



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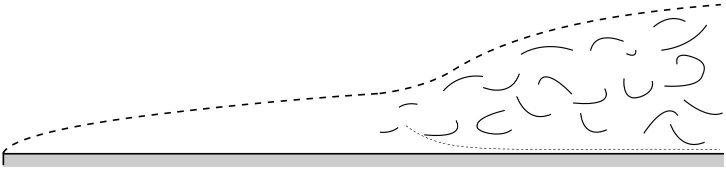

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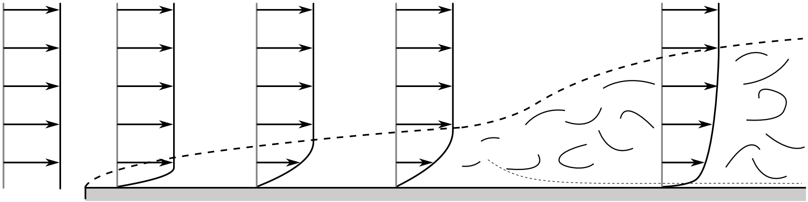

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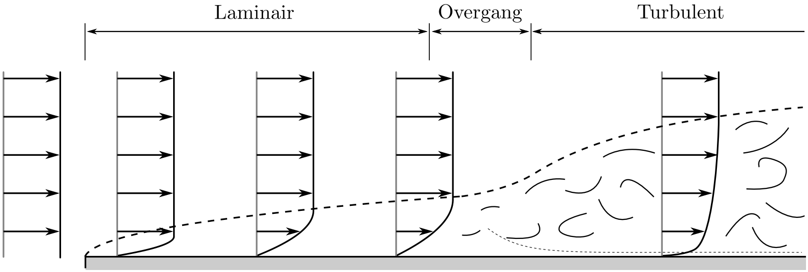
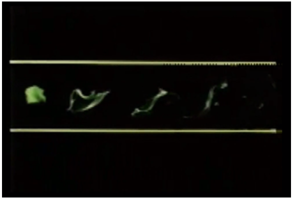
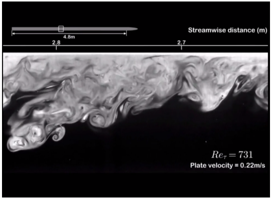
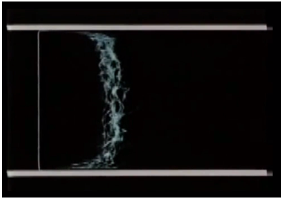
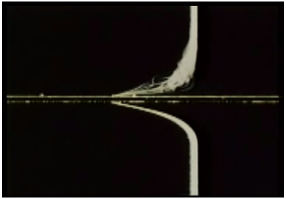


Turbulentie









Bron: https://www.youtube.com/watch?v=1_oyqLOqwnI
Bron: https://www.youtube.com/watch?v=e1TbkLIDWys Bron:
https://www.youtube.com/watch?v=wMxK2GtFFq0

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- Schijnbaar willekeurig

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- Zeer gevoelig aan begincondities
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- Variatie in tijd en lengte schalen
- 3 dimensionaal

Laminaire stroming

Bron: https://www.youtube.com/watch?v=p08_KlTKP50

Energiecascade

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)= -\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)$$

Energiecascade

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Stel:

$$v_x=\sin\omega x$$

Energiecascade

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Stel:

$$v_x = \sin \omega x$$

Dan:

$$v_x \frac{\partial v_x}{\partial x} = \omega \sin(\omega x) \cos(\omega x) = \frac{1}{2} \omega \sin(2\omega x)$$

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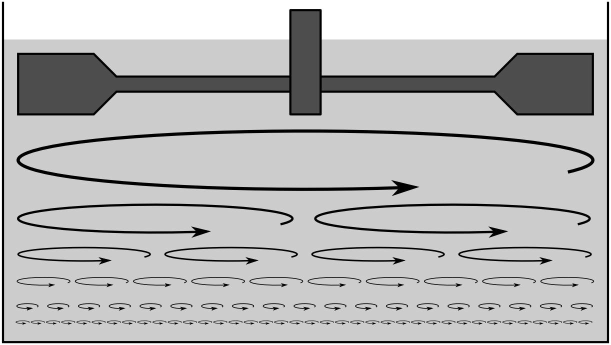

Turbulentie en het Reynoldsgetal

$$\mathrm{Re} = \frac{vx}{\nu} = \frac{\text{traagheidskracht}}{\text{viskeuze krachten}}$$

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$$\text{Re klein} \implies \text{Laminair}$$

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Dus

$$\text{Re klein} \implies \text{Laminair}$$

$$\text{Re groot} \implies \text{Turbulent}$$