Fluïdummechanica

Stroming in leidingen

Brecht Baeten[1]

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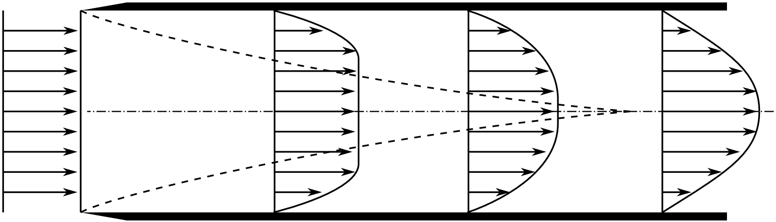
16 september 2016

Inhoud

Voorbeeld

Bron: http://www.etftrends.com/

Ontwikkelende stroming



Inhoud

Dimensieanalyse

$$\Delta p = \phi(L, D, v, \mu, \rho)$$

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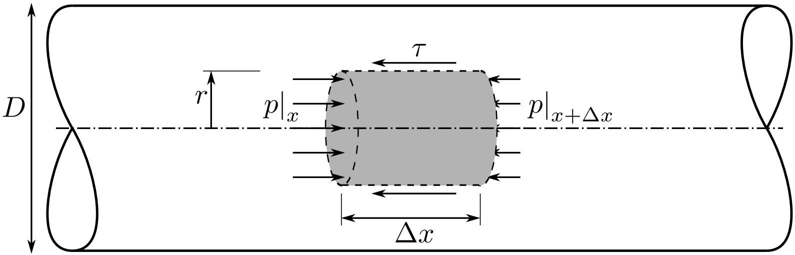
Inhoud

Snelheidsprofiel



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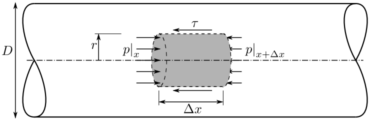

Behoud van impuls in de stromingsrichting:

$$F_x = 0$$

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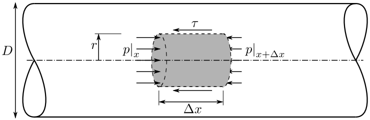

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$$p\pi r^2\big|_x - p\pi r^2\big|_{x+\Delta x} - \tau 2\pi r \Delta x = 0$$

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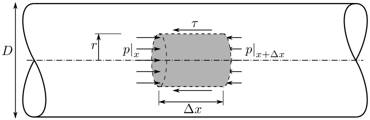

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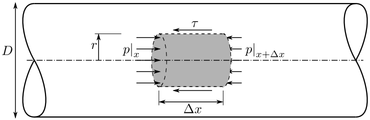

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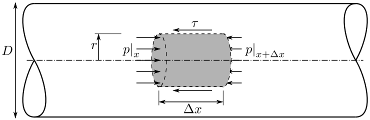
Newtoniaanse vloeistof:

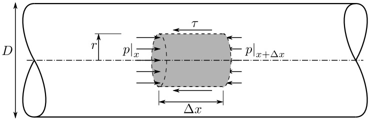
$$\frac{1}{2}\frac{\mathrm{d}p}{\mathrm{d}x} r = \mu \frac{\mathrm{d}v}{\mathrm{d}r}$$

Snelheidsprofiel



$$\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}r$$

Snelheidsprofiel



$$\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}r$$

$$v = \frac{1}{4\mu}\frac{\mathrm{d}p}{\mathrm{d}x}r^2 + C$$

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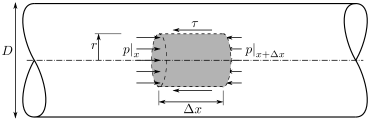

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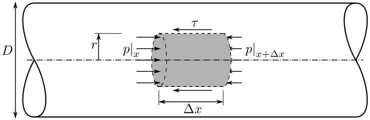

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$$v = -\frac{1}{4\mu}\frac{\mathrm{d}p}{\mathrm{d}x}R^2\left(1 - \frac{r^2}{R^2}\right) \tag{2}$$

Snelheidsprofiel

$$\frac{v}{v_{\max}} = \left(1 - \frac{r^2}{R^2}\right)$$

$$v_{\max} = -\frac{1}{4\mu}\frac{\mathrm{d}p}{\mathrm{d}x}R^2$$

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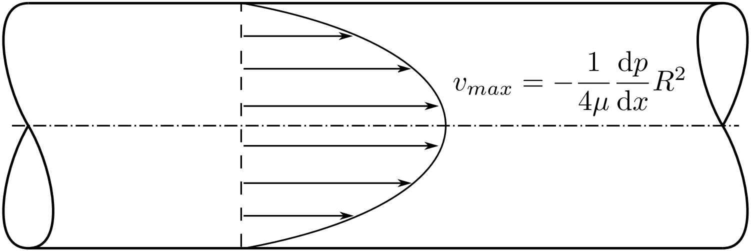
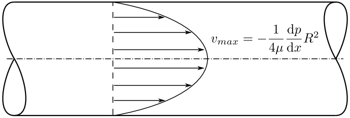


Gemiddelde snelheid



Debiet:

$$\dot{V} = 2\pi \int_0^R v_{\max}\left(1 - \frac{r^2}{R^2}\right) r\mathrm{d}r = v_{\max}\frac{\pi R^2}{2}$$

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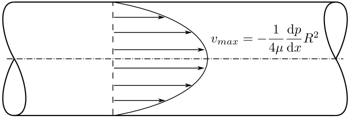

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Gemiddelde snelheid:

$$v_{\text{gem}} = \frac{\dot{V}}{\pi R^2} = \frac{v_{\max}}{2} = -\frac{1}{8\mu}\frac{\mathrm{d}p}{\mathrm{d}x}R^2 \qquad (2)$$

Drukval

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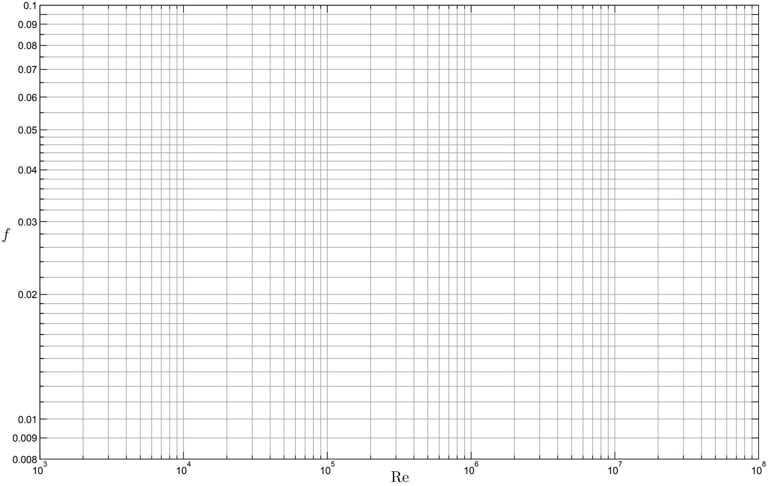
wrijvingsfactor voor laminaire stroming $f = \dfrac{64}{\mathrm{Re}}$

Inhoud

Empirische data



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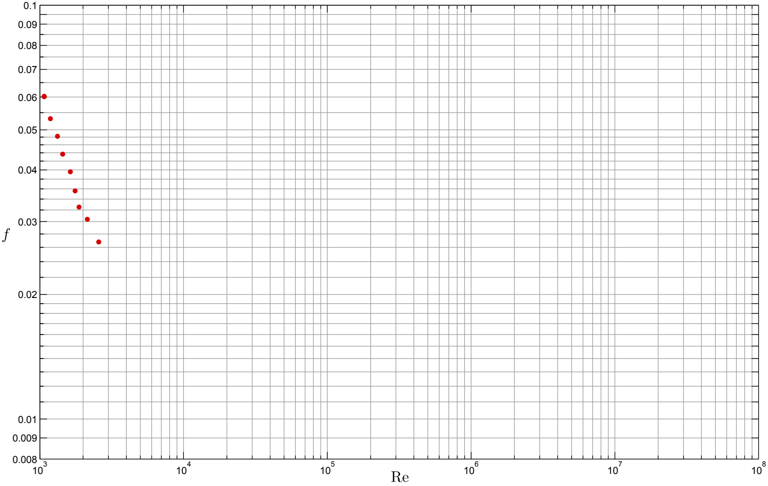

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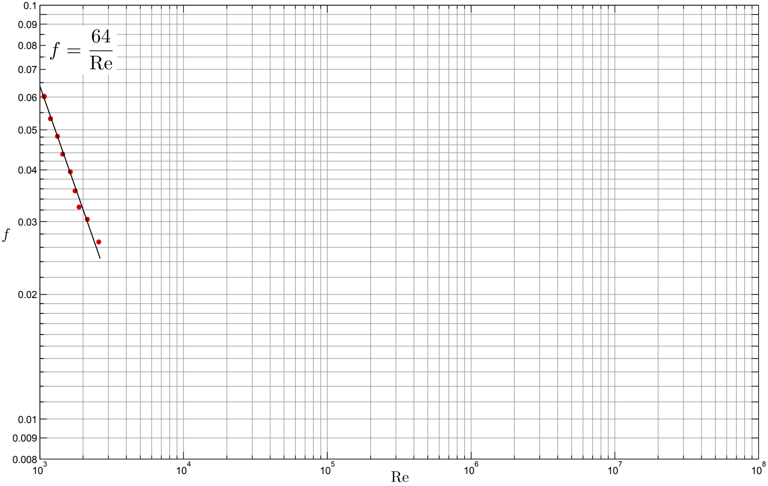

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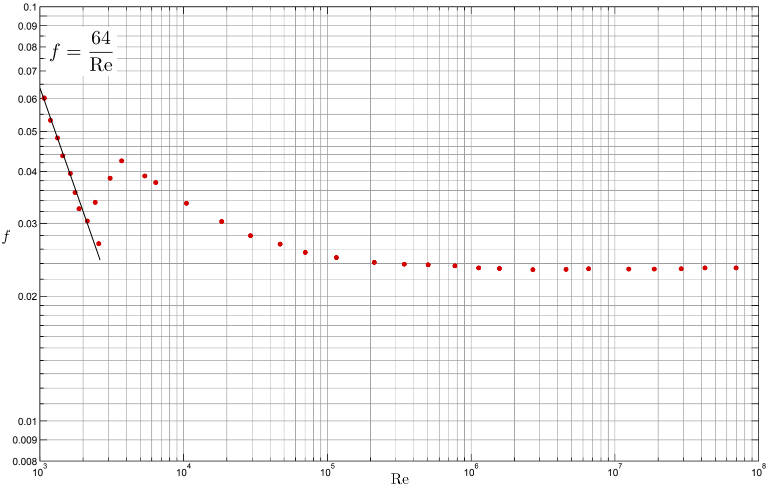

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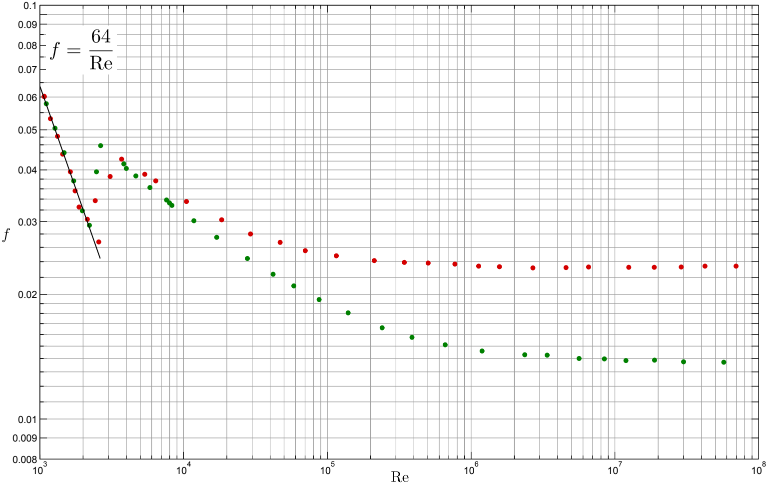

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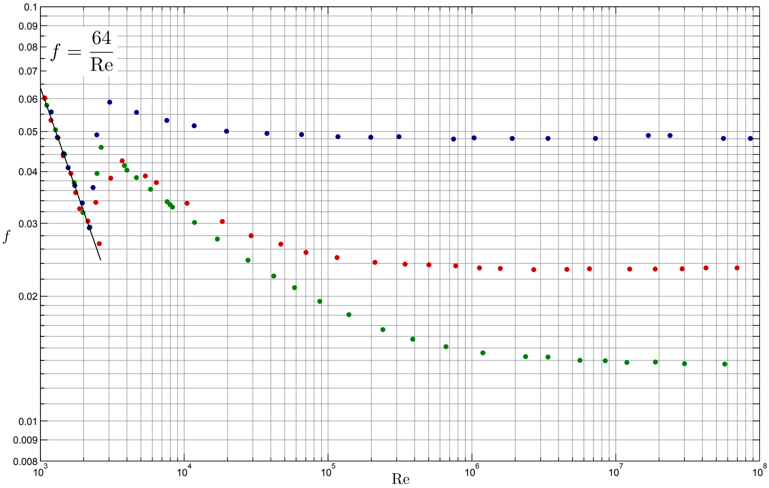

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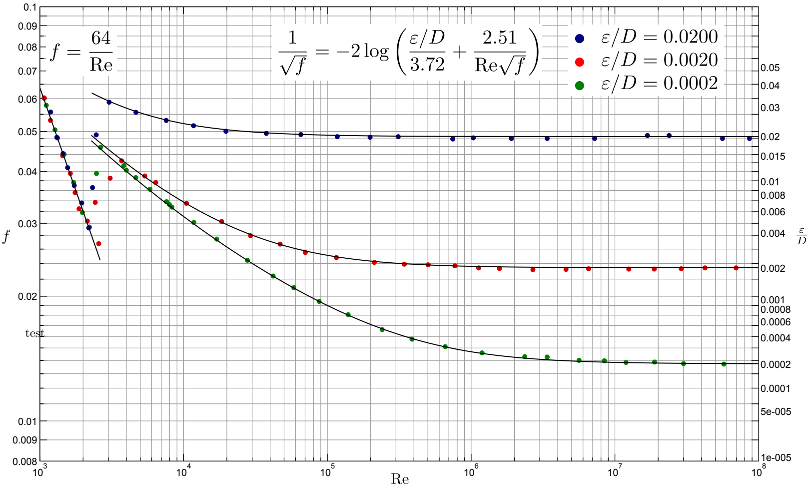

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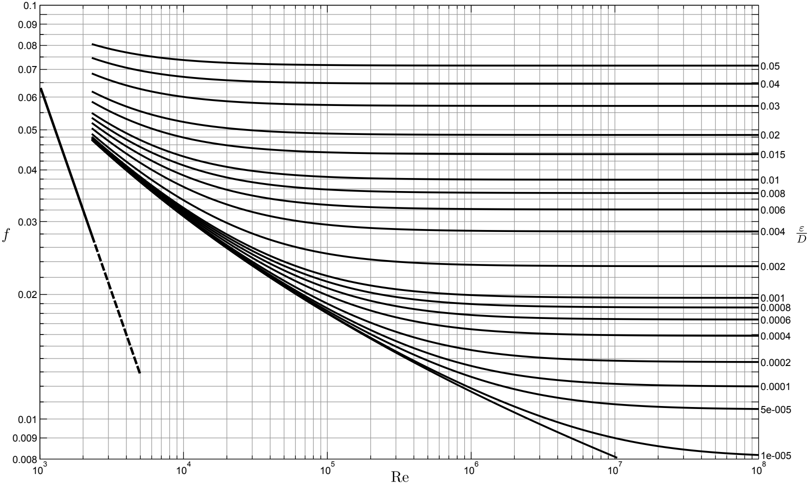

Dimensieanalyse

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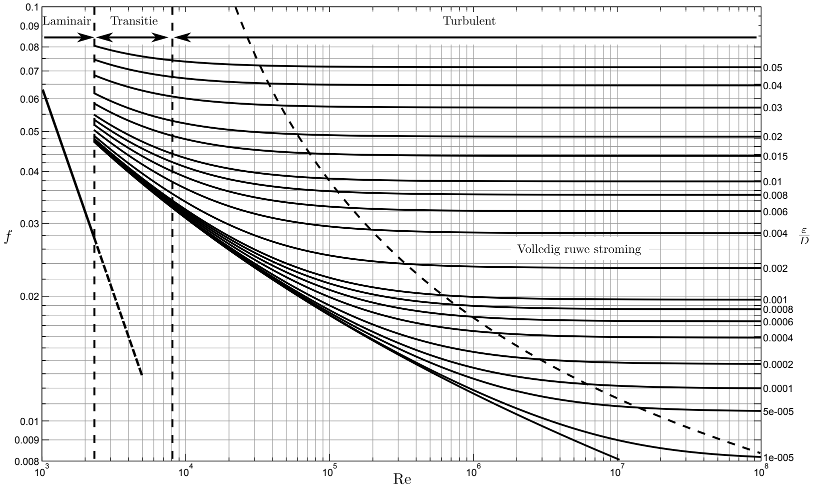
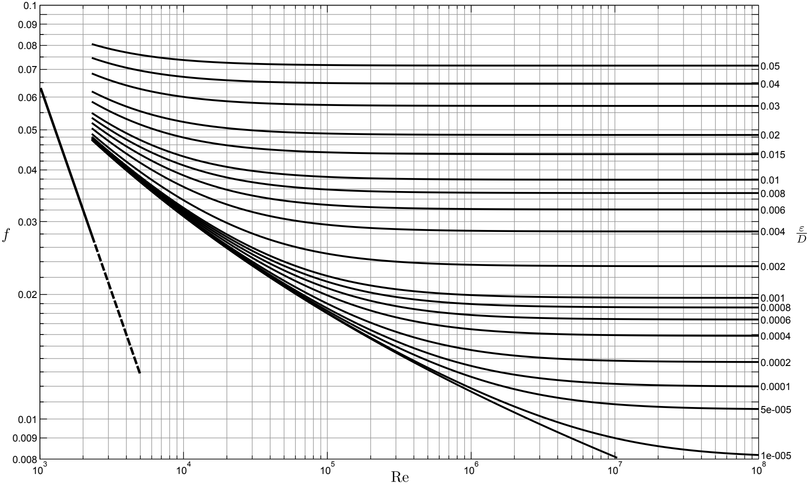
De wrijvingsfactor f voor turbulente stroming moet bepaald worden met behulp van empirische data: Moody diagram

Moody diagram



Gebruik van het Moody diagram



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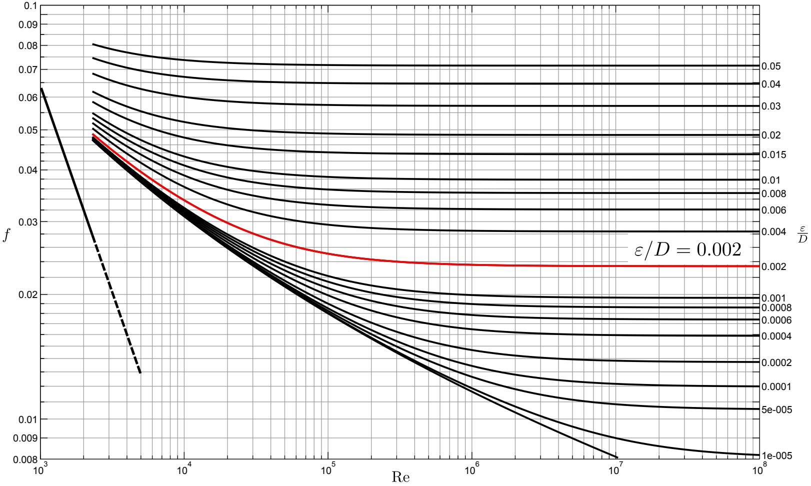

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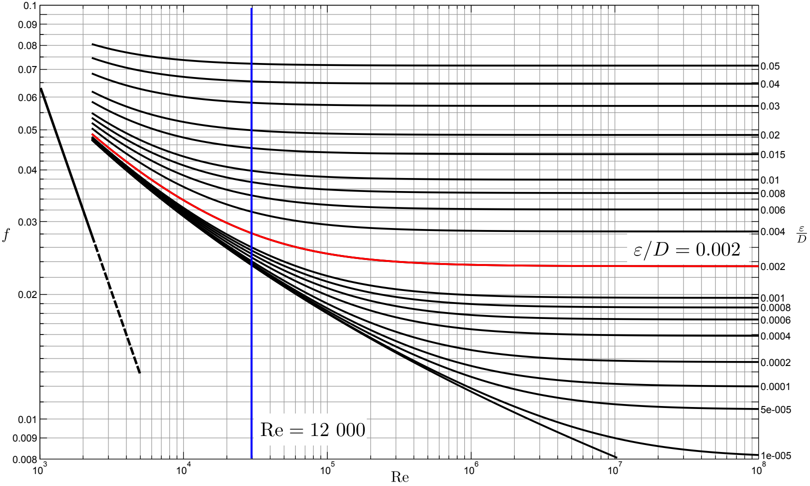

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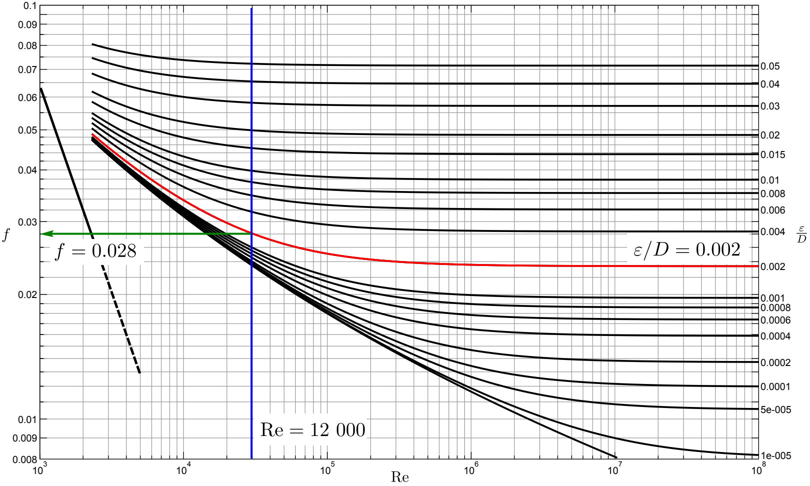
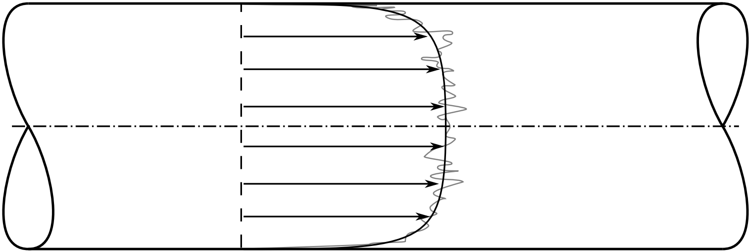


Turbulent snelheidsprofiel



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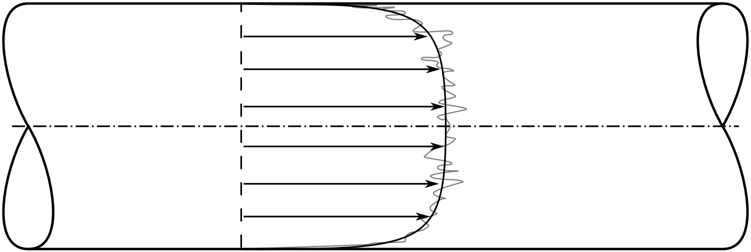
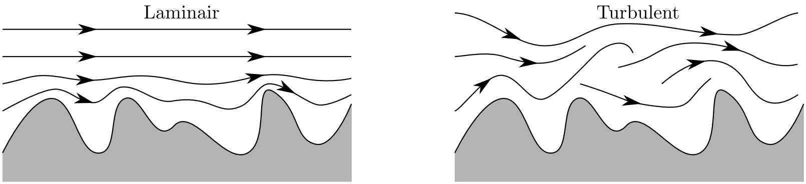


$$\frac{\bar{v}}{v_{\mathsf{max}}} \approx \left(1 - \frac{r}{R}\right)^{1/7}$$

Invloed van ruwheid



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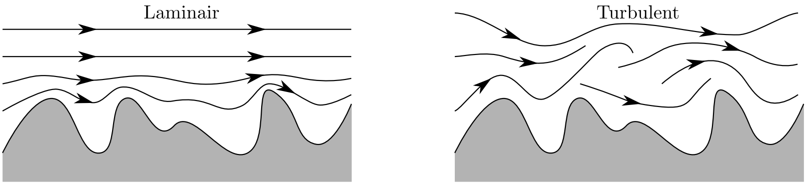

Bij laminaire stroming worden door de ruwheid geïnduceerde fluctuaties door de viskeuze krachten afgevlakt

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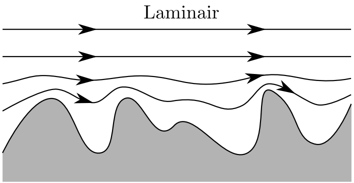

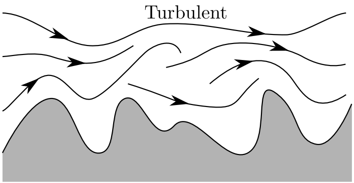

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Bij turbulente stroming hebben door de ruwheid geïnduceerde fluctuaties invloed in de volledige stroming