

Fluidummechanica

Stroming in leidingen

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Inhoud

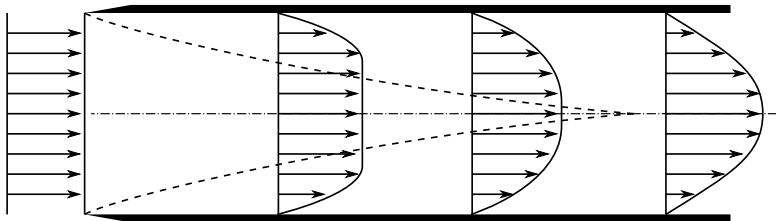
- 1 Inleiding
- 2 Dimensieanalyse
- 3 Laminaire stroming
- 4 Turbulente stroming

Voorbeeld



Bron: <http://www.etftrends.com/>

Ontwikkende stroming



Inhoud

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Dimensieanalyse

$$\Delta p = \phi(L, D, v, \mu, \rho)$$

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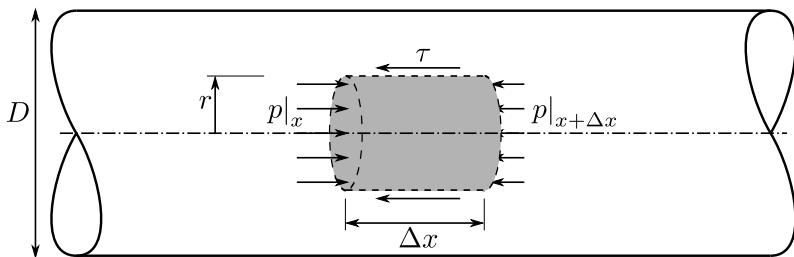
$$\frac{\Delta p}{\frac{1}{2}\rho v^2} = f(Re) \frac{L}{D}$$

$$\Delta p = f(Re) \frac{1}{2}\rho v^2 \frac{L}{D} \quad (1)$$

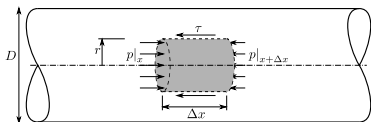
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Snelheidsprofiel



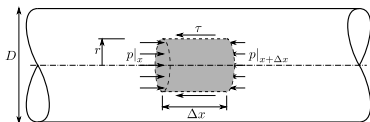
Snelheidsprofiel



Behoud van impuls in de stromingsrichting:

$$F_x = 0$$

Snelheidsprofiel

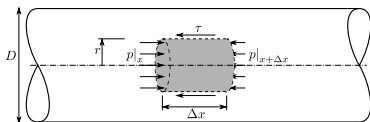


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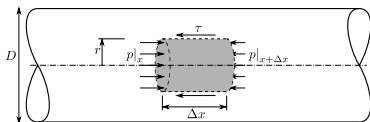
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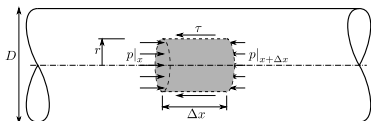
$$p\pi r^2|_x - p\pi r^2|_{x+\Delta x} - \tau 2\pi r \Delta x = 0$$

$$-\frac{1}{2} \frac{dp}{dx} r = \tau$$

Newtoniaanse vloeistof:

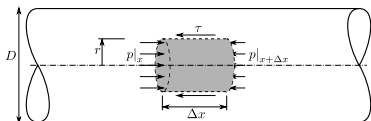
$$\frac{1}{2} \frac{dp}{dx} r = \mu \frac{dv}{dr}$$

Snelheidsprofiel



$$\frac{dv}{dr} = \frac{1}{2\mu} \frac{dp}{dx} r$$

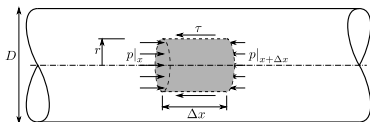
Snelheidsprofiel



$$\frac{dv}{dr} = \frac{1}{2\mu} \frac{dp}{dx} r$$

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Snelheidsprofiel

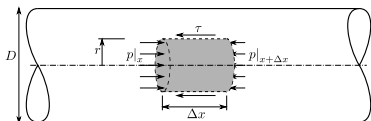


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$$\begin{aligned} \Downarrow \\ v|_{r=R} &= 0 \\ C &= -\frac{1}{4\mu} \frac{dp}{dx} R^2 \end{aligned}$$

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$$v = -\frac{1}{4\mu} \frac{dp}{dx} R^2 \left(1 - \frac{r^2}{R^2} \right) \quad (2)$$

Snelheidsprofiel

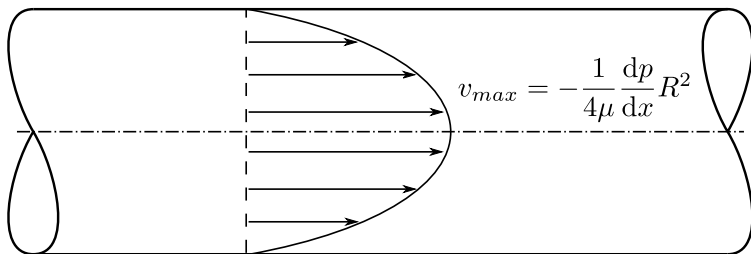
$$\frac{v}{v_{\max}} = \left(1 - \frac{r^2}{R^2}\right)$$

$$v_{\max} = -\frac{1}{4\mu} \frac{dp}{dx} R^2$$

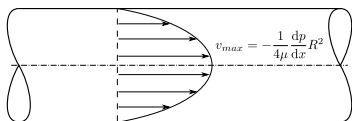
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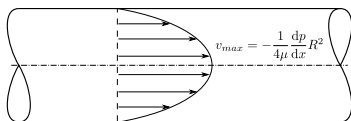
Gemiddelde snelheid



Debiet:

$$\dot{V} = 2\pi \int_0^R v_{\max} \left(1 - \frac{r^2}{R^2}\right) r dr = v_{\max} \frac{\pi R^2}{2}$$

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Gemiddelde snelheid:

$$v_{\text{gem}} = \frac{\dot{V}}{\pi R^2} = \frac{v_{\max}}{2} = -\frac{1}{8\mu} \frac{dp}{dx} R^2 \quad (2)$$

Drukval

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$$\Downarrow \quad \frac{dp}{dx} = -\frac{\Delta p}{L}$$
$$R = D/2$$

Drukval

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$$\frac{dp}{dx} = -8\mu v_{\text{gem}} \frac{1}{R^2}$$

$$\Downarrow \quad \frac{dp}{dx} \frac{R}{L} = -\frac{\Delta p}{D/2}$$

$$\Delta p = 32\mu v_{\text{gem}} \frac{L}{D^2}$$

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$$v_{\text{gem}} = -\frac{1}{8\mu} \frac{dp}{dx} R^2$$

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$$\Delta p = \frac{1}{2} \rho v^2 \frac{64\mu}{\rho v D} \frac{L}{D}$$

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$$\Delta p = \frac{1}{2} \rho v^2 f \frac{L}{D} \tag{3}$$

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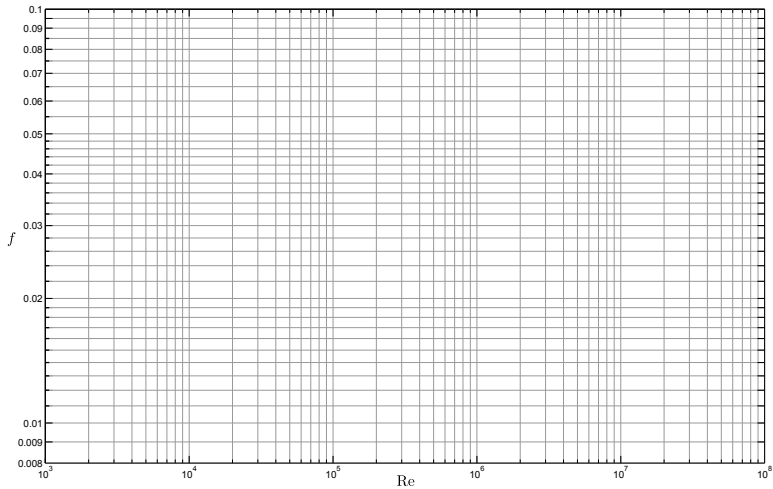
$$\Delta p = \frac{1}{2} \rho v^2 f \frac{L}{D} \tag{3}$$

wrijvingsfactor voor laminaire stroming $f = \frac{64}{\text{Re}}$

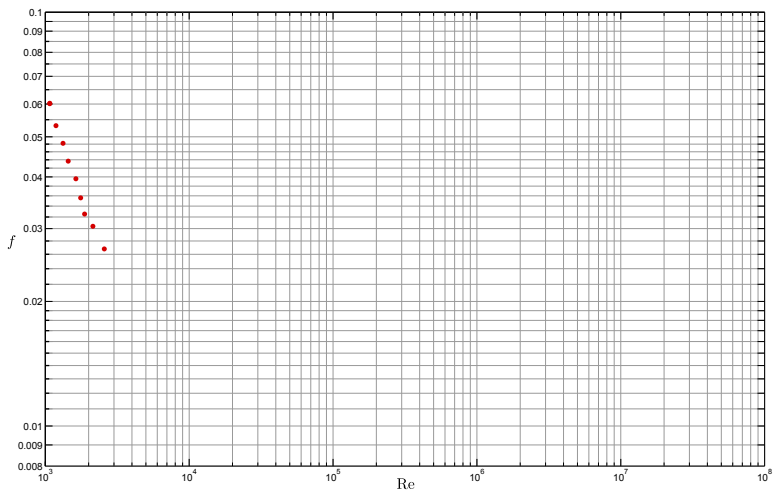
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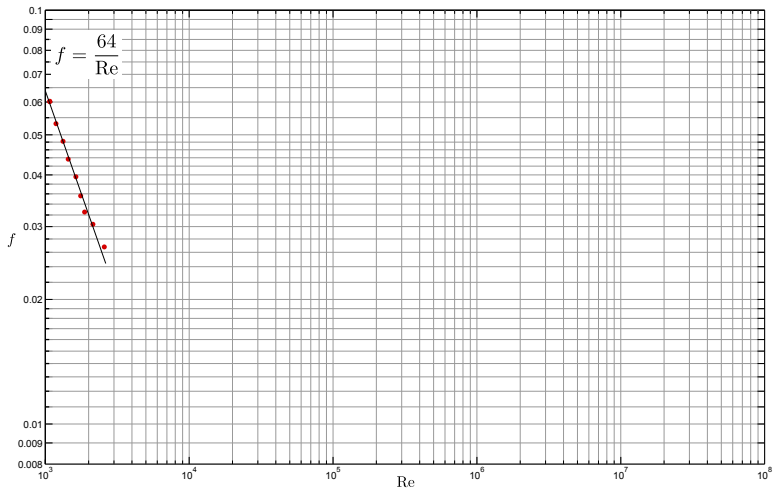
Empirische data



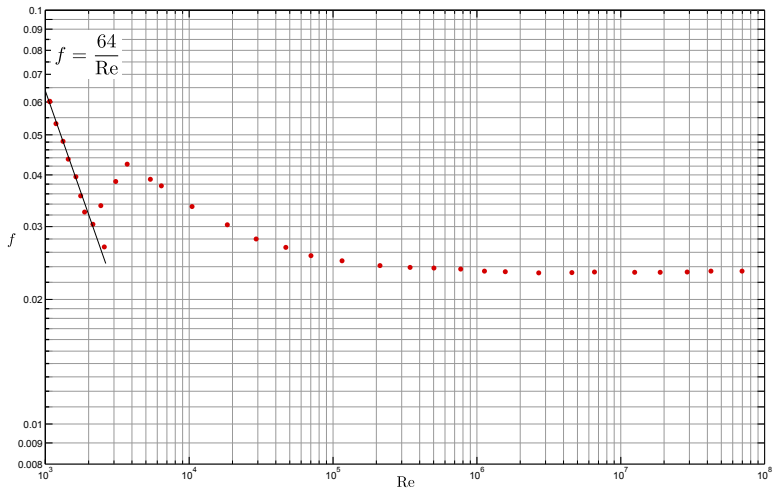
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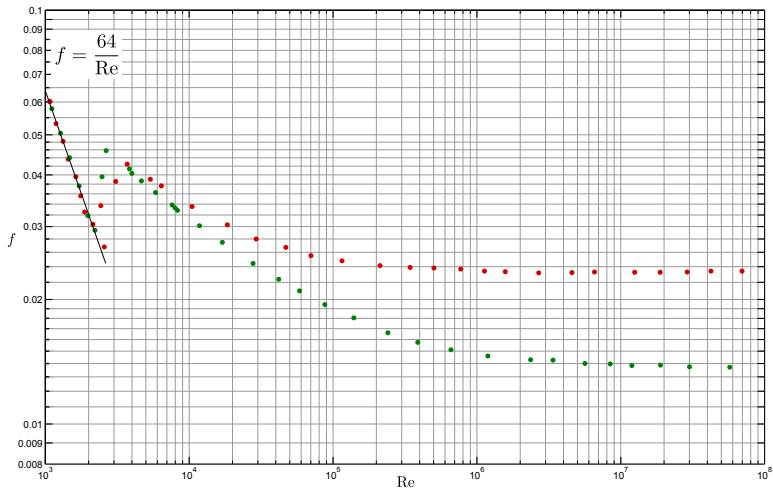
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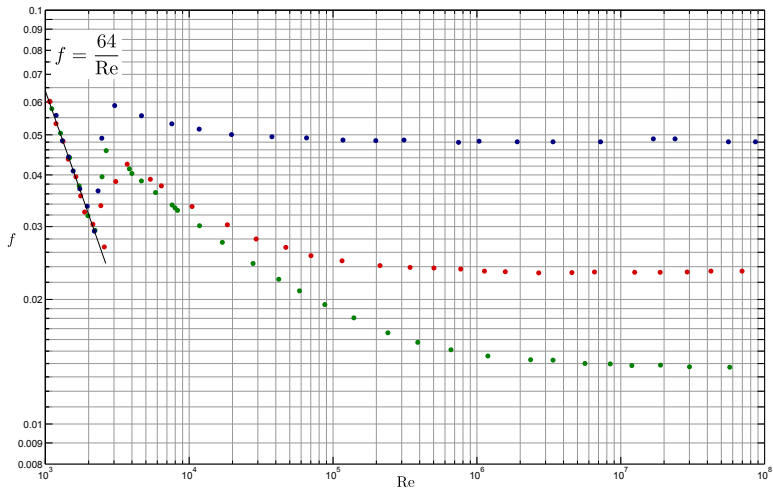
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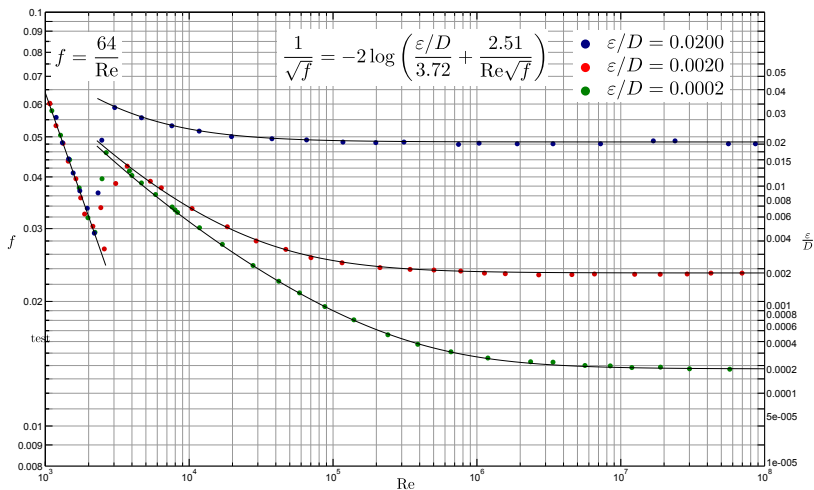
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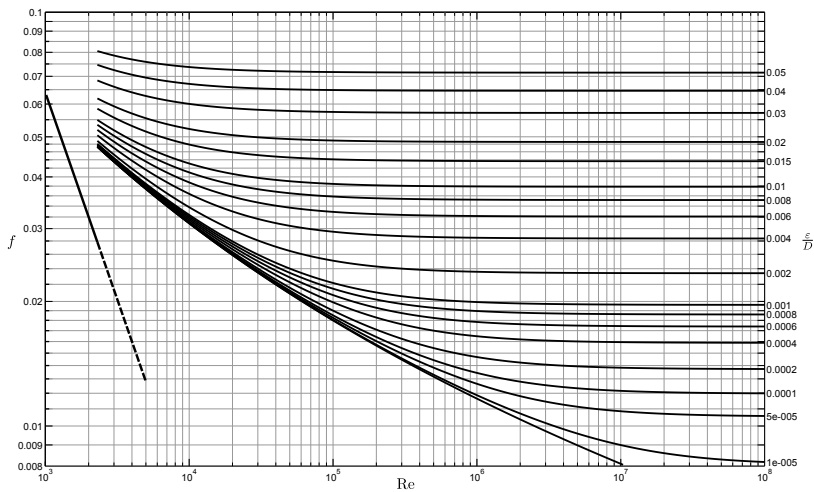
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Dimensieanalyse

$$\Delta p = \phi(L, D, v, \mu, \rho, \varepsilon)$$

$$\Delta p = f(Re, \varepsilon/D) \frac{1}{2} \rho v^2 \frac{L}{D}$$

Dimensieanalyse

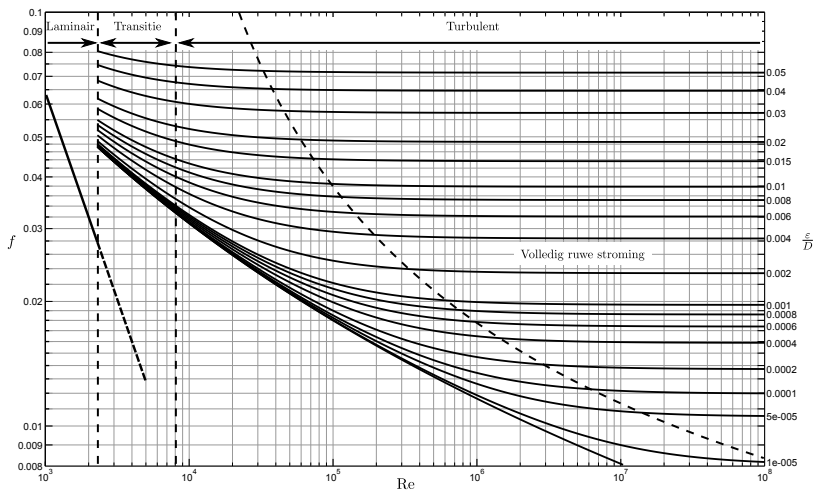
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$$\Delta p = f(Re, \varepsilon/D) \frac{1}{2} \rho v^2 \frac{L}{D}$$

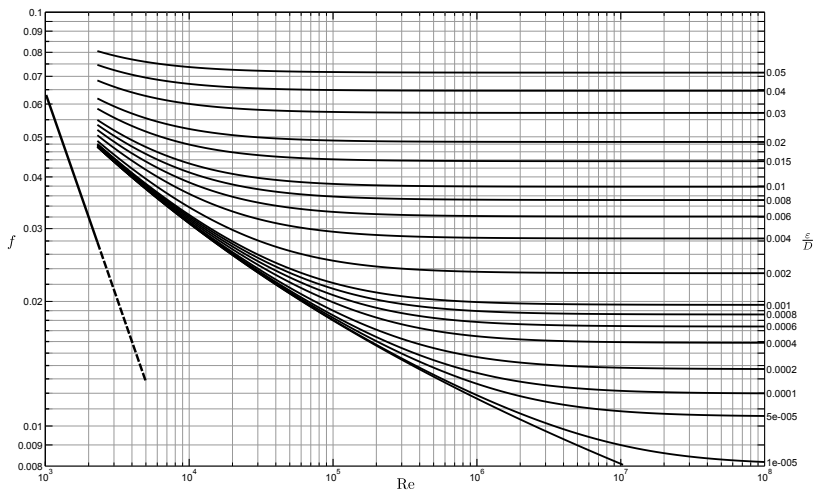
De wrijvingsfactor f voor turbulente stroming moet bepaald worden met behulp van empirische data:

Moody diagram

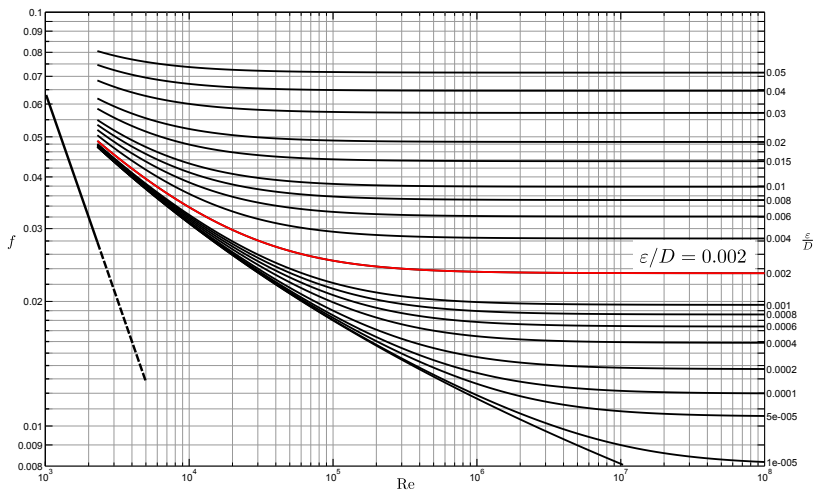
Moody diagram



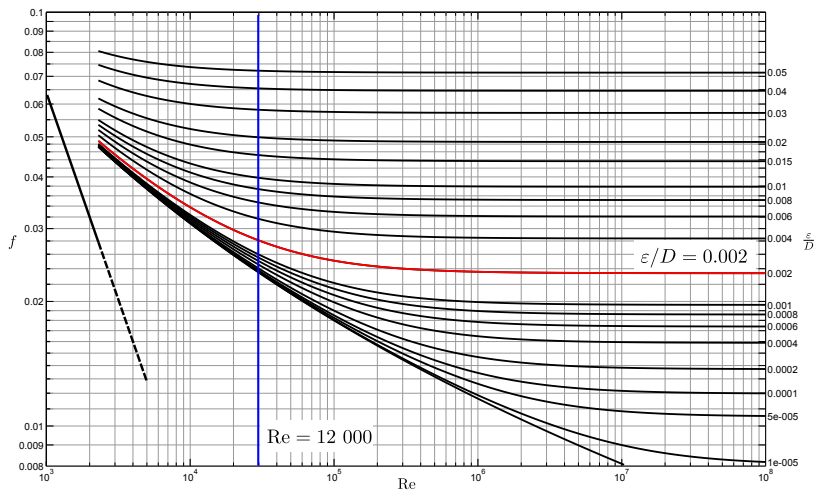
Gebruik van het Moody diagram



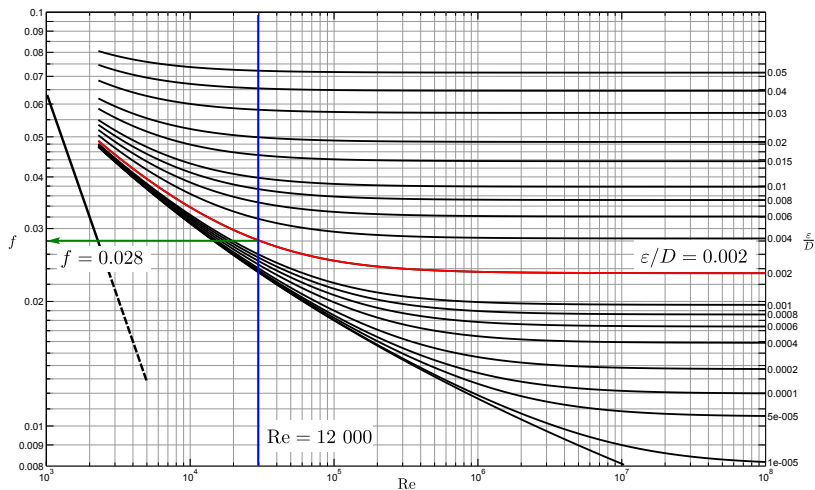
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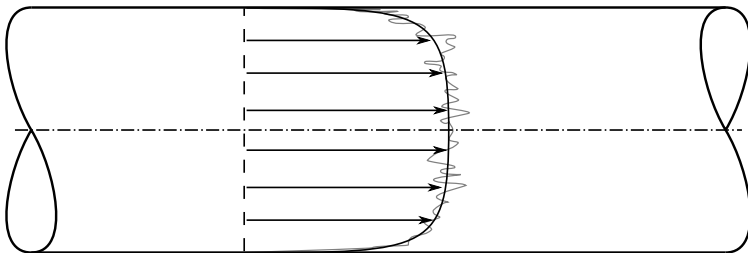
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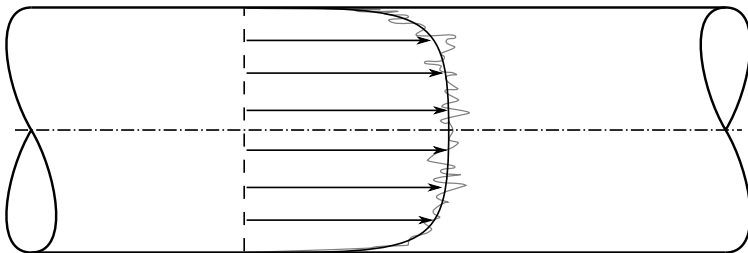
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Turbulent snelheidsprofiel

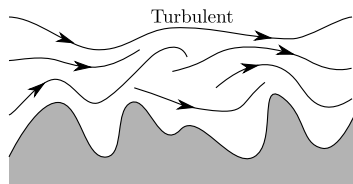
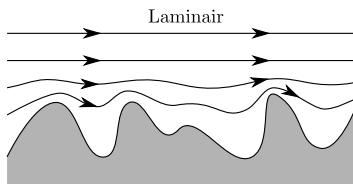


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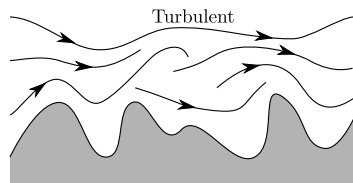
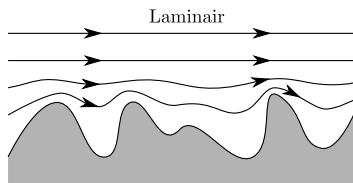


$$\frac{\bar{v}}{v_{\max}} \approx \left(1 - \frac{r}{R}\right)^{1/7}$$

Invloed van ruwheid

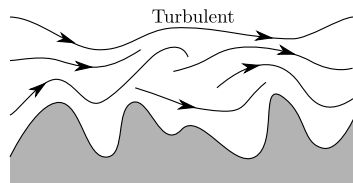
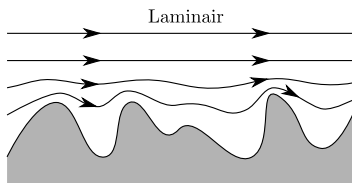


Invloed van ruwheid



Bij laminaire stroming worden door de ruwheid geïnduceerde fluctuaties door de viskeuze krachten afgevlakt

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Bij turbulente stroming hebben door de ruwheid geïnduceerde fluctuaties invloed in de volledige stroming