Fluïdummechanica

Gelijkvormigheid en dimensieloze getallen

Brecht Baeten[1]

[1]KU Leuven, Technologie campus Diepenbeek,
e-mail: brecht.baeten@kuleuven.be

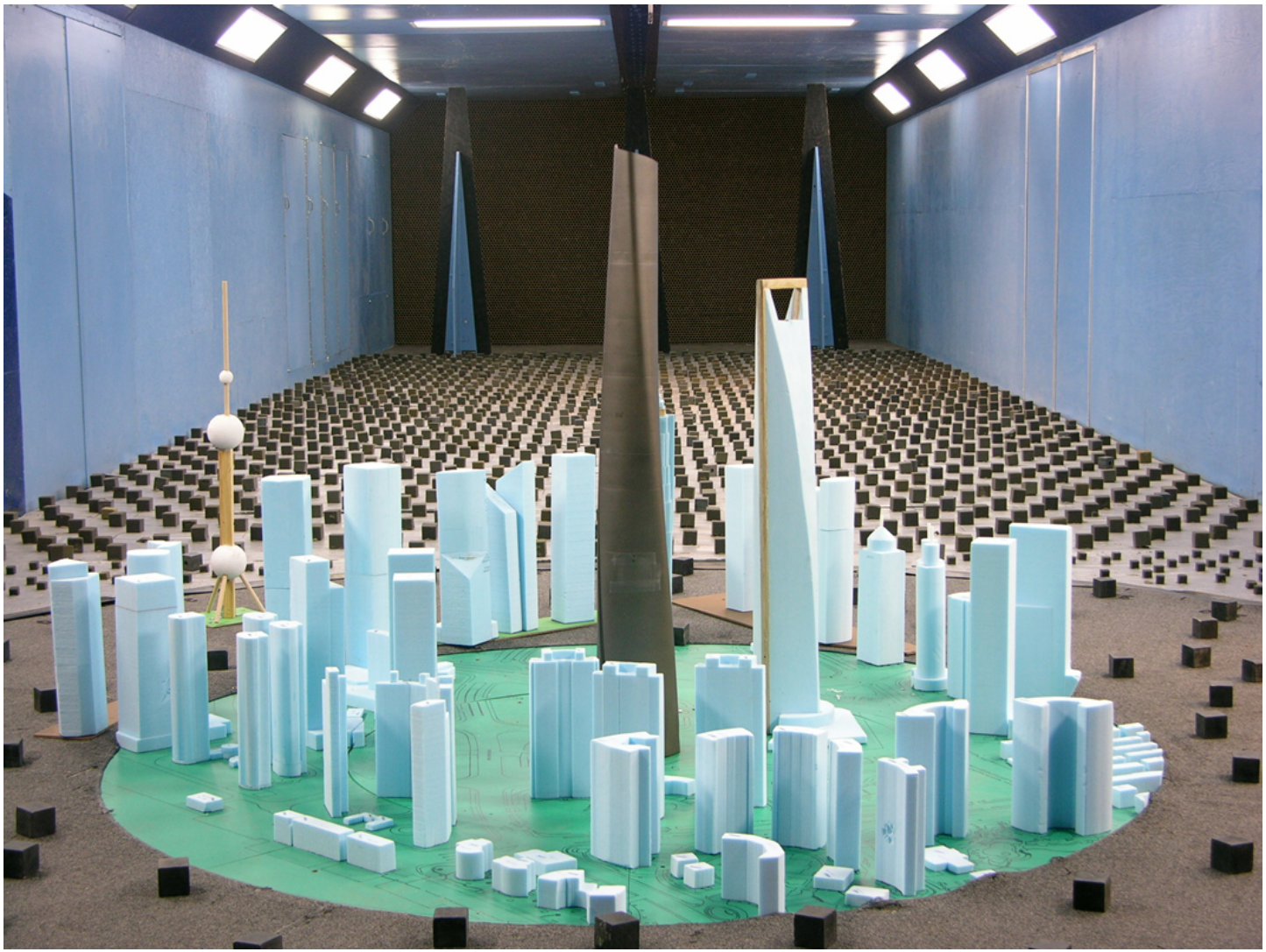
17 oktober 2016

Inhoud

Voorbeeld

Bron: http://www.nasa.gov/

Voorbeeld



Bron: http://www.autodesk.com/

Inhoud

Wat is gelijkvormigheid?

Bron: http://7-themes.com/

Wat is gelijkvormigheid?

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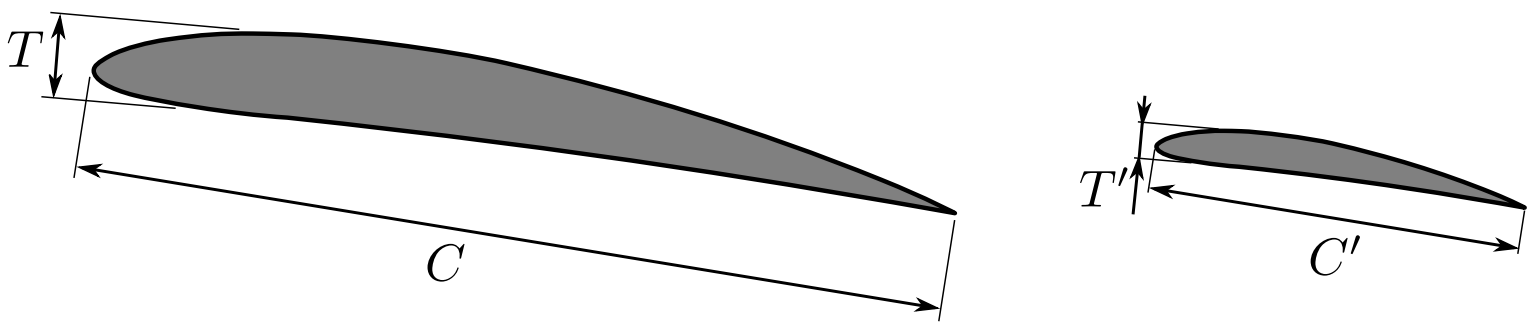
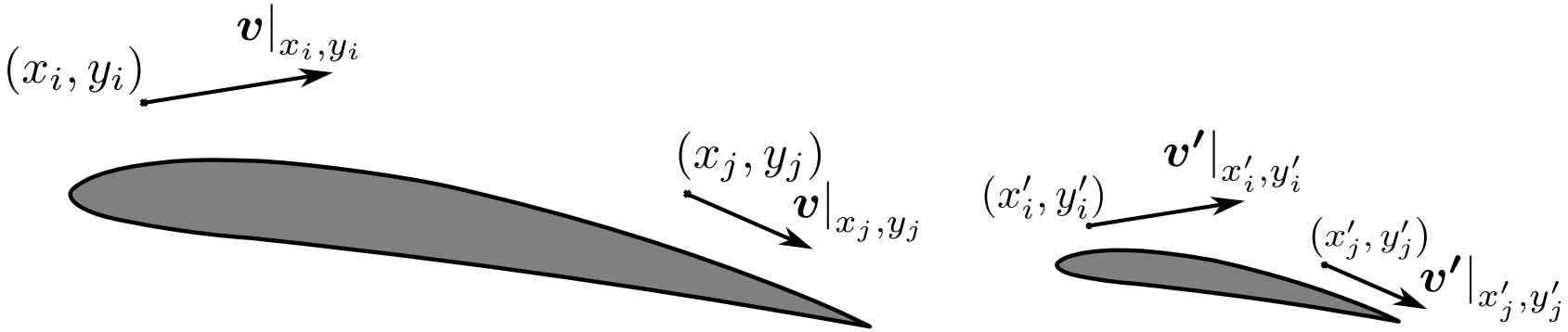
- Gelijke verhoudingen van afstanden



Wat is gelijkvormigheid?

- Gelijke verhoudingen van afstanden
- Gelijke verhoudingen van snelheden



Inhoud

Het dimensieloos maken van de bewegingsvergelijking

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial s} = -\frac{\partial p}{\partial s} + \rho g_s + \mu\frac{\partial^2 v}{\partial s^2}$$

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$$\Downarrow \quad \begin{aligned} s &= s^* D_{\text{ref}} \\ v &= v^* v_{\text{ref}} \\ t &= t^* t_{\text{ref}} \\ p &= p^* p_{\text{ref}} \end{aligned}$$

$$\rho\frac{\partial v^* v_{\text{ref}}}{\partial t^* t_{\text{ref}}}+\rho v^* v_{\text{ref}}\frac{\partial v^* v_{\text{ref}}}{\partial s^* D_{\text{ref}}}=-\frac{\partial p^* p_{\text{ref}}}{\partial s^* D_{\text{ref}}}+\rho g_s+\mu\frac{\partial^2 v^* v_{\text{ref}}}{\partial s^{*2} D_{\text{ref}}^2}$$

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$$\frac{\rho v_{\text{ref}}^2}{D_{\text{ref}}}\frac{\partial v^*}{\partial t^*}+\frac{\rho v_{\text{ref}}^2}{D_{\text{ref}}}v^*\frac{\partial v^*}{\partial s^*}=-\frac{p_{\text{ref}}}{D_{\text{ref}}}\frac{\partial p^*}{\partial s^*}+\rho g_s+\frac{\mu v_{\text{ref}}}{D_{\text{ref}}^2}\frac{\partial^2 v^*}{\partial s^{*2}}$$

Het dimensieloos maken van de bewegingsvergelijking

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial s^*} = -\frac{p_{\text{ref}}}{\rho v_{\text{ref}}^2} \frac{\partial p^*}{\partial s^*} + \frac{g_s D_{\text{ref}}}{v_{\text{ref}}^2} + \frac{\mu}{\rho v_{\text{ref}} D_{\text{ref}}} \frac{\partial^2 v^*}{\partial s^{*2}} \qquad (1)$$

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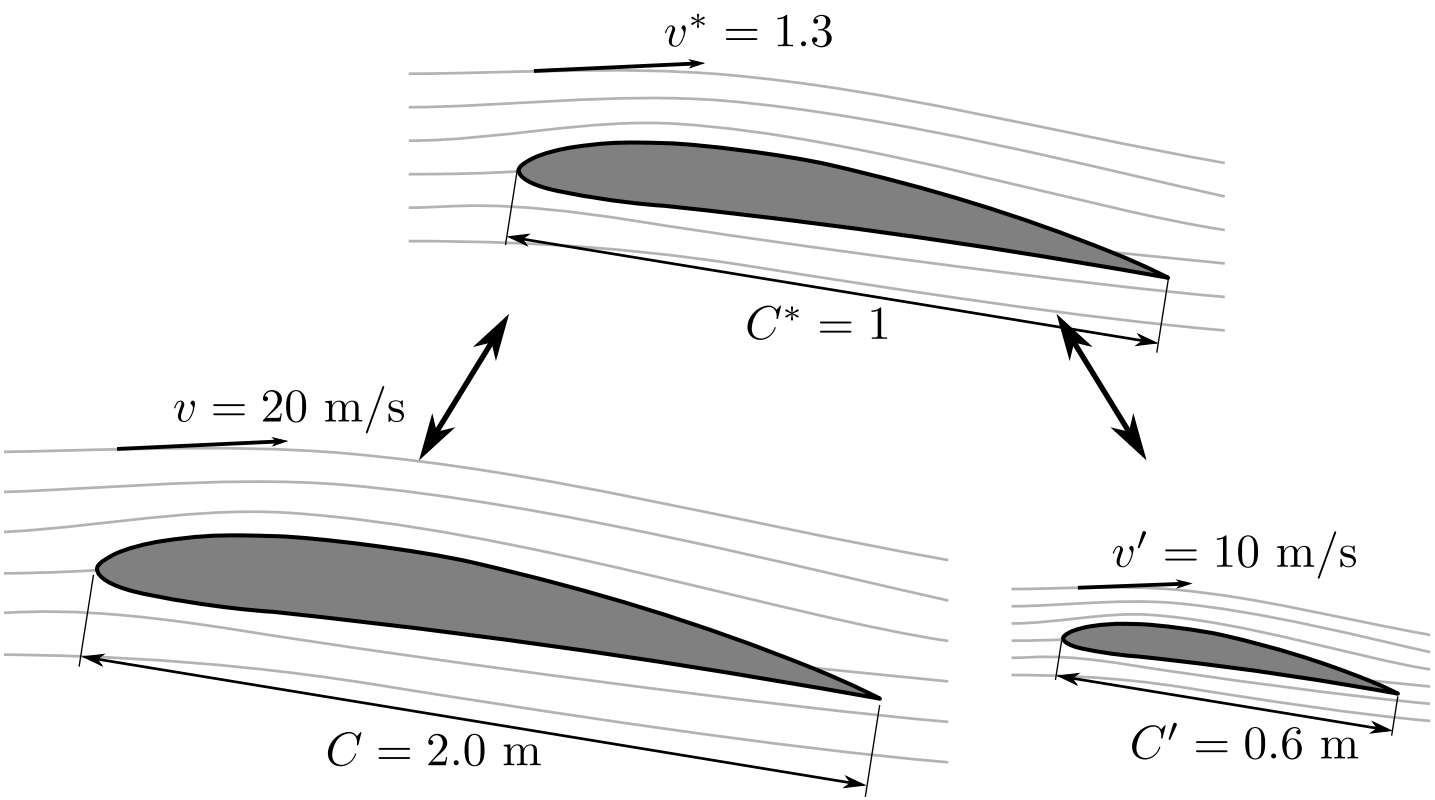

Dimensieloze getallen

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial s^*} = -\mathsf{Eu} \frac{\partial p^*}{\partial s^*} + \frac{1}{\mathsf{Fr}^2} + \frac{1}{\mathsf{Re}} \frac{\partial^2 v^*}{\partial s^{*2}} \tag{2}$$

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Dimensieloze getallen zijn verhoudingen van referentiewaarden voor krachten

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Dimensieloze getallen zijn verhoudingen van referentiewaarden voor krachten

$$\mathrm{Re} = \frac{\rho v D}{\mu} = \frac{v D}{\nu} = \frac{\text{traagheidskracht}}{\text{viskeuze krachten}}$$

$$\mathrm{Eu} = \frac{p}{\rho v^2} = \frac{\text{drukkracht}}{\text{traagheidskracht}}$$

$$\mathrm{Fr} = \frac{v}{\sqrt{gD}} = \sqrt{\frac{\text{traagheidskracht}}{\text{zwaartekracht}}}$$

Dimensieloze getallen en gelijkvormigheid

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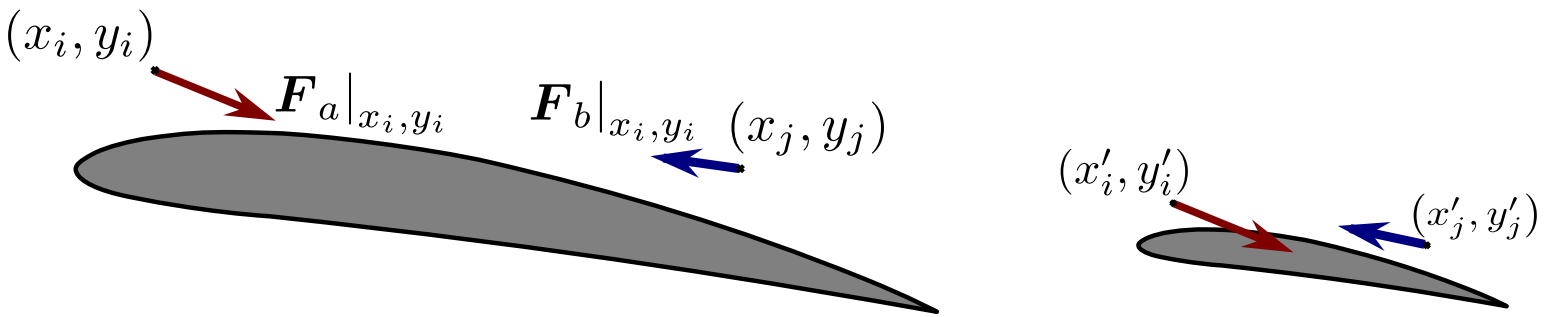

Dimensieloze getallen en gelijkvormigheid

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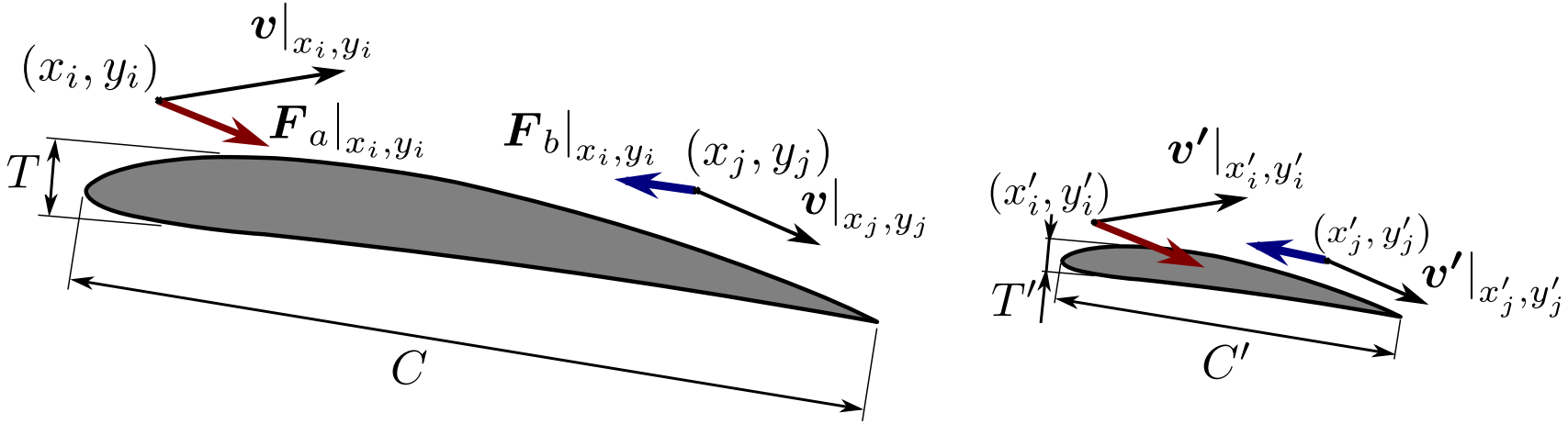

$$\text{Re} = \text{Re'}, \quad \text{Eu} = \text{Eu'}, \quad \text{Fr} = \text{Fr'}$$

Dimensieloze getallen

$$\mathrm{Re} = \frac{\rho v D}{\mu}$$

$$\mathrm{Eu} = \frac{p}{\rho v^2}$$

$$\mathrm{Fr} = \frac{v}{\sqrt{gD}}$$

$$\mathrm{Ma} = \frac{v}{c}$$

$$\mathrm{St} = \frac{fD}{v}$$

Dimensieloze getallen

$$\mathrm{Re} = \frac{\rho v D}{\mu} \qquad \mathrm{Pr} = \frac{\nu}{\alpha}$$

$$\mathrm{Eu} = \frac{p}{\rho v^2} \qquad \mathrm{Nu} = \frac{hD}{k}$$

$$\mathrm{Fr} = \frac{v}{\sqrt{gD}} \qquad \mathrm{Gr} = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}$$

$$\mathrm{Ma} = \frac{v}{c} \qquad \mathrm{Ra} = \mathrm{GrPr}$$

$$\mathrm{St} = \frac{fD}{v}$$

Dimensieloze getallen

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$$\mathrm{St} = \frac{fD}{v}$$

$$C_p = \frac{p}{\frac{1}{2}\rho v^2} \simeq \frac{p}{\rho N^2 D^2}$$

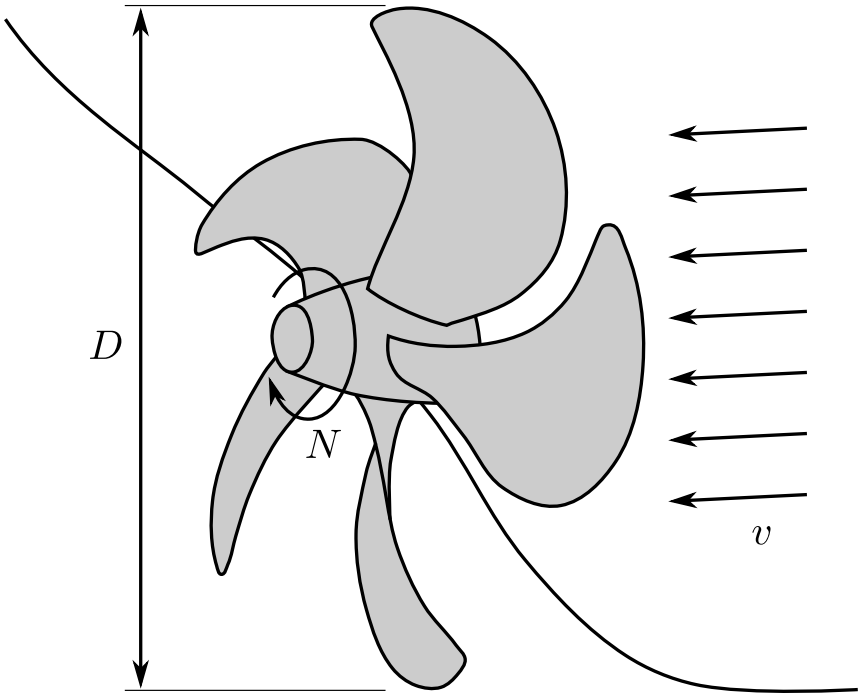
$$C_F = \frac{F}{\frac{1}{2}\rho v^2 A} \simeq \frac{F}{\frac{1}{2}\rho v^2 D^2}$$

$$C_P = \frac{P}{\frac{1}{2}\rho v^3 D^2} \simeq \frac{P}{\rho N^3 D^5}$$

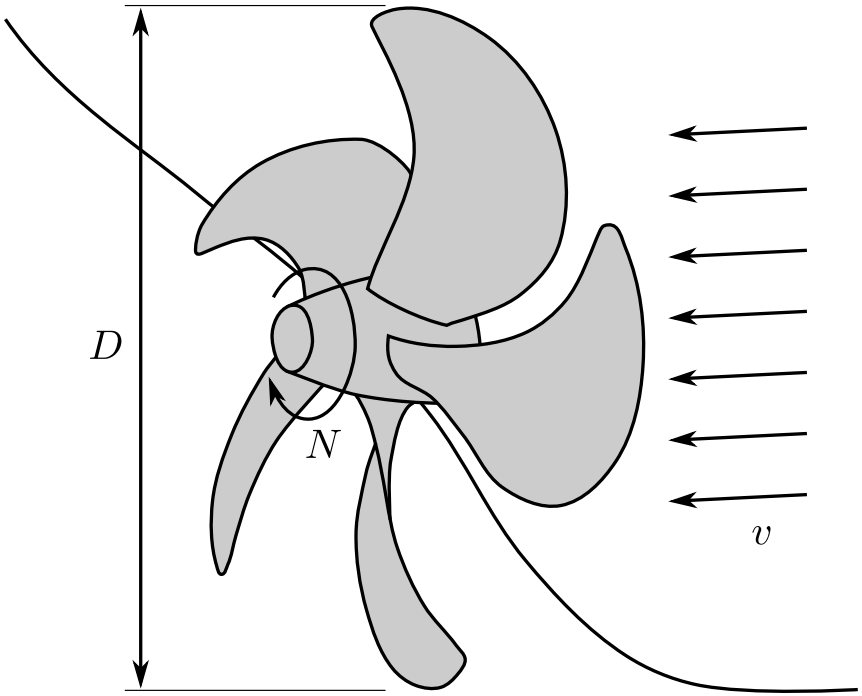
$$C_{\dot{V}} = \frac{\dot{V}}{vD^2} \simeq \frac{\dot{V}}{ND^3}$$

Inhoud

Voorbeeld

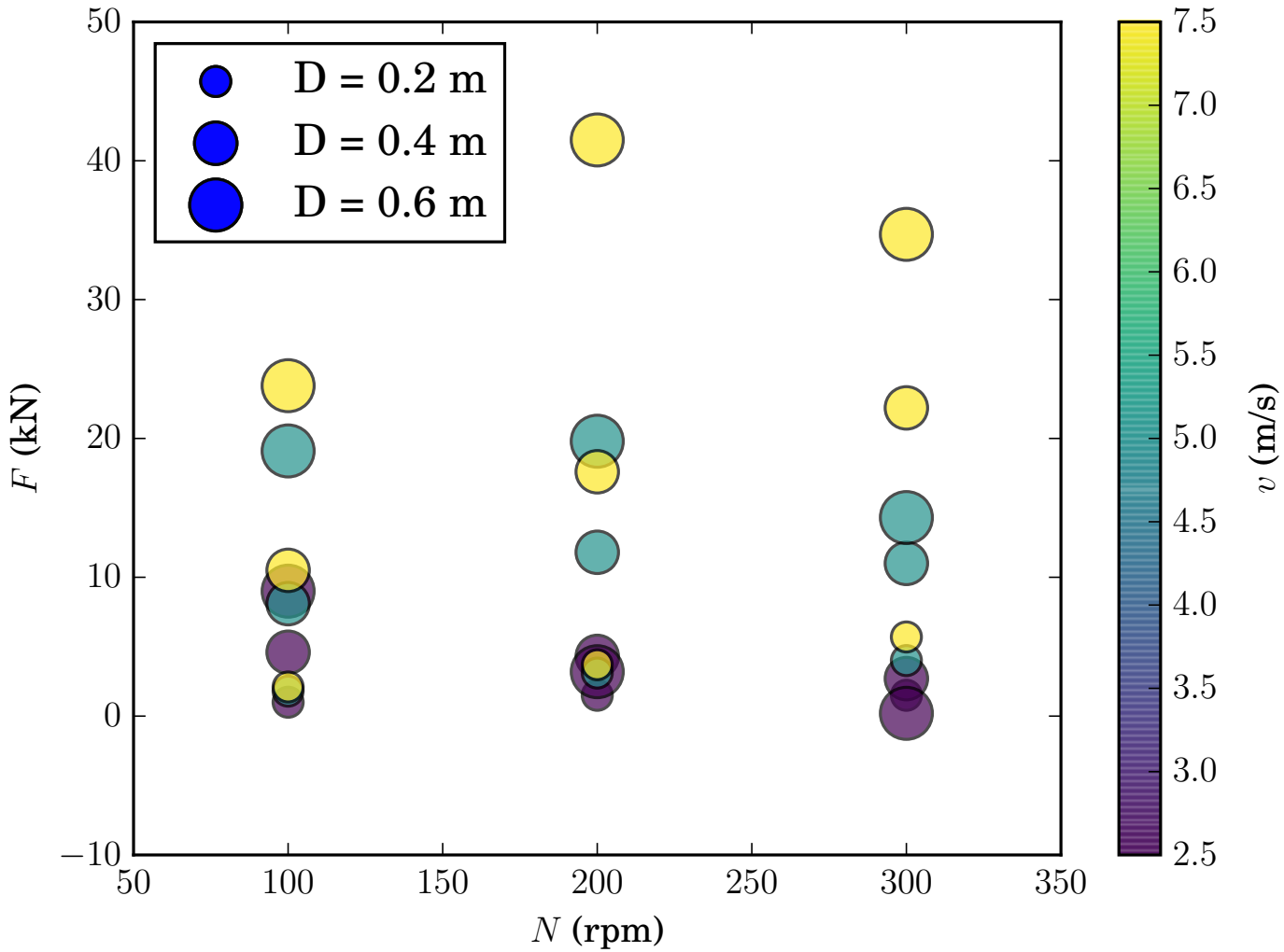


Voorbeeld



Meting	N (rpm)	D (m)	v (m/s)	F (kN)
1	100	0.20	2.5	1.0
2	200	0.20	2.5	1.5
3	300	0.20	2.5	1.5
4	100	0.40	2.5	4.6
5	200	0.40	2.5	4.3
6	300	0.40	2.5	2.7
7	100	0.60	2.5	9.0
8	200	0.60	2.5	3.2
9	300	0.60	2.5	0.2
10	100	0.20	5.0	1.8
11	200	0.20	5.0	3.1
12	300	0.20	5.0	4.0
13	100	0.40	5.0	8.1
14	200	0.40	5.0	11.8
15	300	0.40	5.0	11.0
16	100	0.60	5.0	19.1
17	200	0.60	5.0	19.8
18	300	0.60	5.0	14.3
19	100	0.20	7.5	2.1
20	200	0.20	7.5	3.7
21	300	0.20	7.5	5.7
22	100	0.40	7.5	10.5
23	200	0.40	7.5	17.6
24	300	0.40	7.5	22.2
25	100	0.60	7.5	23.8
26	200	0.60	7.5	41.5
27	300	0.60	7.5	34.7

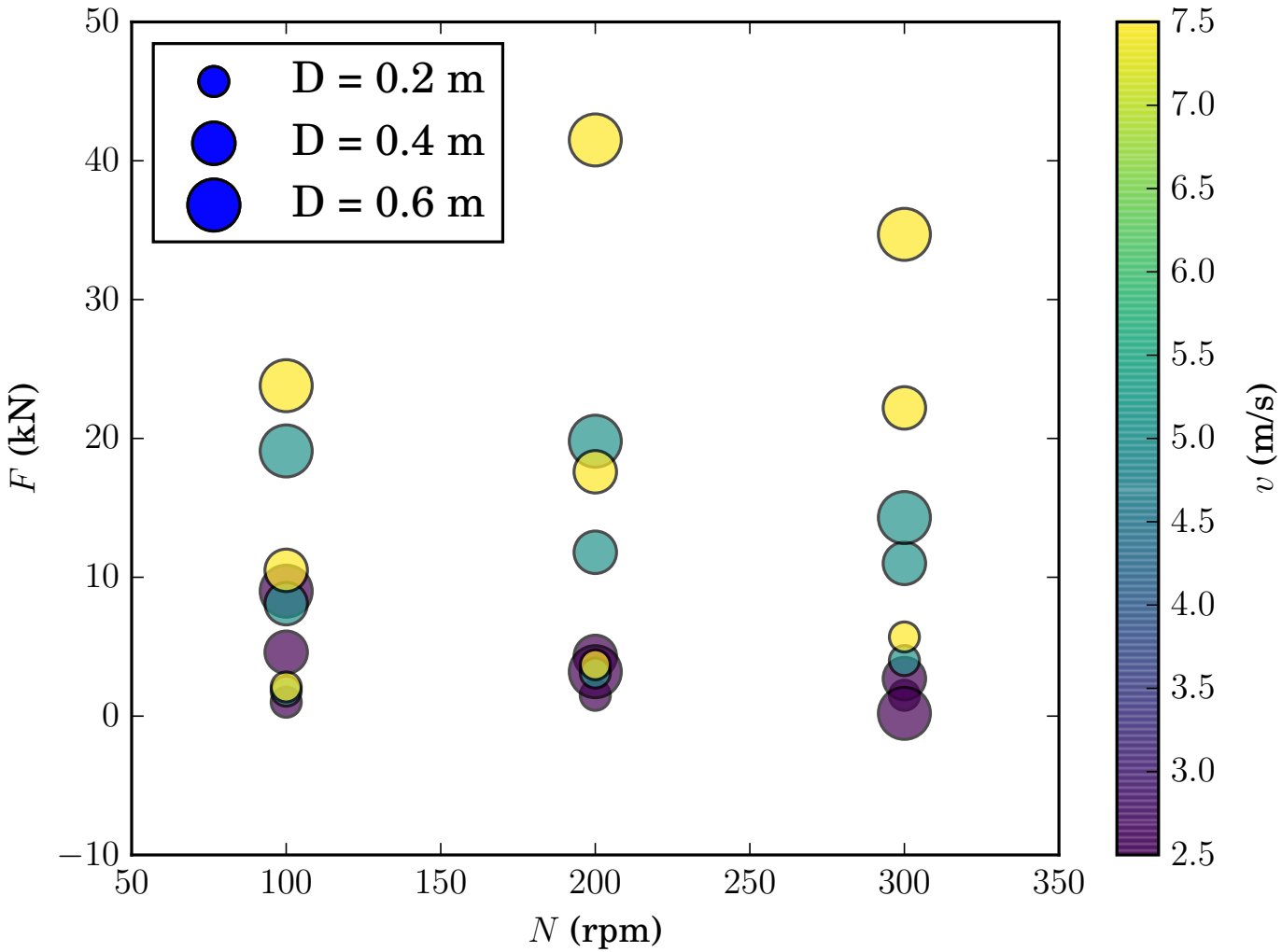
Voorbeeld


D = 0.2 m
D = 0.4 m
D = 0.6 m
50
40
30
20
10
0
−10
F (kN)
50
100
150
200
250
300
350
N (rpm)
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
v (m/s)

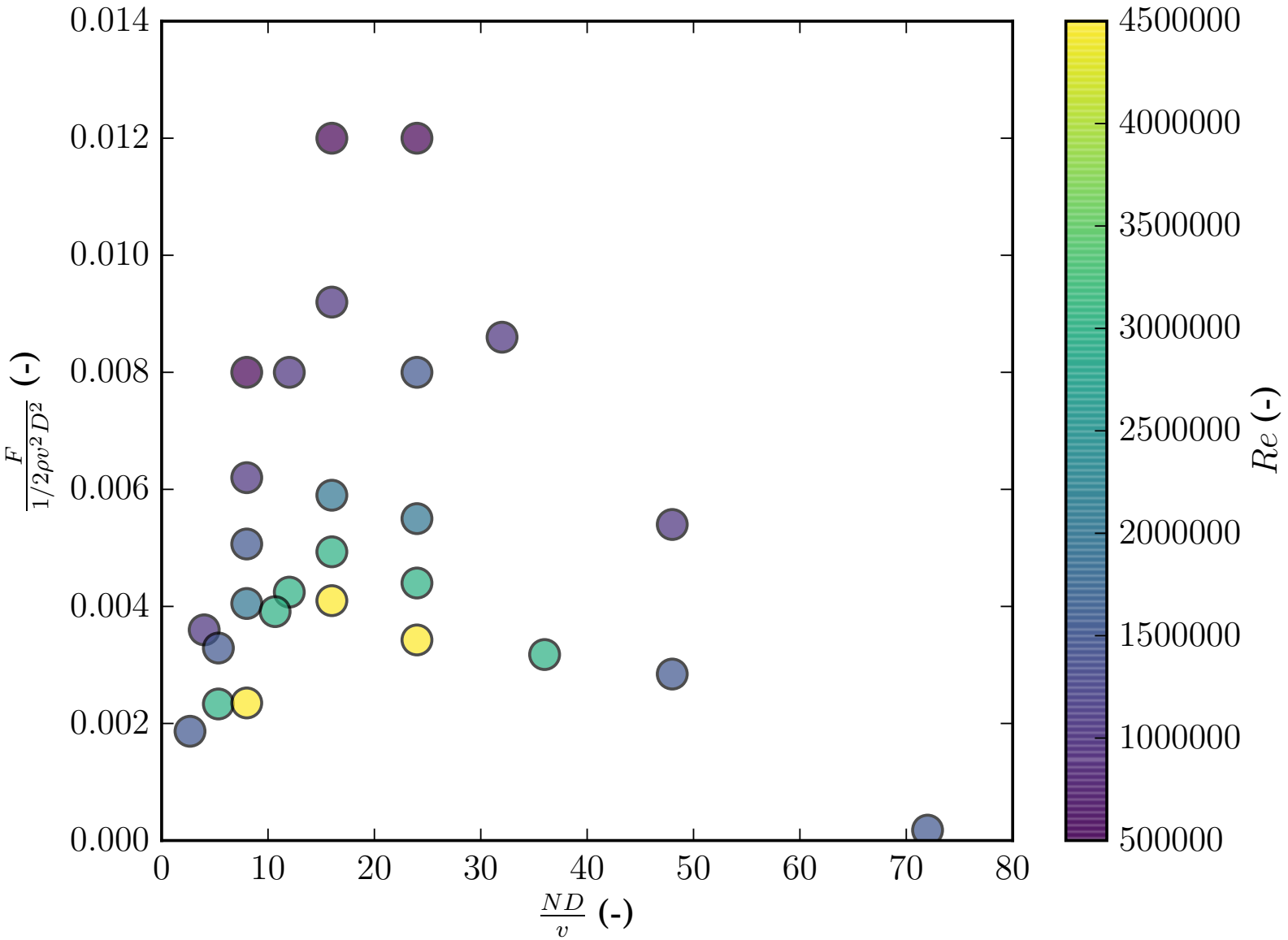
Eenheden

Grootheid	Dimensie	Eenheid
Lengte	L	m
Massa	M	kg
Tijd	T	s
Dichtheid	ML^{-3}	$\mathrm{kg/m}^3$
Druk	$\mathrm{ML}^{-1}\mathrm{T}^{-2}$	$\mathrm{N/m}^2$
Dynamische viscositeit	$\mathrm{MT}^{-1}\mathrm{L}^{-1}$	Pas
Energie (arbeid)	$\mathrm{ML}^2\mathrm{T}^{-2}$	J
Impuls	MLT^{-1}	$\mathrm{kgm/s}$
Kinematische viscositeit	$\mathrm{L}^2\mathrm{T}^{-1}$	m^2/s
Kracht	MLT^{-2}	N
Snelheid	LT^{-1}	$\mathrm{m/s}$
Vermogen	$\mathrm{ML}^2\mathrm{T}^{-3}$	$\mathrm{J/s}$
Versnelling	LT^{-2}	$\mathrm{m/s}^2$
Volume	L^3	m^3

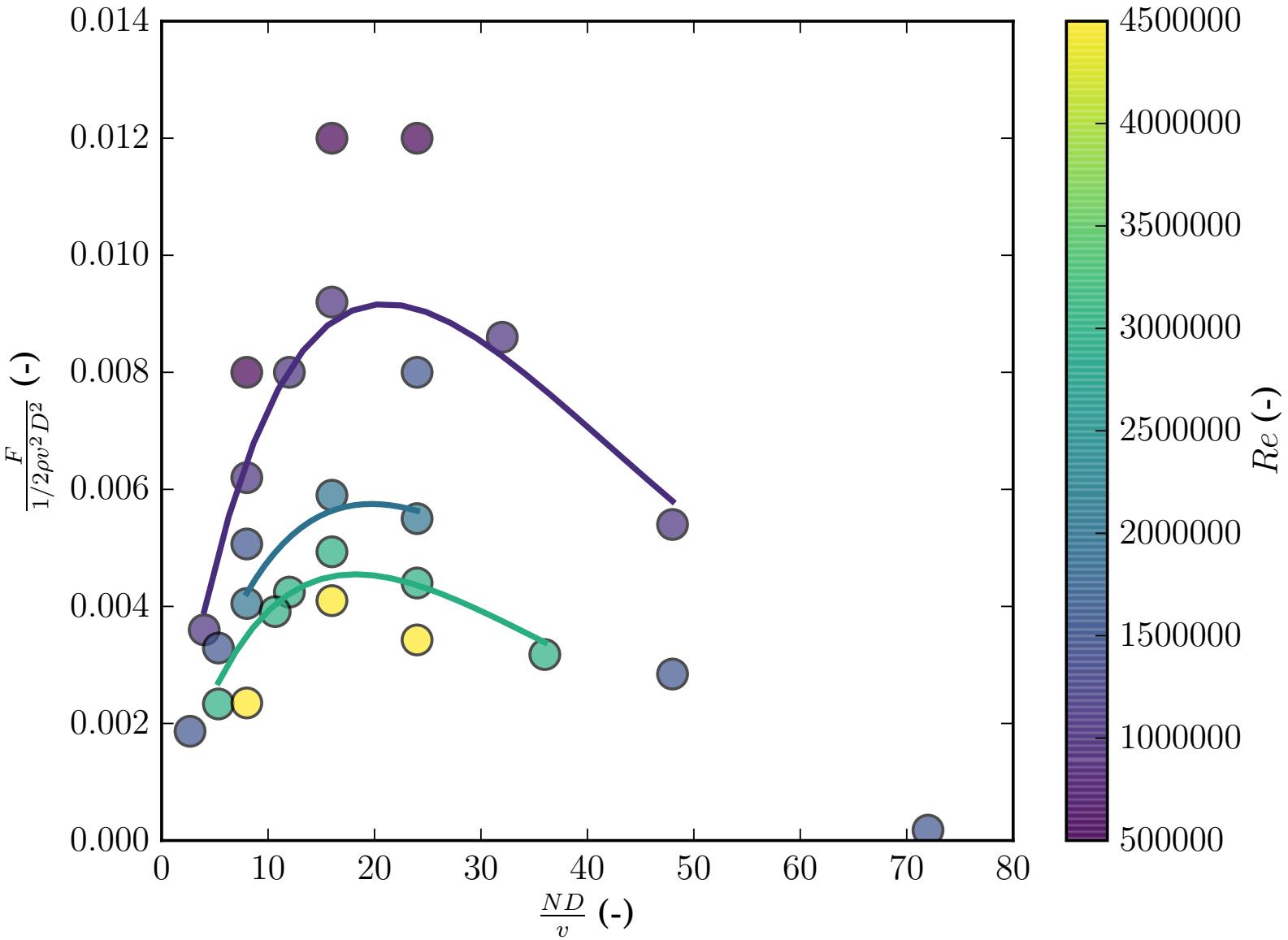
Voorbeeld


D = 0.2 m
D = 0.4 m
D = 0.6 m
F (kN)
N (rpm)
v (m/s)
50
40
30
20
10
0
−10
50
100
150
200
250
300
350
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5

Voorbeeld


0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
$\frac{F}{1/2\rho v^2 D^2}$ (-)
0
10
20
30
40
50
60
70
80
$\frac{ND}{v}$ (-)
4500000
4000000
3500000
3000000
2500000
2000000
1500000
1000000
500000
Re (-)

Voorbeeld


0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
$\frac{F}{1/2\rho v^2 D^2}$ (-)
0
10
20
30
40
50
60
70
80
$\frac{ND}{v}$ (-)
4500000
4000000
3500000
3000000
2500000
2000000
1500000
1000000
500000
Re (-)