Fluïdummechanica

Behoudsvergelijkingen langs stroomlijnen

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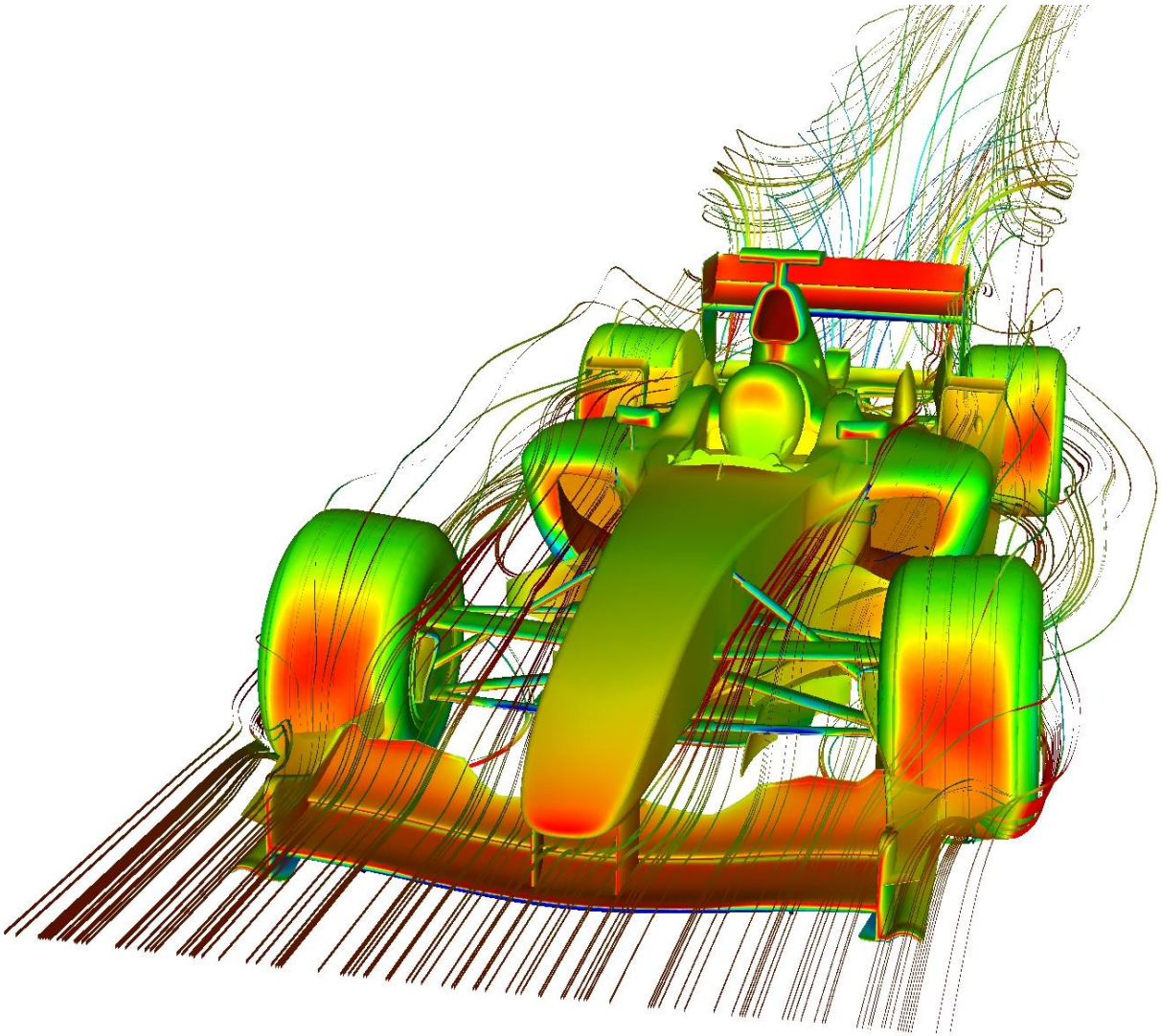
15 september 2015

Inhoud

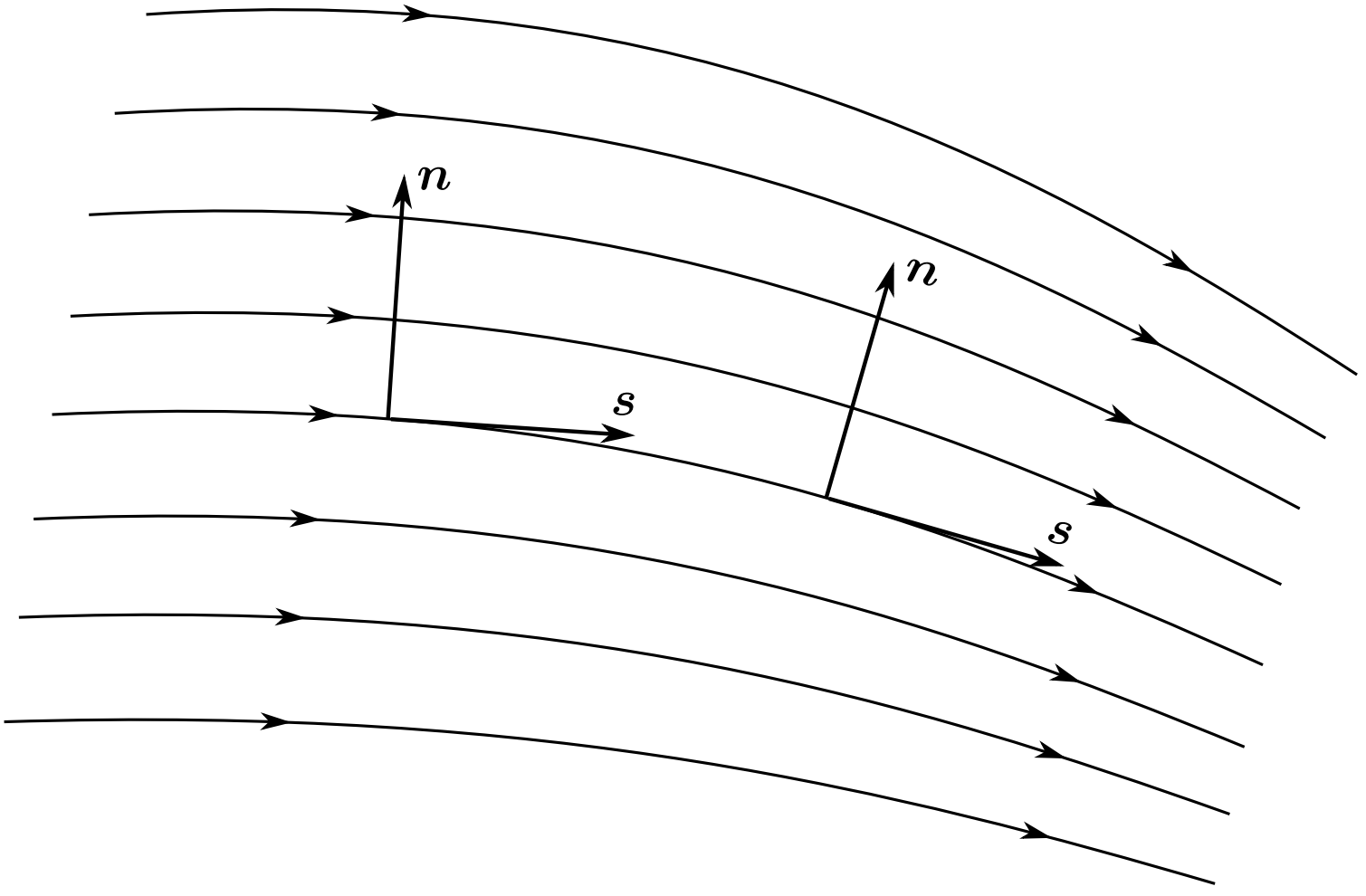
Voorbeeld



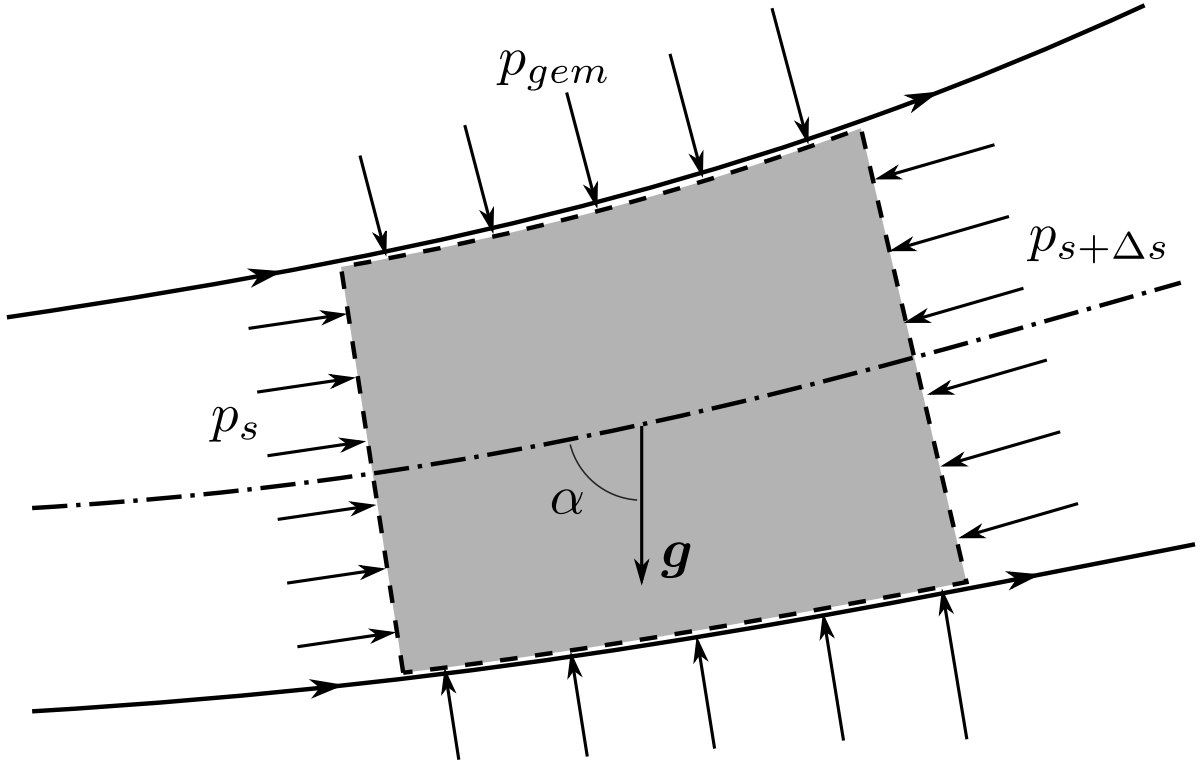
Bron: http://www.dalco.ch/

Inhoud

Stroomlijncoordinaten



Bewegingsvergelijking



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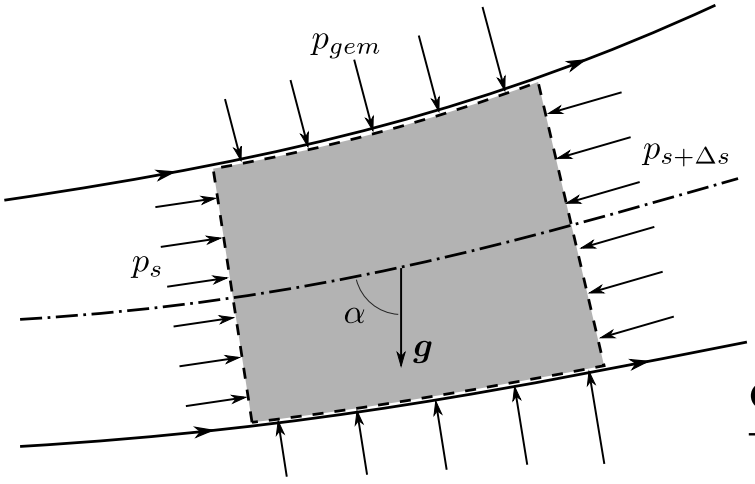

$$\frac{\mathrm{d}\mathbf{P}_{CV}}{\mathrm{d}t} + \dot{\mathbf{P}}_{\partial CV} = \mathbf{F}$$

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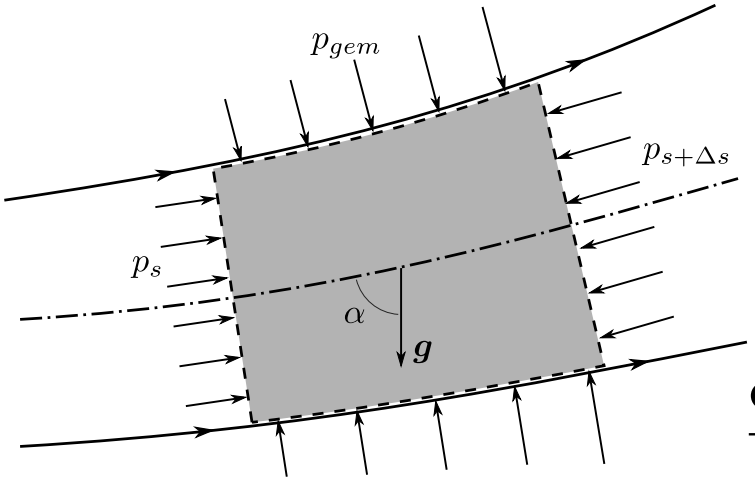
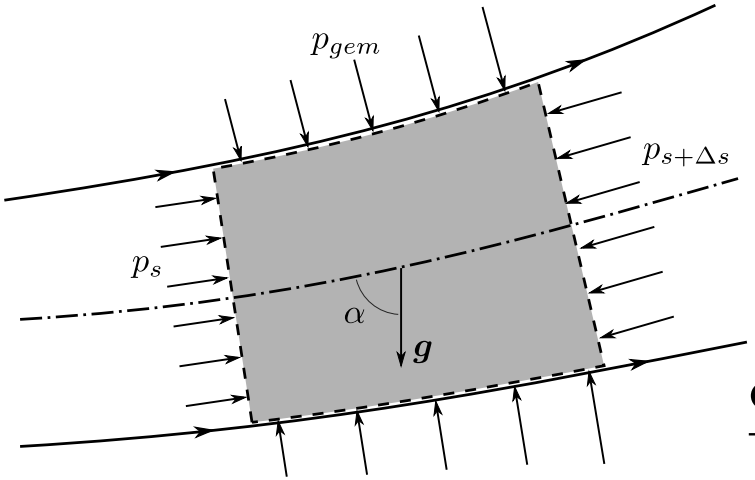


$$\frac{\mathrm{d}\mathbf{P}_{CV}}{\mathrm{d}t} + \dot{\mathbf{P}}_{\partial CV} = \mathbf{F}$$

$$\rho v v_\perp A|_{s+\Delta s} - \rho v v_\perp A|_s = pA|_s - pA|_{s+\Delta s} + p_{gem}(A|_{s+\Delta s} - A|_s) - \rho g A_{gem} \Delta s \cos\alpha$$

Bewegingsvergelijking



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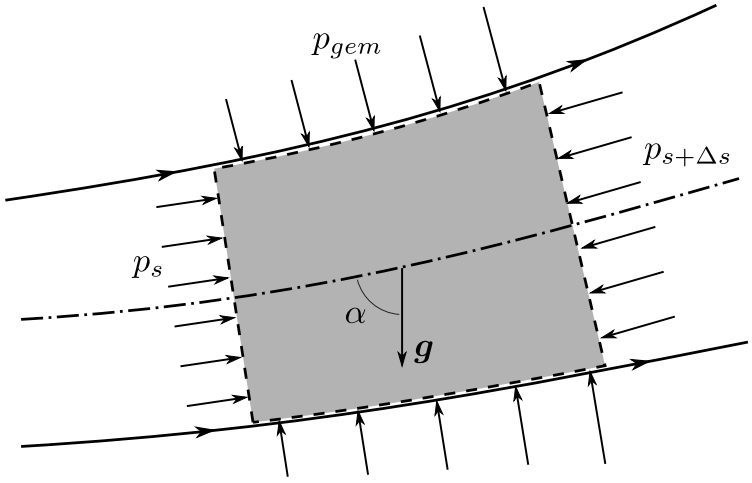
$$\frac{\rho v v A|_{s+\Delta s} - \rho v v A|_s}{\Delta s} = -\frac{pA|_{s+\Delta s} - pA|_s}{\Delta s} + p_{gem}\frac{A|_{s+\Delta s} - A|_s}{\Delta s} - \rho g A_{gem}\frac{z|_{s+\Delta s} - z|_s}{\Delta s}$$

Bewegingsvergelijking



$$\frac{\mathrm{d}\rho v v A}{\mathrm{d}s} = -\frac{\mathrm{d}pA}{\mathrm{d}s} + p\frac{\mathrm{d}A}{\mathrm{d}s} - \rho g A\frac{\mathrm{d}z}{\mathrm{d}s}$$

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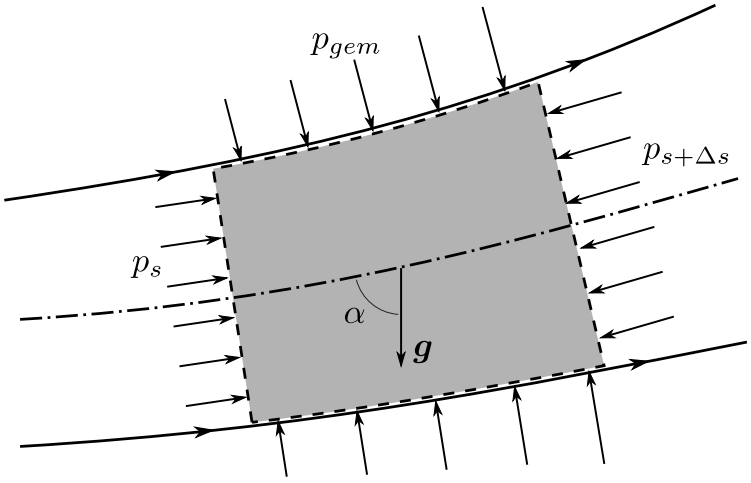

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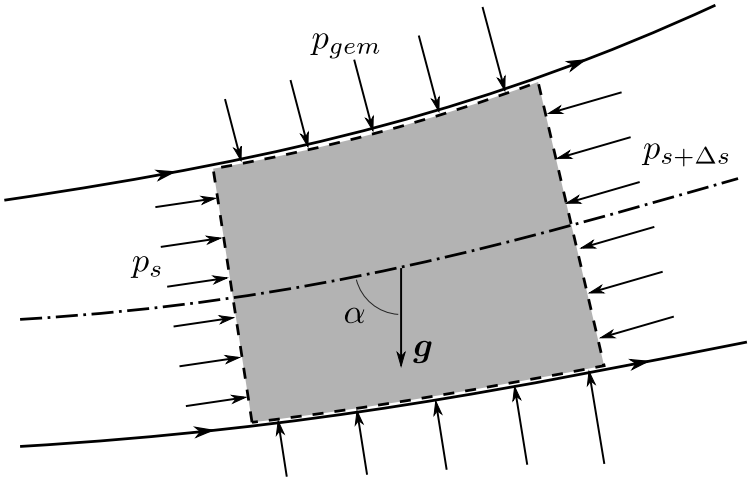
Bewegingsvergelijking



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$$\rho v\frac{\mathrm{d}v}{\mathrm{d}s} + \frac{\mathrm{d}p}{\mathrm{d}s} + \rho g\frac{\mathrm{d}z}{\mathrm{d}s} = 0 \qquad (1)$$

Deeltjesversnelling

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \tag{2}$$

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$$\rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\nabla p + \rho \mathbf{g} \tag{3}$$

Inhoud

Integratie van de bewegingsvergelijking

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}s} + \frac{\mathrm{d}p}{\mathrm{d}s} + \rho g \frac{\mathrm{d}z}{\mathrm{d}s} = 0$$

Integratie van de bewegingsvergelijking

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$$\int \rho v \frac{\mathrm{d}v}{\mathrm{d}s} \mathrm{d}s + \int \frac{\mathrm{d}p}{\mathrm{d}s} \mathrm{d}s + \int \rho g \frac{\mathrm{d}z}{\mathrm{d}s} \mathrm{d}s = \mathrm{Cst}$$

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$$\Downarrow \quad \rho = \mathrm{Cst}$$

$$\frac{1}{2}\rho v^2 + p + \rho g z = \mathrm{Cst} \tag{4}$$

Bernoulli

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- stationaire stroming

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Mechanisceh arbeid van een deeltje

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}s} = -\frac{\mathrm{d}p}{\mathrm{d}s} - \rho g \frac{\mathrm{d}z}{\mathrm{d}s}$$

$$W = \int_1^2 F \mathrm{d}s$$

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$$\Downarrow \quad \rho = \mathrm{Cst}$$

$$\rho \frac{1}{2}(v_2^2 - v_1^2) = -(p_2 - p_1) - \rho g (z_2 - z_1)$$

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$$\rho \frac{1}{2}(v_2^2 - v_1^2) + (p_2 - p_1) + \rho g (z_2 - z_1) = 0 \qquad (5)$$

Energiebeschouwingen en irreversibiliteit

$$\dot{m}(u_u + \frac{p_u}{\rho_u} + \frac{1}{2}v_u^2 + gz_u) - \dot{m}(u_i + \frac{p_i}{\rho_i} + \frac{1}{2}v_i^2 + gz_i) = \dot{Q} - \dot{W}_a$$

$$\Downarrow \quad \rho = \text{Cst}, \dot{Q} = 0, \dot{W}_a = 0$$

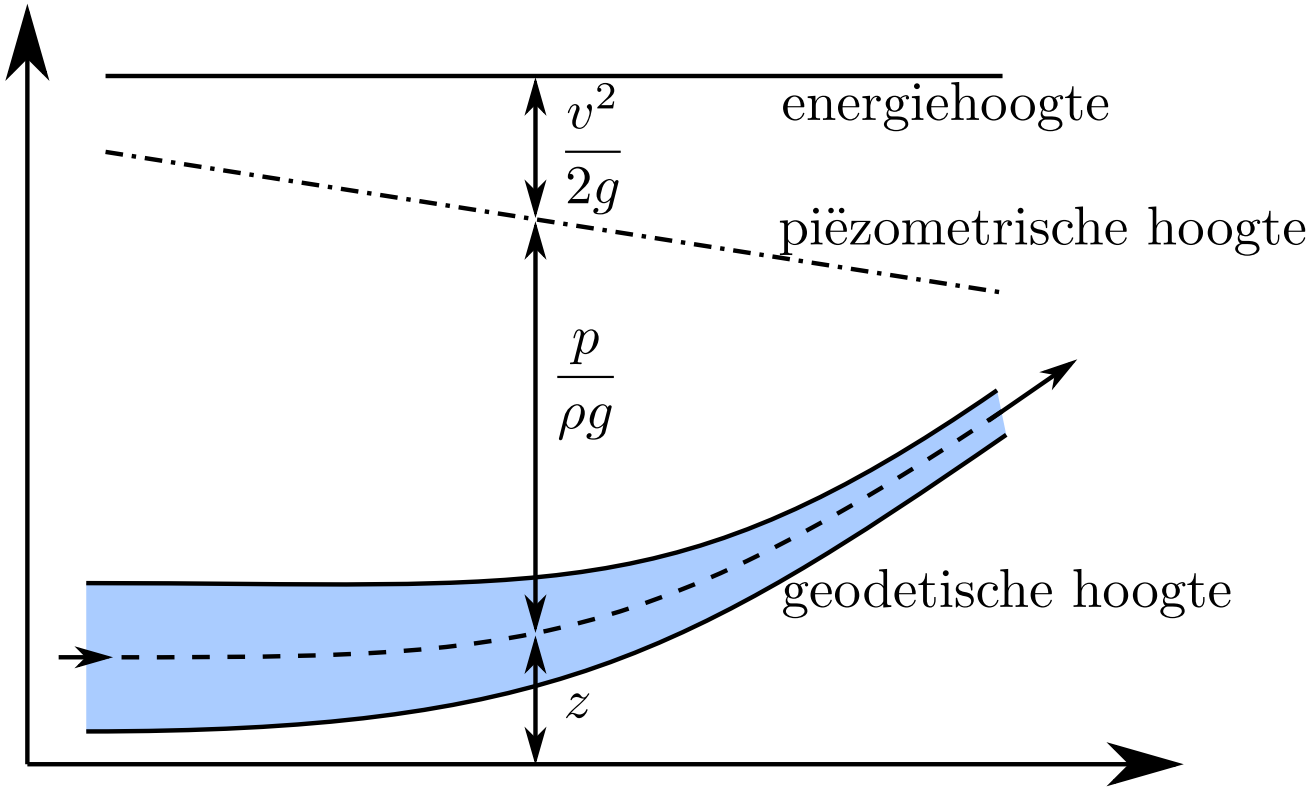
$$u_u + \frac{p_u}{\rho} + \frac{1}{2}v_u^2 + gz_u = (u_i + \frac{p_i}{\rho} + \frac{1}{2}v_i^2 + gz_i)$$

$$\rho u + p + \frac{1}{2}\rho v^2 + \rho gz = \text{Cst}$$

vs

$$p + \frac{1}{2}\rho v^2 + \rho gz = \text{Cst}$$

Grafische voorstelling



Toepassing