MINI PROJECT: BRIDGE DESIGN

Date created: 23/08/2023.

Course/Section: MIET2499/Section 1

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1. Introduction & Theory

1.1 Project Summary

Despite modern technological advances in present society, transportation is always essential as it helps humans to exchange technology and the necessities between two desired areas. Especially in the civil building factor, bridges are commonly constructed as they facilitate the products and technology transfer between two districts, two cities or even two countries. Possessing the knowledge of how to build a workable bridge has become a fundamental skill for any engineer. As for engineering, practical implementation has always been claimed as an effective learning method as it combines real-world examples and theoretical knowledge to introduce a physical product that can work up to the requirements and satisfy all the physics foundations. To gain a deeper understanding of stress analysis and material properties, a project of designing the reinforcement of a bridge has been initiated, whose solution will be presented in detail throughout this report. The bridge deck is provided beforehand, and the dimensions for the reinforcement design are calculated with the scientific method and based on the project's requirements. The testing procedure to examine the behavior of the final product follows the pre-defined setup mentioned in the project statement.

Furthermore, for the resultant design to be considered successful, the following requirements should be met:

- The pre-supplied bridge deck has the dimensions of 1000 x 100 x 3 mm, and it is made of Acrylic material. All parts must be stuck together with glue. Fasteners are not allowed. The bridge must be open for one-way traffic, which consists of two PAScars loaded with weight, whose total weight is 8 kg not including the weight of each car.
- The final bridge design must include two guard rails of about 900 mm long.
- The Safety Factor of this project is given to be 1.5.
- The final design, with a fixed support at one end and simple support at the other, must refrain from breaking when under 8.5 kg- weight and maintain deflection under 7 mm.
- A seamless connection must be made between the bridge and the dynamics tracks.
- Only Acrylic material with a 3-mm thickness is allowed in the design, considering the maximum limit is 2500 cm^2.

1.2 Theoretical Principals

This section displays all the theoretical formulas that will be used in the calculation in order to determine the optimal dimensions and designs for the solution. The equations below will also be leveraged to deduce the necessary values for three load cases: Maximum Shear Stress, Maximum Moment, and Maximum Deflection.

- Equations of Equilibrium in 2D [1]: this piece of knowledge is used to find the reaction forces.

$$\sum F_x = 0; \sum F_y = 0 ; \sum M = 0$$

- Moment of Inertia of a rectangle [2]: this value depends highly on the cross-section of the bridge. The higher the value is, the greater resistance it has toward bending.

$$I_x = \frac{1}{12}bh^3 \; where \; b: the \; cross-sectional \; base; \; h: the \; cross-sectional \; height$$

- Safety Factor [3]: this principle produces the maximum allowable stress that can be put into calculation to find the ideal height for the bridge to work properly in this project.

$$F.S. = \frac{\sigma_y}{\sigma_{allow}} = \frac{\tau_y}{\tau_{allow}}$$

- Bending Stress [2]: this concept helps to find the maximum stress developed in the bridge when it is under 8.5 kg and compare that maximum value to the allowable stress to check for the bridge's feasibility.

$$\sigma_{bending} = -\frac{My}{I}$$

$$where\ M: the\ bending\ moment;\ y: the\ distance\ from\ a\ point\ to\ the\ neutral\ axis;$$
$$I:\ the\ moment\ of\ inertia$$

- The Flexure formula [2]: used to calculate the maximum curvature in the bridge.

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$where\ \rho: the\ radius\ of\ curvature;\ M: the\ bending\ moment; \text{E: Young's Modulus; I: Moment of Inertia}$$

- Elastic Section Modulus [4]: this formula helps to find the bridge's ideal height to meet the project requirements. The maximum moment, allowable stress values can be determined with graph and mathematical analysis, which leaves the bridge's height the only unknown value since c = h / 2.

$$S = \frac{|M|_{max}}{\sigma_{allow}} = \frac{I}{c}$$

$$where\ I: the\ moment\ of\ interia;\ c: the\ distance\ from\ the\ neutral\ axis\ to\ the\ furthest\ point$$

- Shear and Moment diagrams [4]: which display the instantaneous shear and moment value at each point along the horizontal length of the beam, from which the maximum shear and moment values can be deduced.
- Equation of the Slope and Elastic Curve [5]: used to extract the expression for the slope and deflection at any point along the bridge. From the elastic curve, the maximum deflection can be detected and compared to the allowable number (7 mm) to clarify the bridge's workability.

$$Slope\ equation: \theta = \frac{1}{EI}\int_0^x M(x)dx + C_1$$

$$Elastic\ Curve: y = f(x) = \int_0^x \left[\int_0^x M(x)dx + C_1\right] + C_2$$

- Superposition method [5]: used to find the expressions for the reaction forces, from which the shear and moment diagram can be drafted.
- First Moment of Area [6]: this concept supports finding the value for Q, which appears in the longitudinal stress formula to calculate the maximum shear stress developed in the bridge.

$$Q = y'A'$$

- Longitudinal Shear Stress [6]: this formula leads to the result of the maximum shear stress developed in the bridge, whose value will be compared to the allowable shear stress to test the bridge's workability.

$$\tau_{avg} = \frac{VQ}{It}$$

$$where\ V: the\ shear\ stress;\ Q: the\ first\ moment\ of\ area\ about\ the\ neutral\ axis;$$
$$I: the\ moment\ of\ interia;\ t\ the\ cross-section\ width$$

- Slope and Elastic curve equations, maximum deflection of a concentrated load at the free end of the beam [7]: this concept is integrated in the Superposition method to determine the expression for the reaction force R_B.

1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$

Figure 1. Slope and Elastic curve equations, maximum deflection of a concentrated load at the free end of the beam [7]

- Slope and Elastic curve equations, maximum deflection of a concentrated load at an arbitrary point along the beam: this concept is also integrated in the Superposition method to determine the expression for the reaction force R_B.

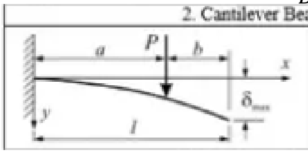
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$

Figure 2. Slope and Elastic curve equations, maximum deflection of a concentrated load at an arbitrary point along the beam [7]

1.3 Potential Sources of Error

Even though careful calculations are implemented and presented in detail in this documentation, there are still some other causes that can lead to the improper behavior of the designed bridge, whose reasons will be discussed in this section.

The first error can be sourced from the tolerances of the manufacturing method, which is laser cutting, which can result in an unfit situation between the parts and an incomplete assembled bridge. In light of the solution, this problem can be minimized by incrementing or reducing the calculated dimensions intentionally while considering the fitting condition between all the parts so that the tolerance cannot have negative influences on the whole system.

The second error can stem from the final assembling phase, where the parts, such as the bridge deck, reinforcement materials, or pillars, are inadvertently misaligned due to the incompetent gluing execution of the assembler. In other cases, not using good quality glue can lead to a weak adhesive connection between the sections and introduce higher chances of failure. This can cause the final product to break down during the demonstration and fail to meet the project's goals. Therefore, extreme carefulness and patience must be put into the assembly and glue selection process.

The final type of error is attributed to the mistakes made in the measurement and calculation procedures. This often emerges when the project participants do not adhere to the standard calculation method or devise a design before justifying it with robust scientific evidence. This error is blamed on the lack of theoretical knowledge and ignorance of the calculating procedures of the participants, which can be solved by paying attention and executing the correct order of the calculation sequence of finding the dimensions for the final design based on the theories delivered in the course.

The design error can be uncovered and solved with the calculation as well as drawing the graphs of the maximum shear stress, normal stress and deflection developed within the bridge based on the modified cross-section area in the suggested solution and comparing them with the allowable quantities defined in the project statement. If the calculated results turn out to be greater than the allowable ones, the solution design must be revised and adjusted accordingly. In order to prevent this type of error, careful calculations and precise simulations following the certified scientific knowledge must be carried out.

1.4 Project Constraints

When implementing a project, the constraints must always be considered since they significantly impact the dimensions and physical looks of the solution design, influence the results of the theoretical calculations and what values should be put into 3D drawings., which make the whole finalize system working properly. This section will discuss three types of constraints (safety factor, material, client's requirement) in detail.

- Safety factor is the element which provides the value for the allowable stresses of various types (normal stress, shear stress, longitudinal shear stress, bending stress, etc.), that need to be put into consideration in order to increase safety and mitigate the risk of failure of the final design. Therefore, this value is the most essential factor that cannot be changed; hence, by applying the formular in section 1.2 and given S.F. of 1.5, we can figure out the allowable stress by:

$$S.F. = \frac{\sigma_y}{\sigma_{allow}} = 1.5 \Longrightarrow \sigma_{allow} = \frac{\sigma_y}{1.5} = \frac{24.4}{1.5} = 16.26\ (MPa).$$

Paying attention to those values can ensure qualified performance of the finalize bridge under different maximum stress conditions.

- Material constraints are also critical to the final product since they dictate how much and what type of material can be used to be integrated into the final solution for best performance with lowest cost. Based on each situation, the requirement for material can be modified. In this project, the ideal load of material required is lower than 2500 (cm^2) with scalar thickness of 3mm. From this, the moment of inertia I and cross-section area A in the theoretical principal need to be changed to meet the requirement with appropriate bridge height. Different materials or different thicknesses can lead to a different bridge design. Paying extreme caution to this constraint can ensure the expected behavior of the physical product and meet the project's requirements.

- The material has a Young's Modulus of 1.1 (GPa) and a Yield Strength of 24.4 (MPa). This information will be considered when calculating Flexural Rigidity.

- Client's requirement helps to control how the testing experiment will be carried out to judge whether the design works as expected. The numbers supplied in these constraints are based on the project's requirements, which will be applied to the theoretical formula to deduce the optimal dimensions for the bridge's cross-section, also for the engineer to strictly follow the client's need. Therefore, these constraints play an important role in moving from the calculation in theories to the actual finite product.

- A level and seamless connection must be made between the bridge and 1.2 (m) the PASCO dynamics track.
- The bridge must withstand the total weight of 8 kg (8.5 kg when considering the weight of two PAScars) without breaking while maintaining deflection under 7 (mm).
- The bridge will have a fixed support at one end, whereas a simple support will be applied to the remaining end. This information is essential for calculating the reactional forces and moments.
- A pair of guard rails, which span about 900 (mm), should be designed and assembled into the bridge to prevent the cars from going off-track.

1.5 Solution sketch

The sketch of the solution denoted with explaining texts is illustrated in Figure 3.

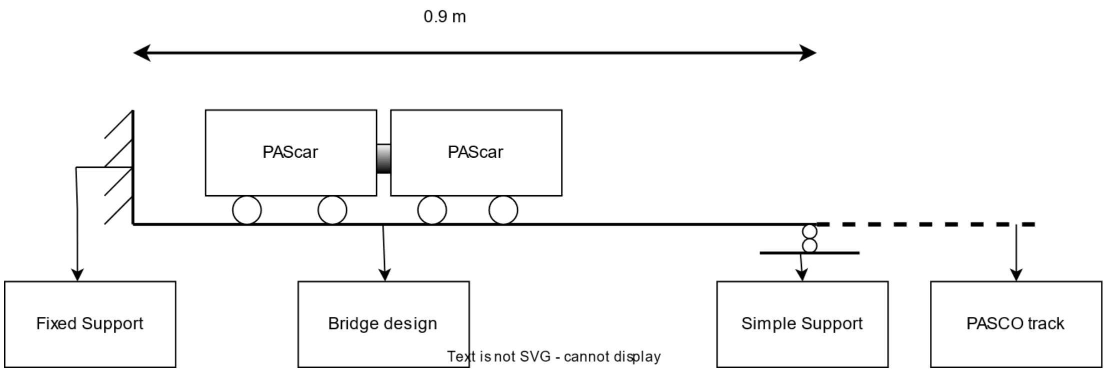


Figure 3. The general sketch of the bridge solution design and testing

To test the stiffness and failure conditions of the bridge, it will be fixed at one end and simply supported at the other end (which resembles the behavior of a roller). The bridge end that has simple support will be connected to a PASCO track. Each car weighs about 250g and will carry 4 kg as a load, leading to a total of 8.5 kg in weight.

Three load cases are presented: Maximum Shear Stress, Maximum Moment, and Maximum Deflection. In this report, the letter a is defined as the distance from the fixed support to the wheel of a PAScar, whose value runs from 0 to 0.9 (m). The safety of this project is 1.5, which is the base for calculating the maximum allowable shear and normal stress used in finding the bridge height afterwards. Regarding the Maximum Moment and Maximum Deflection cases, the calculation procedure begins with the Superposition Method (explained in Appendix A) to compute the value for the reaction force R_B, from which other reaction forces R_A and M_A will also be found. After that, the Section Method is applied to divide the whole bridge system into five sections. In each, the internal shear force and moment will be deduced as a function with respect to a; those expressions will be plotted with the range of a (the length of the bridge) using MATLAB. After extracting the maximum shear force and maximum moment from the Shear and Moment graphs and the locations where they occur, the bridge's height can be calculated. The thickness of the bridge will be assumed based on the remaining materials amount available and the optimized moment of inertia.

For the Maximum Deflection case, the reaction force R_B function, which has been deduced in the Superposition approach used at the beginning, will be reapplied in the summation of the subsystem ($\delta_{total} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5$). This summation (more details in Appendix A) expression will be plotted with the range of 0 to 0.9 m, from which the maximum deflection and its occurring location will be reported. The deflection requirement (7 mm) will be used to find another value for the design height. The two heights from all three cases will be compared, where the smallest value will be put into the solution design.

2.Design Proposal and Analysis

2.1 Design Overview

In order to meet the requirement by providing the best performance for the project, our team have the idea of adding these features into the bridge deck for stability during the demonstration procedure.

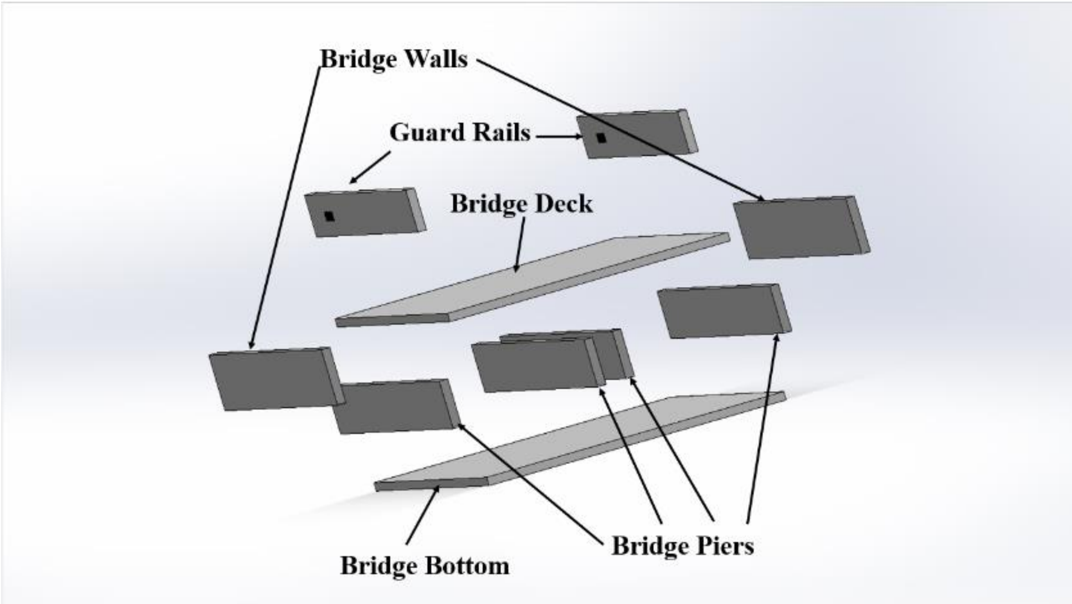


Figure 4. Bridge's Part Names Notation

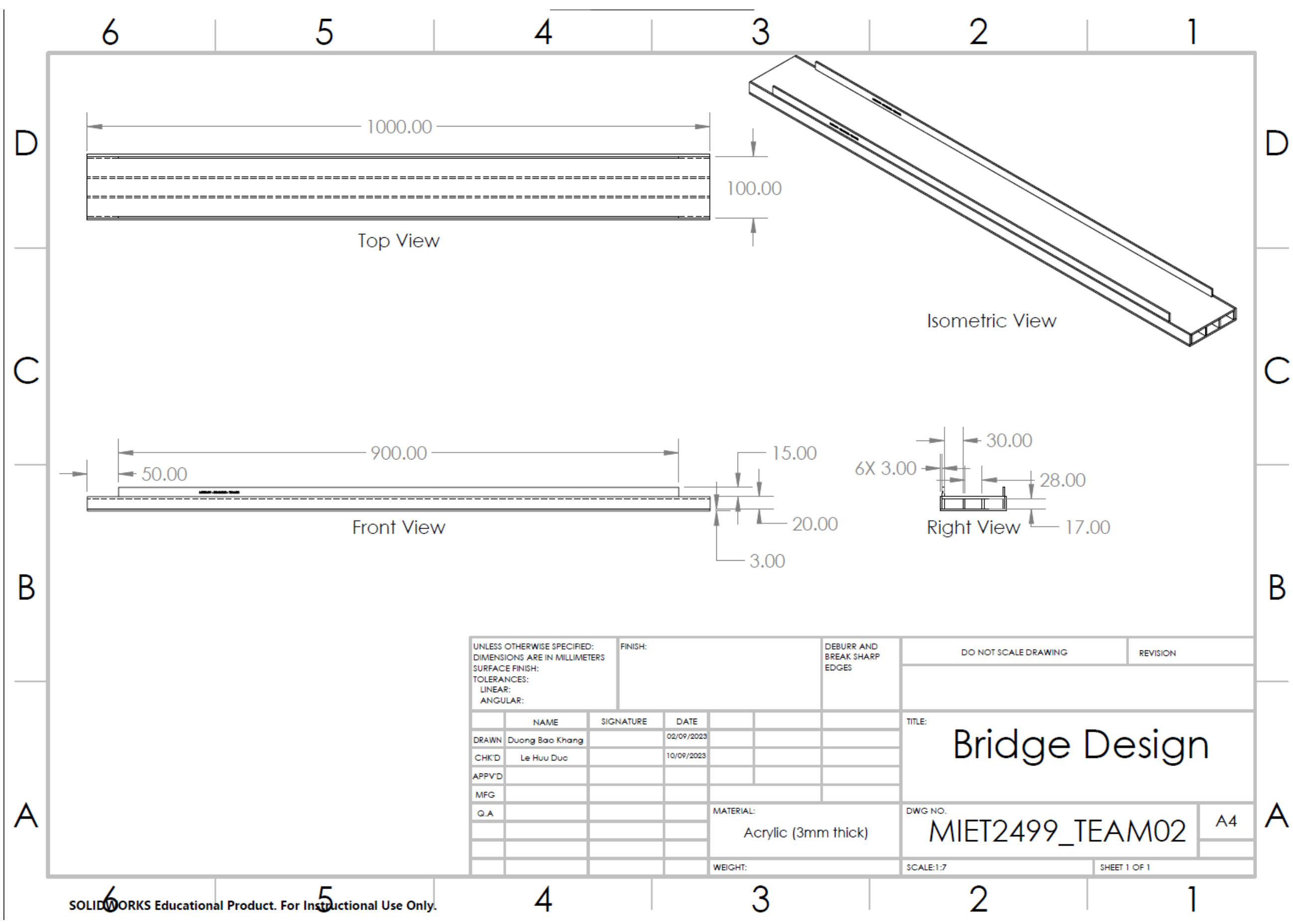


Figure 5. 2D Drawing of the proposed bridge design.

Figure 5 outlines the proposed bridge design for this project with four primary views and their essential dimensions. Figure 4 demonstrates how each part is assembled to implement the suggested solution. The only material used in the project is 3-mm thick acrylic. The cross-section bears a generally rectangular shape), similar to what has been listed in the project requirements, with 1000 x 106 x 23 mm (Length x Width x Height). The general height of the bridge is 23 mm, which has been selected carefully with the calculating scientific method presented in parts 2.2 to 2.3 and adjusted to meet the seamless connection requirement between the bridge and the PASCO track. Two guard rails as high as 15 mm and as long as 900 mm are installed on the bridge surface to prevent the cars from going off track and simulate one-way traffic. Two 20-mm high acrylic parts added to the outer edges encapsulating four 17-m high ones ensure that the forces are distributed equally under the effects of 8-kg PAScars. The insertion of two 17-mm columns inside the rectangular cross-section equips the whole system with the behaviour of an I-beam, which means moving the most material area far away from the neutral axis and to the location receiving the most extensive amount of stress. As a result, this structure will help to reduce the maximum bending and shear stress the bridge bears and its maximum deflection. To dive into deeper details of this design, Table 1 displays the Bill Of Materials for this project. The amount of material spent in the solution is 2410 cm^2, adhering to the 2500 cm^2 restriction.

Table 1. Bill Of Materials for the bridge design

ITEM NO.	PART NAME	PART NUMBER	DIMENSIONS	DESCRIPTION	MANUFACTURING METHOD	MATERIAL	QTY.
1	Bridge Deck	MIET2499_TEAM02_001	1000 x 100 x 3 mm (Length x Width x Thickness) Bridge Deck	Provided by the course coordinator. A slight straight rectangular plate	Provided by the course coordinator	Acrylic (3 mm-thick)	1
2	Bridge Bottom	MIET2499_TEAM02_002	1000 x 106 x 3 mm (Length x Width x Thickness) Bridge Bottom	Surface Area: 1060 cm^2 A slight straight rectangular plate	Laser cutting	Acrylic (3 mm-thick)	1
3	Guard Rail	MIET2499_TEAM02_003	900 x 15 x 3 mm (Length x Width x Thickness) Guard Rail	Surface Area: 135 cm^2 There are 2 small slight straight rectangular plates	Laser cutting	Acrylic (3 mm-thick)	2
4	Bridge Pier	MIET2499_TEAM02_004	1000 x 17 x 3 mm (Length x Width x Thickness) Bridge Pier	Surface Area: 170 cm^2 There are four I beam put in the middle of the bridge	Laser cutting	Acrylic (3 mm-thick)	4
5	Bridge Wall	MIET2499_TEAM02_005	1000 x 20 x 3 mm (Length x Width x Thickness) Bridge Wall	Surface Area: 200 cm^2 It has the same shape as Bridge Pier.	Laser cutting	Acrylic (3 mm-thick)	2
Total Material Used				2410 cm^2			

2.2 Design Load Cases

First of all, the maximum allowable normal and shear stress should be computed based on the project requirements and the safety factor:

$$\sigma_{allow} = \frac{\sigma_y}{S.F.} = \frac{24.4\ MPa}{1.5} \approx 16.267\ (MPa);\ \tau_{allow} = \frac{\tau_y}{1.5} = \frac{14.1\ MPa}{1.5} = 9.4\ (MPa)$$

As stated in section 1.5, the allowable values would become the essential ingredients to find out what is the ideal bridge height in the solution design.

Regarding the "the worst-case scenarios" as previously mentioned above in section 1.5, three variants of the maximum will be calculated: Maximum Shear Force, Maximum Moment, and Maximum Deflection. The general free body diagram of the bridge with the reaction forces is illustrated in Figure 6 below.

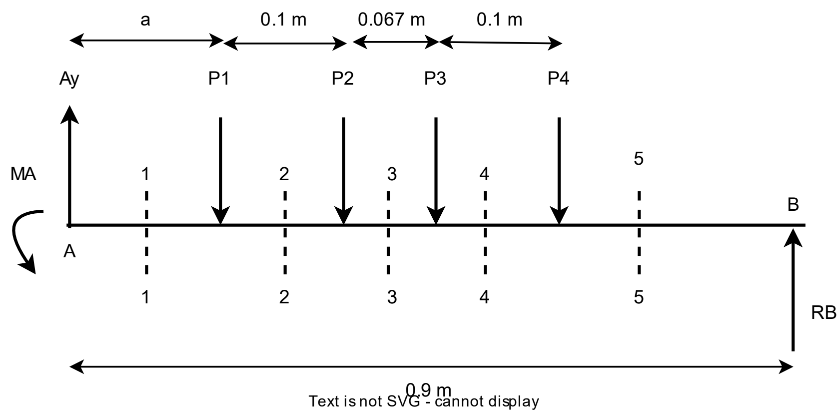


Figure 6. General Free-Body Diagram of the Project Problem

In order to find the values for the reaction forces, the expression of R_B with respect to "a" (distance from first end of the bridge to the wheel 1) should be uncovered first. Then, after applying the equations of equilibrium in the y direction and moment around point A, A_y and M_A can be discovered as well. The detailed procedure to find B_y with the superposition method is presented in Appendix A; in general, the expression of R_B will be:

$$R_B = [3.474a^2(2.7 - a) + 3.474(a + 0.1)^2(2.6 - a) + 3.474(a + 0.167)^2(2.533 - a) + 3.474(a + 0.267)^2(2.433 - a)\,] / 0.243\ where\ 0 \leq a \leq 0.9\ m$$

The values for $P_1, P_2, P_3, and\ P_4$ can be obtained as:

$$P_1 = P_2 = P_3 = P_4 = \frac{(8.5\ kg)(9.81\ m/s^2)}{4} \approx 20.846\ (N)$$

With the application of the equations of equilibrium, $M_A\ and\ A_y$ can also be found:

$$\uparrow + \sum F_y = 0 \Rightarrow A_y - P_1 - P_2 - P_3 - P_4 + R_B = 0 \Longrightarrow A_y = 4P_1 - R_B = 4(20.846) - R_B$$

$$\Rightarrow A_y = 83.384 - R_B\ (N)$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow M_A - (P_1)(a) - (P_2)(a + 0.1) - (P_3)(a + 0.167) - (P_4)(a + 0.267) + (0.9)R_B = 0 \Rightarrow M_A = (20.846)(4a + 0.534) - 0.9R_B\ (N \cdot m)$$

After that, the expressions for the internal forces of each section (1-1 to 5-5) can be obtained by following the same concept as finding A_y *and* M_A. The whole process and the results of those forces will be dived into in the Maximum Stress Analysis and Appendix B. If x is called as an arbitrary cutting position in each section, each expression will now depend on x, whose value depends on "a" (detailed calculation in Appendix B). By providing a variety of numbers to x and "a" as the test condition, the graphs outlining the changes in the internal shear forces and moments can be viewed in Figures 7 and 8.

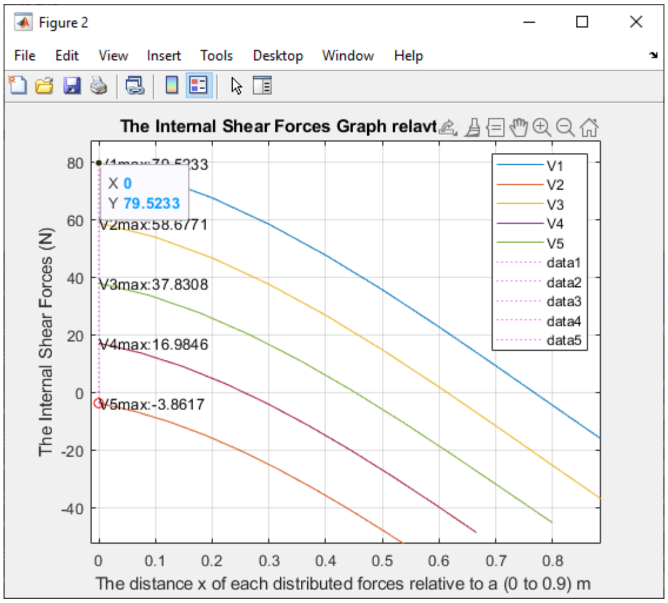


Figure 7 Maximum Shear Stress and its location

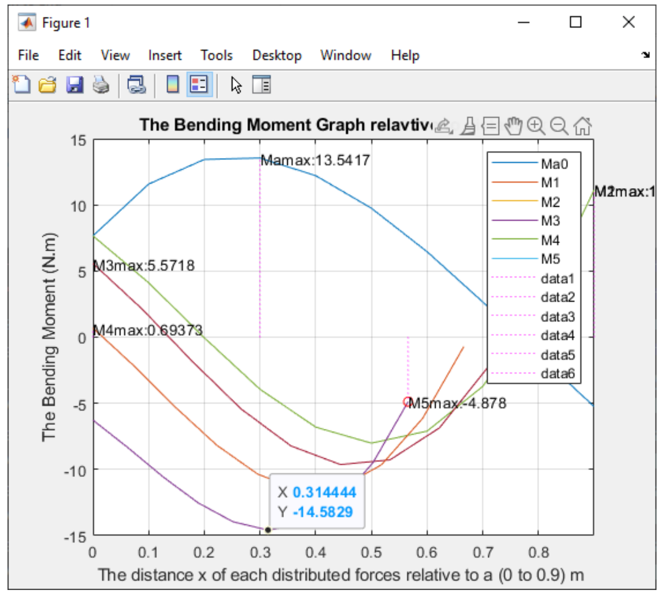


Figure 8. Maximum Bending Moment and its location

As can be seen in the internal shear force graph (Figure 7) and the internal moment graph (Figure 8), it is concluded that the maximum moment occurs when the value "a" reaches 0.314 (m); the moment will reach its peak as $-14.583\ (N \cdot m)$ (Compressive bending moment). The internal shear force achieves the most significant value when the PAScars are at the beginning of the bridge (a = 0 m); the highest number recorded is $V_{max} = 79.5233\ (N)$. The free-body diagrams of both the Maximum Shear Force (Figure 9) and Maximum Moment (Figure 10) load cases are shown below.

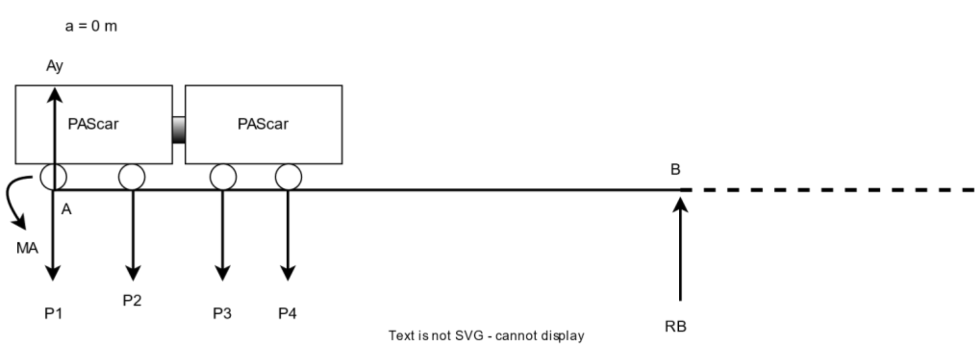


Figure 9. Sketch of Maximum Shear Stress case

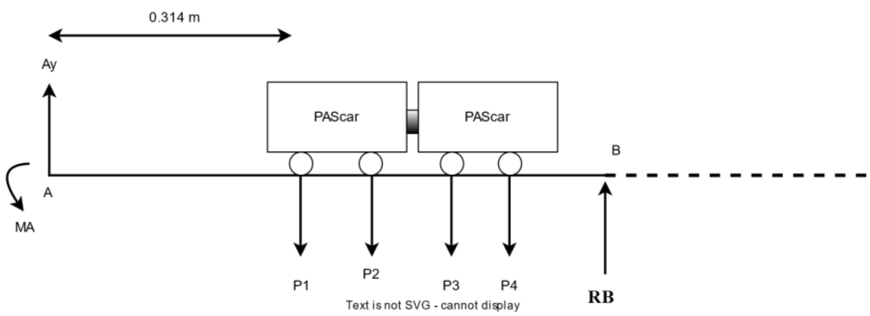


Figure 10. Sketch of Maximum Bending Moment case

The Maximum Deflection case is also solved with the Superposition Method and the help of Elastic Curve Drawing Software (details in Appendix C). The result is that when a = 0.525 (m), where the deflection will peak at -6.92 (mm). The free-body diagram for this worst case is displayed in Figure 11.

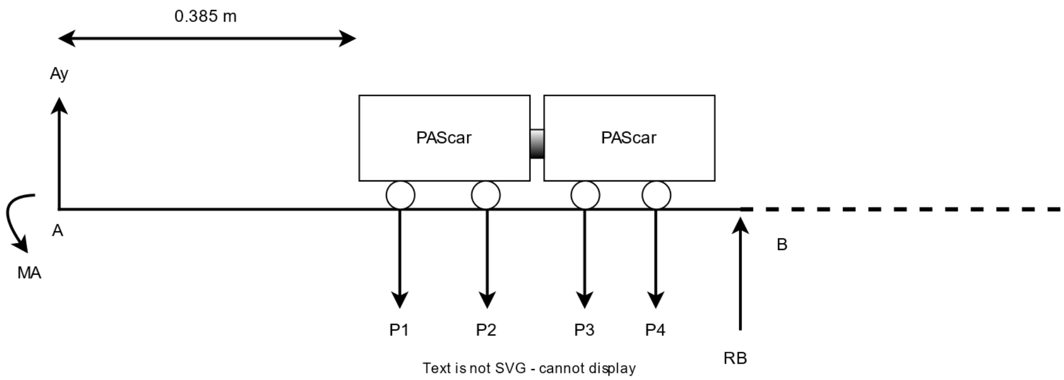


Figure 11. Sketch of Maximum Deflection case

2.3 Maximum Stress Analysis

As elaborated in the previous part, the whole system is treated as five smaller systems with their distinct internal forces. With the leverage of the equations of equilibrium, the equations are shown as follows:

$$V_1 = A_y\ (N); M_1 = A_y x - M_A\ (N \cdot m)$$

$$V_2 = A_y - 20.846\ (N);\ M_2 = (20.846 - P_1)(x + a) + 20.846(a) - M_A(N \cdot m)$$

$$V_3 = A_y - 41.692(N);\ M_3 = (A_y - 41.692)(x + a + 0.1) + (20.846)(2a + 0.1) - M_A(N \cdot m)$$

$$V_4 = A_y - 62.538(N);\ M_4 = (A_y - 62.538)(x + a + 0.167) + (20.846)(3a + 0.267) - M_A\ (N \cdot m)$$

$$V_5 = A_y - 83.384\ (N);\ M_5 = (A_y - 83.384\ )(x + a + 0.267) + (20.846)(4a + 0.534) - M_A\ (N \cdot m)$$

Based on Figures 7 to 8 above, when the value “a” reaches 0.314 m, the bending stress will climb to its peak at $-14.583\ (N \cdot m)$. The shear force will also record the highest number with 79.5233 (N) when “a” equals 0 (at the fixed support). With these maximum values, the bridge’s height can be computed with the elastic section modulus formula:

$$S = \frac{|M_{max}|}{\sigma_{allow}} = \frac{I}{c} \Rightarrow \frac{14.583\ (N \cdot m)}{16.267\ MPa} = \frac{\frac{1}{12}(0.1\ m)h^3}{\frac{h}{2}} \Longrightarrow h \approx 7.3\ (mm)$$

Therefore, the ideal minimum height dimension for the bridge would be 7.3 (mm). However, this height assumes that the rectangular cross-section is not hollow, which is not feasible in this project due to material restrictions. Taking that number into design consideration produces the solution whose details are discussed in the Design Overview (2.1) part. To minimize the bridge's deflection and ensure a seamless connection between the bridge and the PASCO track, a height of 23 (mm) is chosen. Reapplying the new moment of inertia of the proposed solution bridge into the same section modulus formula to calculate the maximum normal stress will introduce another conclusion:

$$I_{new} = \frac{1}{12}(0.1 + 0.006)(0.023)^3 - \frac{1}{12}(0.028)(0.017)^3 - \frac{2}{12}(0.03)(0.017)^3 \approx 7.145\ \times 10^{-8}(m)$$

$$\sigma_{max} = \frac{|M_{max}| \times c_{new}}{I_{new}} = \frac{14.583\ (N \cdot m) \times \left(\frac{0.023\ m}{2}\right)}{7.145\ \times 10^{-8}(m)} \approx 2.35\ (MPa)$$

$$\Rightarrow \sigma_{max} = 2.35\ (MPa)\ < \sigma_{allow} = 16.267\ (MPa)$$

The maximum longitudinal shear stress occurs at the neutral axis so that:

$$\tau_{max} = \frac{V_{max}Q_{new}}{I_{new}t_{new}} = \frac{(79.5233)\left[2\left(\frac{23}{2}\right)(3)\left(\frac{23}{4}\right) + 4\left(\frac{17}{2}\right)(3)\left(\frac{17}{4}\right) + (100)(3)\left(\frac{17}{2} + \frac{3}{2}\right)\right](10^{-9})}{[7.145\ \times 10^{-8}](0.003 \times 6)}$$

$$\Rightarrow \tau_{max} = 0.24\ (MPa) < \ \tau_{allow} = 9.4\ (MPa)$$

Based on the two deductions above, it is safe to confirm that the suggested bridge design (shown in part 2.1) qualifies for the failure condition since both the maximum shear stress and normal stress are less than the allowable ones.

Figures 12 to 14 below outline the Shear and Moment Diagram for each worst-case scenario. Overall, a few important points are noted (x is an arbitrary position along the beam):

- For Maximum Shear Case (a = 0):
  $V = V_{max} = 79.5233\ (N)\ at\ x\ = 0;\ M = |M_{max}| =\ 9.02\ (N \cdot m)\ at\ x\ =\ 0$
- For Maximum Moment Case (a = 0.314 m):
  $V = V_{max} = 56.8537\ (N)\ at\ \ 0 \leq x \leq 0.31\ (m);\ M = |M_{max}| =\ 13.4371\ (N \cdot m)\ at\ x\ =\ 0$
- For Maximum Shear Case (a = 0.385 m):
  $V = V_{max} = 49.2\ (N)\ when\ 0 \leq x \leq 0.39\ (m);\ M = |M_{max}| =\ 12.05\ (N \cdot m)\ when\ x\ =\ 0$

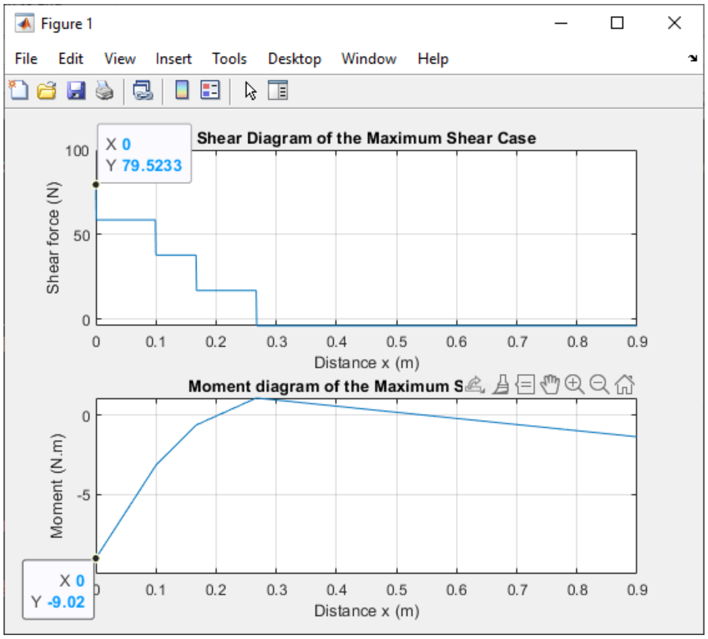


Figure 12. Shear and Moment Diagram of Maximum Shear Case

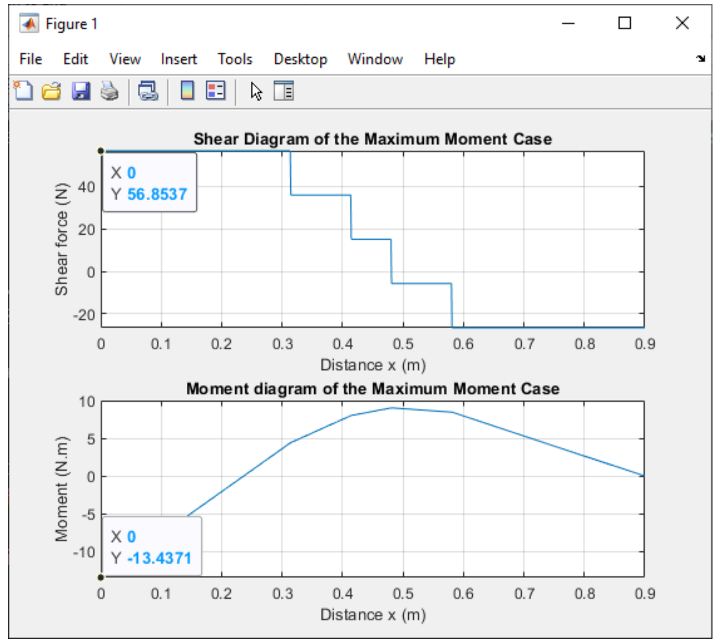


Figure 13. Shear and Moment Diagram of Maximum Moment Case

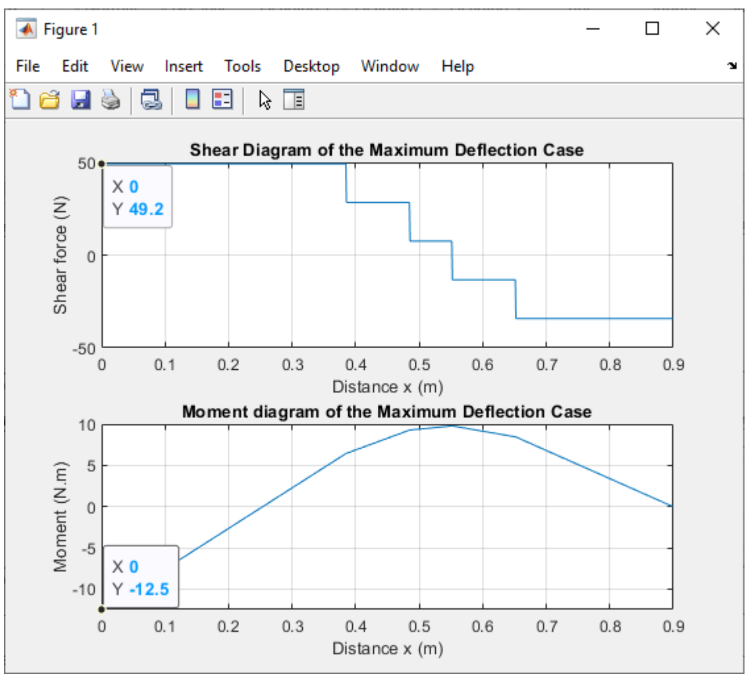


Figure 14. Shear and Moment Diagram of Maximum Deflection Case

In terms of the amount of bending stress that each component bears to contribute to the total, the greatest amount of stress will likely be moved to the bridge deck, on which surface the PASCars drive. This is because the bridge's cross-section is designed to be primarily hollow inside, which means that the area with the most use of materials will be moved to the location furthest from the neutral axis, in this case, the bridge deck and the bridge bottom (visualization details in section 2.1). However, since the cars only move along the bridge deck, it is prone to more bending stress than other components. Therefore, when the PASCars are a = 0.314 (m) far away from the fixed support where the maximum bending stress is 2.35 (MPa), the bridge deck will share the most significant percentage of that number.

Regarding the longitudinal shear stress, it is highly speculated that the area where the glues are applied to attach the bridge deck and the bridge piers, the bridge piers and the bridge bottom, between the bridge piers and bridge walls (visualization image in section 2.1) will suffer the most significant proportion; since the longitudinal shear stress is the type primarily leading to slipping between layers where the fasteners or glues are applied. In the suggested design analyzed in Design Overview, the maximum shear stress developed within the system is 0.24 (MPa), so the assemblers must ensure that the glue is suitable for this application and applied sufficiently to create a strong attachment between the parts. Another stress worth mentioning is the transverse shear stress, which distorts the materials along with the direction of the applied forces (the weight of the loads and PAScars). Being aware of that, this stress type usually reaches its peak at the neutral axis. Therefore, the bridge piers are predicted to receive the most transverse shear stress.

2.4 Deflection Analysis

This deflection aspect of the project helps to pose another reason why, even though the ideal minimum height $h = 7.3\ (mm)$ is found, it is not brought into the final design dimensions. When using the software [8] to draw the elastic curve using the 7.3 (mm) value, the maximum deflection is reported to be greater than 7 (mm) (Figure 15) given $a = 0.385\ (mm)\ (where\ the\ highest\ maximum\ deflection\ is\ likely\ to\ occur)$, failing the stiffness condition. This is because the moment of inertia designed with this height does not promise a high enough value to prevent the bridge from bending down less than 7 (mm). Furthermore, the 7-mm design does not make a smooth connection between the bridge and the PASCO track (23 mm high), blamed for the noticeable height difference. Thus, the suggested height of the bridge, which is 23 (mm), not only increases the overall bridge moment of inertia substantially but also ensures the PAScars can travel from and to the bridge seamlessly.

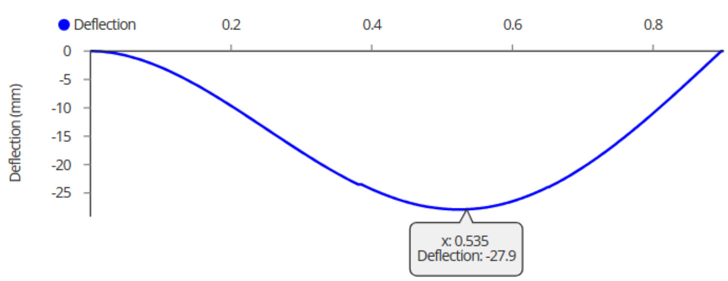


Figure 15. Maximum deflection when designing with 7.3 mm height.

The procedure of finding the position where the bridge deflects downwards the most has been presented in detail in Appendix C with the support of Elastic Curve Drawing software [8]. To summarize the result, it is deduced that when a = 0.385 (m), the peak deflection value will be $-6.92\ (mm)$ at x = 0.525 (m), which is less than the requirement (7 mm). It is enough to reflect that the bridge has been proven scientifically to pass the Failure and Stiffness Conditions. Therefore, all the goals of the project statements have been satisfied with the suggested bridge design in part Design Overview (2.1). Below (Figures 16 to 18) are the elastic curves for all three loading cases (Maximum Shear, Maximum Moment, and Maximum Deflection).

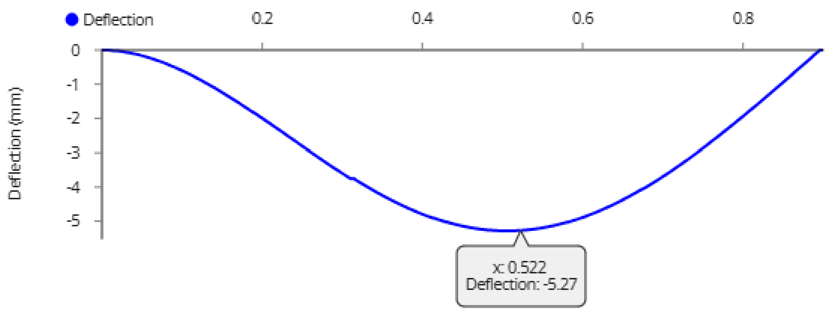


Figure 16. Elastic Curve for Maximum Shear Case (a = 0)

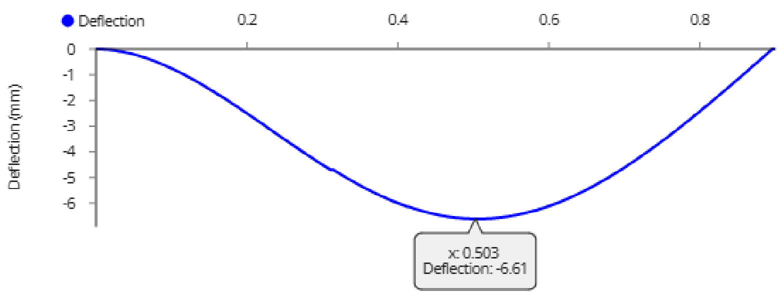


Figure 17. Elastic Curve for Maximum Moment Case (a = 0.314 m)

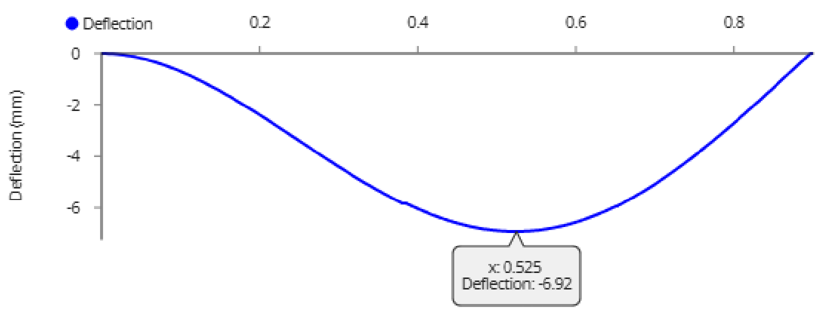


Figure 18. Elastic Curve for Maximum Deflection Case (a = 0.385 m)

Based on the graphs above, the greatest value and its location for each loading case is recorded:

- For Maximum Shear Case (a = 0): $\delta = |\delta_{max}| = 5.27\ (mm)\ at\ x\ =\ 0.522\ (m)$
- For Maximum Moment Case (a = 0.314 m): $\delta = |\delta_{max}| = 6.61\ (mm)\ at\ x\ =\ 0.503\ (m)$
- For Maximum Deflection Case (a = 0.385 m): $\delta = |\delta_{max}| = 6.92\ (mm)\ at\ x\ =\ 0.525\ (m)$

The maximum curvature in the bridge for each worst-case scenario can be calculated as:

- For Maximum Shear Case ($|M_{max}| = 9.02\ (N \cdot m)\ found\ in\ section\ 2.3$):

$$\left(\frac{1}{\rho}\right)_{max} = \frac{|M_{max}|}{EI} = \frac{9.02\ (N \cdot m)}{(1.1 \times 10^9\ Pa)(7.145\ \times 10^{-8} m)} \approx 0.115$$

- For Maximum Moment Case ($|M_{max}| = 13.44\ (N \cdot m)\ found\ in\ section\ 2.3$):

$$\left(\frac{1}{\rho}\right)_{max} = \frac{|M_{max}|}{EI} = \frac{13.44\ (N \cdot m)}{(1.1 \times 10^9\ Pa)(7.145\ \times 10^{-8} m)} \approx 0.171$$

- For Maximum Deflection Case ($|M_{max}| = 12.5\ (N \cdot m)\ found\ in\ section\ 2.3$):

$$\left(\frac{1}{\rho}\right)_{max} = \frac{|M_{max}|}{EI} = \frac{12.5\ (N \cdot m)}{(1.1 \times 10^9\ Pa)(7.145\ \times 10^{-8} m)} \approx 0.159$$

Generally, despite different distances from the fixed support to the PAScars in various worst-case scenarios, the elastic curves of all three cases resemble in terms of shape and trend. All curves start at 0 in deflection and begin curving down to reach the peak value at roughly x = 0.5 (m) before concaving up to 0 in deflection at x = 0.9 (m) again. The negative slope value in the first half of the bridge stems from the internal negative moment value, which is deduced from the equations of equilibrium presented in Appendix B. At the location where the maximum deflection occurs, the slope will become zero and be in a positive state since then. The concentration of the peak deflection values around the middle point can be attributed to the increase of the internal moment, which increments along with the length of the cut section. That explains why the internal moment tends to reach the highest value at the middle point (where the slope is zero) before decreasing when the cut-section length is longer than 0.5 (m) despite the more extensive sliced portion. Additionally, the deflection stays at 0 when x = 0 and x = 0.9 (m) since there are supports (fixed and simple types) at two ends of the bridge, restricting the movement of the structure. The differences in the maximum deflection reported numbers are due to the distinct locations of the PAScars in different worst-case scenarios, leading to diversity in the reaction forces values.

3. Conclusion

In conclusion, for each design worst case, the highlight information (in absolute values) is:

- For Maximum Shear Case (a = 0): maximum shear force (79.5233 N) at x = 0, maximum moment ($9.02\ N \cdot m$) at x = 0, maximum deflection (5.27 mm) at x = 0.55 (m), maximum curvature (0.115)
- For Maximum Moment Case (a = 0.314 m): maximum shear force ($56.8537\ N$) at $0 \le x \le 0.31\ (m)$, maximum moment ($13.4371\ N \cdot m$) at x = 0, maximum deflection (6.61 mm) at x = 0.503 (m), maximum curvature (0.171)
- For Maximum Shear Case (a = 0.385 m): maximum shear force ($49.2\ N$) at $0 \le x \le 0.39\ (m)$, maximum moment ($12.05\ N \cdot m$) at x = 0, maximum deflection (6.92 mm) at x = 0.525 (m), maximum curvature (0.159)

All in all, the bridge suffers the maximum longitudinal shear stress of $79.5233\ N$ when the PAScars are at the beginning of the bridge (the distance from the fixed support to the first wheel is 0). To eradicate the destructive consequences of this stress type, the calculation of the maximum shear stress based on the proposed design and comparing the result to the allowable number must be implemented. Other measures can be increasing the moment of inertia (I), cross-sectional width (t) or selecting the material with a smaller Poisson's ratio. The peak value for the bending moment is $-14.583\ (N \cdot m)$ when the first wheel is 0.314 mm away from the fixed support. The methods to increase the bridge's resistance to bending stress can also be incrementing the moment of inertia by making the design higher or wider; the other way can be opting for the material possessing a higher Young's Modulus (E).

When the bridge is under the effects of two simple supports, the maximum shear stress will remain unchanged since the vertical reaction force (A_y) will retain its position, as compared to the fixed support. However, because the moment at the fixed support (M_A) is no longer included in the free-body diagram, the bridge will suffer a higher maximum bending moment, leading to a more significant maximum deflection. In the context of this project, the maximum shear stress is predicted to stay at 79.5233 (N), as calculated previously. The maximum shear stress will soar over the peak of 14.4371 ($N \cdot m$), and the maximum deflection is also higher than 6.92 (mm). Besides, if the bridge is supported by two simple supports at two ends and under the load of 8.5 (kg), the whole system is speculated to be unstable and prone to overall failure compared to when there is a fixed support at one end.

4. References

[1] T. T. Thanh. *Lecture 1 - Introduction + Static Review (Presentation Slidies) v3.pdf: Stress Analysis (instructure.com).* [Online]. Available: https://rmit.instructure.com/courses/122157/files/32207742?module_item_id=5280919 (accessed August 10, 2023).

[2] T. T. Thanh. *Lecture 5 - Bending (Presentation Slides).pdf: Stress Analysis (instructure.com)* [Online]. Available: https://rmit.instructure.com/courses/122157/files/32206056?module_item_id=5280758 (accessed August 10, 2023).

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[4] T. T. Thanh. *Lecture 7 - Shear and Moment Diagrams (Presentation Slides).pdf: Stress Analysis (instructure.com)* [Online]. Available: https://rmit.instructure.com/courses/122157/files/33350829?module_item_id=5508831 (accessed August 10, 2023).

[5] T. T. Thanh. Lecture 8 - Deflection of Beams (Presentation Slides).pdf: Stress Analysis (instructure.com) [Online]. Available: https://rmit.instructure.com/courses/122157/files/32206067?module_item_id=5280891 (accessed August 10, 2023).

[6] T. T. Thanh. Lecture 6 - Combined Loading & Review (Presentation Slides).pdf: Stress Analysis (instructure.com) [Online]. Available: https://rmit.instructure.com/courses/122157/files/33246155?module_item_id=5494321 (accessed August 10, 2023).

[7] A. Srivastava. “Deflection of Beams: Deflection Formula, Deflection Unit.” BYJU’S EXAM PREP. https://byjusexamprep.com/deflection-of-beams-i (accessed August 10, 2023).

[8] ClearCalcs. “Free Beam Calculator.” ClearCalcs. https://clearcalcs.com/freetools/beam-analysis/au#instructions (accessed August 10, 2023).

Appendix A – Calculating the reaction forces using the superposition method.

The superposition method is illustrated in Figure 19 below. The whole system consists of four concentrated forces and one reaction force and is divided into five subsystems if each one force is kept in each case while the others are deemed redundant and removed. Following the division procedure, each case will be solved as a statically determinate problem, which can be treated with the maximum deflection formula discussed in the Theoretical Principals section. Regarding the known parameters, the expression of each case will be displayed with respect to a $(0 \leq a \leq 0.9\,m)$, $L = 0.9\,m$ (the total length of the beam), and $P_1 = P_2 = P_3 = P_4 = 20.846\,N$ (calculated in the Design Load Cases section).

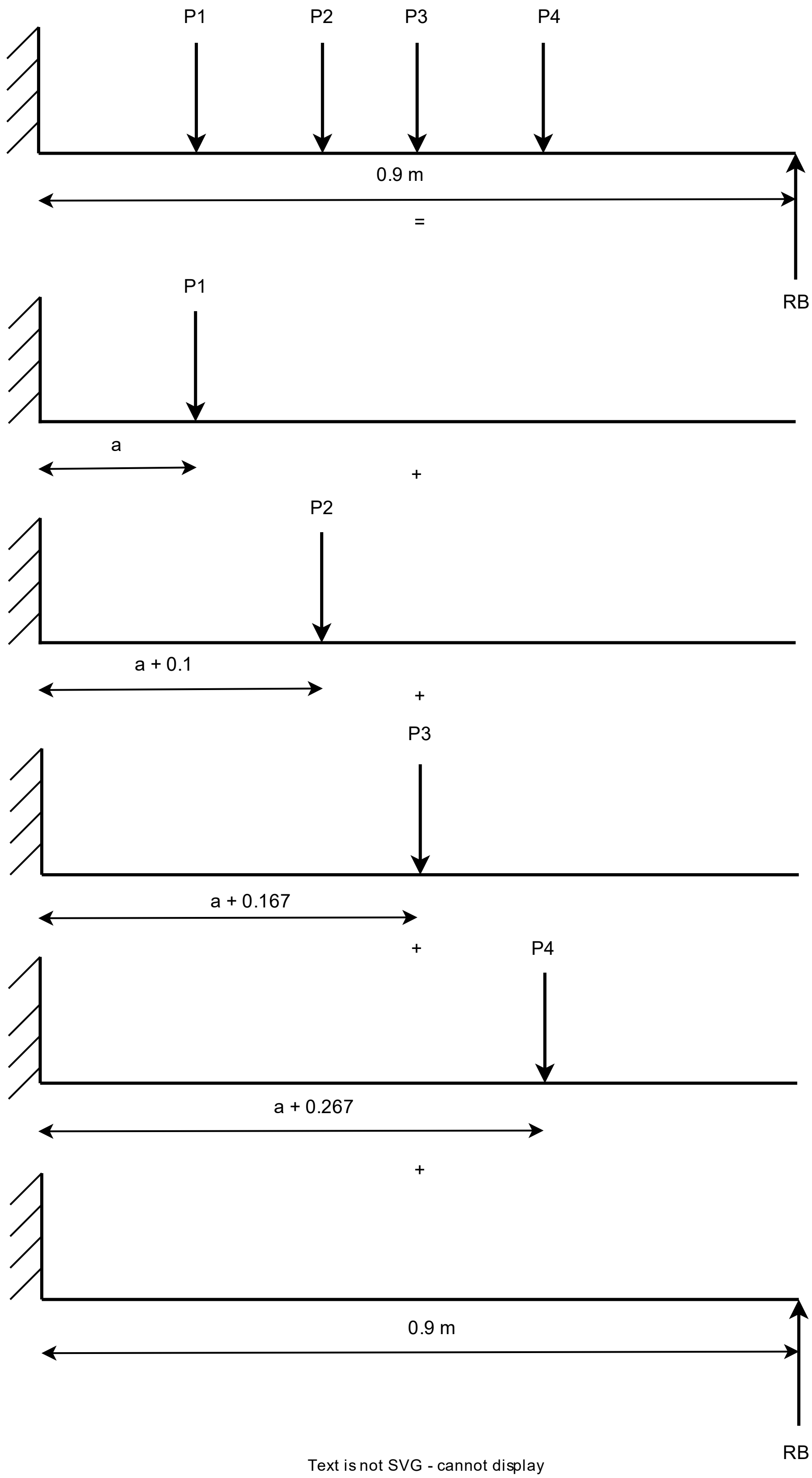


Figure 19. Using Superposition Method to find reaction force RB.

For the case when P_1 is the only concentrated force:

$$\delta_{max1} = \frac{Pa^2}{6EI}(3L - a) = \frac{(20.846\ N)a^2}{6EI}[3\ x\ (0.9\ m) - a] = \frac{3.474a^2}{EI}(2.7 - a)\ (m)$$

For the case when P_2 is the only concentrated force:

$$\delta_{max2} = \frac{P(a + 0.1)^2}{6EI}[3L - (a + 0.1)] = \frac{(20.846\ N)a^2}{6EI}[3\ x\ (0.9\ m) - (a + 0.1)]$$
$$= \frac{3.474(a + 0.1)^2}{EI}(2.6 - a)\ (m)$$

For the case when P_3 is the only concentrated force:

$$\delta_{max3} = \frac{P(a + 0.167)^2}{6EI}[3L - (a + 0.167)] = \frac{(19.62\ N)(a + 0.167)^2}{6EI}[3\ x\ (0.9\ m) - (a + 0.167)]$$
$$= \frac{3.474(a + 0.167)^2}{EI}(2.533 - a)\ (m)$$

For the case when P_4 is the only concentrated force:

$$\delta_{max4} = \frac{P(a + 0.267)^2}{6EI}[3L - (a + 0.267)] = \frac{(19.62\ N)(a + 0.267)^2}{6EI}[3\ x\ (0.9\ m) - (a + 0.267)]$$
$$= \frac{3.474(a + 0.267)^2}{EI}(2.433 - a)\ (m)$$

For the case when R_B is the only concentrated force:

$$\delta_B = \frac{-PL^3}{3EI} = \frac{-R_B(0.9\ m)^3}{3EI} = \frac{-0.243R_B}{EI}$$

Applying the kinetic condition with the summation of five cases above, we can derive the expression for R_B:

$$\delta_B = \delta_{max1} + \delta_{max2} + \delta_{max3} + \delta_{max4} + \delta_{max5} = 0$$

$$\Rightarrow \frac{3.474a^2}{EI}(2.7 - a) + \frac{3.474(a + 0.1)^2}{EI}(2.6 - a) + \frac{3.474(a + 0.167)^2}{EI}(2.533 - a)$$
$$+ \frac{3.474(a + 0.267)^2}{EI}(2.433 - a) - \frac{0.243R_B}{EI} = 0$$

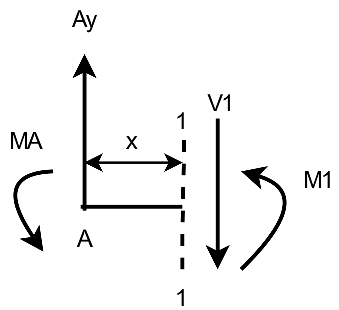
$$\Longrightarrow R_B = [3.474a^2(2.7 - a) + 3.474(a + 0.1)^2(2.6 - a) + 3.474(a + 0.167)^2(2.533 - a)$$
$$+ 3.474(a + 0.267)^2(2.433 - a)\ ]\ /\ 0.243\ where\ 0 \le a \le 0.9\ m$$

Appendix B – Calculating the internal forces using the section method.

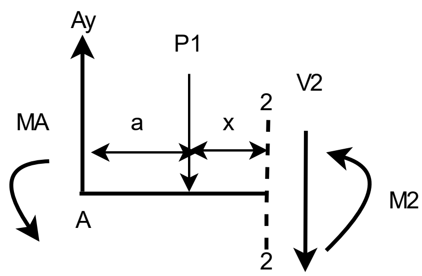
This appendix depicts how the section method can help extract the expressions for the internal shear force and moment. Figure 20 demonstrates how the internal forces act in each cut section. The fixed support forces (A_y, M_A) have previously been found in writing part 2.2 using the equilibrium equations. The computation of the internal forces also involves using equilibrium and delivering the expressions with respect to “a” ($0 \le a \le 0.9\ m$). The only difference is the appearance of the variable x, representing the arbitrary cutting position within a specified range. For example, regarding the 1-1 section in the figure below, x will range from 0 to “a”; for the 2-2 section, x will vary from “a” to “a” + 0.1.

SECTION METHOD

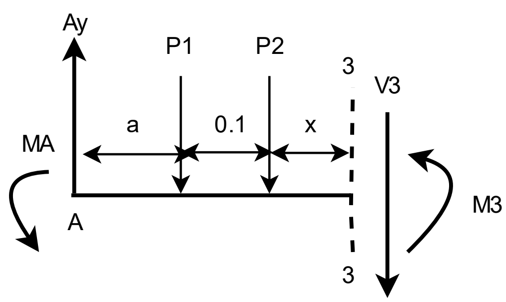
Section 1-1:



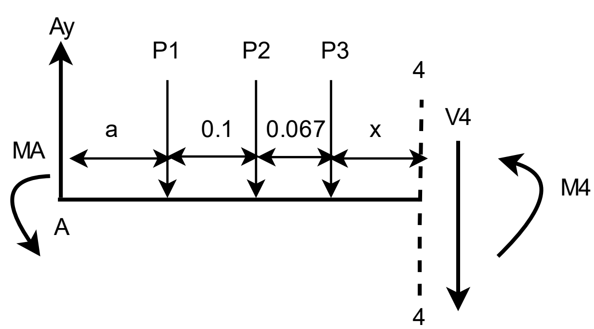
Section 2-2:



Section 3-3:



Section 4-4:



Section 5-5:

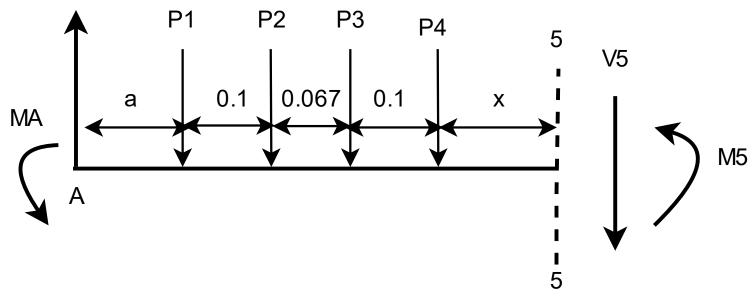


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Figure 20. Using Section Method to find reaction forces.

For section 1-1, the calculations can be done as:

$$\uparrow + \sum F_y = 0 \Rightarrow A_y - V_1 = 0 \implies V_1 = A_y \; (N)$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow M_A - V_1(x) + M_1 = 0 \Rightarrow M_1 = A_y x - M_A \; (N \cdot m)$$

The same procedure will be applied to the remaining sections.

For section 2-2:

$$\uparrow + \sum F_y = 0 \Rightarrow A_y - P_1 - V_2 = 0 \implies V_2 = A_y - P_1 = A_y - 20.846 \; (N)$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow M_A - V_2(x + a) - P_1(a) + 2 = 0 \Rightarrow M_2 = \left(A_y - P_1\right)(x + a) + P_1(a) - M_A$$

$$\Rightarrow M_2 = (20.846 - P_1)(x + a) + 20.846(a) - M_A(N \cdot m)$$

For section 3-3:

$$\uparrow + \sum F_y = 0 \Rightarrow A_y - P_1 - P_2 - V_3 = 0 \implies V_3 = A_y - P_1 - P_2 = A_y - 41.692 \; (N)$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow M_A - V_3(x + a + 0.1) - P_1(a) - P_2(a + 0.1) + M_3 = 0 \Rightarrow M_3$$
$$= V_3(x + a + 0.1) + P_1(a) + P_2(a + 0.1) - M_A$$

$$\implies M_3 = (A_y - 41.692)(x + a + 0.1) + (20.846)(2a + 0.1) - M_A(N \cdot m)$$

For section 4-4:

$$\uparrow + \sum F_y = 0 \Rightarrow A_y - P_1 - P_2 - P_3 - V_4 = 0 \implies V_4 = A_y - P_1 - P_2 - P_3 = A_y - 62.538 \; (N)$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow M_A - V_4(x + a + 0.167) - P_1(a) - P_2(a + 0.1) - P_3(a + 0.167) + M_4 = 0 \Rightarrow M_4$$
$$= V_4(x + a + 0.1) + P_1(a) + P_2(a + 0.1) + P_3(a + 0.167) - M_A$$

$$M_4 = (A_y - 62.538)(x + a + 0.167) + (20.846)(3a + 0.267) - M_A \; (N \cdot m)$$

For section 5-5:

$$\uparrow + \sum F_y = 0 \Rightarrow A_y - P_1 - P_2 - P_3 - P_4 - V_5 = 0 \implies V_5 = A_y - P_1 - P_2 - P_3 - P_4$$
$$= A_y - 83.384 \; (N)$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow M_A - V_5(x + a + 0.267) - P_1(a) - P_2(a + 0.1) - P_3(a + 0.167) - P_4(a + 0.267) + M_5$$
$$= 0 \Rightarrow M_5$$
$$= V_4(x + a + 0.267) + P_1(a) + P_2(a + 0.1) + P_3(a + 0.167) + P_4(a + 0.267) - M_A$$

$$\implies M_5 = (A_y - 83.384)(x + a + 0.267) + (20.846)(4a + 0.534) - M_A \; (N \cdot m)$$

Appendix C – Calculating the elastic curve using the superposition method.

This part demonstrates how to deduce the elastic curve of the system with the superposition method. The entire procedure resembles what has been presented in Appendix A with only the formula differences. This appendix also used the same figure as in Appendix A to solve the problem.

For the case when P_1 is the only concentrated force:

$$\delta_1 = \begin{cases} \dfrac{-(20.846) \times x^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}(3a - x) = -0.044(x^2)(3a - x)\ for\ 0 < x < a \\ \dfrac{-(20.846) \times a^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}(3x - a) = -0.044(a^2)(3x - a)\ for\ a < x < 0.9 \end{cases}$$

For the case when P_2 is the only concentrated force:

$$\delta_2 = \begin{cases} \dfrac{-(20.846) \times x^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}[3(a + 0.1) - x)] = -0.044(x^2)[3(a + 0.1) - x]\ for\ 0 < x < a \\ \dfrac{-(20.846) \times (a + 0.1)^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}[3x - (a + 0.1)] = -0.044(a + 0.1)^2[3x - (a + 0.1)]\ for\ a < x < 0.9 \end{cases}$$

For the case when P_3 is the only concentrated force:

$$\delta_3 = \begin{cases} \dfrac{-(20.846) \times x^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}[3(a + 0.167) - x)] = -0.044(x^2)[3(a + 0.167) - x]\ for\ 0 < x < a \\ \dfrac{-(20.846) \times (a + 0.167)^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}[3x - (a + 0.167)] = -0.044(a + 0.167)^2[3x - (a + 0.167)]\ for\ a < x < 0.9 \end{cases}$$

For the case when P_4 is the only concentrated force:

$$\delta_4 = \begin{cases} \dfrac{-(20.846) \times x^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}[3(a + 0.267) - x)] = -0.044(x^2)[3(a + 0.267) - x]\ for\ 0 < x < a \\ \dfrac{-(20.846) \times (a + 0.267)^2}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}[3x - (a + 0.267)] = -0.044(a + 0.267)^2[3x - (a + 0.267)]\ for\ a < x < 0.9 \end{cases}$$

For the case when R_B is the only concentrated force:

$$\delta_5 = \frac{-R_B(0.9)^3}{6(1.1 \times 10^9)(7.145 \times 10^{-8})}(3 \times 0.9 - x) = (1.546 \times 10^{-3})(2.7 - x)$$

Applying the kinetic condition with the summation of five cases above, a general elastic curve can be found:

$$\delta_{total} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + R_B$$

$$\Rightarrow \delta_{total} = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5$$

An elastic curve can be drawn with the expression of δ_{total} deduced above, and with the use of derivation, the maximum deflection and its location can be recorded. However, this method requires a high degree of hand calculation accuracy and tends to have errors. Therefore, instead of following the traditional mathematical approach, this project implements deflection calculation software [8] to draw an elastic curve based on a Free-body diagram, from which the maximum value can be observed. By testing with numerous values of a ($0 \leq a \leq 0.9\ m$), a conclusion is reached that the deflection value crawls nearer to maximum values when a is in the range of 0.35 m to 0.4m (Figures 21 to 31).

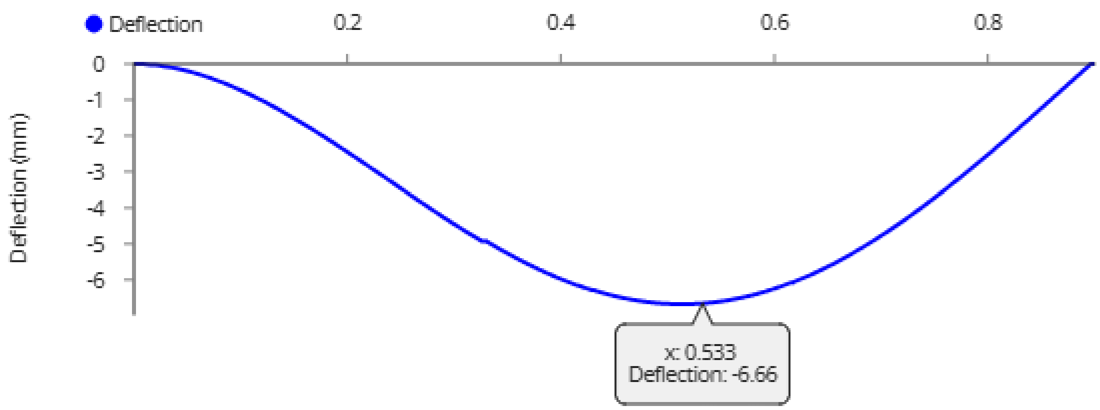

Figure 21. Elastic Curve when a = 0.33 (m)

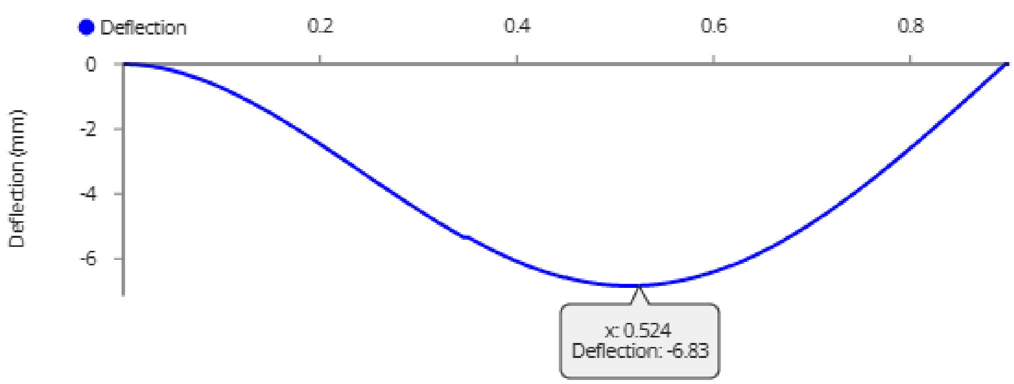

Figure 22. Elastic Curve when a = 0.35 (m)

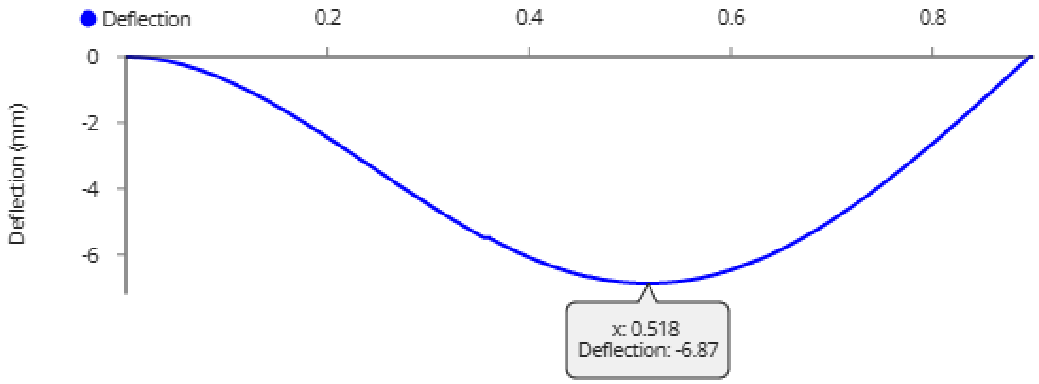

Figure 23. Elastic Curve when a = 0.36 (m)

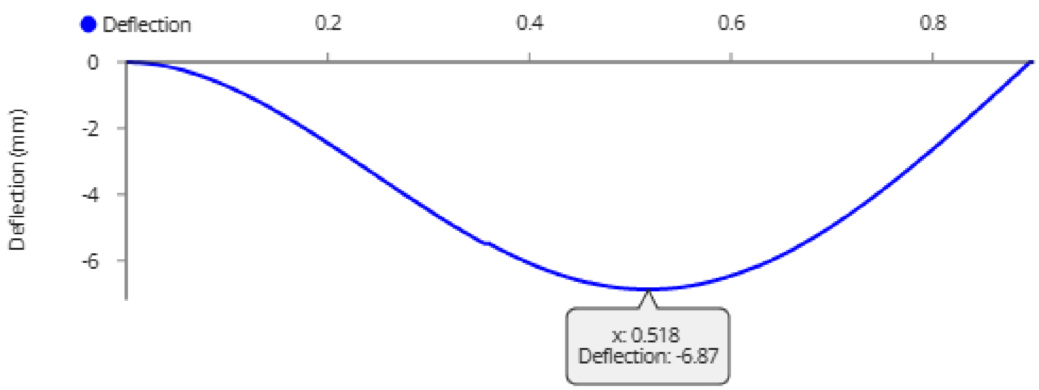

Figure 24. Elastic Curve when a = 0.37 (m)

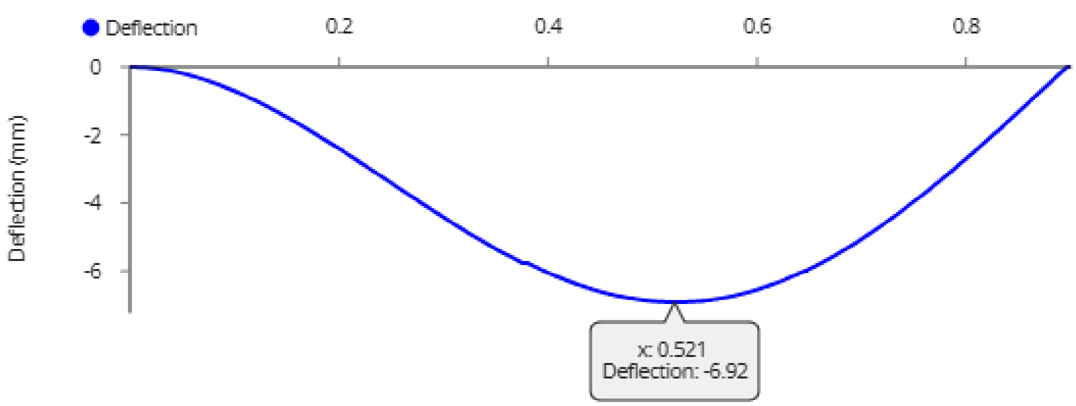

Figure 25. Elastic Curve when a = 0.38 (m)

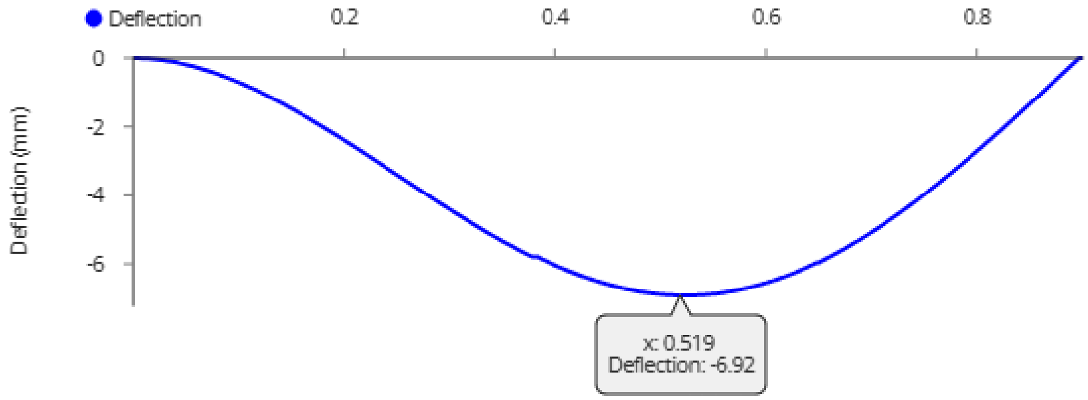

Figure 26. Elastic Curve when a = 0.383 (m)

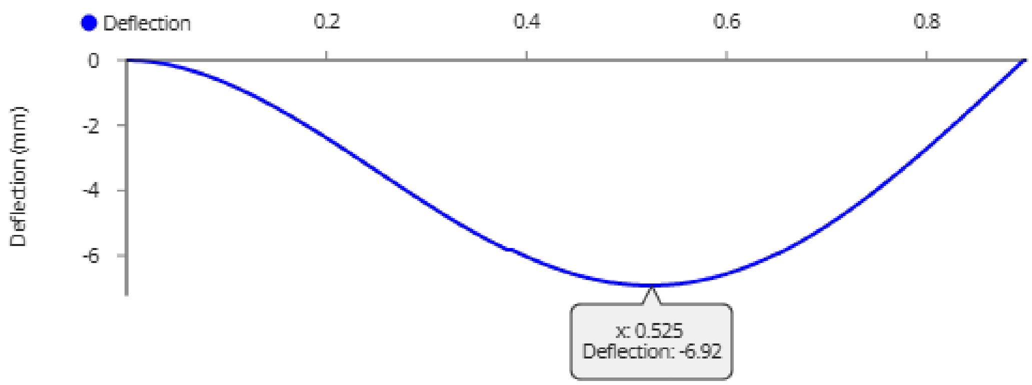

Figure 27. Elastic curve when 0.385 (m)

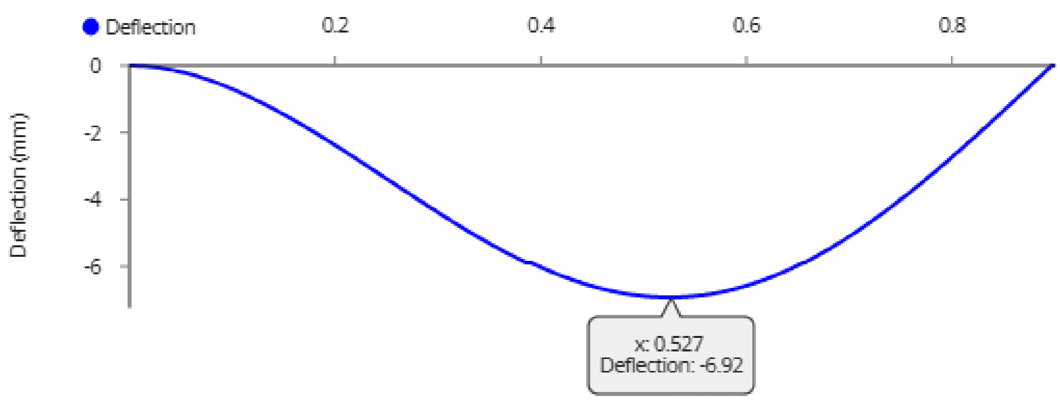

Figure 28. Elastic Curve when a = 0.39 (m)

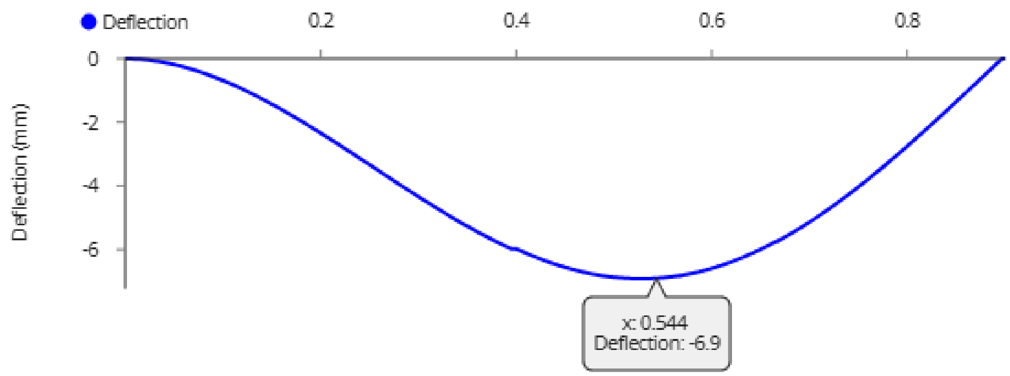

Figure 29. Elastic Curve when a = 0.4 (m)

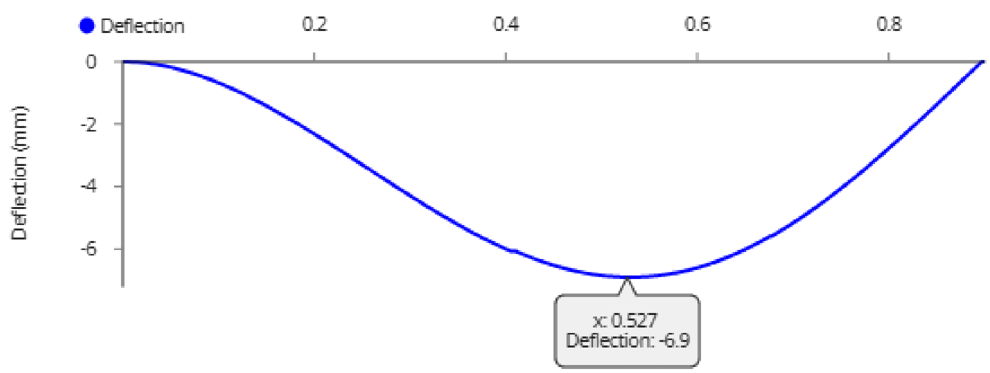

Figure 30. Elastic Curve when a = 0.41 (m)

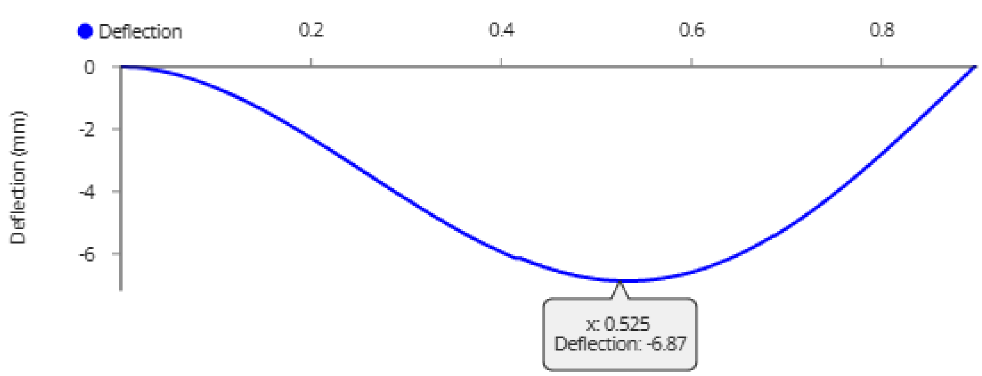

Figure 31. Elastic Curve when a = 0.42 (m)

With a wide range of testing values for a, it can be observed that the maximum deflection continues increasing from a = 0.33 (m) to 0.38 (m), where it reaches the peak high of $-6.92\ (mm)$. After a = 0.39 (mm), the deflection decreases continuously. Therefore, it is speculated that the highest maximum deflection lies between a = 0.38 (m) and a = 0.39 (m), whose range reports $-6.92\ (mm)$ of deflection. For a better approximation, a = 0.385 (m) is selected as the result.

Figures 32 and 33 demonstrate how software is set up to draft the elastic curve. The input for this software is the Length of the Beam (stated in the project statement), Young's Modulus (given by the project statement), the moment of inertia (calculated in the 2.3 section), and surface area ($A_{surface} = 106 \times 23 - 17 \times (100 - 3 \times 4) = 942\ (mm^2)$). After that, the supports are inserted in terms of their types and positions, while the forces are added with their values and locations. The outputs of this software, based on the entered inputs, are the free-body diagram, shear and moment diagrams and the elastic curve. Since the shear and moment diagrams have already been drawn in MATLAB, only the elastic curve output is reported.

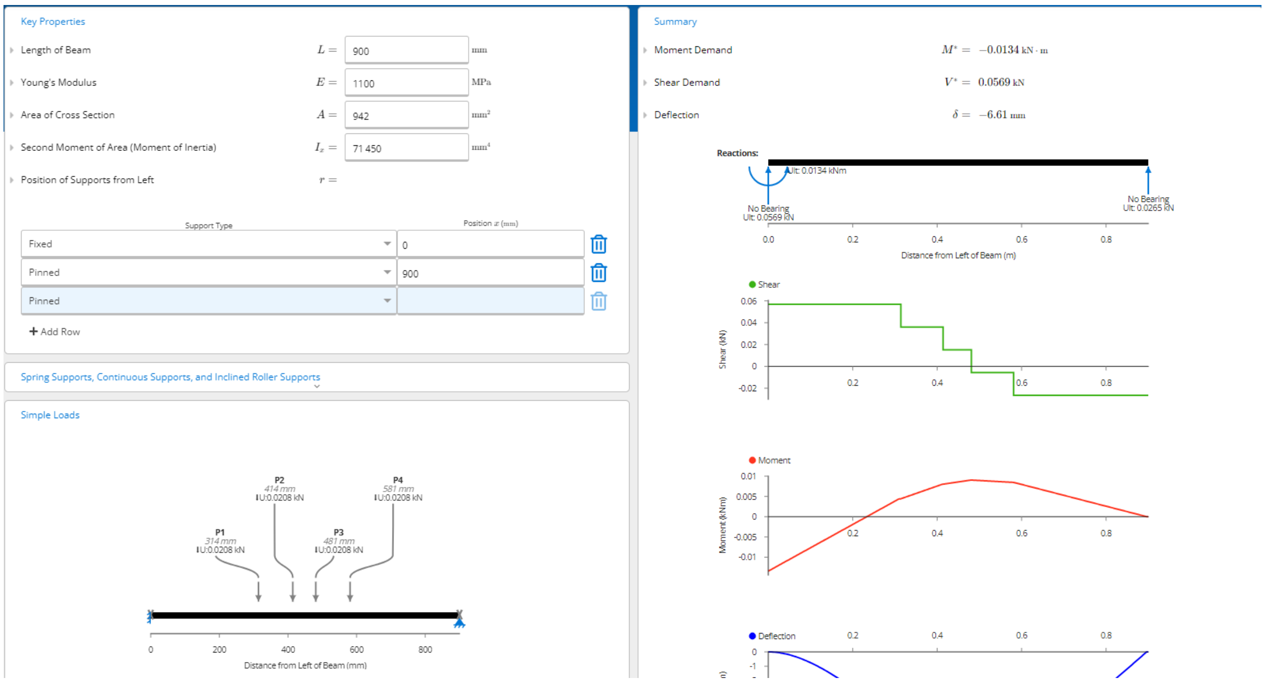

Figure 32. Elastic Curve Drawing Software Setup

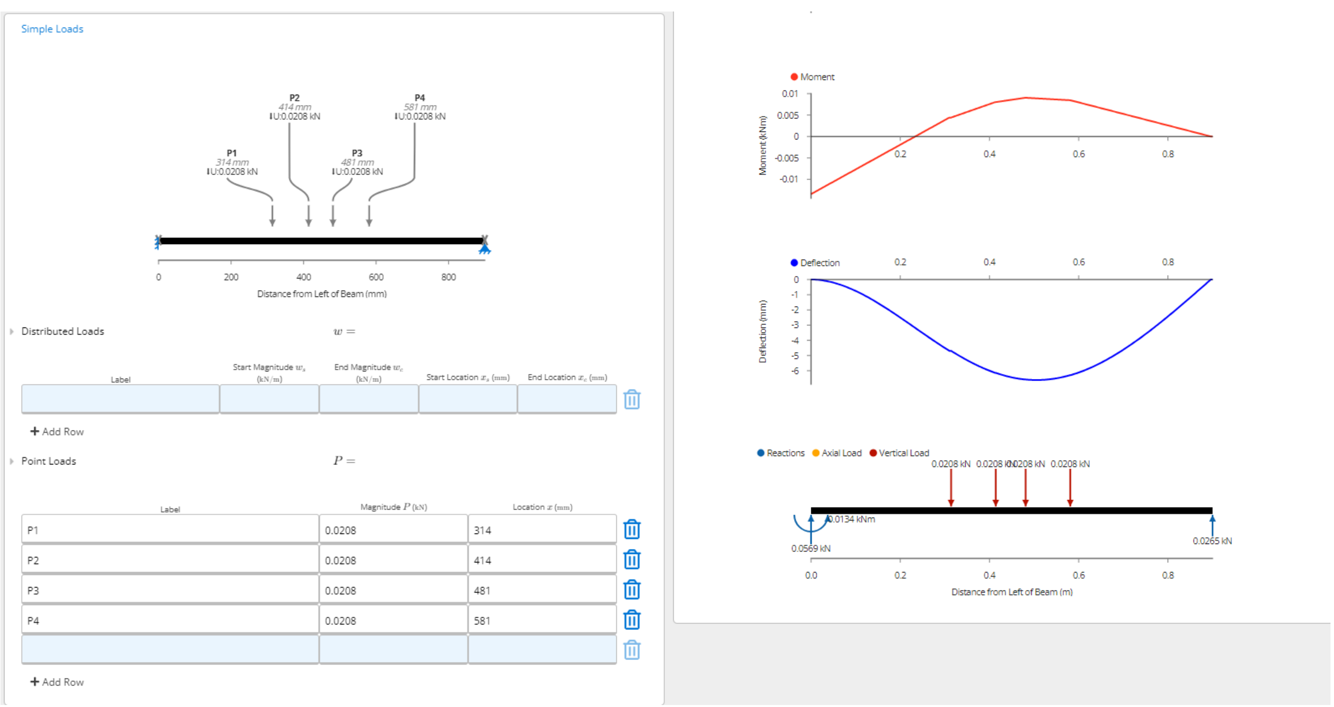

Figure 33. Elastic Curve Drawing Software Setup (continued)