# Mind the gap! Stylized facts and structural models.

Fabio Canova, Norwegian Business School and CEPR
September 2022

from joint work with Filippo Ferroni, Chicago Fed

Solution to the problem is not a VAR but VARMA model (a state space).

## Introduction

• Common in macroeconomics to compare dynamics induced by disturbances using SVAR/LP and DSGE; see e.g. Gali (1999); Christiano et al. (2005); Iacoviello (2005), Basu and Bundik (2017), etc.

• Non-invertibility/truncation problems: Ravenna (2007), Fernandez et al. (2007), Giacomini (2013), Plagborg-Moller (2018), Pagan and Robinson (2021), Chahrour and Jurado (2018) well known.

• Typically, if a VAR has $q$ shocks, use a theory with $q_1$ or less disturbances in the match. What happens if the true DGP has more than $q_1$ disturbances? Does the matching exercise work?

• Assume DGP has $q$ disturbances; empirical model $q_1 < q$ variables. **Deformation distortions may make identified shocks and identified dynamics mongrels with little economic interpretation. Comparison exercise hard.**

May end up mixing not only shocks with different meaning (in theory), but also shocks with different location in time.

Invertibility = fundamentalness

• *Cross sectional deformation*:

- Identified shocks need not combine "classes" of structural disturbances.

- Valid theoretical restrictions may be insufficient for shock identification.

**Difficult to match e.g., identified technology shocks to TFP disturbances.**

• *Time deformation*:

- Identified shocks are linear combinations of current and **past** structural disturbances.

**Perceived internal transmission stronger than in the DGP**. Include in the theory frictions which are artificial - absent from DGP.

Take the first order condition until everything is in terms of that one variable of interest (?)

## Punchline

• **Empirical models can not be too small**: difficult to make sense of identified shocks. If the empirical model can not be sufficiently large (sample restrcitions or identification problems) , two options:

• If one trusts the theory, reduce optimality conditions to the same observables as the empirical model and perform the matching exercise. In this case **theory should guide the choice of empirical variables and tell us the disturbances which could be identified.**

• If you do not trust the theory, for a given small empirical model, add to the basic theory interesting disturbances and perform the matching exercise again. **Empirical facts should be confronted with a favorite and an enlarged theory**.

• Deformation vs. invertibility.

- Problems distinct.

- Long lags do not eliminate cross sectional deformation.

• Early literature: Lutkepohl (1984), Hansen and Sargent (1991), Marcet (1991), Braun and Mittnik (1993), Faust and Leeper (1998), Forni and Lippi (1999).

• Related literature: Canova and Sahneh (2018), Wolf (2020), Miranda-Agrippino and Ricco (2021).

## Intuition for the results

technology shock

• Growth model with log preferences, full depreciation, iid shocks to TFP ($Z_t$), investment ($V_t$), preferences ($B_t$). Solution:

$$
\begin{aligned}
K_{t+1} &= \alpha\beta V_t Z_t K_t^{\alpha} & (1)\\
C_t &= (1-\alpha\beta) B_t Z_t K_t^{\alpha} & (2)\\
Y_t &= Z_t K_t^{\alpha} & (3)
\end{aligned}
$$

• System invertible if $0 \leq \alpha < 1$.

Solve for Z, then B, then V.

• An empirical model with $(\log K_{t+1}, \log C_t, \log Y_t)$ allows to identify the three disturbances via recursive scheme.

**If the empirical model has two variables: can I recover a "supply" and a "demand" disturbance? Can I recover the preference disturbance?**

- System 1: $(\log K_{t+1}, \log C_t)$. Solution:

May screw up midly.

$$\begin{aligned} \log K_{t+1} &= \log(\alpha\beta) + \alpha \log K_t + u_{1t} && (4) \\ \log C_t &= \log(1 - \alpha\beta) + \alpha \log K_t + u_{2t} && (5) \end{aligned}$$

$\boxed{u_{1t} = \log V_t + \log Z_t,\ u_{2t} = \log B_t + \log Z_t.}$ Cross sectional deformation. $u_t$ mix demand and supply disturbances. Recursivity lost; identification fails.

K eliminated

- System 2: $(\log C_t, \log Y_t)$: Solution:

May screw up big time.

$$\begin{aligned} \log C_t &= \alpha \log(\alpha\beta) + \alpha \log C_{t-1} + u_{1t} && (6) \\ \log Y_t &= \alpha \log(\alpha\beta) + \alpha \log Y_{t-1} + u_{2t} && (7) \end{aligned}$$

$\boxed{u_{1t} = \log B_t - \alpha \log B_{t-1} + \log Z_t + \alpha \log V_{t-1};\ u_{2t} = \log Z_t + \alpha \log V_{t-1}.}$ Time and cross sectional deformations. Impossible to go from $u_{jt}, j = 1, 2$ to demand and supply disturbances. Persistence distortions.

- System 3: $(\log K_{t+1} - \log Y_t, \log C_t - \log Y_t)$. Solution:

$$\log K_{t+1} - \log Y_t = \log(\alpha\beta) + \log V_t \tag{8}$$

$$\log C_t - \log Y_t = \log(1 - \alpha\beta) + \log B_t \tag{9}$$

Here only one disturbance enters each equation in the "reduced" theory. Can recover $V_t$, $B_t$.

## Conclusions

• The variables entering the empirical model determine which disturbances can be identified.

• Theoretically valid conditions may be insufficient for identification.

• Eliminating states more problematic than eliminating controls.

• Only a disturbance that enters the decision rule of a variable in the "reduced" theory has the potential to be identified.

## Formalization

- DGP:

$$x_t = A(\theta)x_{t-1} + B(\theta)e_t \tag{10}$$

$$y_t = C(\theta)x_{t-1} + D(\theta)e_t \tag{11}$$

$x_t$ is $k \times 1$ vector of endogenous and exogenous states, $e_t \sim (0, \Sigma)$, $\Sigma$ diagonal, is $q \times 1$ vector of disturbances, $y_t$ is $m \times 1$ vector of endogenous controls. $A(\theta)$ is $k \times k$, $B(\theta)$ is $k \times q$, $C(\theta)$ is $m \times k$, $D(\theta)$ is $m \times q$, $\theta$ structural parameters.

- Observables $z_{it} = S_i[x_t, y_t]'$, $S_i$ is $q_i \times q$ matrix.

the monetary policy shock is robust for this problem for 99% of the time

**Case 1: Empirical system eliminates some controls**

- $S_1 = [I, S_{12}]$

- Innovations $u_{1t}$ generated by

$$u_{1t} = z_{1t} - E[z_{1t}|\Omega_{1t-1}] \equiv z_{1t} - \tilde{F}_1 z_{1t-1} \tag{12}$$

**Proposition 1**

lambda is not a square matrix

i) $u_{1t} = \lambda_1(\theta)e_t$, where $\lambda_1(\theta)$ is $q_i \times q$.

ii) To identify a "class" of $e_t$ disturbances, it is sufficient that $\cancel{G_1}(\theta) \equiv \begin{pmatrix} B(\theta) \\ S_{12}D(\theta) \end{pmatrix}$ is block diagonal.

lambda(theta)

iii) To identify one $e_j$ it is sufficient that the k-th row of $G_1(\theta)$ has at most one non-zero element in the j-th position.

- Related to Faust and Leeper (1998).

- **Cases 2-3: The empirical system eliminates/repackages states**

- $S_2 = [S_{21}, S_{22}]; S_3 = [S_{31}, 0]$.

- Innovations $u_{it}, i = 2, 3$ generated by

$$u_{it} = z_{it} - E[z_{it}|\Omega_{it-1}] \equiv z_{it} - \tilde{F}_i z_{it-1} \tag{13}$$

**Proposition 2**

i) $u_{it} = \lambda_i(\theta, L)e_t$, $\lambda_i$ is $q_i \times q$, each $L$, i=2,3.

ii) $u_{it} = \psi_i(\theta, L)u_{1t}$, $i = 2, 3$.

## Dynamics

**Proposition 3**

i) If a shock can be identified from $u_{1t}$ and $\tilde{F}_1 = \begin{pmatrix} A(\theta) \\ S_{12}C(\theta) \end{pmatrix}$, structural dynamics in the empirical system proportional to those of the DGP.

ii) With $u_{it}, i = 2, 3$ responses to identified shocks are distorted at all horizons.

- Braun and Mittnik (1991): expression for response biases in VARs.

## Given a model, what disturbances can one recover?

euler equation

phillips rule

preference shock

taylor rule

monetary policy shock

$$\chi_t = \chi_{t+1} - \frac{1}{1-h} g_{t+1} + \frac{h}{1-h} g_t + r_t - \pi_{t+1} \tag{14}$$

$$\pi_t = \pi_{t+1} \beta + k_p \left( \frac{h}{1-h} g_t + (1+\sigma_n)\, n_t \right) + k_p \left( \mu_t - \chi_t \right) \tag{15}$$

$$o_t = \zeta_t + (1-\alpha)\, n_t \tag{16}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r) \left( \phi_y\, g_t + \phi_p\, \pi_t \right) + \varepsilon_t \tag{17}$$

$$g_t = a_t + o_t - o_{t-1} \tag{18}$$

- (14) is an Euler equation, (15) is a Phillips curve, (16) is a production function, (17) is a Taylor rule and (18) is the definition of output growth.

- Shocks: $\zeta_t = \rho_z\, \zeta_{t-1} + \varepsilon_{zt}$; $a_t = \rho_a\, a_{t-1} + \varepsilon_{at}$; $\chi_t = \rho_\chi\, \chi_{t-1} + \varepsilon_{\chi t}$; $\mu_t = \rho_\mu\, \mu_{t-1} + \varepsilon_{\mu t}$; $\epsilon_t = \varepsilon_{mpt}$

- Set: $\alpha = 0.33; \beta = 0.99; \sigma_n = 1.5; h = 0.9; k_p = 0.05; \phi_y = 0.1; \phi_p = 1.5; \rho_r = 0.8; \rho_z = 0.5; \rho_a = 0.2; \rho_\chi = 0.5; \rho_\mu = 0.0$.

- Minimal state vector $x_{t-1} = [o_{t-1}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$ $(6 \times 1)$

- Control vector $y_t = [g_t, o_t, \pi_t, n_t, r_t]'$ $(5 \times 1)$.

- Shock vector $e_t = [\varepsilon_{z_t}, \varepsilon_{a_t}, \varepsilon_{\chi_t}, \varepsilon_{\mu_t}, \varepsilon_{mp_t}]'$ $(5 \times 1)$

- **Compare shocks/dynamics of an empirical system with $q_1 < 5$ variables with those of the original model.**

- System with $z_t = (o_t, \pi_t, n_t, r_t)$.

$$\chi_t = \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + \frac{h}{1-h}(a_t + o_t - o_{t-1}) + r_t - \pi_{t+1} \quad (19)$$

$$\pi_t = \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\,n_t\right) + k_p\,(\mu_t - \chi_t) \quad (20)$$

$$o_t = \zeta_t + (1-\alpha)\,n_t \quad (21)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(\phi_y(a_t + o_t - o_{t-1}) + \phi_p \pi_t\right) + \varepsilon_{mp_t} \quad (22)$$

^ Can't identify these, as they only enter this equation (meaning; to separate these two)

- State vector: $x_{t-1} = [o_{t-1}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

- Law of motion of the states (A, B matrices) unaltered.

- **Cross sectional deformation, no time deformation distortions**.

- System with $z_t = (o_t, \pi_t, n_t)$.

$$
\begin{aligned}
((1+\rho_r) - \rho_r L)\chi_t &= \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + (\frac{h+\rho_r}{1-h} + (1-\rho_r)\phi_y)(a_t + o_t - o_{t-1}) \\
&- (\frac{h\rho_r}{1-h})(a_{t-1} + o_{t-1} - o_{t-2}) + (\rho_r + (1-\rho_r)\phi_p)\pi_t + \varepsilon_{mp_t} - \pi_{t+1} \quad (23) \\
\pi_t &= \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \quad (24) \\
o_t &= \zeta_t + (1-\alpha)n_t \quad (25)
\end{aligned}
$$

- State vector: $\hat{\mathrm{x}}_{t-1} = [o_{t-1}, \mathbf{o_{t-2}}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

- Law of motion of the states (A, B matrices) altered.

- **Cross-sectional and time deformation distortions**.

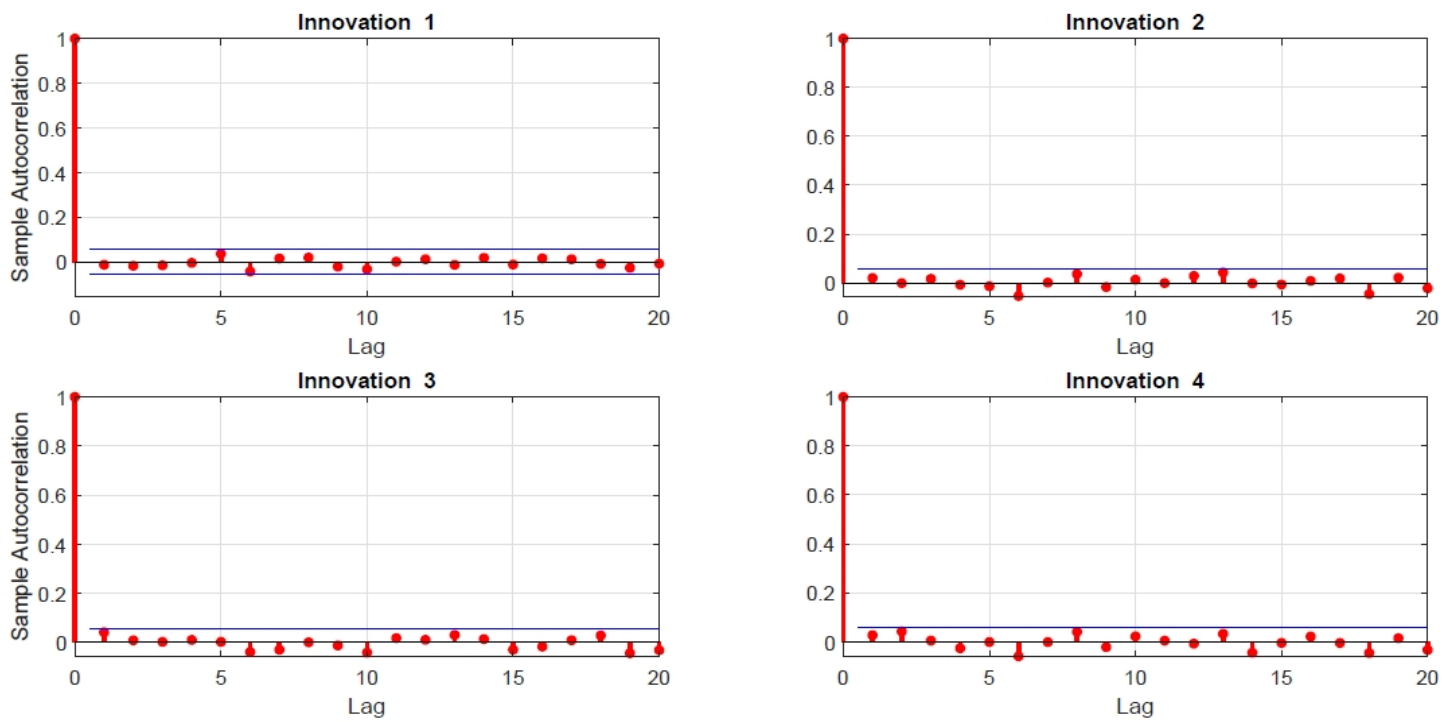
- System with $z_t = (\pi_t, n_t, r_t)$.

$$
\begin{aligned}
\chi_t &= \chi_{t+1} - \frac{1}{1-h}\left(a_{t+1} + \zeta_{t+1} - \zeta_t + (1-\alpha)\left(n_{t+1} - n_t\right)\right) \\
&+ \frac{h}{1-h}\left(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\left(n_t - n_{t-1}\right)\right) + r_t - \pi_{t+1} \qquad (26) \\
\pi_t &= \pi_{t+1}\beta + k_p\left(\frac{h}{1-h}\left(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\left(n_t - n_{t-1}\right)\right) + (1+\sigma_n)\, n_t\right) \\
&+ k_p\left(\mu_t - \chi_t\right) \qquad (27) \\
r_t &= \rho\, r_{t-1} + (1-\rho)\left(\phi_y\left(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\left(n_t - n_{t-1}\right)\right) + \phi_p\, \pi_t\right) + \varepsilon_{mp,t} \qquad (28)
\end{aligned}
$$

- State vector: $\hat{\mathrm{x}}_{t-1} = [\mathbf{n_{t-1}}, r_{t-1}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

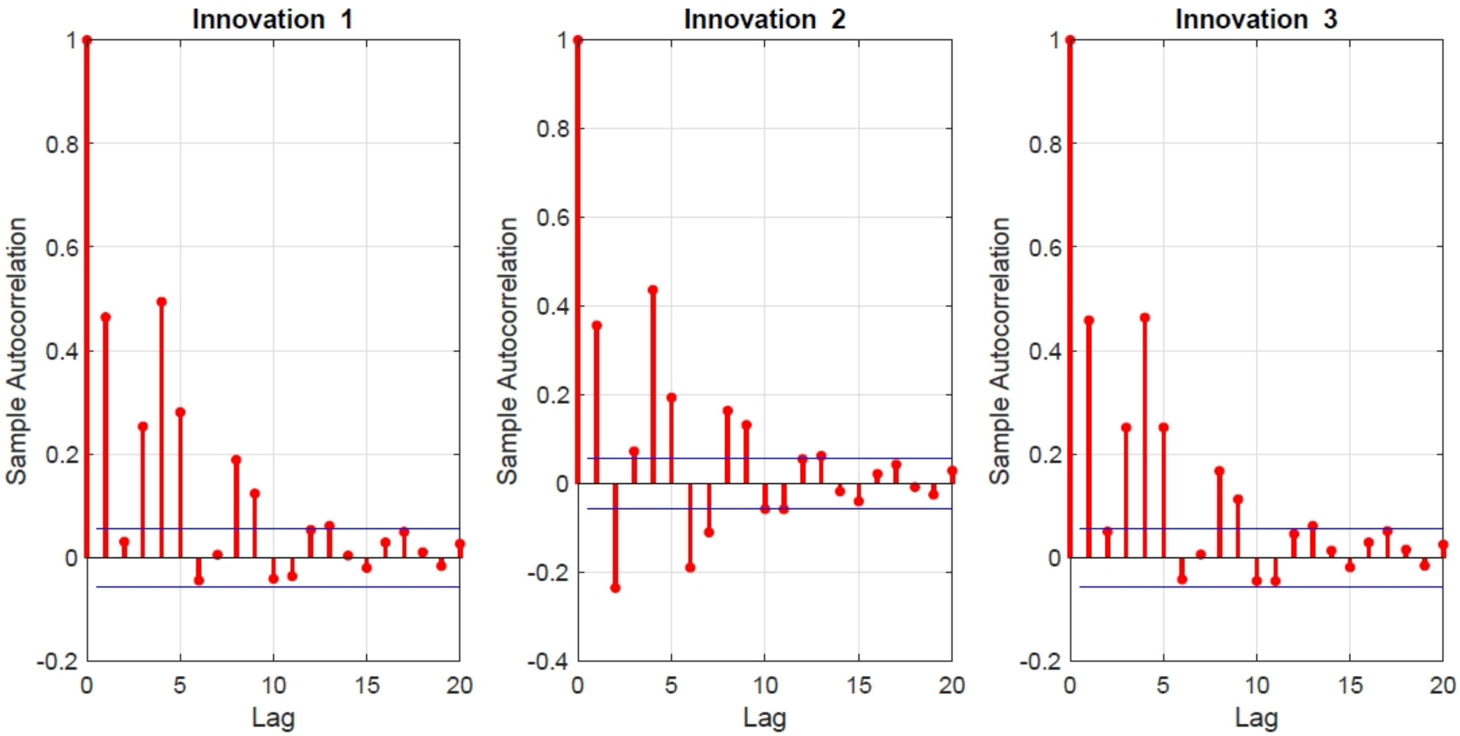
- Law of motion of the states unchanged (given production function $n_{t-1}$ proxies for $o_{t-1}$).

- **Cross-sectional deformation, limited time deformation distortions**.

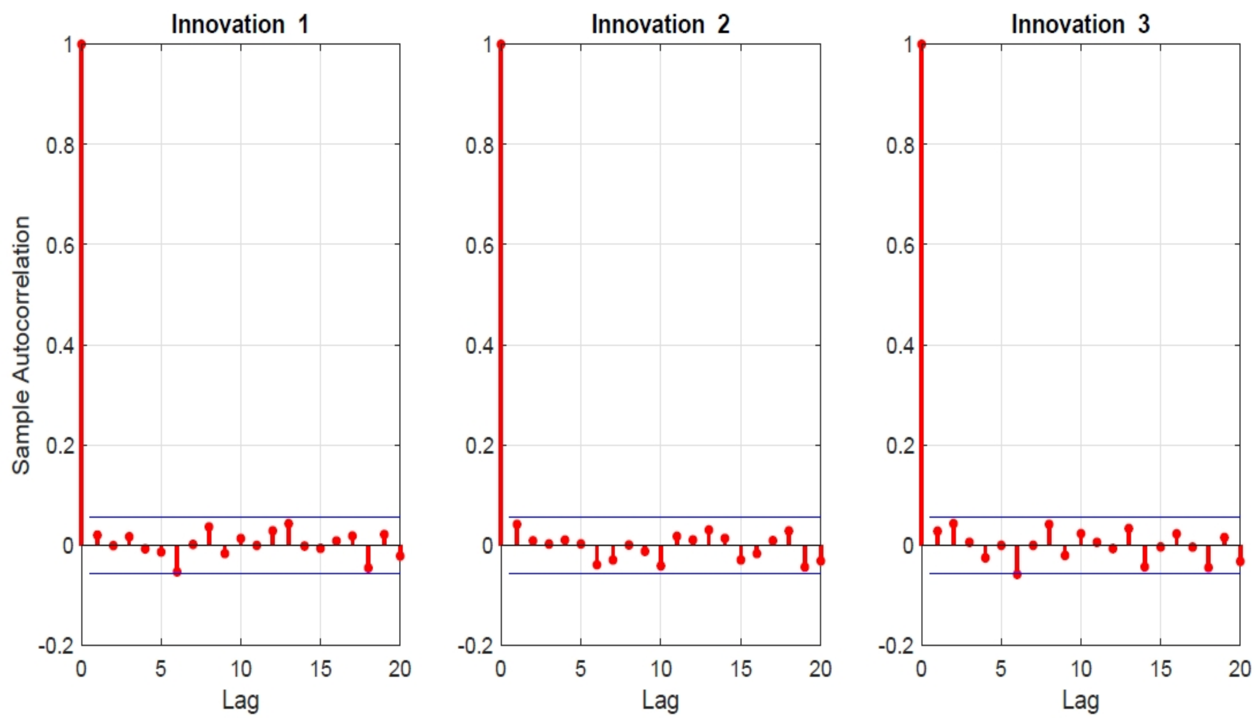



Autocorrelation function innovations in $z_t = (o_t, \pi_t, n_t, r_t)$.

Corresponds to the first equation, where serially correlated.

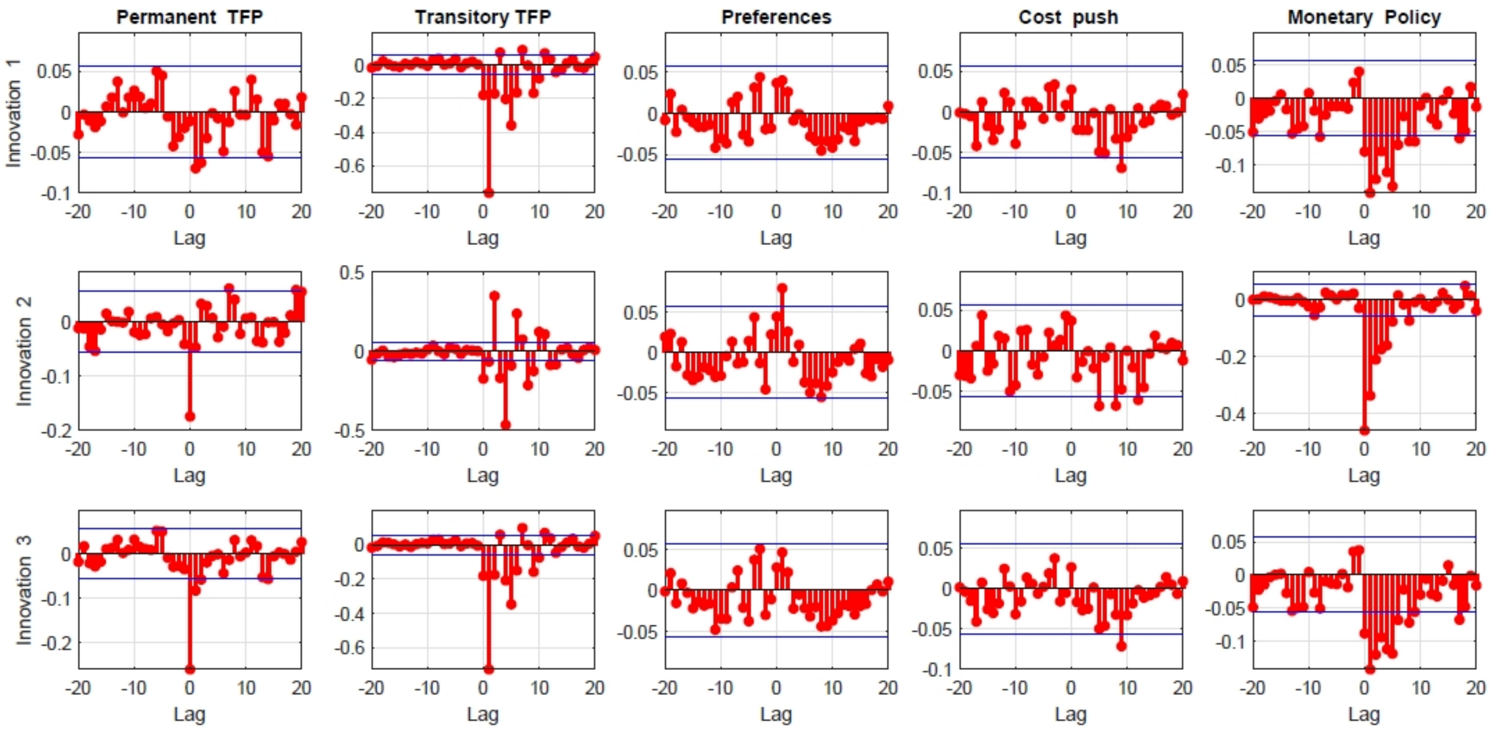



Autocorrelation function innovations in $z_t = (o_t, \pi_t, n_t)$




Autocorrelation function innovations in $z_t = (\pi_t, n_t, r_t)$.

lead 20 to lag 20




$\lambda(\theta, L)$ system with $z_t = (o_t, \pi_t, n_t)$

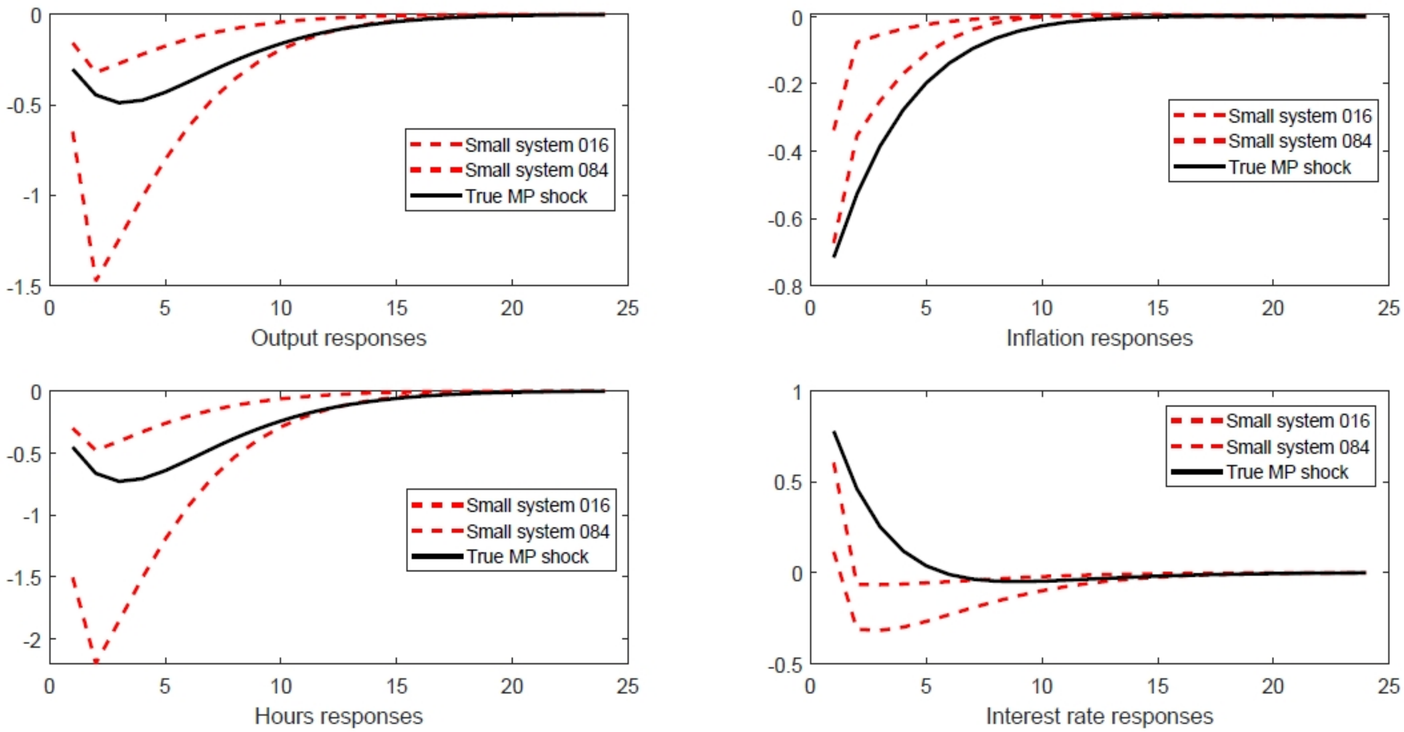
| | | | Structural disturbances | | | | |
|---|---|---|---|---|---|---|---|
| | | | $a_t$ | $\zeta_t$ | $\chi_t$ | $\mu_t$ | $\epsilon_t$ |
| $(o_t, \pi_t, n_t, r_t)$ innovations | $\lambda_0$ | $u_{1t}$ | 0.018 | -0.722 | 0.087 | -0.005 | -0.303 |
| | | $u_{2t}$ | -0.158 | -0.306 | 0.042 | 0.042 | -0.716 |
| | | $u_{3t}$ | -1.464 | -1.078 | 0.131 | -0.007 | -0.452 |
| | | $u_{4t}$ | -0.047 | -0.086 | 0.014 | 0.012 | 0.778 |
| $(o_t, \pi_t, n_t)$ innovations | $\lambda_0$ | $u_{1t}$ | -0.05 | 0.71 | 0.11 | 0.03 | -0.29 |
| | | $u_{2t}$ | -0.19 | -0.30 | 0.05 | 0.05 | -0.70 |
| | | $u_{3t}$ | -1.57 | -1.06 | -0.17 | 0.05 | -0.43 |
| | $\lambda_1$ | $u_{1t}$ | -0.07 | -0.92 | 0.12 | 0.04 | -0.41 |
| | | $u_{2t}$ | -0.01 | -0.28 | 0.03 | 0.01 | -0.52 |
| | | $u_{3t}$ | -0.25 | -1.37 | 0.18 | 0.06 | -0.61 |
| | $\lambda_2$ | $u_{1t}$ | -0.05 | -0.90 | 0.11 | 0.04 | -0.46 |
| | | $u_{2t}$ | -0.01 | 0.28 | 0.03 | -0.01 | -0.52 |
| | | $u_{3t}$ | -0.09 | -1.35 | 0.16 | -0.07 | -0.69 |

## Cholesky factors

| | Original system | | | | Smaller system | | | |
|---|---|---|---|---|---|---|---|---|
| $(o_t, \pi_t, n_t, r_t)$ | 0.75 | | | | 0.78 | | | |
| | 0.68 | 0.26 | | | 0.55 | 0.57 | | |
| | 1.06 | 1.14 | 0.95 | | 1.14 | 0.44 | 1.14 | |
| | -0.42 | -0.13 | 0.16 | 0.07 | -0.22 | -0.70 | 0.26 | 0.07 |
| $(o_t, \pi_t, n_t)$ | 0.75 | | | | 9.55 | | | |
| | 0.68 | 0.26 | | | 5.16 | 1.50 | | |
| | 1.06 | 1.14 | 0.95 | | 15.36 | -0.02 | 1.52 | |
| $(\pi_t, n_t, r_t)$ | 0.26 | | | | 0.79 | | | |
| | 1.14 | 0.95 | | | 1.11 | 1.50 | | |
| | -0.13 | 0.16 | 0.07 | | -0.65 | 0.36 | 0.23 | |

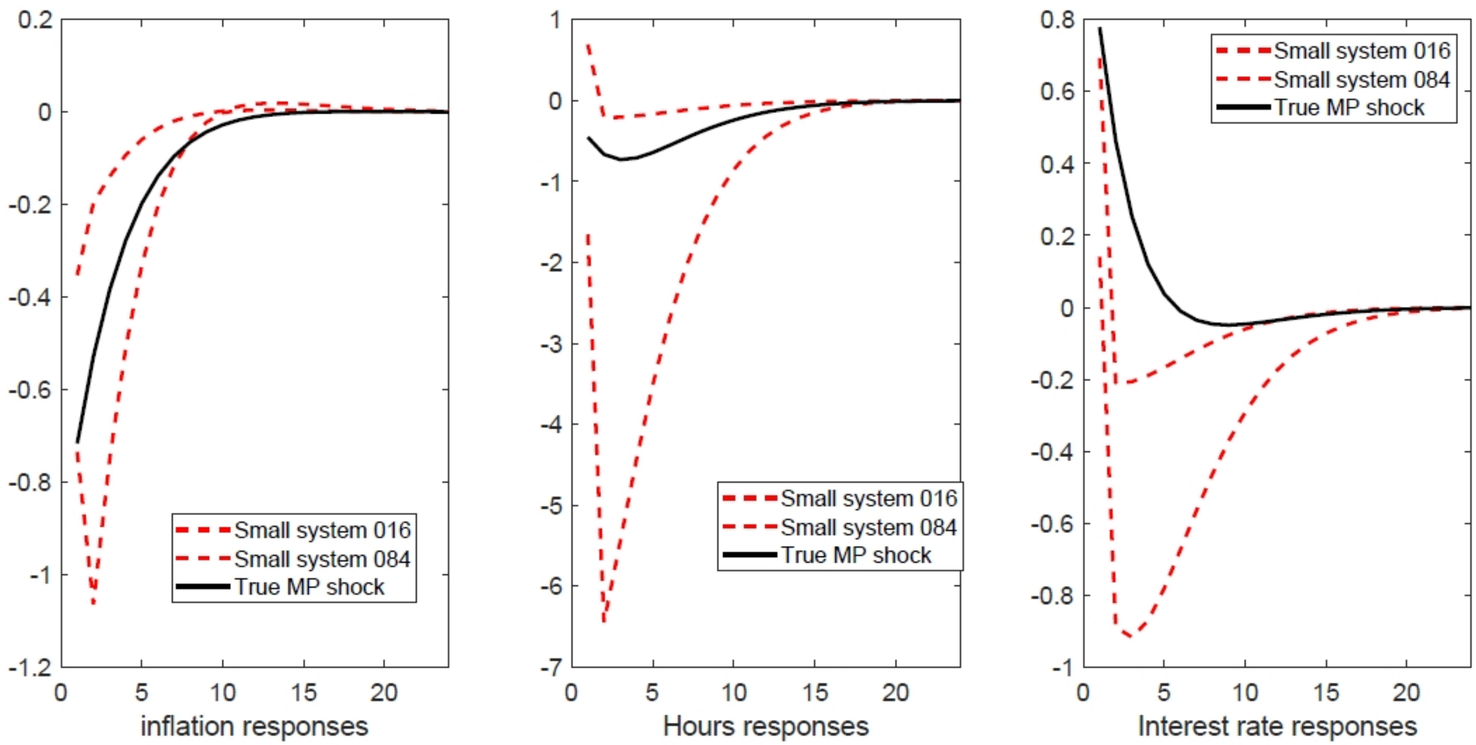
reduced with 4 shocks / four variable system

# Impulse responses

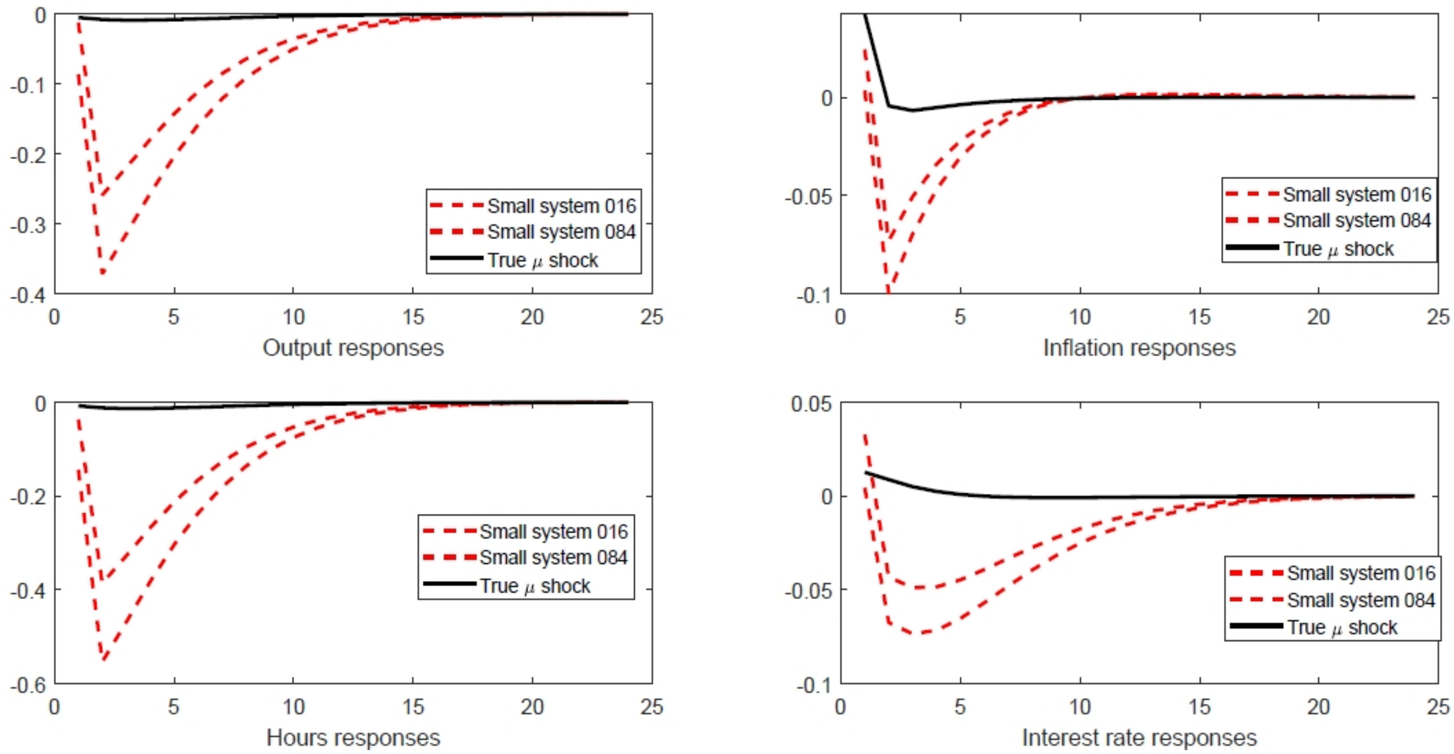



Monetary shocks, $z_t = (o_t, \pi_t, n_t, r_t)$

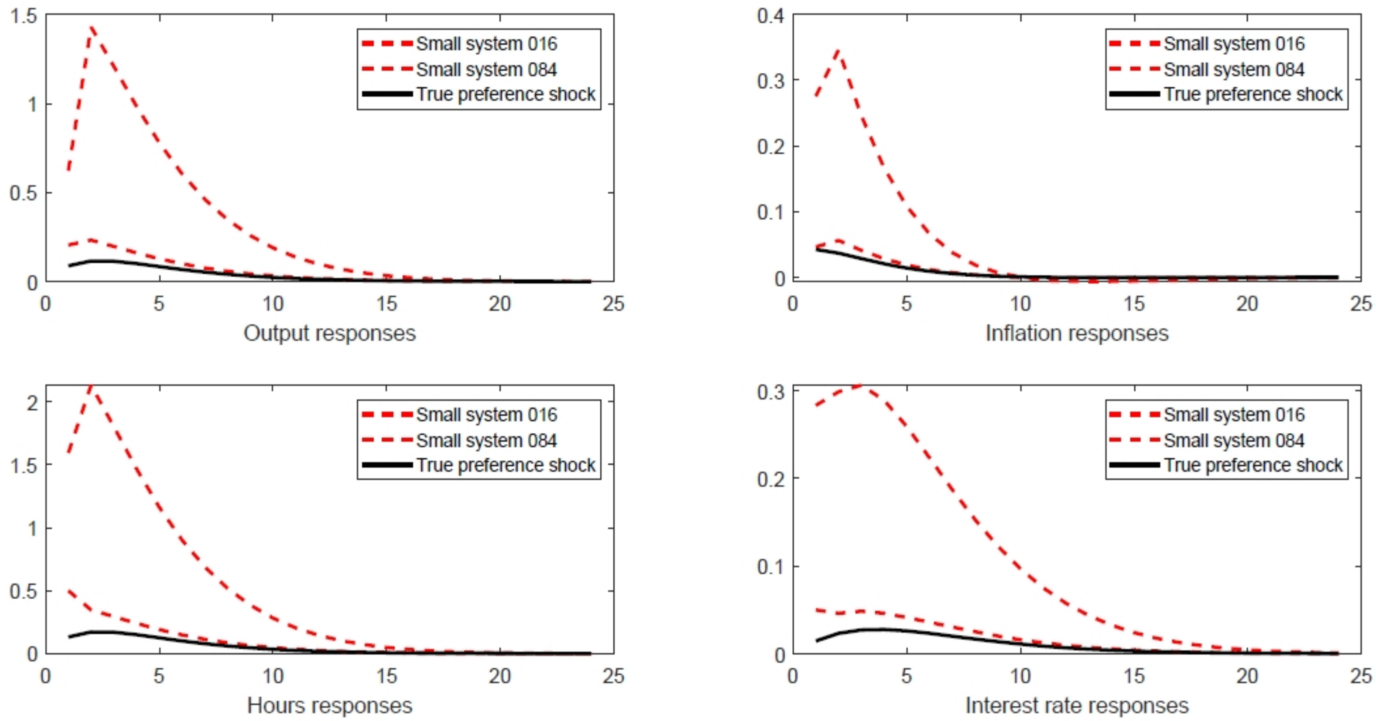
three variable system




Monetary shocks, $z_t = (\pi_t, n_t, r_t)$.




Cost push shocks, $z_t = (o_t, \pi_t, n_t, r_t)$




Preference shocks, $z_t = (o_t, \pi_t, n_t, r_t)$.

A system with only the endogenous states: $z_t = (o_t, r_t)$

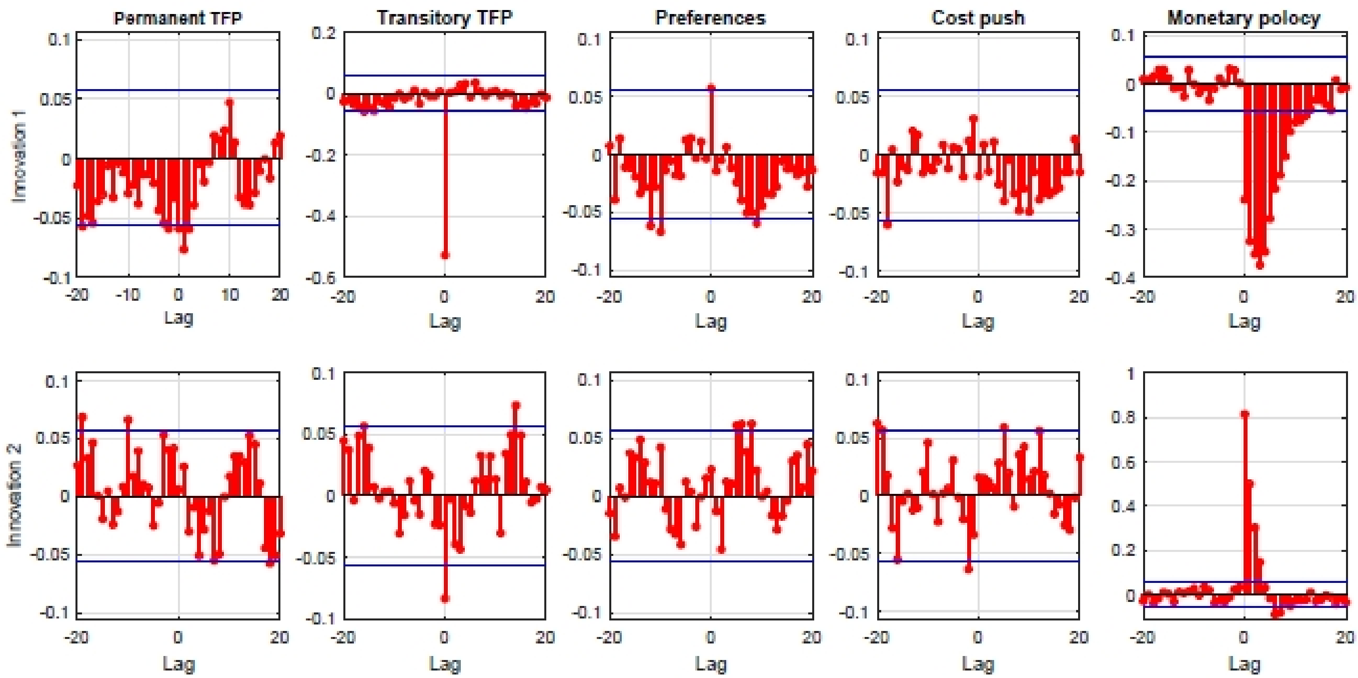
$$
\begin{aligned}
\chi_t &= (1+\beta)\chi_{t+1} - \beta\chi_{t+2} - \frac{1}{1-h}(a_{t+1} + o_{t+1} - o_t) + \frac{\beta}{1-h}(a_{t+2} + o_{t+2} - o_{t+1}) \\
&+ (\frac{h}{1-h})(a_t + o_t - o_{t-1}) - (\frac{h\beta}{1-h})(a_{t+1} + o_{t+1} - o_t) + r_t - \beta r_{t+1} \\
&- k_p\left(\frac{h}{1-h}(a_{t+1} + o_{t+1} - o_t) + (1+\ \sigma_n)\ \frac{1}{1-\alpha}(o_{t+1} - \zeta_{t+1})\right) - k_p\ (\mu_{t+1} - \chi_{t+1})
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
r_t &= \beta r_{t+1} + \rho_r r_{t-1} - \beta\rho_r r_t\ + (1-\rho_r)\phi_y((a_t + o_t - o_{t-1}) - \beta(a_{t+1} + o_{t+1} - o_t)) \\
&+ (1-\rho_r)\phi_\pi\left(k_p\left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\ \frac{1}{1-\alpha}(o_t - \zeta_t)\right) + k_p\ (\mu_t - \chi_t)\right) \\
&+ \epsilon_{mp_t} - \beta\epsilon_{mp_{t+1}}
\end{aligned} \tag{30}
$$

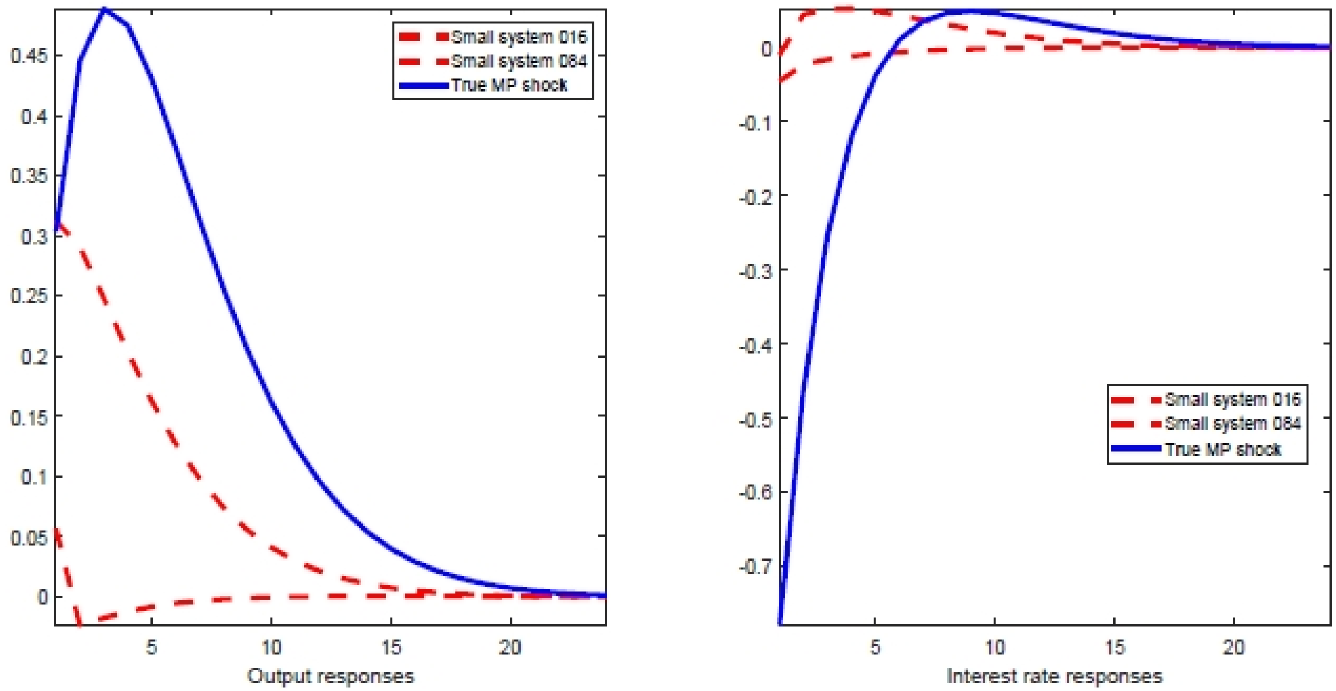
- $x_{t-1} = \hat{x}_{t-1}$.

- Law of motion of the states changed (($\bar{A}, \bar{B}$) differ from the (A,B)).

- **Cross sectional and time deformation distortions.**



Permanent TFP
Transitory TFP
Preferences
Cost push
Monetary polocy
Innovation 1
Innovation 2
Lag




Monetary policy shocks: $z_t = (o_t, r_t)$.

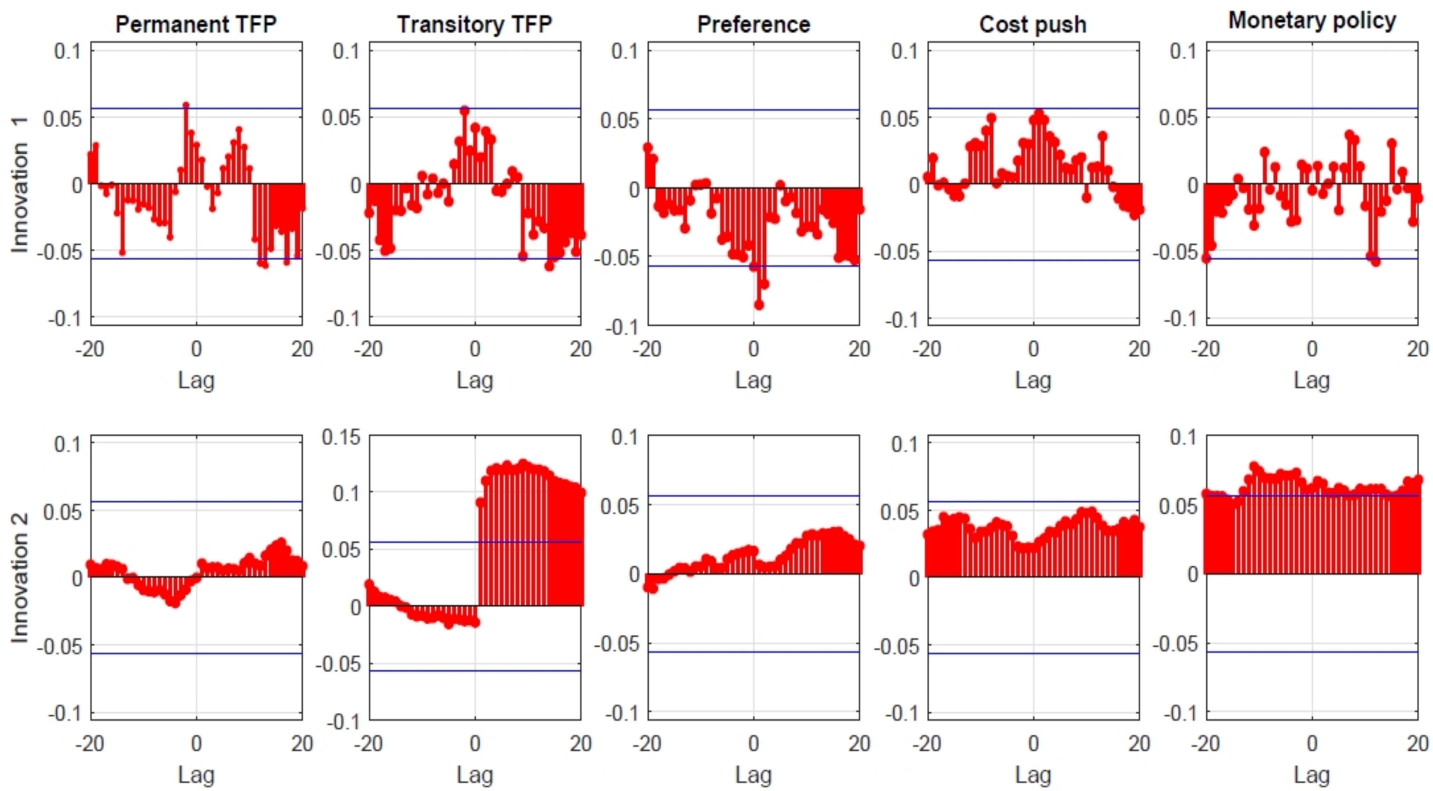
**Permanent TFP shocks: $(g_t, n_t)$.**

- Assume $\beta = (\rho_r + (1-\rho_r)\phi_p)^{-1}$.

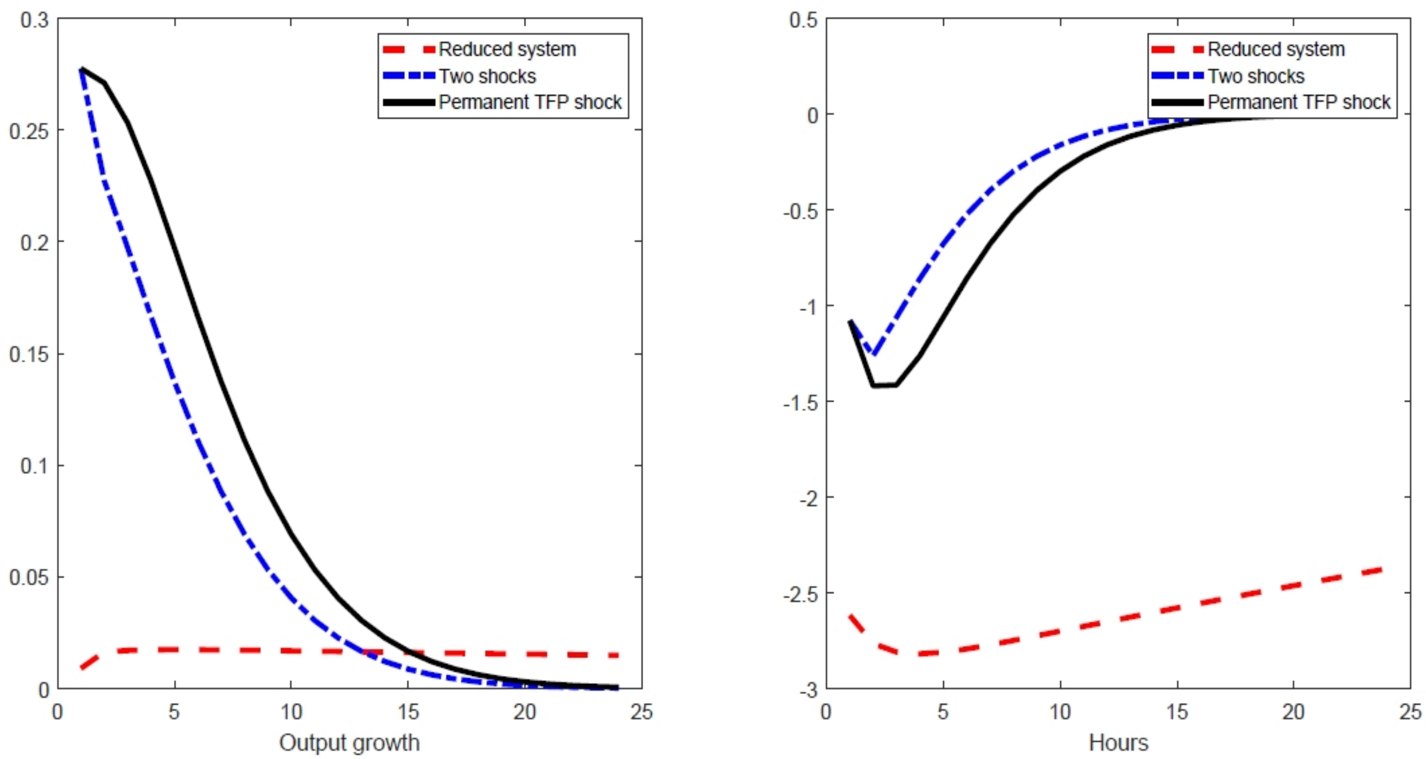
$$
\begin{aligned}
(1+\rho_r)\chi_t &= \rho_r\chi_{t-1} + \chi_{t+1} + \frac{1}{1-h}g_{t+1} + (\frac{\rho_r + h}{1-h} + (1-\rho_r)\phi_y)g_t \\
&- \frac{h\rho_r}{1-h}g_{t-1} + \epsilon_{mp_t} + \kappa_p(\frac{h}{1-h}g_t + (1+\sigma_n)n_t) + \kappa_p(\mu_t - \chi_t) \qquad (31) \\
g_t &= a_t + \zeta_t + (1-\alpha)n_t - \zeta_{t-1} - (1-\alpha)n_{t-1} \qquad (32)
\end{aligned}
$$

- $\hat{\mathrm{x}}_{t-1} = [\mathbf{g_{t-1}}, \mathbf{n_{t-1}}, \zeta_{t-1}, a_{t-1}, \mu_{t-1}, \chi_{t-1}]'$.

- Identify permanent TFP shocks via long run restrictions.



Permanent TFP
Transitory TFP
Preference
Cost push
Monetary policy
Innovation 1
Innovation 2
Lag



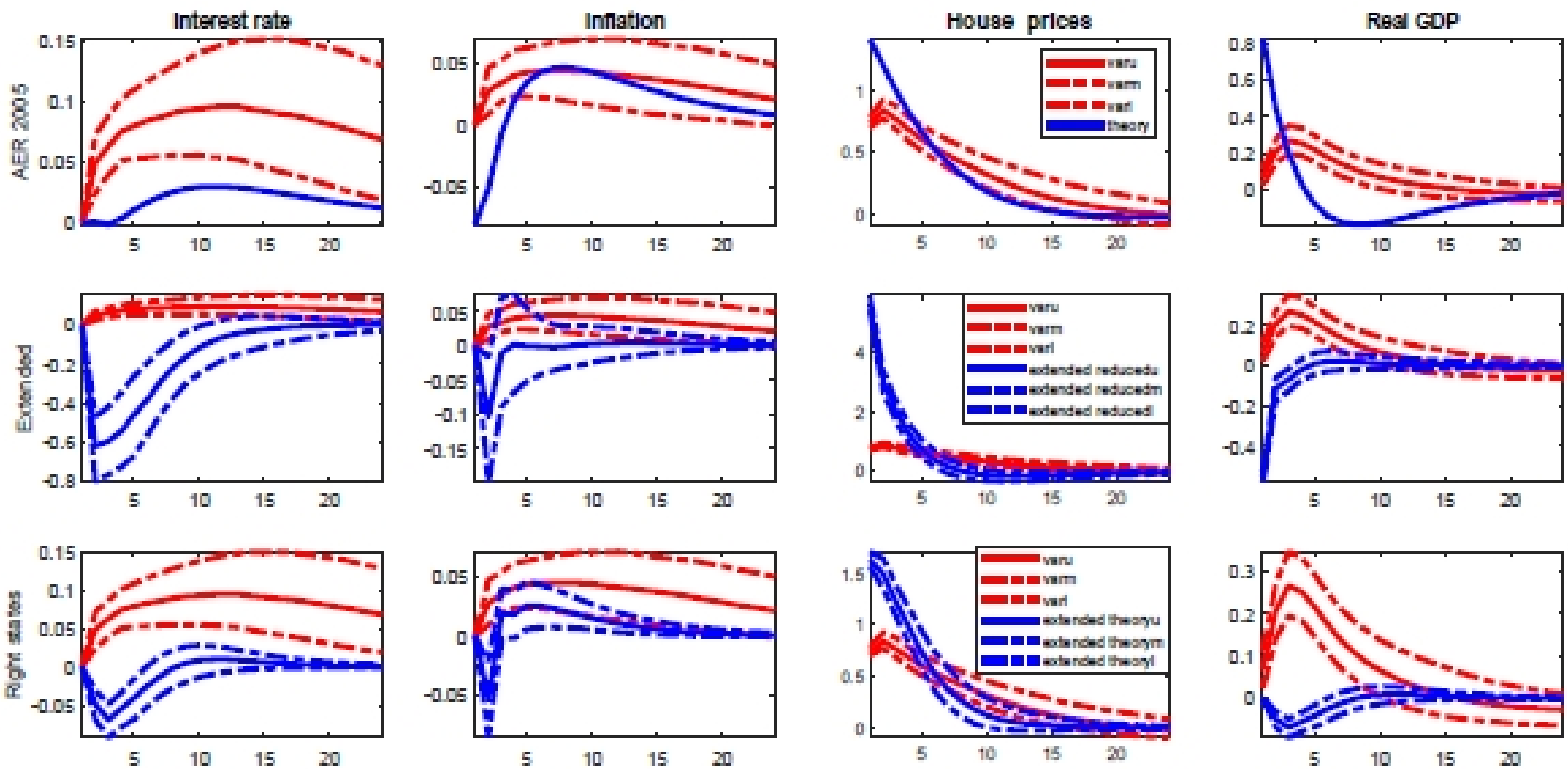

Permanent TFP shocks, $z_t = (g_t, n_t)$

**Given an empirical model, does the theory match stylized facts?**

• Iacoviello (2005): VAR with 4 variables (Y, $\pi$ R, q).

• Theory has preferences, technology, cost push, monetary policy disturbances. Map preference disturbances into $q$ shocks.

• Are there only 4 disturbances in DGP? Add LTV constraint and impatient consumers wealth shocks (Rabanal,2018; Linde',2018).


Interest rate
Inflation
House prices
Real GDP
AER 2005
Extended
Right states
varu
varm
varl
theory
extended reducedu
extended reducedm
extended reducedl
extended theoryu
extended theorym
extended theoryl

- Cross sectional distortions

| Innovations | Disturbances | | | | | | |
|---|---|---|---|---|---|---|---|
| | $e_R$ | $e_j$ | $e_u$ | $e_a$ | $e_{has}$ | $e_{i1}$ | $e_{i2}$ |
| $R_t$ | 1.0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\pi_t$ | -0.53 | -0.003 | 1.43 | -0.11 | -0.13 | 0.18 | 0.24 |
| $q_t$ | **-1.83** | **0.05** | -0.80 | 0.13 | 0.33 | **-1.27** | -0.51 |
| $y_t$ | -3.92 | 0.03 | -1.14 | -0.02 | -0.09 | 2.46 | 0.92 |

- Cross correlation coefficients:

- $q$ innovations - preference disturbances $e_j$: 0.66 (0.60, 0.72).

- $q$ innovations - LTV disturbances $e_{i1}$: -0.63 (-0.67,-0.58).

- $q$ innovations - preference disturbances $e_j$: 0.92 (0.88-0.96). (DGP with 4 disturbances)

**Higher order models**

- Pruned solution has a linear state space representation (Andreasan, et al., 2018).

- State and shock vectors very large.

- Deformation potentially more important.

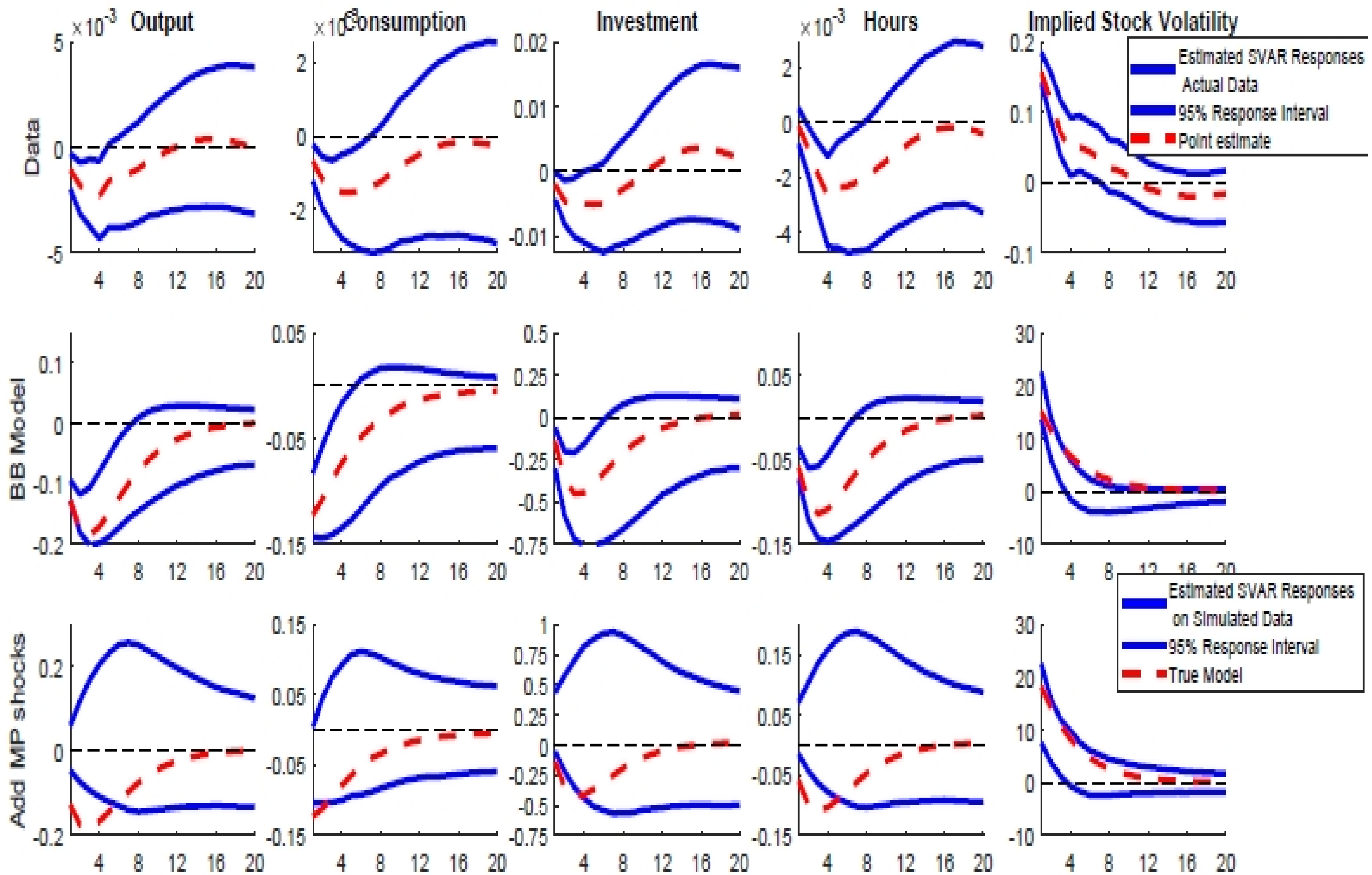
**Transmission of uncertainty shocks: Basu and Bundick (2017)**

• Theory has TFP, preference and preference volatility disturbances. Solved with third order perturbation.

• Pruned solution has a huge state and shock vector (432 states, 1112 shocks).

• Use a linear VAR with 8 variables. Potentially huge deformation distortions.

• Monetary policy disturbance is missing from the theory. What happens to the (informal) matching exercise if a MP disturbance included?


Output
Consumption
Investment
Hours
Implied Stock Volatility
Data
BB Model
Add MP shocks
Estimated SVAR Responses
Actual Data
95% Response Interval
Point estimate
Estimated SVAR Responses
on Simulated Data
95% Response Interval
True Model

- Cross correlation coefficients:

- $VIX$ innovations - uncertainty disturbances: 0.63 (0.50, 0.74).

- $VIX$ innovations - monetary disturbances: -0.46 (-0.50,-0.41).

- $VIX$ innovations - uncertainty disturbances: 0.77 ( 0.68, 0.84). (DGP with 3 disturbances)

For four variable VAR there might be more than four shocks? What shocks will be included and why some others are not?

## What empirical models should one use?

- Large scale BVARs?

- FAVARs?

- IV-Local Projections? SVAR-IV? (Miranda-Agrippino and Ricco, 2019).

## Impulse response matching with deformation

- $u_t = \lambda_0(\theta)e_t + \lambda_1(\theta)e_{t-1} + \lambda_2(\theta)e_{t-2} + \dots$

- Generous lag length need in empirical model to take care of time deformation.

- To match $e_{1t}$ when cross sectional deformation is present need:

- $\lambda_{01}(\theta) = [\lambda, 0, 0, \dots, 0]$.

- sign$(\lambda_{01}(\theta)) = sign(G_{11}) \neq sign(\lambda_{0j}(\theta)),\ j \neq 1$, where $G_{11}$ first row of $G_1(\theta) \equiv \begin{pmatrix} B(\theta) \\ S_{12}D(\theta) \end{pmatrix}$

- Check these conditions for each model prior to estimation.

## Conclusions

• Deformation may make identified shocks and dynamics mongrels with little economic interpretation. Magnitude, sign and persistence distortions. Formal impulse response matching problems.

• Recovered shocks do not necessarily aggregate structural disturbances of the same type (cross sectional deformation). Valid theory restrictions may be insufficient.

• Recovered shocks are, in general, linear combinations of current and past structural disturbances (time deformation).

- Potential solutions:

i) use empirical models with larger information set (BVAR, FAVARs, LP-IV). No guarantee it will work.

ii) use a model to select the empirical variables and the shocks to be identified.

iii) given an empirical model, enlarge the shock space of the theory and recheck the quality of the match.