Skolkovo Institute of Science and Technology

Experimental Data Processing
MA060238

Development of optimal smoothing to increase the estimation accuracy

Authors:

Luca Breggion
Irina Yareshko

October 17, 2022

1 Introduction

The aim of this work is to develop algorithms to improve Kalman filter estimates, that is of prime importance for many practical control and forecasting problems.

2 Development of backward smoothing algorithm for Kalman filter

The smoothing is performed in backward in time:

$$X_{i,N} = X_{i,i} + A_i(X_{i+1,N} - \Phi i + 1, iX_{i,i}), \quad where\ i = N-1, N-2, ..., 1 \tag{1}$$

where the coefficient A_i is evaluated as:

$$A_i = P_{i,i}\Phi^T_{i+1,i}P^{-1}_{i+1,i} \tag{2}$$

The smoothing error covariance matrix P:

$$P_{i,N} = P_{i,i} + A_i(P_{i+1,N} - P_{i+1,i})A_i^T \tag{3}$$

According to the procedure described above, the Kalman filter was supplemented with backward smoothing algorithm. In **Figure 1** below, the True Data, Measurements, Filtered Data, Smoothed Data are plotted for a trajectory of a moving object disturbed by normally distributed unbiased random acceleration with variance $\sigma_a^2 = 0.2^2$. It is observed that the result of smoothing algorithm is the closest curve to the initial true data. Also, unlike filtered data, smoothed data is free from perceptible sharp projections.

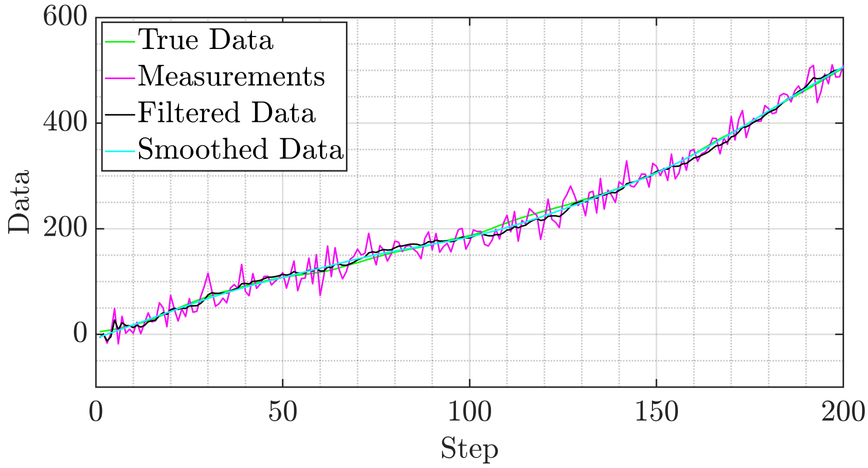

Figure 1: True Data, Measurements, Filtered Data and Smoothed Data

To analyse the effect of smoothing more deeply, the comparison between true estimation, true smoothed estimation, smoothing algorithm error and filtration error was performed both for coordinate x_i and velocity V_i.

The error of true estimation with and without smoothing was calculated as followed:

$$FinalError(i) = \sqrt{\frac{1}{M-1}\sum_{k=1}^{M}(x_i - \widehat{x}_{i,i})^2} \tag{4}$$

The error of smoothing algorithm is obtained from the Smoothing Error Covariance Matrix $P_{i,N}$ and the error of filtration algorithm is obtained from the Filtration Error Covariance Matrix $P_{i,i}$. The results are presented in **Figure 2** and **Figure 3**.

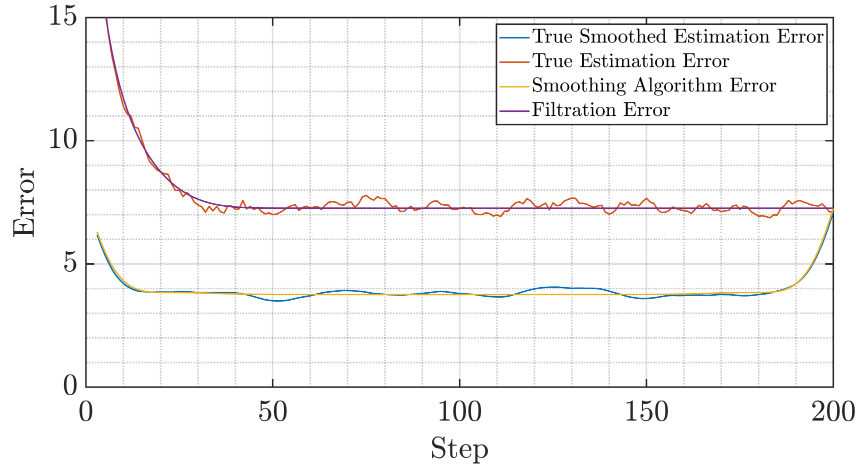

Figure 2: Comparison of True Estimation Error, True Smoothed Estimation Error, Error of Smoothing Algorithm, Filtration Error for coordinate

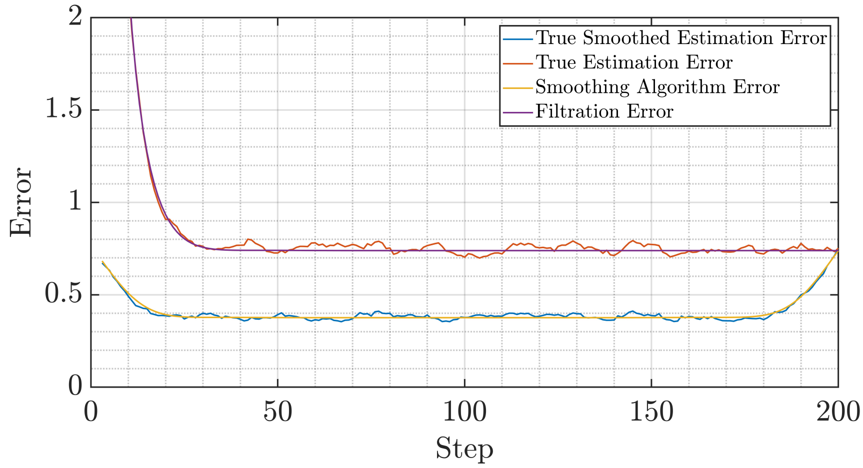

Figure 3: Comparison of True Estimation Error, True Smoothed Estimation Error, Error of Smoothing Algorithm, Filtration Error for velocity

It is observed both for velocity and for coordinate, that the result of smoothing procedure provided the error 1.5 times smaller than the standard Kalman filter. In addition to that, smoothing allows to obtain an even smooth curve without any sharp components.

3 Conclusion

By the end of this assignment, we learnt how to improve Kalman filter with quite an easy to implement backwards smoothing algorithm that allows for getting more accurate result compared to standard Kalman filter. We showed in **Figure 1** that backward smoothed data is much closer to the true data.

As well as that, we learnt by experience, how to assess the performance of the filter numerically. With figures **Figure 2** and **Figure 3** we proved numerically that the backwards smoothing reduced the prediction mistakes. We learnt that in smoothing algorithm there are both current and future measurements are taken into account, that provides better estimation results.

Individual contribution:

- Luca Breggion: Matlab code, report
- Irina Yareshko: Matlab code, report