Skolkovo Institute of Science and Technology

Experimental Data Processing
MA060238

Joint assimilation of navigation data coming from different sources

Authors:

Luca Breggion
Irina Yareshko

October 24, 2022

1 Introduction

The objective of this laboratory work is to develop a navigation filter by assimilating data coming from different sources.

2 Generate a true trajectory X_i of an object motion disturbed by normally distributed random acceleration Q1 - Q4

At the first step, the actual trajectory X_i is determined considering:

- Observation interval $N = 500$ steps;
- Interval between measurements $T = 2$;
- Initial conditions $x_0 = 1000$, $y_0 = 1000$;
- Initial components of velocity $V_x = 100$, $V_y = 100$;
- Variance of noise $\sigma_a^2 = 0.3^2$ for both a_i^x and a_i^y

A true trajectory was generated by following equations:

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2} \tag{1}$$

$$V_i^x = V_{i-1}^x + a_{i-1}^x T \tag{2}$$

$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2} \tag{3}$$

$$V_i^y = V_{i-1}^y + a_{i-1}^y T \tag{4}$$

True values of range D and azimuth β are determined by:

$$D_i = \sqrt{x_i^2 + y_i^2} \tag{5}$$

$$\beta_i = \arctan \frac{x}{y} \tag{6}$$

Measurements D^m and β^m, of distance D and azimuth β, provided by the first observer are available every 4 seconds.

$$D_i^m = D_i + \eta_i^D \quad \beta_i^m = \beta_i + \eta_i^\beta \quad where\ i\ = 1, 3, 5, ..., N-1\ (odd\ steps) \tag{7}$$

Variances of measurement noises η_i^D, η_i^β are given by $\sigma_D = 50^2$, $\sigma_\beta = 0.004^2$.
In order to generate more accurate measurements of azimuth β^m, the second observer provides measurement between those of the first observer:

$$\beta_i^m = \beta_i + \eta_i^\beta \quad where\ i = 4, 6, 8, ..., N\ (even\ steps) \tag{8}$$

Variances of measurement noises η_i^β, for this case, is given by $\sigma^2_{\beta add} = 0.001^2$.

3 Extended Kalman filter development (Q5 - Q7)

Initial filtered estimate of state vector $X_{0,0}$:

$$X_0 = \begin{vmatrix} x_3^m \\ \frac{x_3^m - x_1^m}{2T} \\ y_3^m \\ \frac{y_3^m - y_1^m}{2T} \end{vmatrix} \tag{9}$$

where

$$x_1^m = D_1^m \sin\beta_1^m \qquad x_3^m = D_3^m \sin\beta_3^m \qquad y_1^m = D_1^m \sin\beta_1^m \qquad y_3^m = D_3^m \sin\beta_3^m \tag{10}$$

Initial filtration error covariance matrix $P_{0,0}$:

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix} \tag{11}$$

The transition matrix $\Phi_{i,i-1}$ with $T = 1$:

$$\Phi_{i,i-1} = \begin{vmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{12}$$

The state noise covariance matrix Q :

$$Q = GG^T\sigma_a^2 \tag{13}$$

The measurement noise covariance matrix R varies:

- Observer 1, odd time steps

$$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix} \tag{14}$$

- Observer 2, even time steps

$$R = \sigma^2_{\beta_{add}} \tag{15}$$

At every filtration step in the algorithm the measurements is linearized by determining:

$$\frac{dh(\widehat{X}_{i+1,i})}{dX_{i+1}} \tag{16}$$

making a distinction to the measurements of the first and second observer.

After running the Kalman filter over $M = 500$ runs, the true estimation errors of range D and azimuth β are determined and shown in **Figure 2 Figure 3**

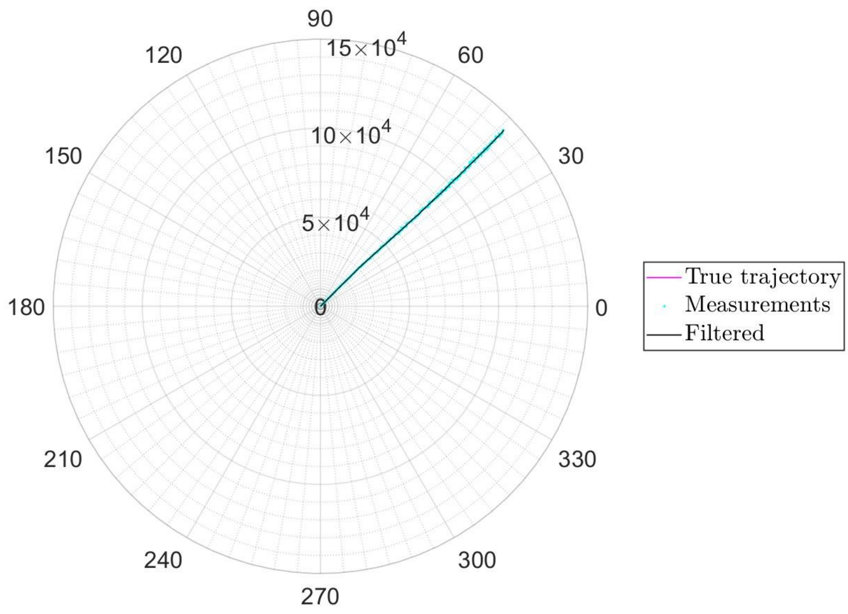

Figure 1: True trajectory, Measurements and Filtered

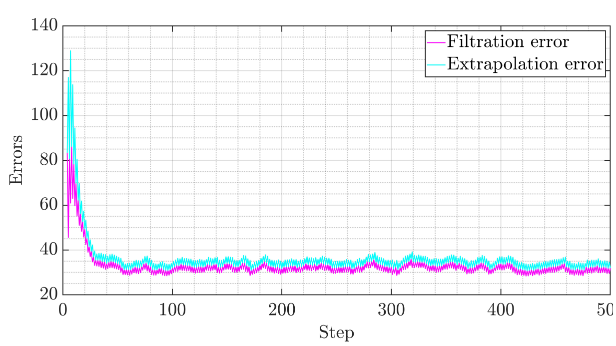

Figure 2: Range D errors

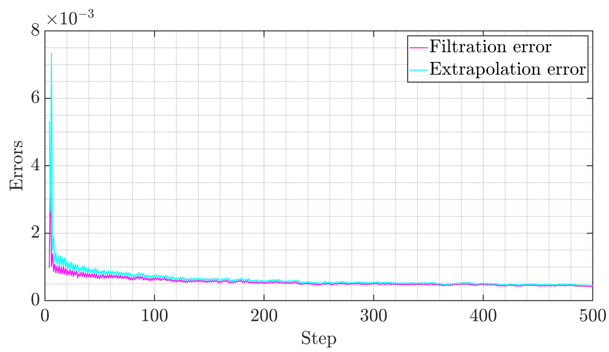

Figure 3: Azimuth β errors

As it can be seen from the plots, as the number of steps increase, the azimuth errors decrease considerably, approaching the zero value **Figure 3**. This is due to the fact that for range values, we only have measurements from a single observer, while for azimuth, measurements for odd and even steps are used, hence the azimuth errors are more accurate than the range ones **Figure 2**.

Furthermore, since Kalman's algorithm consists of a first stage of *extrapolation* and a second stage of adjustment of the predicted estimate, called *filtration*, the improved estimated provides smaller errors than the first stage. Indeed, it can be appreciated how the filtered estimates are closer to the true trajectory depicted in **Figure 4**.

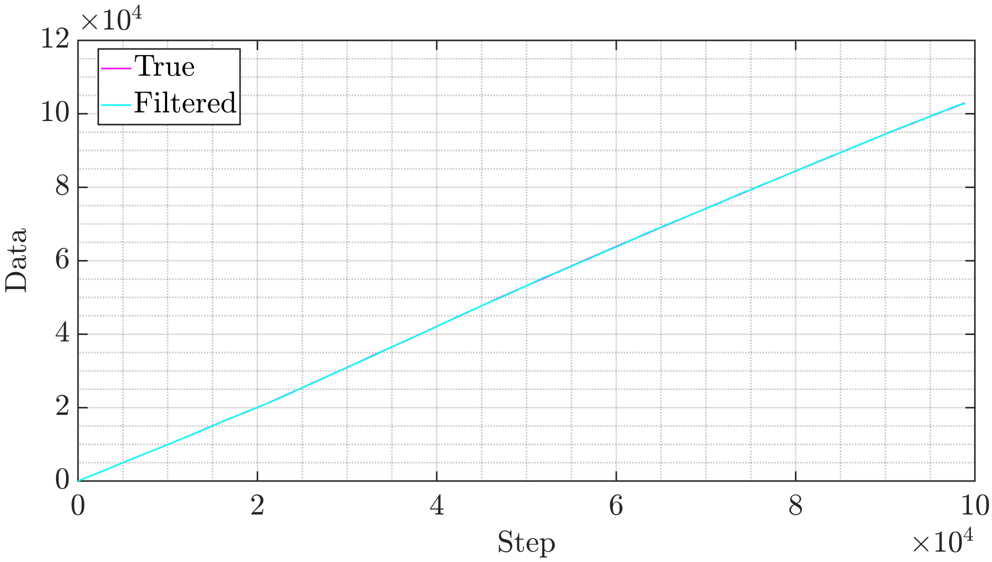

Figure 4: True trajectory with *cartesian coordinates* and Filtered trajectory

4 Compare estimation results with measurement errors of D and β Q8 - Q9

The true trajectory and filtered estimations coincides very closely. Considering also the measurements, we can not observe any noise in the range **Figure 5**. The noise is present, but but is it small and one of the reasons could be that the order of magnitude of range is 10^4, while the variance of measurement noise for range was given as 50^2. Therefore, the variance is small compared to the order of magnitude of range.

Regarding the azimuth β, two different observers with different variances of measurement noises are included. Hence, this could fluctuate the measurements. Moreover, if we check the order of magnitudes, β is in the order of 10^{-1} while the variance is in the order of 10^{-3}. This difference in orders of magnitude could explain why it is possible to observe the noise in the azimuth β.

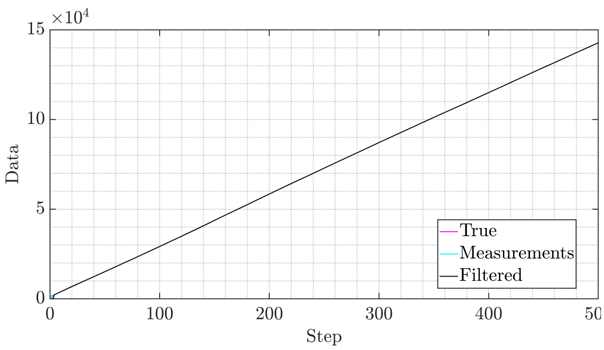

Figure 5: True trajectory, Measurements and Filtered estimates for *Range* in Polar Coordinates

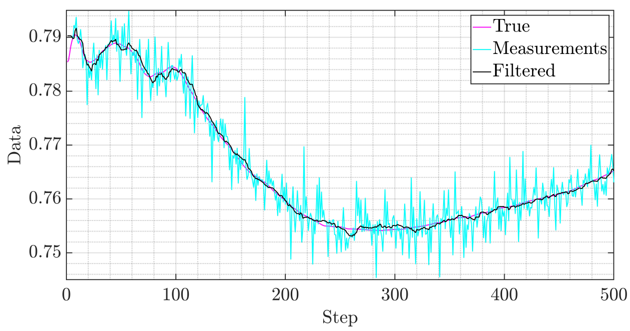

Figure 6: True trajectory, Measurements and Filtered estimates for *Azimuth* in Polar Coordinates

For the first observer, odd steps for both azimuth and range are considered. In addition to the first observer, the second one also analyzes even time steps to obtain more accurate azimuth measurements, making possible to obtain knowledge between the odd time steps. By adding a second observer to check even time steps besides odd ones, we have increased the accuracy of our results for filtering and extrapolating azimuth.

5 Conclusion

In this assignment we implemented the *Extended Kalman filter* and we analyzed results obtained for azimuth β and range D. The measurement of azimuth where provided by two observers: the first with *odd* time steps and the second with *even* time steps. Whereas, the range measurements are given only by the first observer. Precisely for this reason, the azimuth errors estimations are more accurate with respect to the range values. Furthermore, we have learnt the effect of using odd and even steps together on prediction of errors. in fact, to obtain information between odd-numbered time steps a second observer were available to check the even time steps in addition to odd ones. This increases the accuracy of the filtered and extrapolation azimuth.

Individual contribution:

- Luca Breggion: Matlab code, report
- Irina Yareshko: Matlab code, report