Skolkovo Institute of Science and Technology

Experimental Data Processing
MA060238

Extended Kalman filter for navigation and trackin

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1 Introduction

The aim of this laboratory work is to develop Extended Kalman filter for tracking a moving object when measurements and motion model are in different coordinate systems. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation for nonlinear models.

2 Generation of the true trajectory and measurements (Q1-Q3)

At the first step, the actual trajectory X_i is determined considering:

- Observation interval $N = 500$ steps;
- Interval between measurements $T = 1$;
- Initial conditions $x_0 = 1000$, $y_0 = 1000$;
- Initial components of velocity $V_x = 10$, $V_y = 10$;
- Variance of noise $\sigma_a^2 = 0.3^2$ for both a_i^x and a_i^y

A true trajectory was generated by following equations:

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2} \tag{1}$$

$$V_i^x = V_{i-1}^x + a_{i-1}^x T \tag{2}$$

$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2} \tag{3}$$

$$V_i^y = V_{i-1}^y + a_{i-1}^y T \tag{4}$$

True values of range D and azimuth β are determined by:

$$D_i = \sqrt{x_i^2 + y_i^2} \tag{5}$$

$$\beta_i = \arctan\frac{x}{y} \tag{6}$$

Measurements D^m and β^m are generated of range D and azimuth β:

$$D_i^m = D_i + \eta_i^D \tag{7}$$

$$\beta_i^m = \beta_i + \eta_i^\beta \tag{8}$$

Variances of measurement noises η_i^D, η_i^β are given by $\sigma_D = 50^2$, $\sigma_\beta = 0.004^2$.

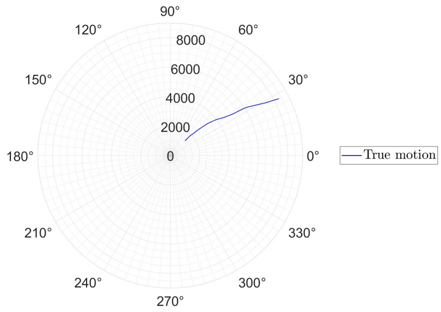

Figure 1: True Trajectory

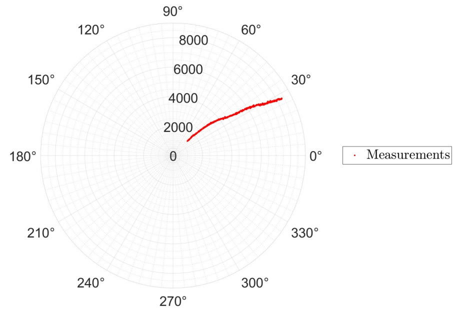

Figure 2: Measurements

3 Extended Kalman filter development (Q4 - Q11)

Initial filtered estimate of state vector $X_{0,0}$:

$$X_0 = \begin{vmatrix} D_i^m(1)sin\beta_i^m(1) \\ 0 \\ D_i^m(1)sin\beta_i^m(1) \\ 0 \end{vmatrix} \tag{9}$$

Initial filtration error covariance matrix $P_{0,0}$:

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix} \tag{10}$$

The transition matrix $\Phi_{i,i-1}$ with $T = 1$:

$$\Phi_{i,i-1} = \begin{vmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{11}$$

The state noise covariance matrix Q :

$$Q = GG^T\sigma_a^2 \tag{12}$$

The measurement noise covariance matrix R :

$$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix} \tag{13}$$

At every filtration step in the algorithm the measurements is linearized by determining:

$$\frac{dh(\widehat{X}_{i+1,i})}{dX_{i+1}} = \begin{vmatrix} \frac{x_{i+1,i}}{\sqrt{x_{i+1,i}^2+y_{i+1,i}^2}} & 0 & \frac{y_{i+1,i}}{\sqrt{x_{i+1,i}^2+y_{i+1,i}^2}} & 0 \\ \frac{y_{i+1,i}}{x_{i+1,i}^2+y_{i+1,i}^2} & 0 & -\frac{x_{i+1,i}}{x_{i+1,i}^2+y_{i+1,i}^2} & 0 \end{vmatrix}$$

(14)

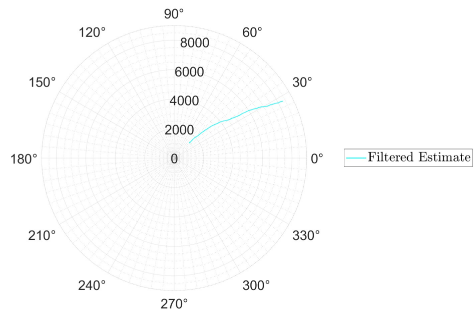

Figure 3: Filtered Estimates

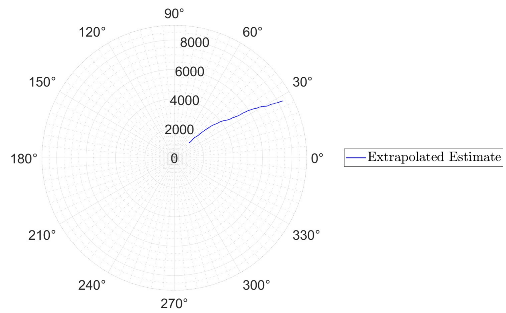

Figure 4: Extrapolation Estimates

In this part, Kalman filter estimation **Figure 3** and extrapolated estimation **Figure 4** are plotted by calculating both range D and azimuth β. It is observed that even though extrapolated estimate is pretty close to the true trajectory, filtration is even closer as it is expected.

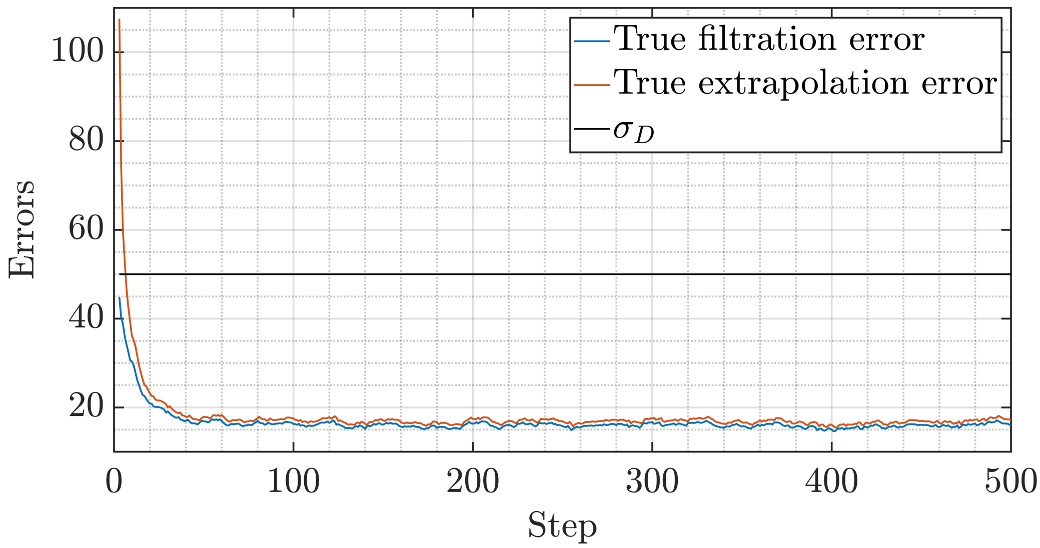

Figure 5: Errors of range

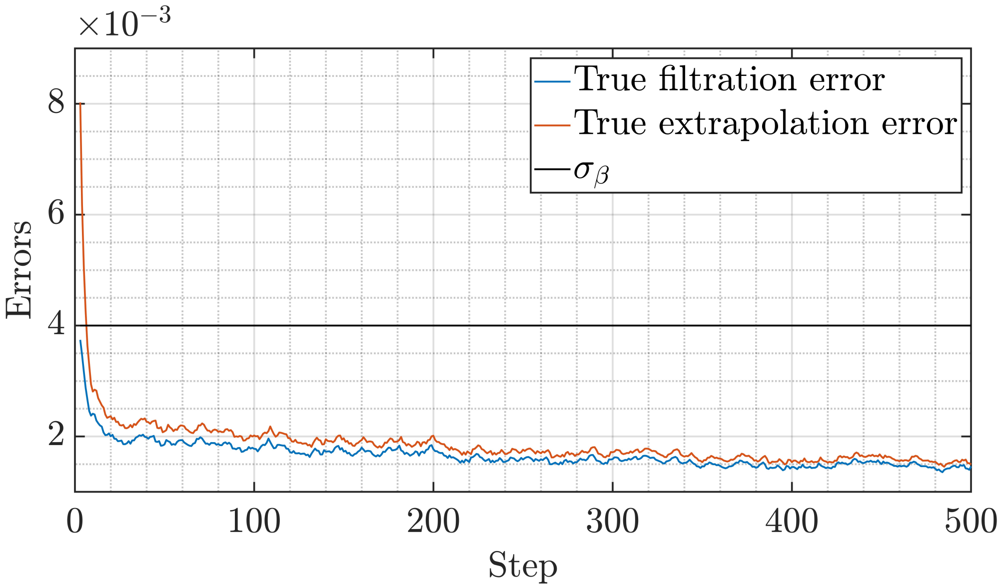

Figure 6: Errors of azimuth

In order to have a deeper understanding of the results obtained, errors of extrapolation and filtration of both range D and azimuth β are visualised in **Figure 5** and **Figure 6** respectively. According to these plots, we can conclude that both errors of filtration and extrapolation have the same trend, decreasing over the observation interval. It can be seen that the errors of filtration over the whole observation interval have slightly smaller values. Therefore, filtration is slightly more accurate than the extrapolation. The other thing that stands out is that the value of the errors of azimuth angle β is too small. It can be explained by the distance. Being far enough from the observer allows reducing the affect on the accuracy of the azimuth angle. Therefore, it is normal to expect these values.

4 Conclusion

By the end of this assignment, we have learnt the Kalman filter can serve as a powerful tool for navigating and tracking. We learnt by experience that the filtration provides slightly better results than the extrapolation. We observed that over long distances error of the azimuth β decreases significantly and does not change much by the change in range. We have critically reflected upon the development of a Kalman filter and calculation the range D and azimuth β at every step. We experimented with different values of number of runs M and decided that it is better when it is 500 or greater. We calculated and compared the errors in order to see the accuracy of estimations.

Individual contribution:

- Luca Breggion: Matlab code, report
- Irina Yareshko: Matlab code, report