Experimental Data Processing
MA060238

Determining and removing drawbacks of exponential and running mean. Task 2

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1 Introduction

The aim of this work is to determine conditions for which running and exponential mean methods provide effective solution and conditions under which they break down.

2 Comparison of the traditional 13-month running mean with the forward-backward exponential smoothing for approximation of 11-year sunspot cycle

2.1 Make smoothing of monthly mean data by 13-month running mean

The 13-month running mean method was used (**Equation 1**) to smooth the given monthly mean sunspot number, furthermore, the deviation and variability indicator are determined and listed in **Table 1**.

$$\mathbf{R} = \frac{1}{24}R_{i-6} + \frac{1}{12}(R_{i-5} + R_{i-4}... + R_{i-1} + R_i + R_{i+1} + ... + R_{i+5}) + \frac{1}{24}R_{i+6} \tag{1}$$

Indicator	Running mean
I_d	$1.677 \cdot 10^5$
I_v	$1.307 \cdot 10^3$

Table 1: Deviation and Variability indicators for 13-month Running mean

2.2 Make forward-backward exponential smoothing of monthly mean sunspot number

Forward-backward exponential smoothing was performed according to **Equation 2** and **Equation 3**.

$$X_i^f = X_{i-1}^f + \alpha(z_i - X_{i-1}^f), \quad for\ i = 2, ..., N \tag{2}$$

$$X_i^b = X_{i+1}^b + \alpha(X_i^f - X_{i+1}^b), \quad for\ i = N-1, ..., 1 \tag{3}$$

In order to find the smoothing constants α, that provides better results than the 13-month month running mean based on the indicators of deviation and variability, several values are considered.
The results are shown in **Table 2**.

α	0.05	0.10	0.15	0.20	0.25	0.30
I_d	$4.798 \cdot 10^5$	$2.611 \cdot 10^5$	$1.866 \cdot 10^5$	$1.494 \cdot 10^5$	$1.252 \cdot 10^5$	$1.066 \cdot 10^5$
I_v	$0.708 \cdot 10^3$	$2.970 \cdot 10^3$	$7.008 \cdot 10^3$	$1.306 \cdot 10^4$	$2.139 \cdot 10^4$	$3.227 \cdot 10^4$

Table 2: Deviation and Variability indicators for Forward-Backward Exponential Smoothing with different coefficients α

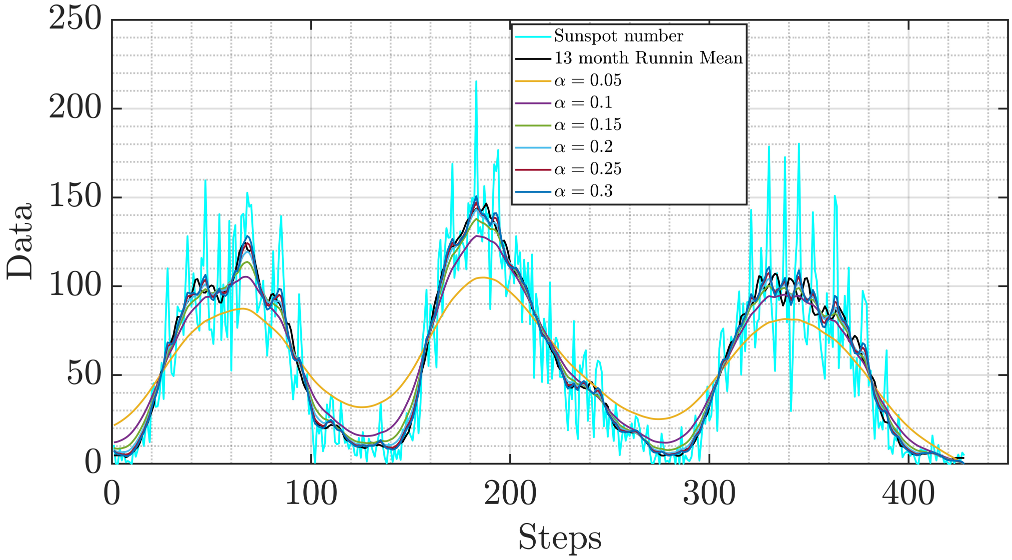

Figure 1: Comparison of Running Mean and Exponential Smoothing with different values of α.

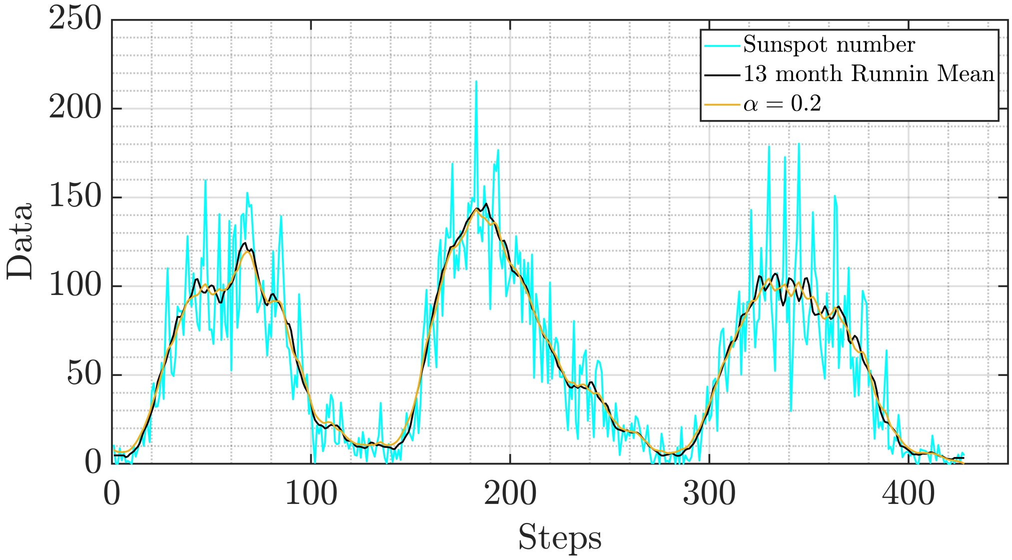

Figure 2: Comparison of Running Mean and Exponential Smoothing with $\alpha = 0.2$.

Comparing the performance of Running Mean and Exponential Smoothing in terms of indicators (**Table 1** and **Table 2**), it is observed that for exponential smoothing the deviation indicator is smaller for $\alpha = 0.2$, $\alpha = 0.25$, $\alpha = 0.3$. However, the variation indicator of exponential smoothing is smaller for the case when $\alpha = 0.05$. For better understanding of smoothing performance **Figure 1** is built for comparing all the cases.
As the initial function is far from straight line, it is reasonable not to consider too small values of α, as the smoothing effect is too strong. In other words, although the variation indicator, which describes the offset, is smaller for the low $\alpha = 0.05$, the deviation is not acceptable.
Among $\alpha = 0.2$, $\alpha = 0.25$, $\alpha = 0.3$ it is reasonable to choose $\alpha = 0.2$, as in this case we get better smoothing performance in terms of deviation and acceptable variation indicator value. To demonstrate more clearly that with $\alpha = 0.2$ Exponential Smoothing performs better than Running Mean, **Figure 1** is built.

3 3D surface filtration using forward-backward smoothing

3.1 Determine the variance of deviation of noisy surface from the true one

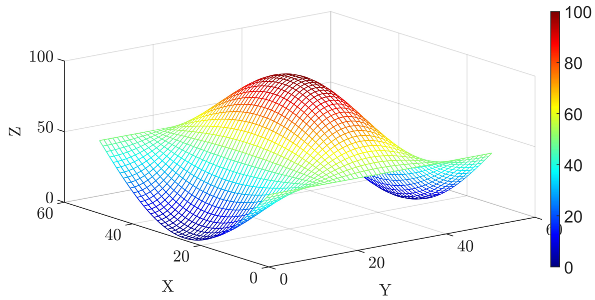

Figure 3: True surface

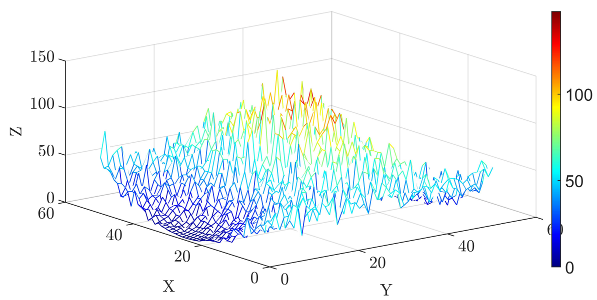

Figure 4: Noisy surface

In **Figure 3** and **Figure 4** are shown the true and noisy surfaces, respectively. It can be observed that the noisy surface does not have smoothness and it is reflected on the variance of deviation, calculated according to the following formula:

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X - \bar{X})^2 \tag{4}$$

Thus, the variance value between true and noisy surface is $\sigma^2 = 122.062$, proving that it is not accurate.

3.2 Apply forward-backward exponential smoothing to filter noisy surface measurements. The smoothing constant can be $\alpha = 0.335$

Forward-backward exponential smoothing was performed among rows and columns according to **Equation 2** and **Equation 3**. The results are presented in **Figure 5** and **Figure 6**

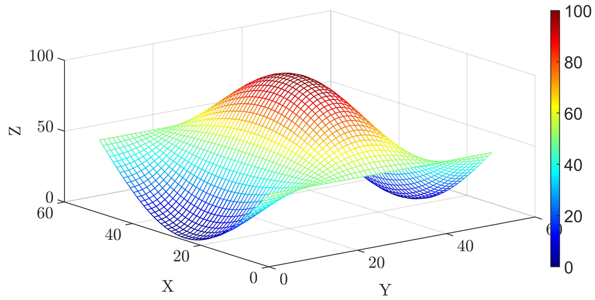

Figure 5: True surface

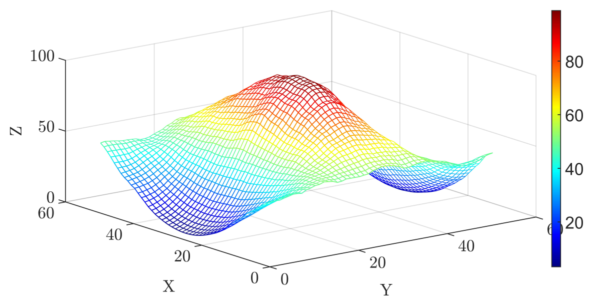

Figure 6: Reconstructed surface

After applying the exponential smoothing on noisy data, we can definitely say that the distracting noise was reduced and the result of smoothing is closer to the true surface given. The reconstruction was performed step by step among columns and rows. In order to prove the effectiveness of smoothing not only visually but mathematically, the variance between smoothed noisy surface and the true one was calculated according to **Equation 4**. The variance after smoothing constituted $\sigma^2 = 6.795$, which is significantly smaller than the variance for noisy surface $\sigma^2 = 122.062$.

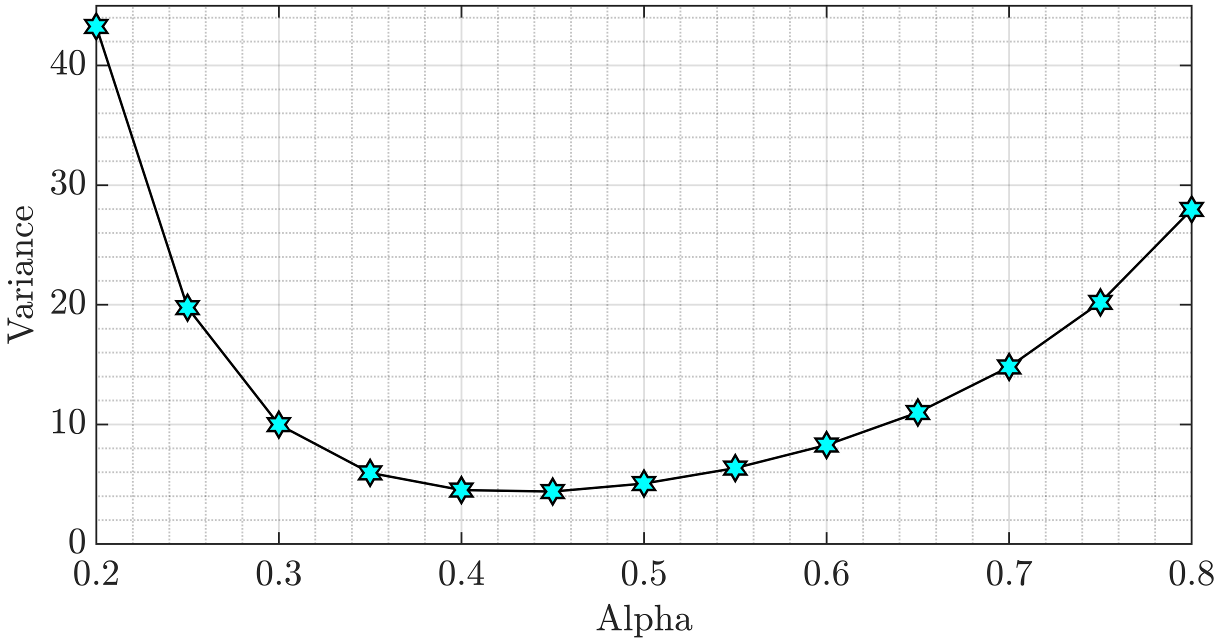

Figure 7: The dependency of variance on the smoothing coefficient α

In addition, different values of smoothing coefficient α were tried for exponential smoothing. The result of these experiments is presented in **Figure 7**. It is observed that the best smoothing effect is achieved for values of $\alpha = 0.45$. For too small values of α the smoothing effect is too strong, causing big variance, and for values of $\alpha > 0.45$ the smoothing is not enough.

4 Conclusion

By the end of this assignment, we compared the traditional 13-month running mean smoothing algorithm with forward-backward exponential smoothing and performed 3D surface filtration using the forward-backward exponential smoothing. We learned how to reshape the matrix and plot 3D surfaces in MATLAB. We learned the importance of smoothing the noisy data and its effects. In particular, we learnt a way to find an optimum value of smoothing coefficient α that would provide the best smoothing. We found out a way to apply forward-backward exponential smoothing to 3D data, and how it affects the noise step by step. In additions to this, we learnt the way to estimate if the smoothing is good or not not only by visual representation, but calculating the variance and deviation and variation indicators.

Individual contribution:

- Luca Breggion: Matlab code, report
- Irina Yareshko: Matlab code, report