Control engineering

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The Root Locus Technique

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Introduction

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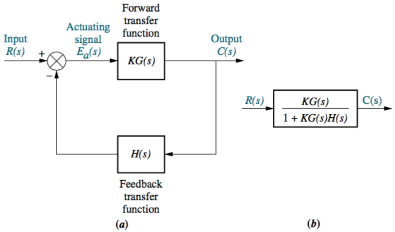
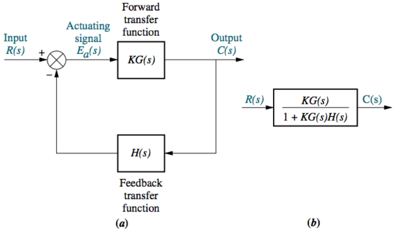
- The characteristics of a closed-loop system is closely related to the location of the closed-loop poles.
- If the system has a variable loop gain, then the location of the closed-loop poles depends on the value of the loop gain chosen
- It is important, therefore, that the designer know how the closed-loop poles move in the s plane as the loop gain is varied



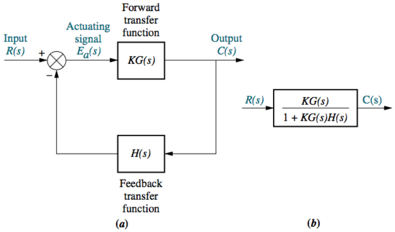
Introduction

- If a control system does not satisfy the requirements, a simple gain adjustment may move the closed-loop poles to desired locations.
- Then the design problem may become the selection of an appropriate gain value
- If the gain adjustment alone does not yield a desired result, addition of a compensator to the system will become necessary



Introduction

The root-locus diagram is essentially a graphical plot of the loci of the roots of the characteristic equation of a system as a function of a real parameter K which varies from 0 to $+\infty$.
It gives an indication of the absolute stability and the dynamic behaviour of a control system with respect to the variation of the system parameter K.



Introduction

By using the root-locus method the designer can predict the effects on the location of the closed-loop poles of varying the gain value or adding open-loop poles and/or open-loop zeros.

In designing a linear control system, we find that the root-locus method proves to be quite useful, since it indicates the manner in which the open-loop poles and zeros should be modified so that the response meets system performance specifications. This method is particularly suited to obtaining approximate results very quickly.

Properties of Root Loci

Properties of Root Loci

Properties of Root Loci

Characteristic equation

Consider the system shown in the figure beneath. The closed loop transfer function is written as :

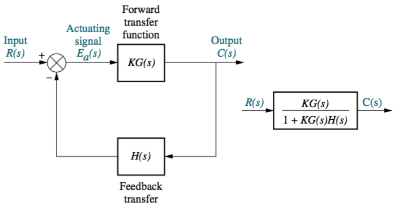
$$T(s) = \frac{KG(s)}{1 + KH(s)G(s)} \tag{1}$$

The roots of the characteristic equation must satisfy the equation ;

$$1 + KG(s)H(s) = 0 \tag{2}$$

or

$$G(s)H(s) = -\frac{1}{K} \tag{3}$$



Properties of Root Loci

Angle and Magnitude conditions

In order to satisfy the Equation (3), the following two conditions must be satisfied simultaneously *Angle Condition* :

$$\angle G(s)H(s) = 180(2\nu + 1) \quad \nu = 0, 1, 2 \ldots \tag{4}$$

Magnitude Condition

$$\|G(s)H(s)\| = \|\frac{1}{K}\| \tag{5}$$

The values of s that satisfy both the angle and the magnitude conditions are the roots of the characteristic equation, or the closed-loop poles (Eq. 2).

Properties of Root Loci

1- Points of the loci corresponding to $K = 0$

The points on the root loci corresponding to the value of $K = 0$ are at the poles of $G(s)H(s)$

2- Points of the loci corresponding to $K = \infty$

The points on the root loci corresponding to the value of $K = \infty$ are at the zeros of $G(s)H(s)$, including those at the infinity.

3- Number of branches of separate root loci

The number of root loci is equal to the number of finite poles or zeros of $G(s)H(s)$ whichever is greater.
This is apparent, since the root loci must start at the poles and terminate at the zeros of $G(s)H(s)$, the number of branches of loci is equal to the maximum of the two numbers finite poles or zeros.

Properties of Root Loci

4- Symmetry of root loci

The root loci are symmetric with respect to the real axis, since the complex roots occur in complex conjugate pairs.

5- Root loci on the real axis

Root loci are found on a given section of the real axis of the s-plane only if the total number of real poles and real zeros of $G(s)H(s)$ to the right of the section is odd for $K > 0$.

6- Asymptotes of root loci

For large values of s the root loci are asymptotic to the straight lines with angles given by :

$$\theta_k = \frac{(2\nu + 1)180^o}{\|n - m\|} \tag{6}$$

where $\nu = 0, 1, 2, ...\|n - m\| - 1$; n is the number of finite poles of $G(s)H(s)$, and m is the number of finite zeros of $G(s)H(s)$.

Properties of Root Loci

7- Intersection of the asymptotes on the real axis

The intersection of the asymptotes lies only on the real axis of the s-plane. The points of intersection of the asymptotes on the real axis is given by :

$$\sigma_1 = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n-m} \tag{7}$$

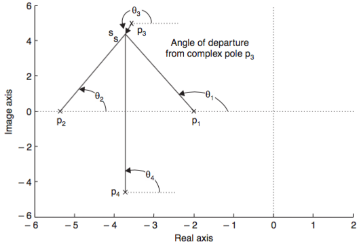
Where n is the number of finite poles of $G(s)H(s)$, and m is the number of finite zeros of $G(s)H(s)$.
The center of the linear asymptotes is often called the asymptote centroid

Properties of Root Loci

8- Angles of departure from complex poles and angles of arrival at complex zeros

The angle of departure of the root loci from a complex pole or the angle of arrival at a complex zero of G(s)H(s) can be determined by considering a search point ss very close to the pole, or zero that satisfy the phase condition of relation $\angle G(s)H(s) = (2K+1)180^o$
For illustration, the figure beneath shows the determination of angle of departure from complex poles, where

$$G(s)H(s) = \frac{K}{s(s+4)(s+2+j4)(s+2-j4)} \quad (8)$$



Properties of Root Loci

9- Intersection of the root loci with the imaginary axis in the s-domain

The value of K at the point of intersection of the root loci with the imaginary axis $s = j\omega$ may be determined by using the Routh-Hurwitz test.

10- Breakaway points

The breakaway points on the root loci are points at which multiple-order roots lie, and are then determined from the roots of the equation obtained by setting

$$\frac{dK}{ds} = 0 \tag{9}$$

Properties of Root Loci

11- Values of K on the root loci

The value of K at any point s_1 on the root loci is determined from the following equation

$$\|K\| = \left\| \frac{1}{G(s_1)H(s_1)} \right\| \tag{10}$$

or

$$\|K\| = \frac{\text{Product of lengths of vectors from poles of } G(s)H(s) \text{ to } s_1}{\text{Product of lengths of vectors drawn from zeros of } G(s)H(s) \text{ to } s_1}$$

With reference to the figure beneath $K = \frac{L_{p1} \cdot L_{p2}}{L_{z1}}$