Project proposals for Jules Verne

Random walk on large genus triangulations

Markov chains are mathematical models that describe how a system evolves when randomness plays a role. At each step, the system moves from one state to another according to certain probabilities. In many situations, as time passes, the system tends to settle into a stable pattern known as an equilibrium. This is a state where the overall behavior no longer changes significantly, even though individual changes may still occur. A central question in this field is: how long does it take for the system to get close to this equilibrium? The time needed for this to happen is called the mixing time. This project will focus on understanding the mixing time for a particular type of Markov chain known as the random walk on large genus triangulations. This is a complex mathematical structure, which can serve as an abstract model for real-world networks. Studying this can improve our understanding of the behavior of very large and well-connected systems, such as communication networks, transportation systems, or even the internet. By exploring this model, we aim to gain insights into how such systems evolve and how quickly they stabilize, which has implications in both theoretical and applied contexts.

Speed of a random walk on supercritical percolation on hyperbolic random triangulations.

Random planar maps are classical models of random surfaces. Hyperbolic random triangulations form a special class of random planar maps, which exhibit unusual behaviour and are currently less well understood than their "uniform" (non-hyperbolic) counterparts. We propose to study supercritical percolation on these maps and answer an open question that was posed by Ray in 2014: Does a random walk on a supercritical percolation cluster have positive speed? Answering this question will give information about the geometry of this percolation cluster. We propose to study this question using classical results relating this property to another known as "anchored expansion", and exploration techniques that allow us to encode the percolation cluster using random walks. This follows a similar strategy to that used to answer the same question for the map itself in a paper by Angel, Nachmias, and Ray in 2016.

Scaling limits of critical decorated stable trees

In a previous collaboration, DS, SÖS, and a further coauthor considered a model of decorated stable trees, in which they constructed scaling limits of models of trees endowed with an additional metric structure. A special case not treated by their paper concerns a so-called critical regime. A natural model which falls into this category is the induced triangulation of a critical percolation cluster on uniform planar maps. In this project, we aim to obtain the scaling limit of the general critical model and also apply the result to the aforementioned induced triangulation. We conjecture that the scaling limit is a random fractal tree known as the stable tree.

Massive spanning forests on Z^d

A spanning tree of a graph G is a subgraph that includes all the vertices and contains no loops. Spanning trees have been shown to be crucial in computer science, for pathfinding algorithms in particular. We consider a generalization of spanning trees, called massive spanning forests, where we allow the components to be disconnected. We are interested in studying this model on a large scale using so-called scaling limits. SÖS and BP are currently studying this model on K_n, the complete graph, and have found a sharp phase transition in the size of the components, and we expect similar results in the case of Z^d.

Colour-avoiding percolation.

If we color each edge of a graph G with k colors, we then say that two vertices u and v are connected if they can always be connected using any subset of k-1 colors. This recent model was first introduced in 2016 by Krause, Danziger, and Zlatić, and has many unanswered questions that have been answered for the Erdős–Rényi model, such as the size of connected components, isolation of vertices, and percolation thresholds. The model can be applied to network analysis, such as internet infrastructure, to ensure that the connection does not stop in the case of a discovered fault.