On Wilson's algorithm with a stopping parameter

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University of Iceland

Université Paris-Dauphine-PSL, France.
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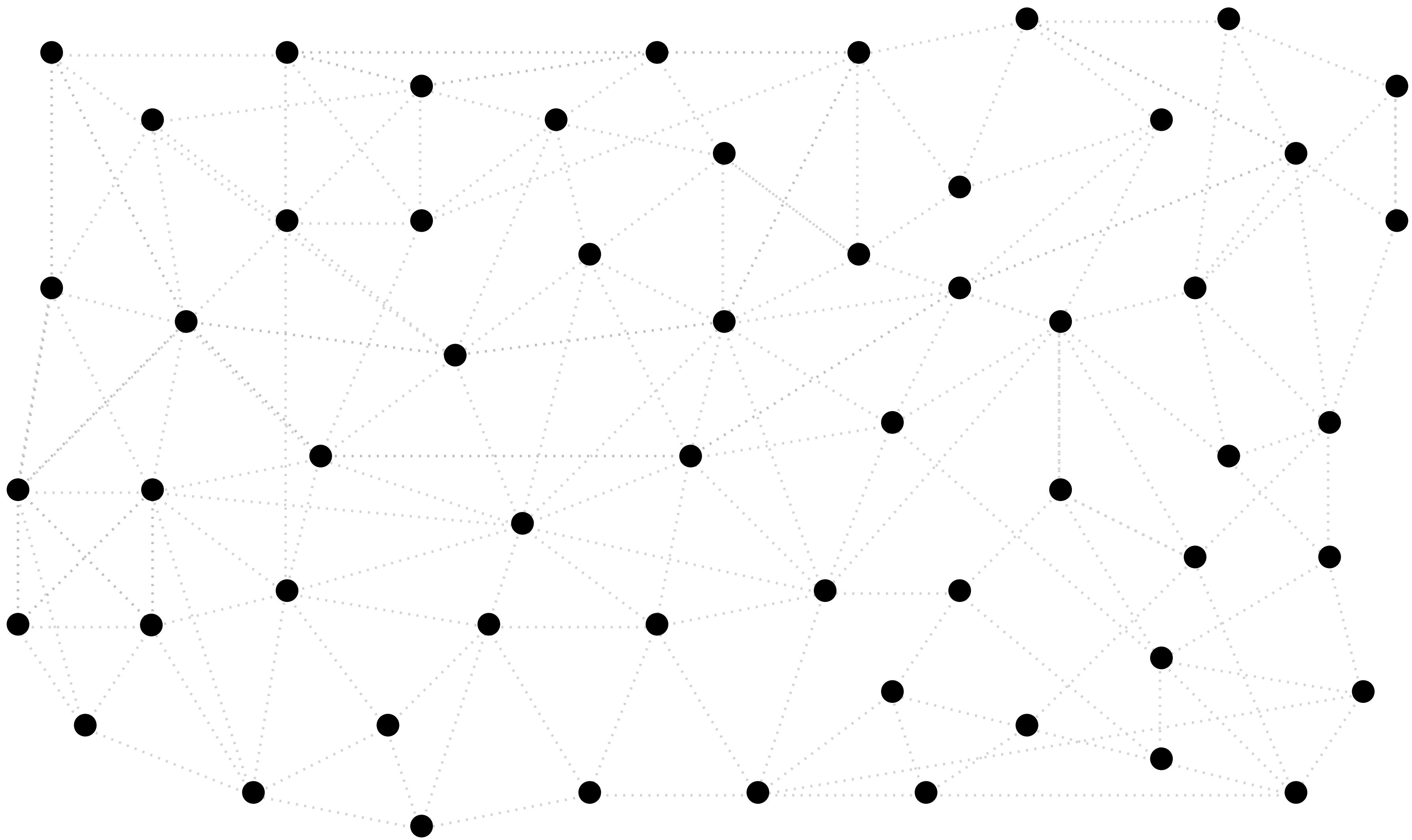
Wilson's algorithm



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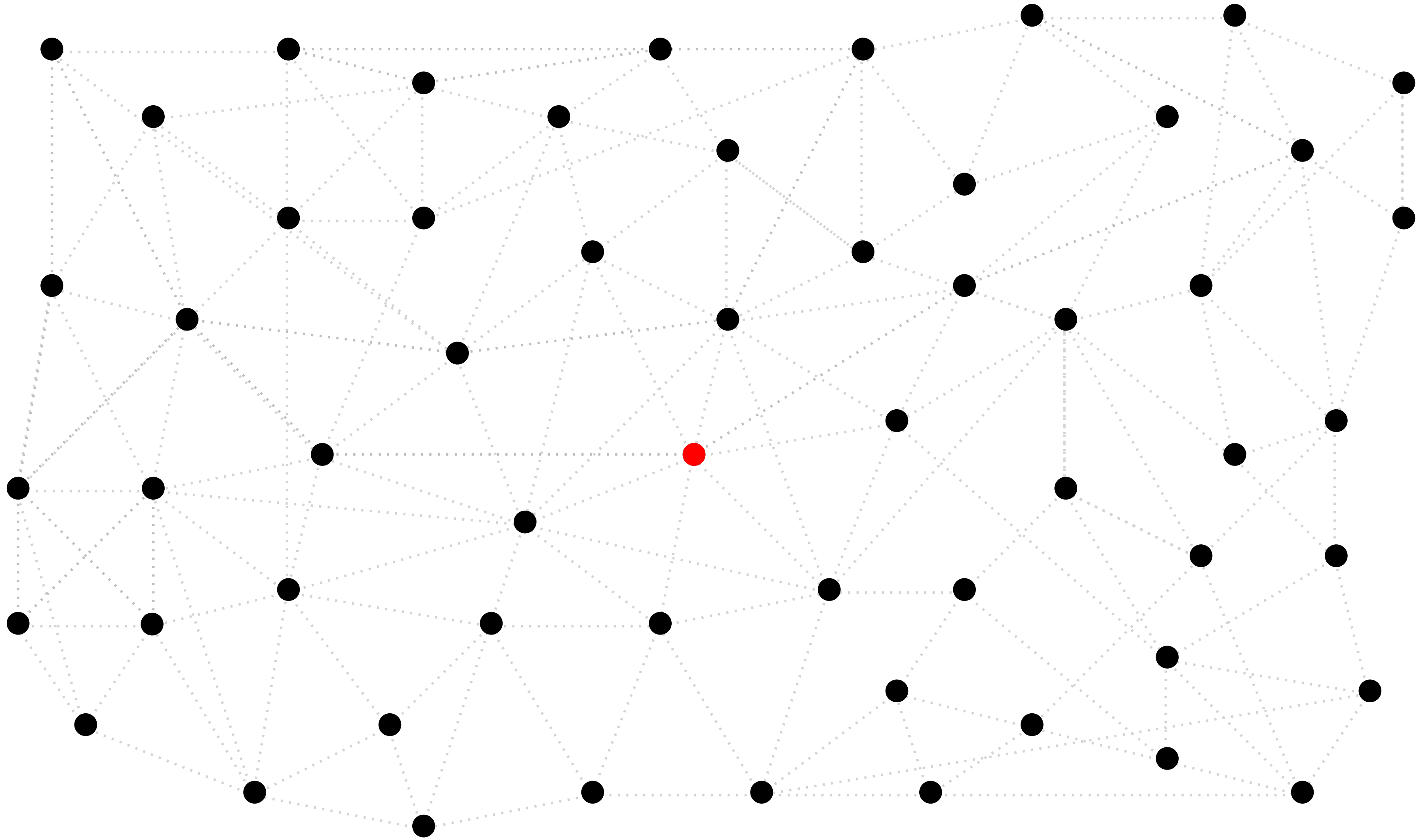

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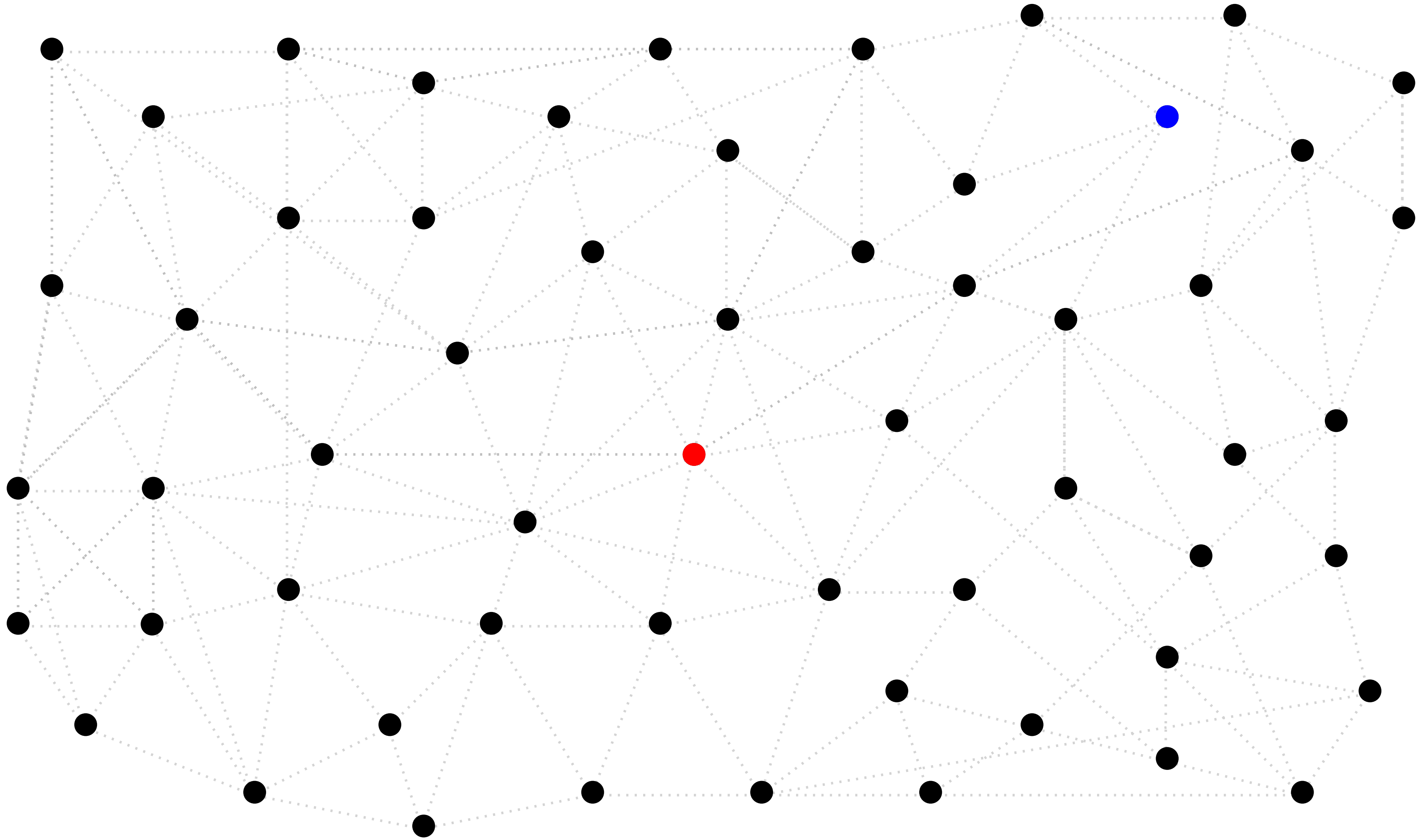

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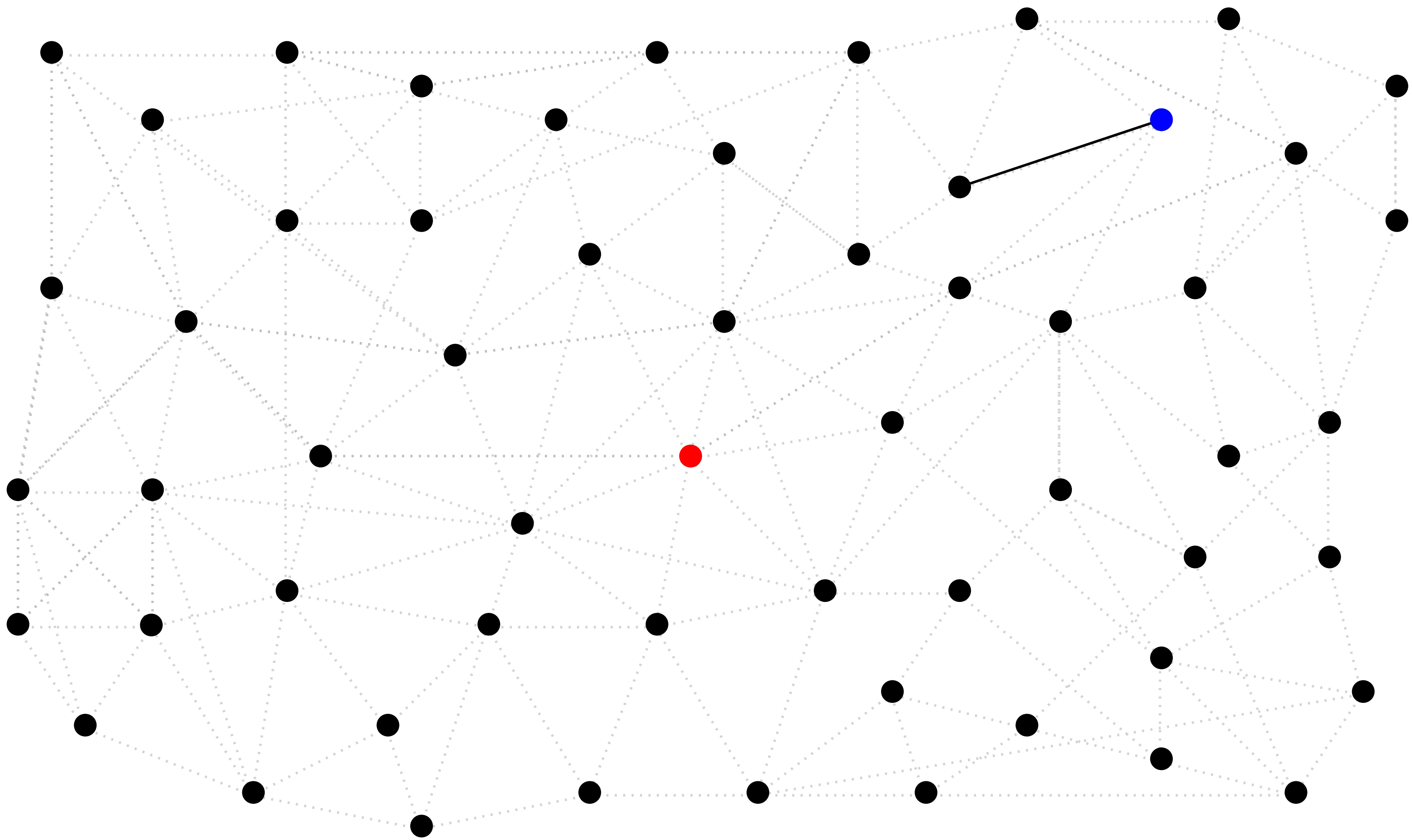

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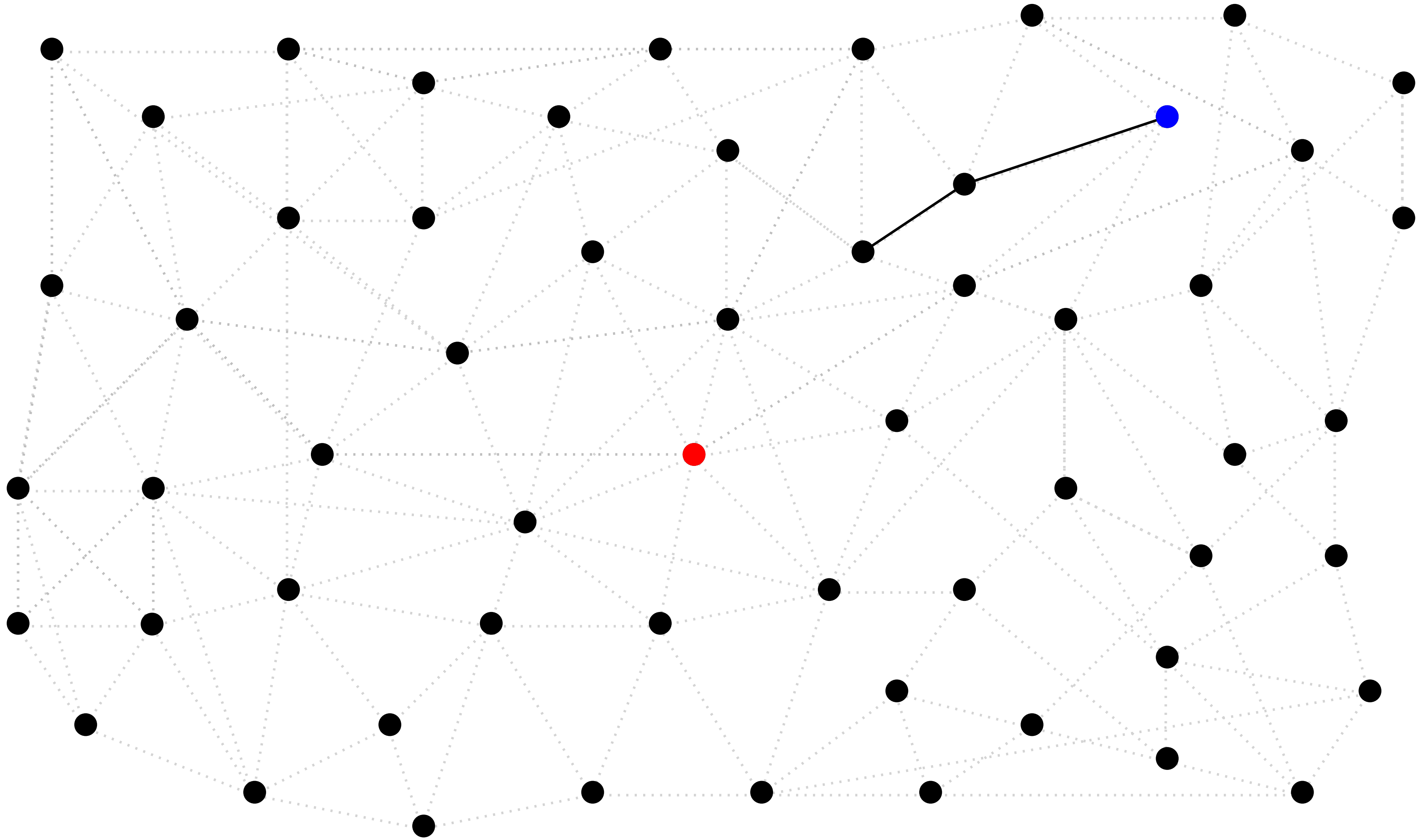

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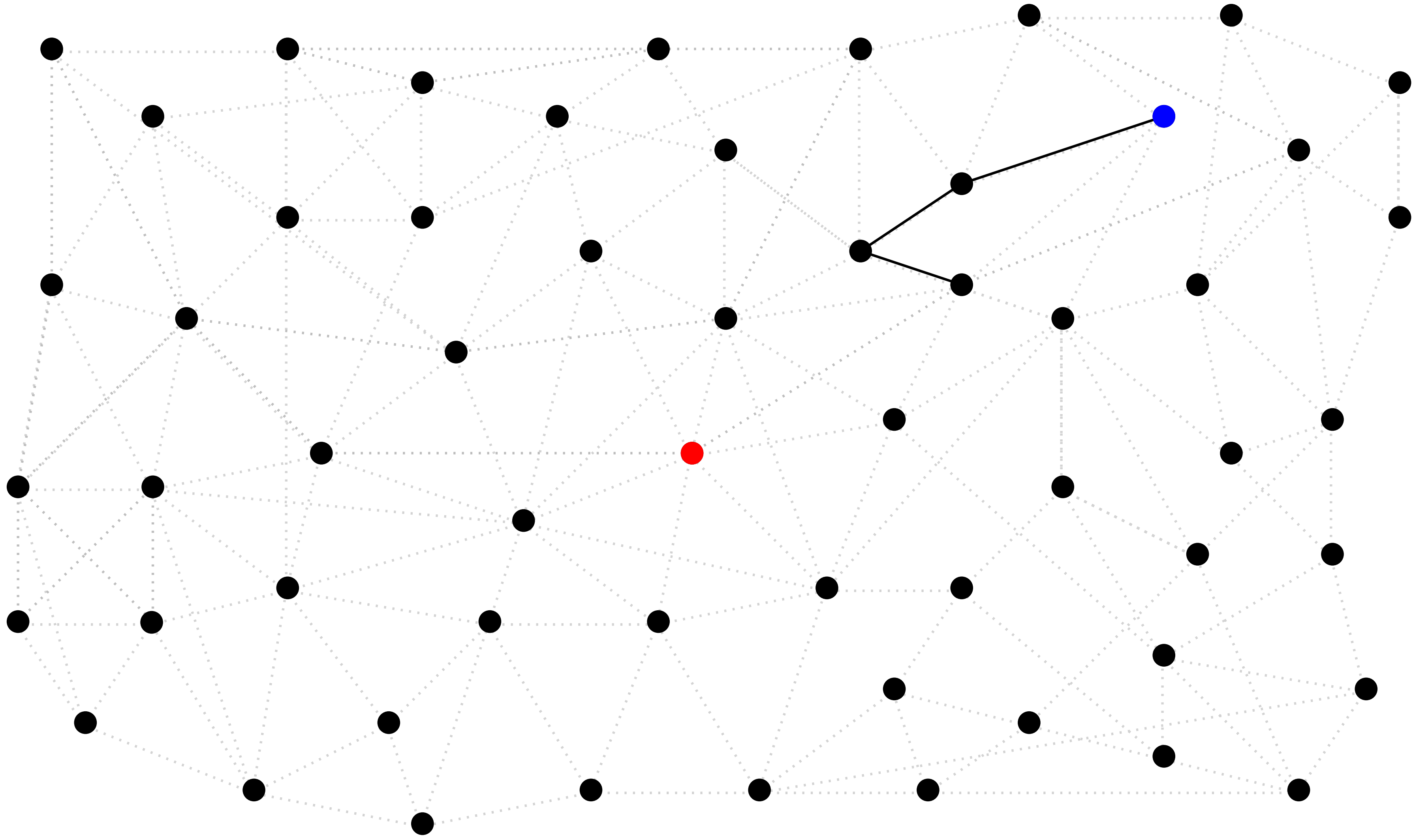

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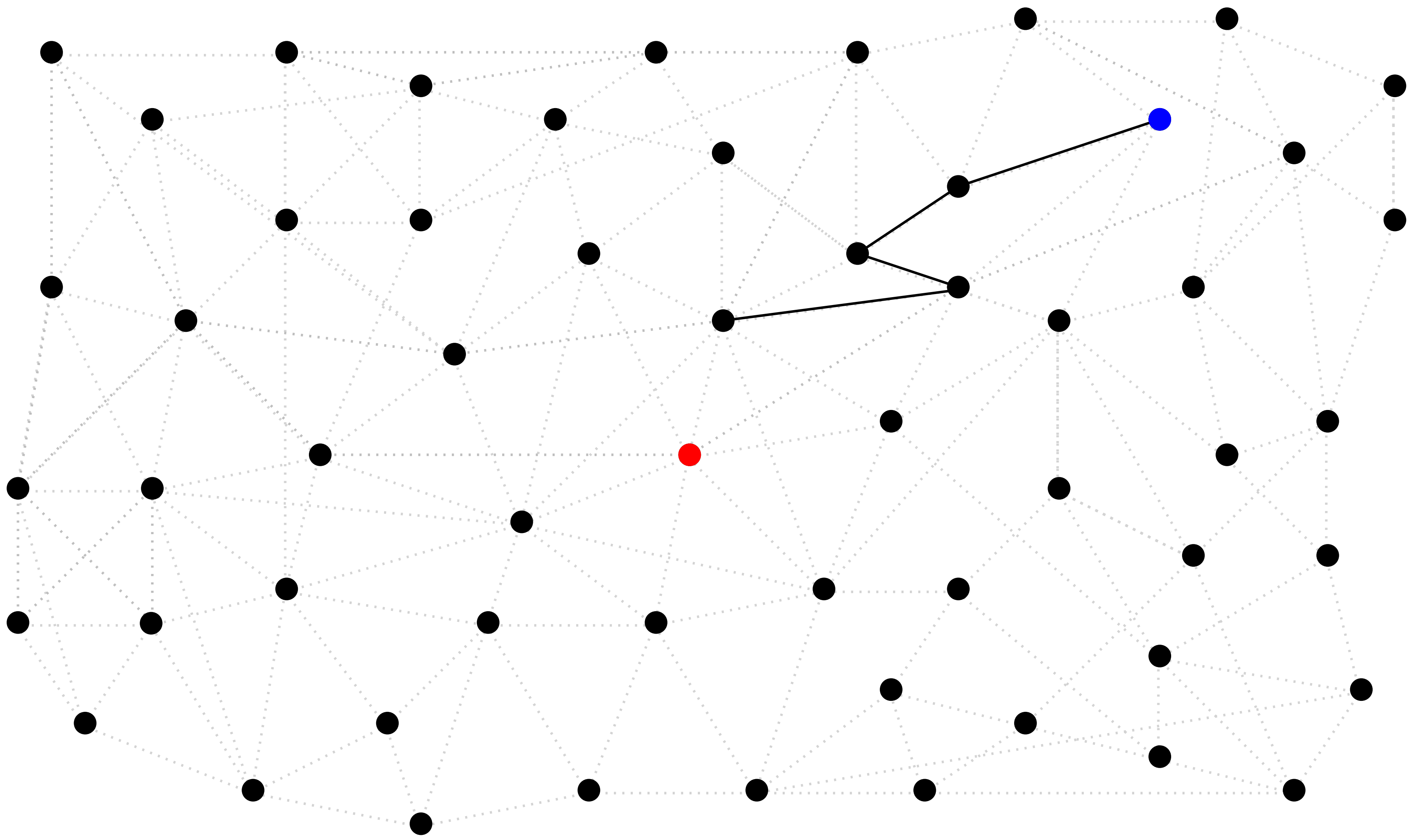

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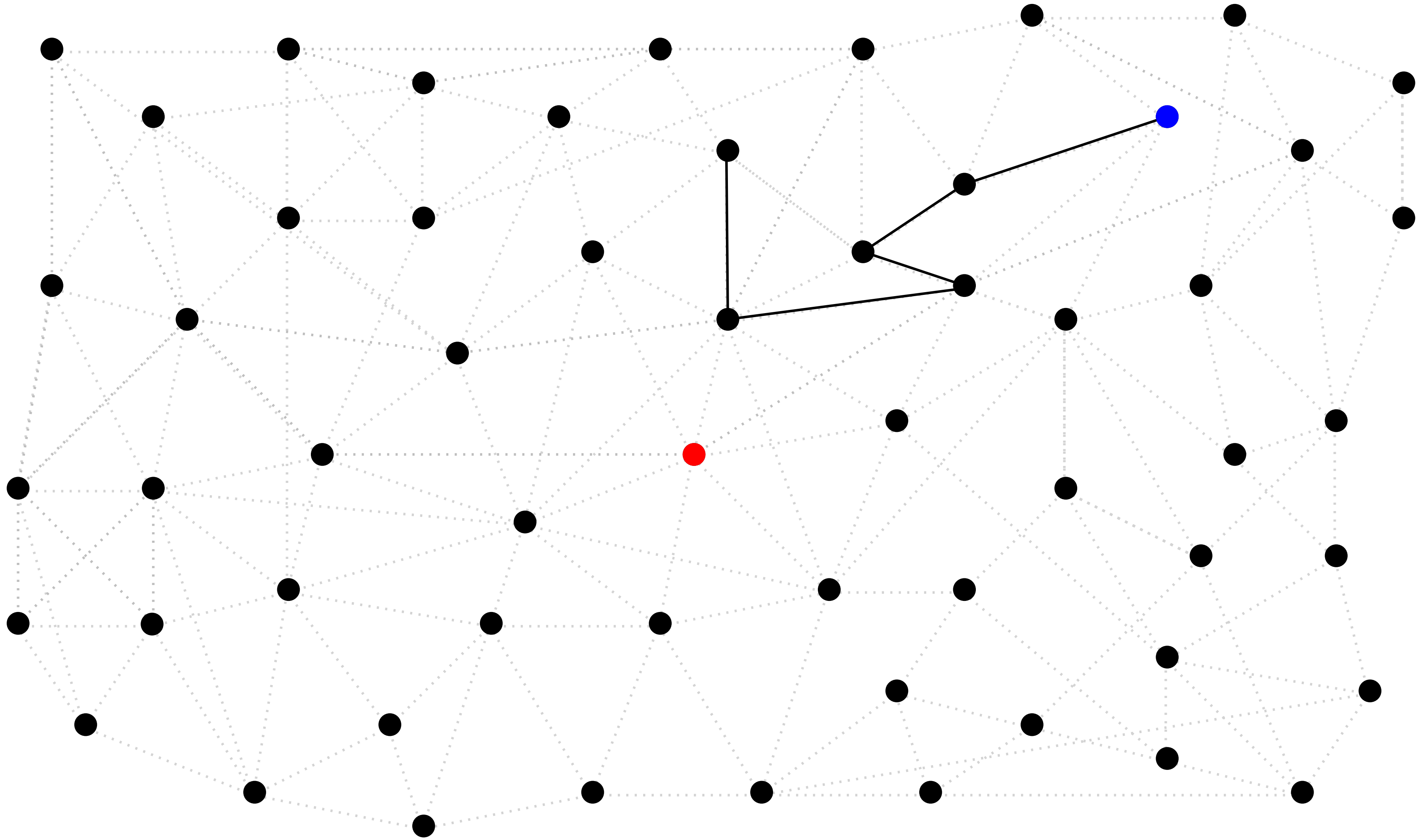

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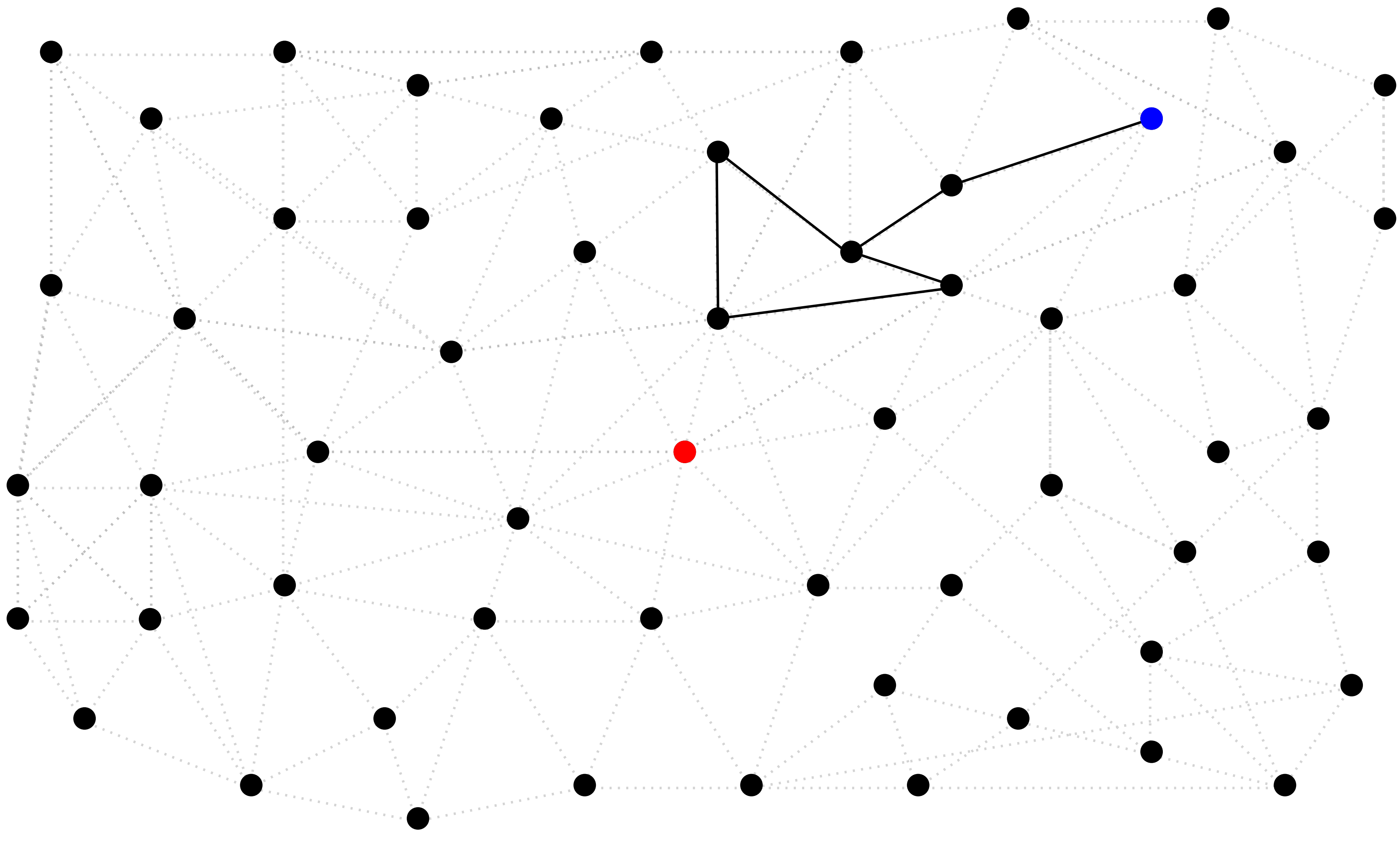

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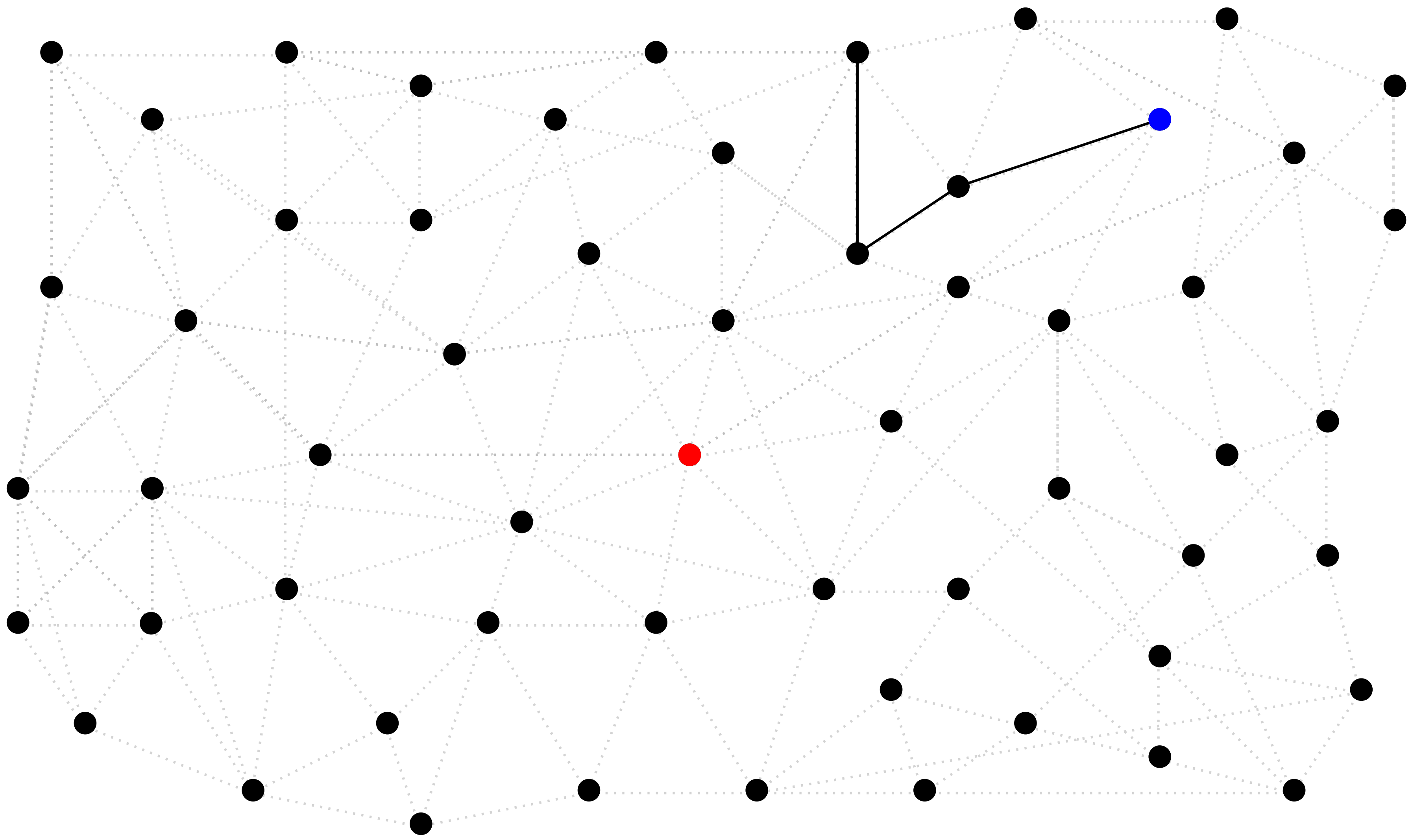

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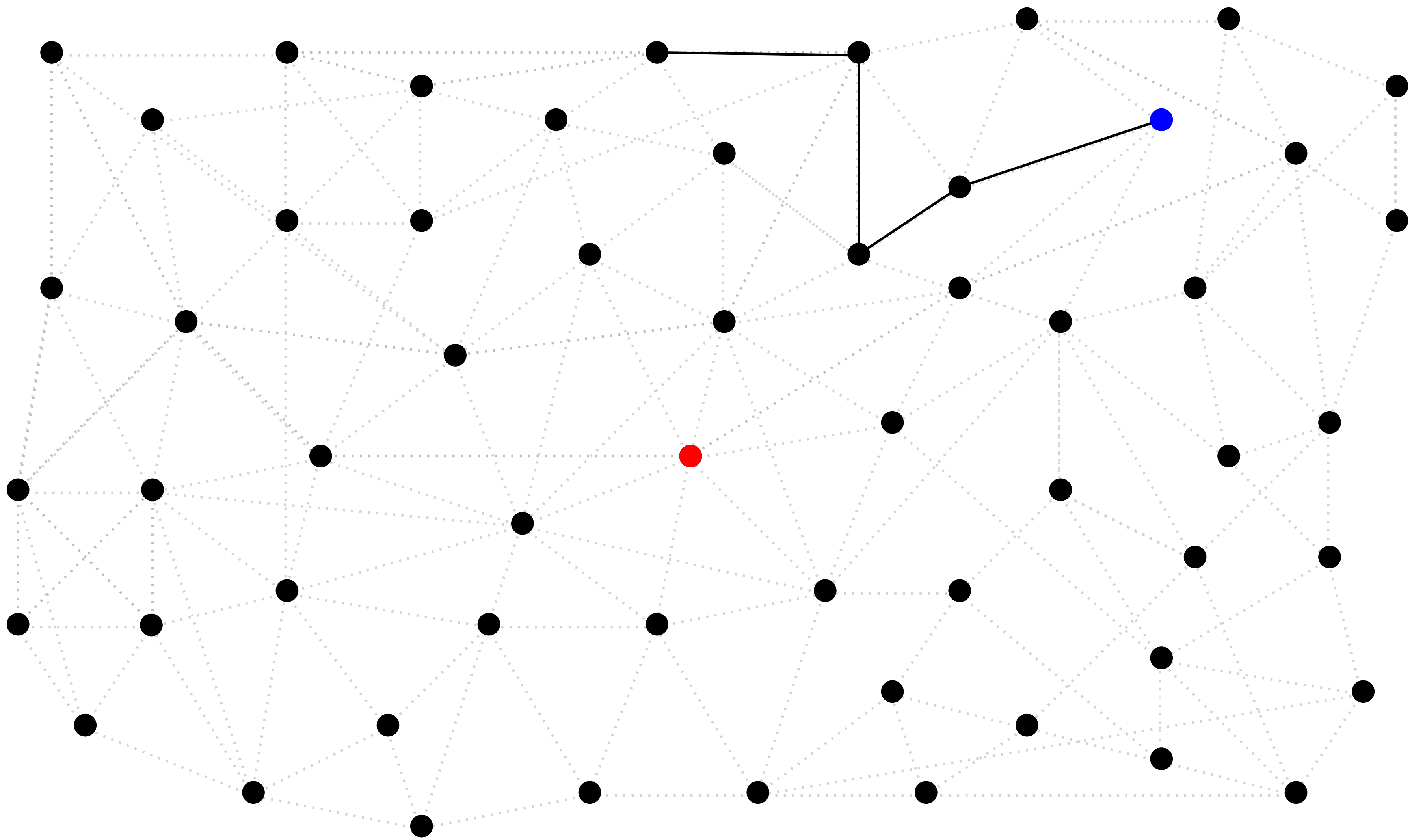

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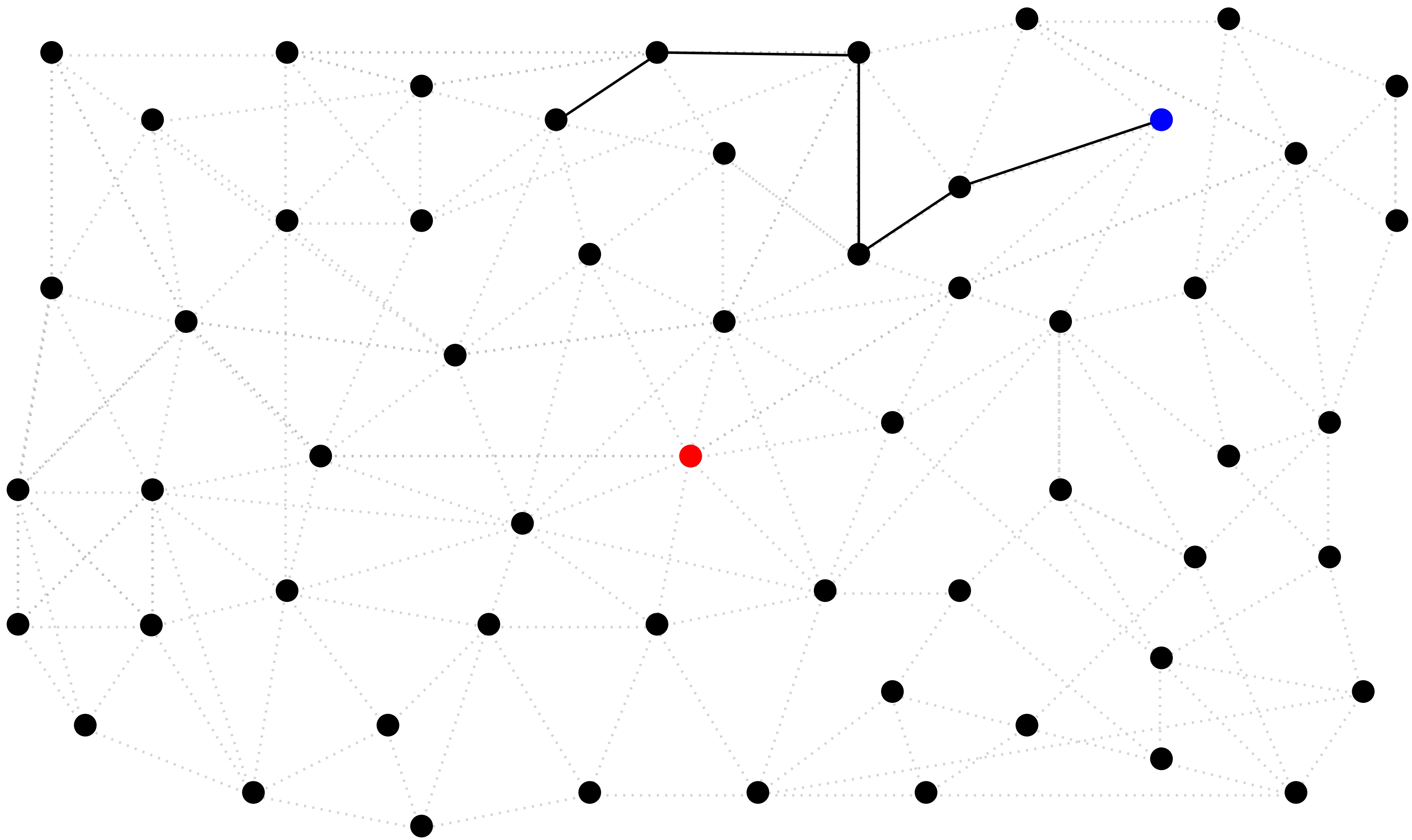

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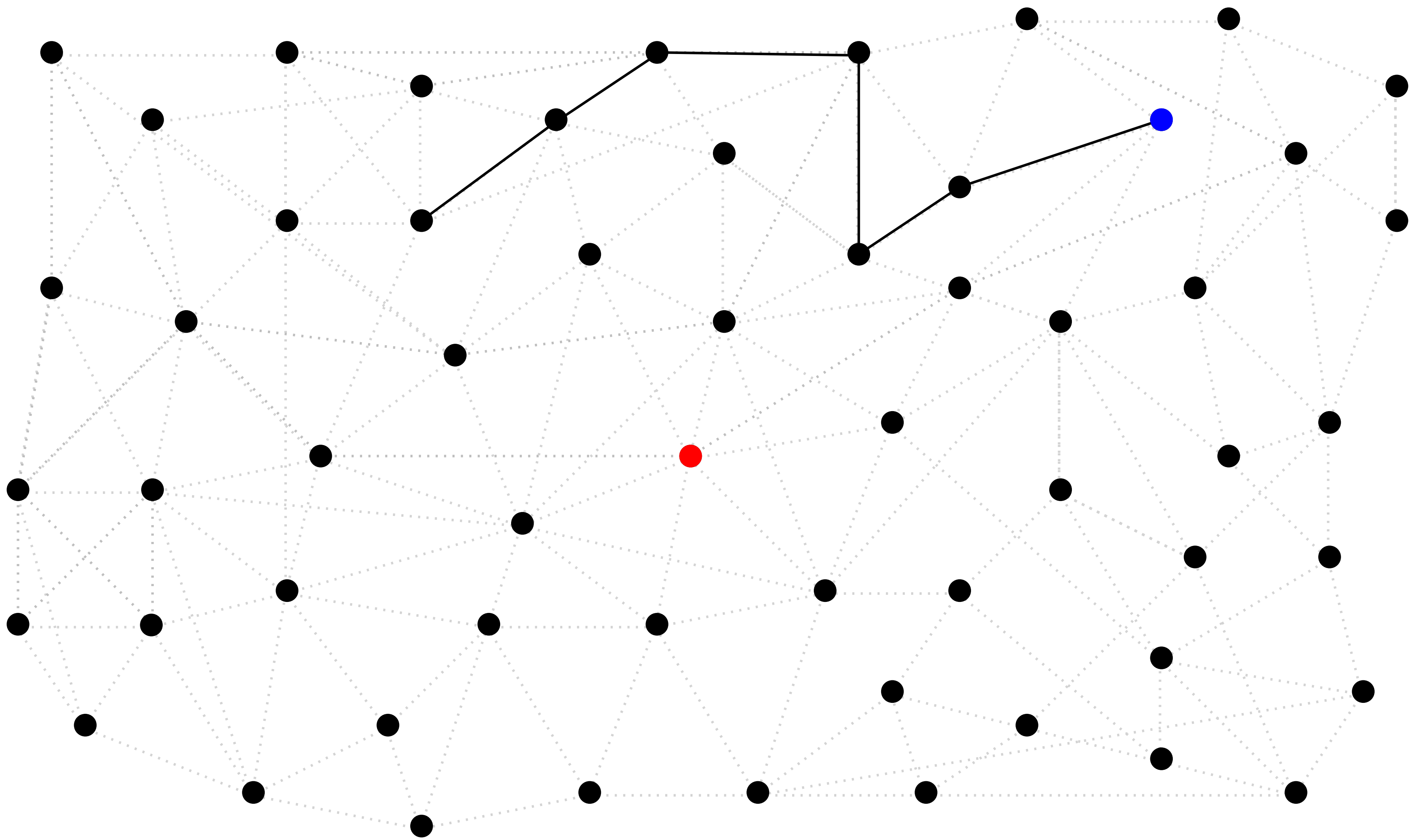

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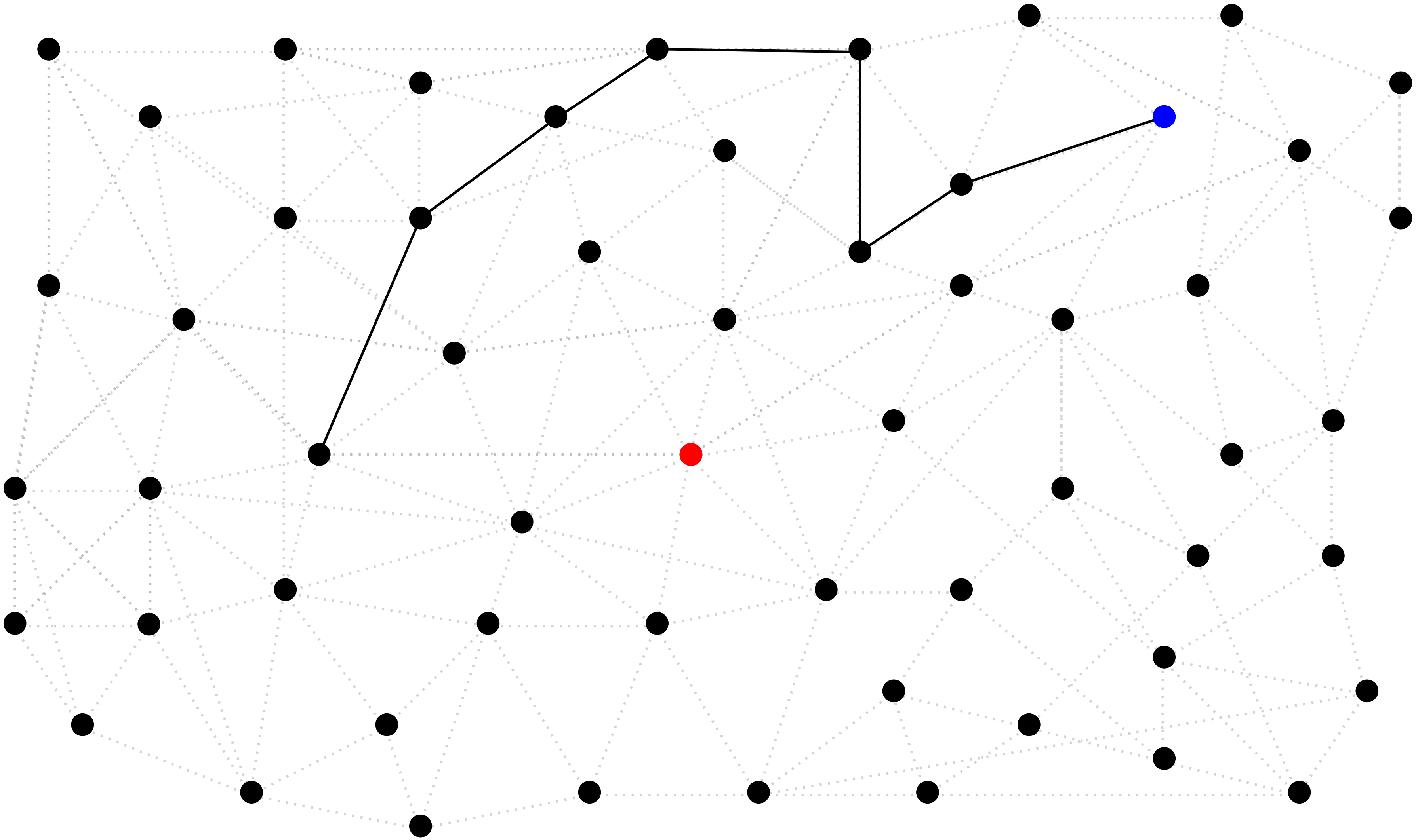

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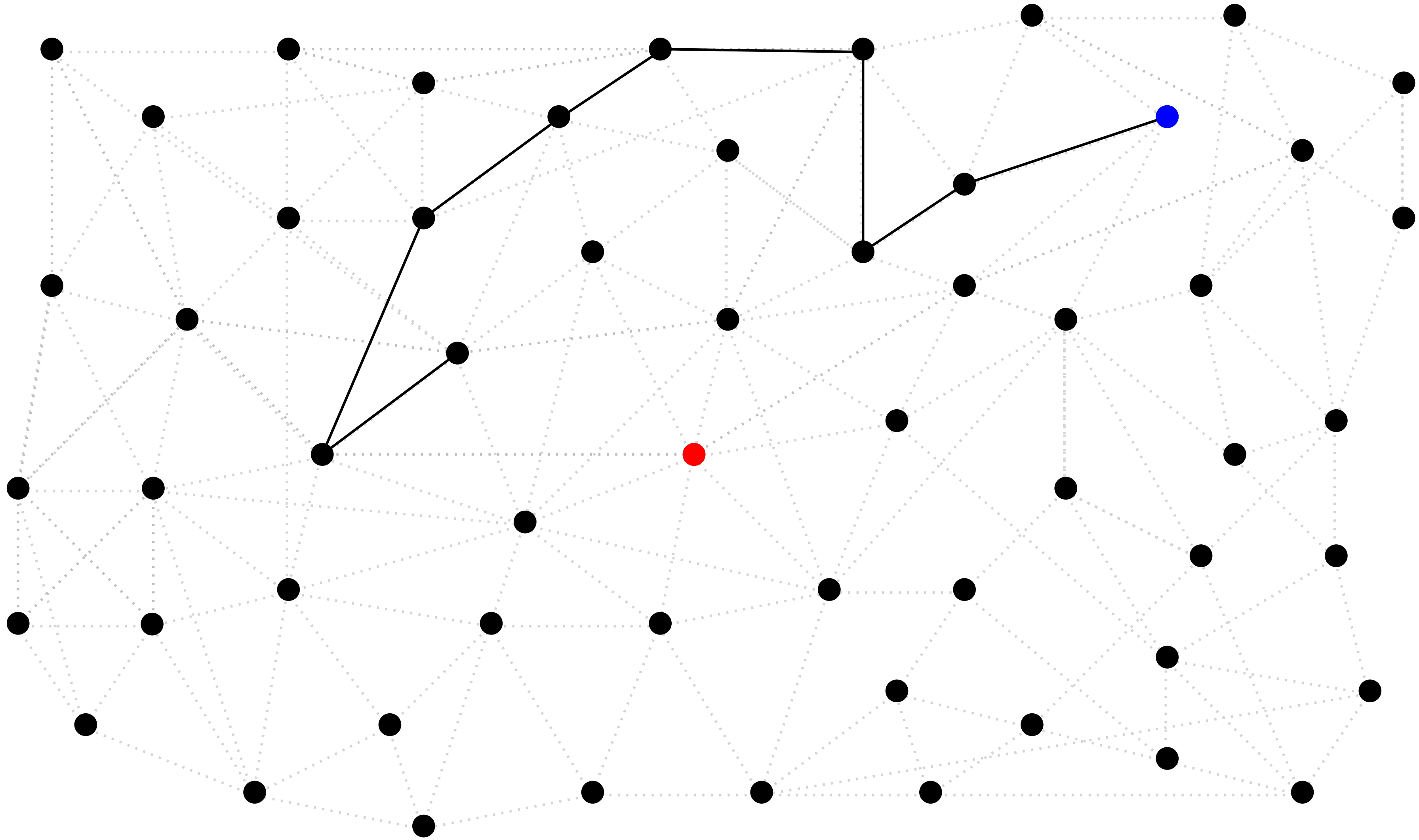

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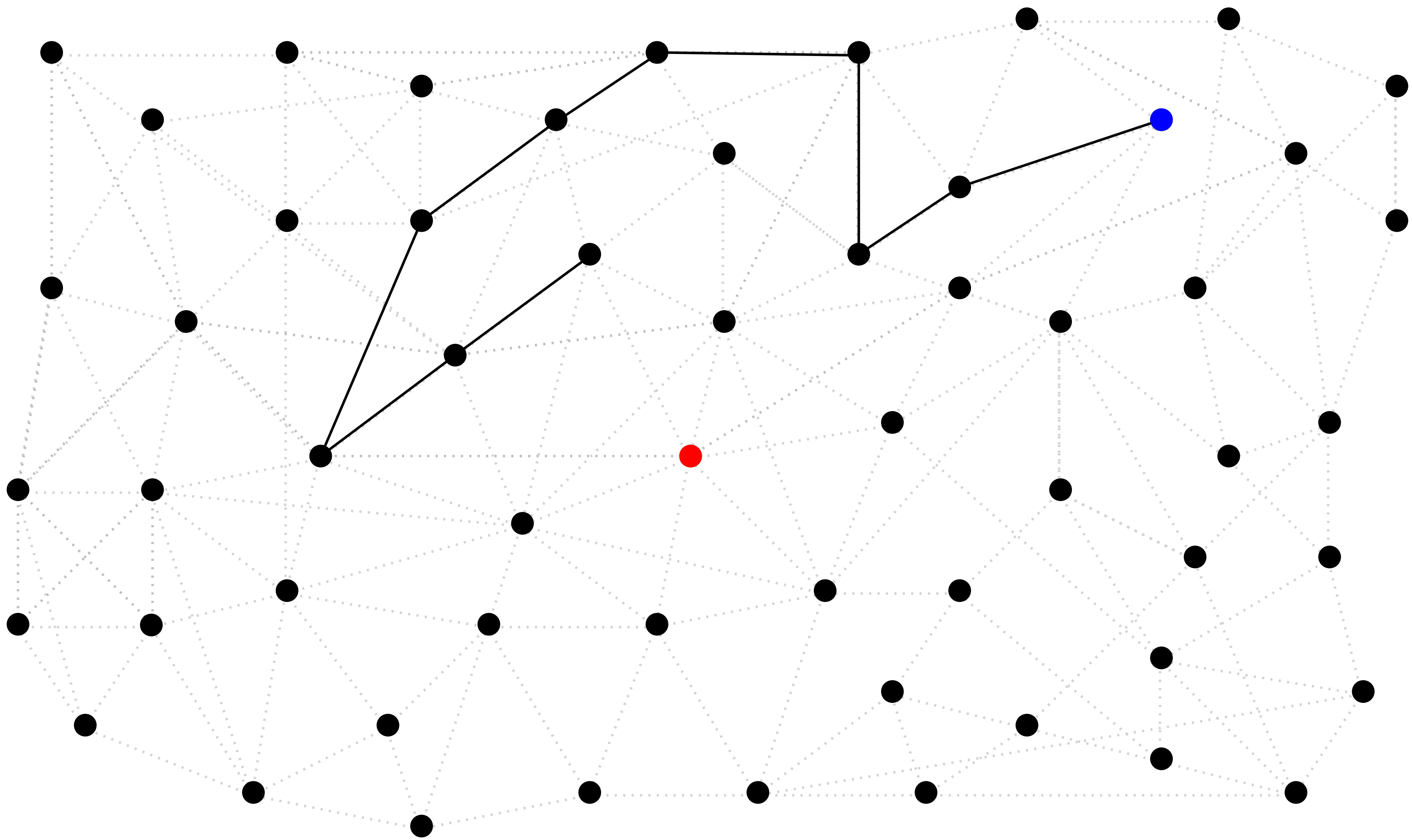

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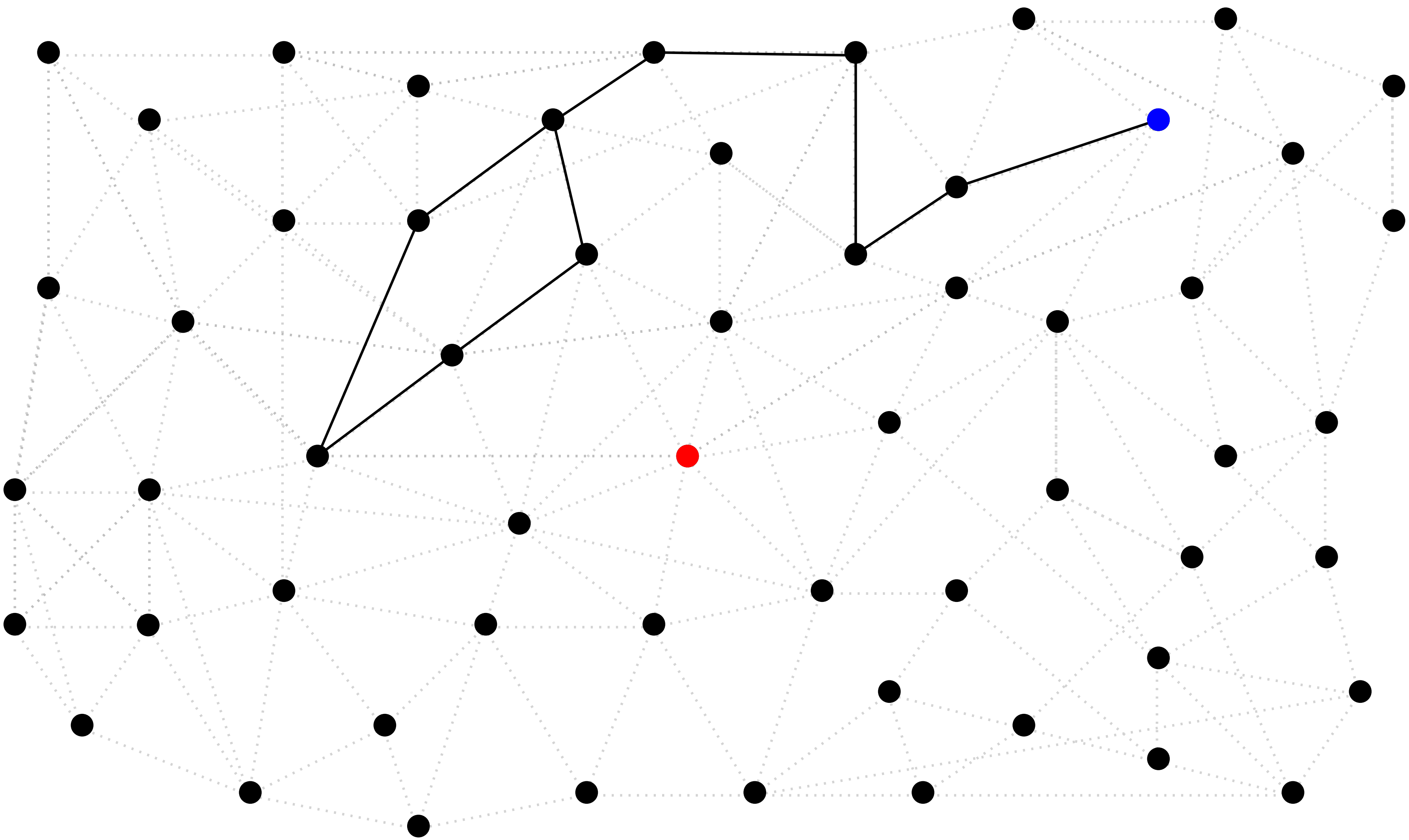

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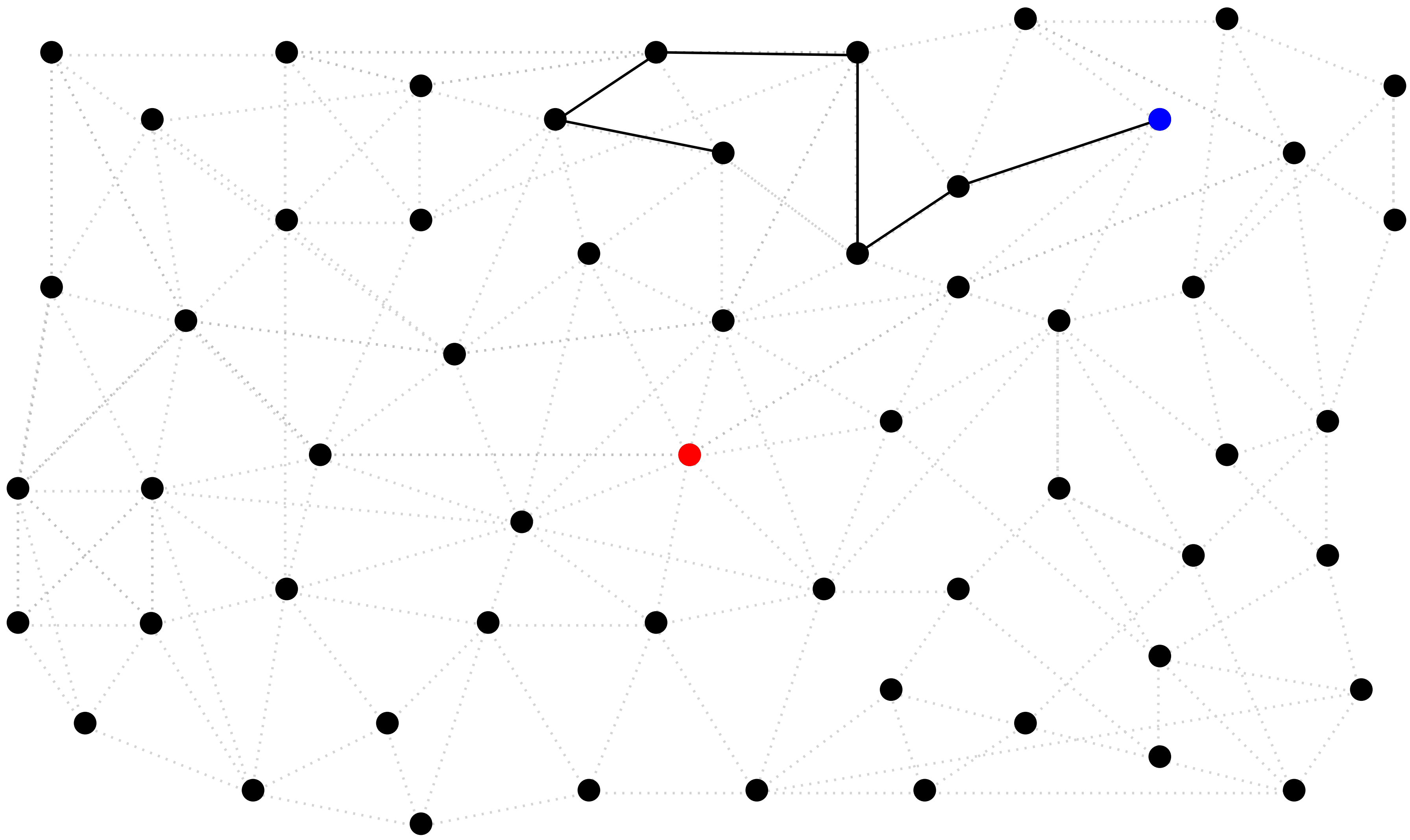

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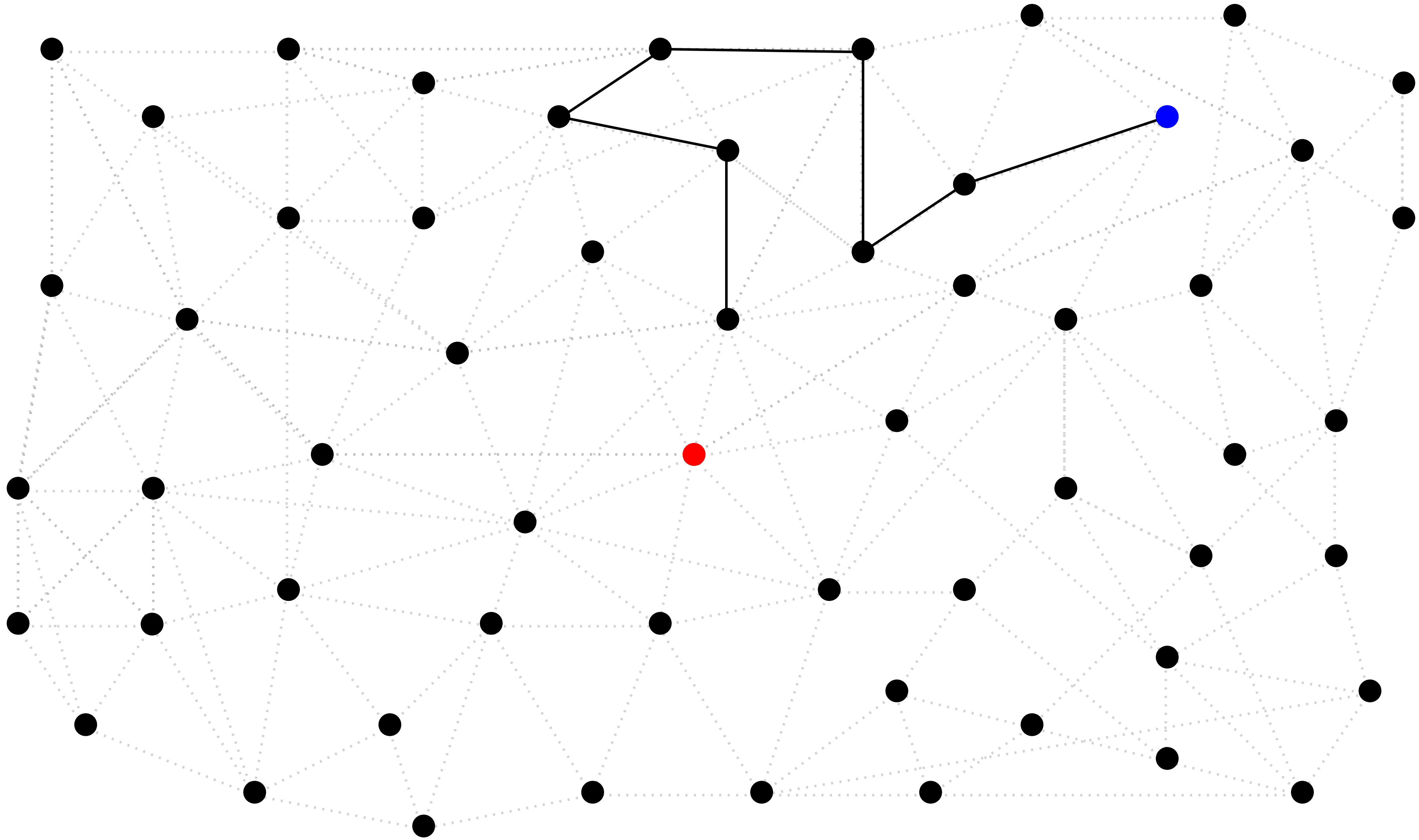

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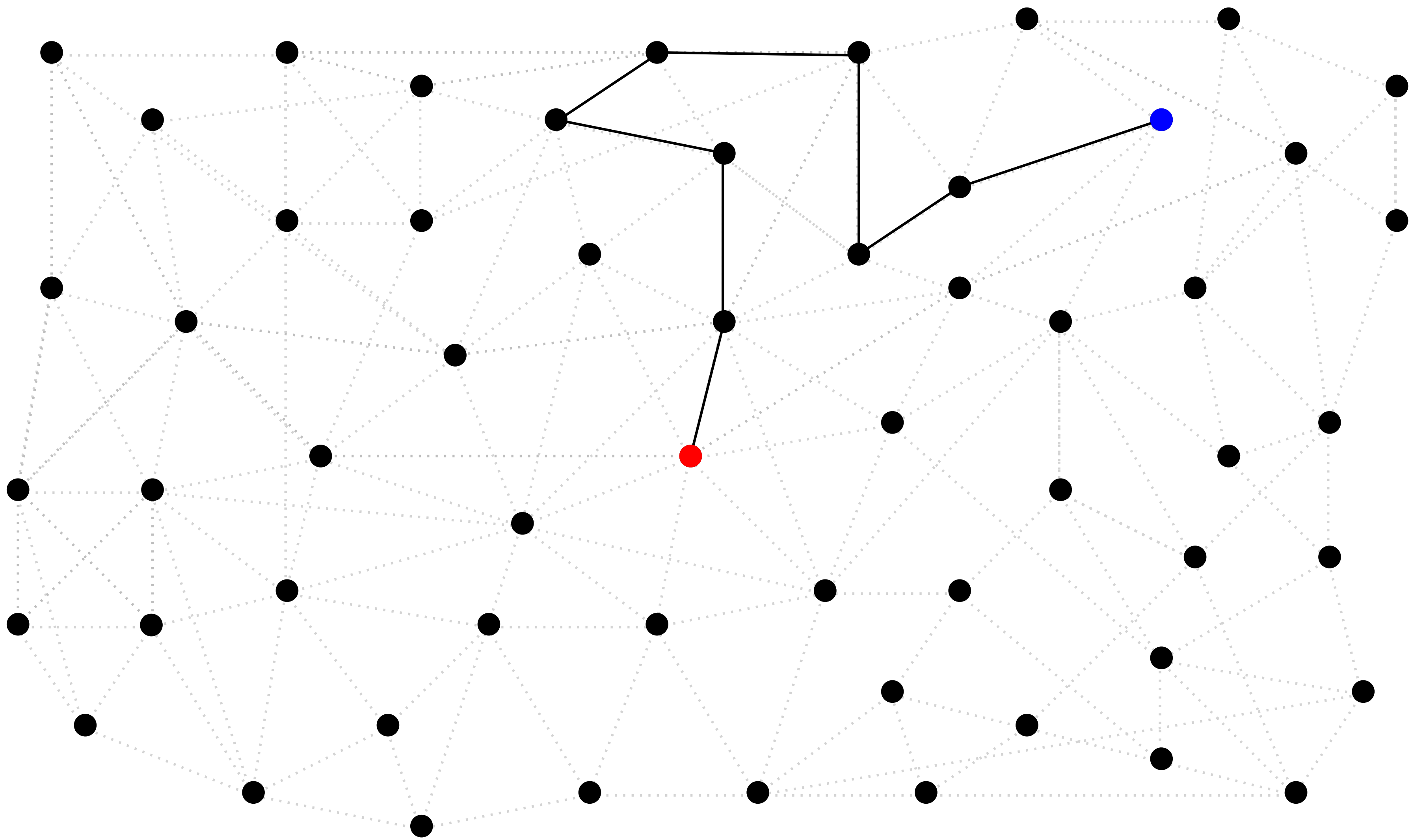

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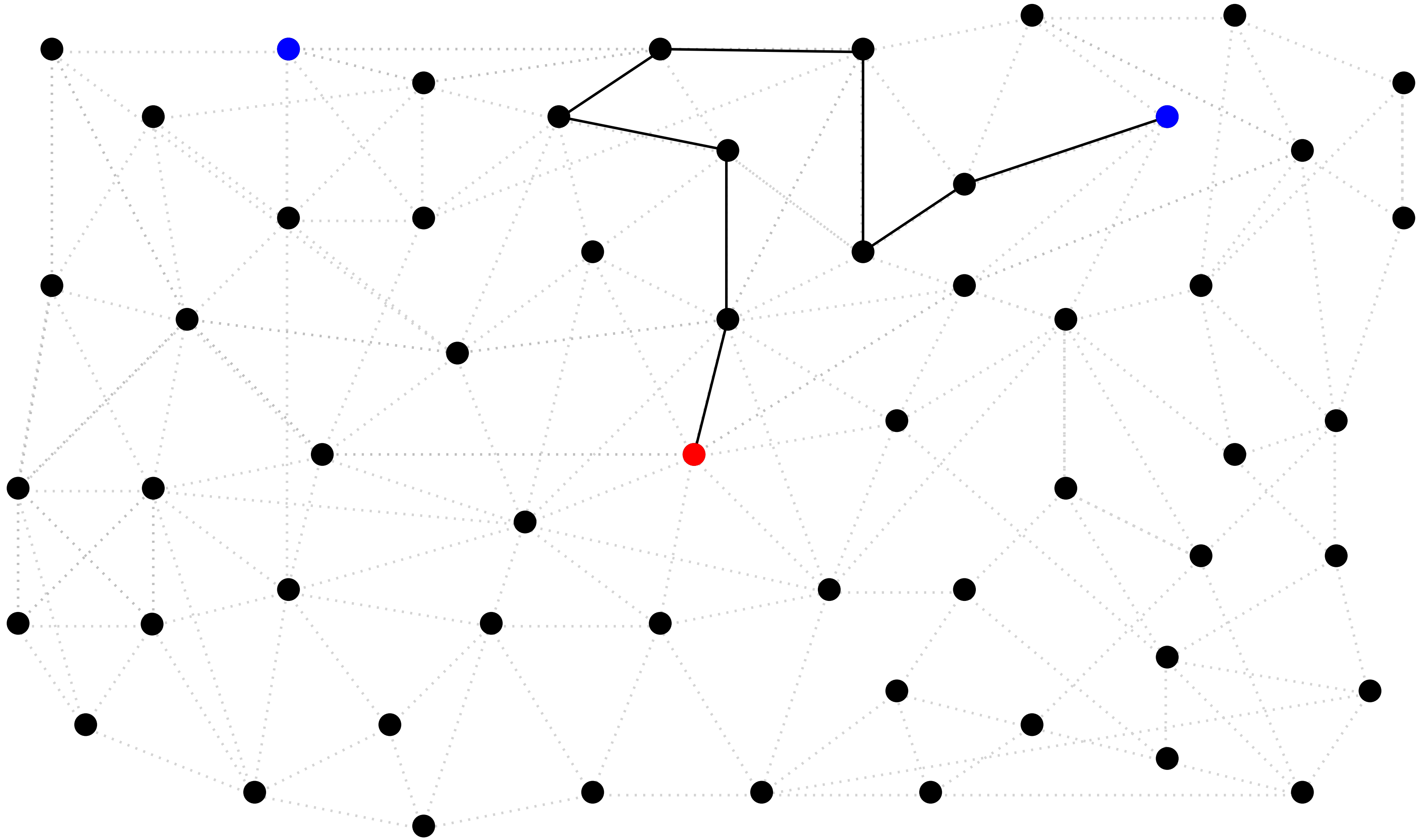

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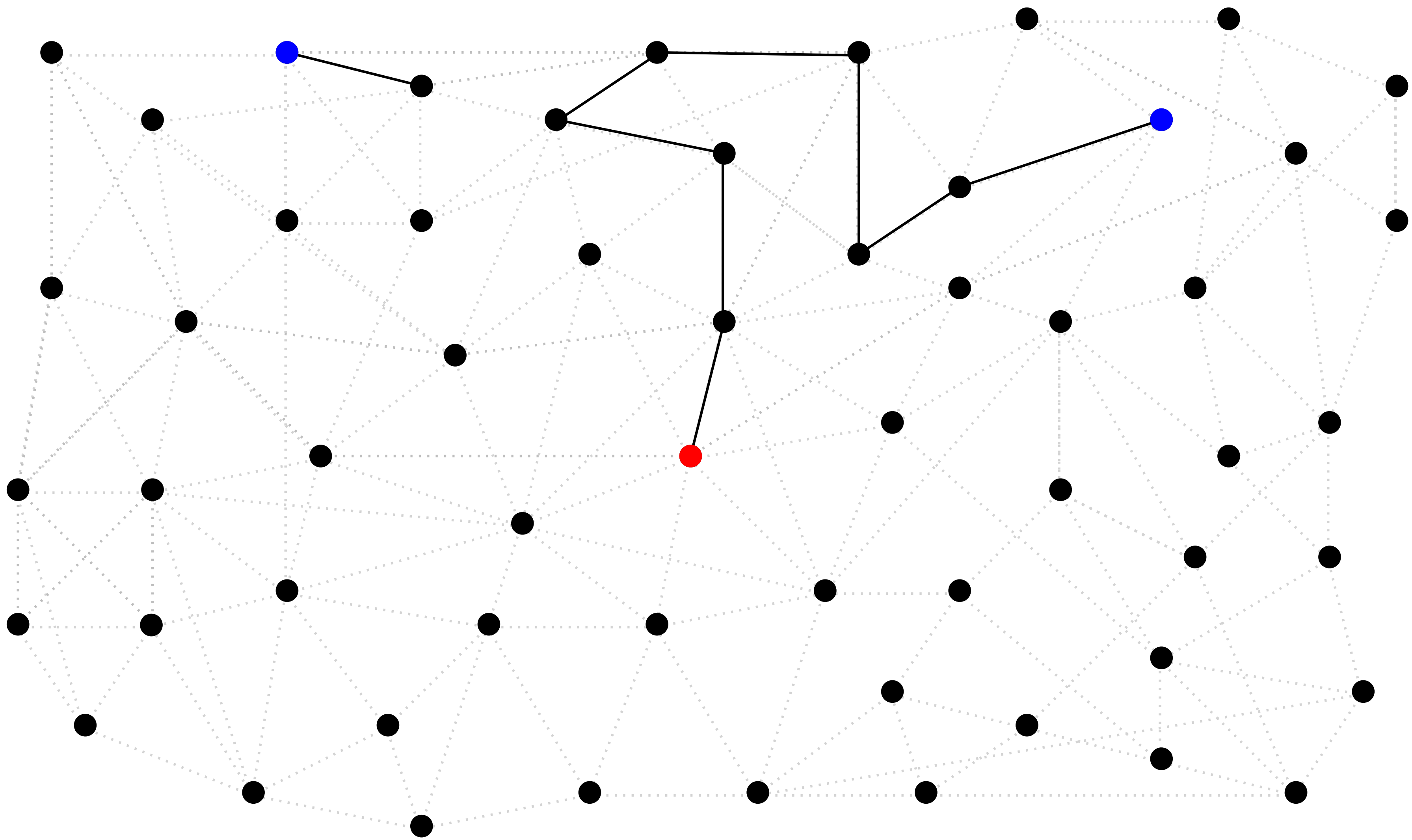

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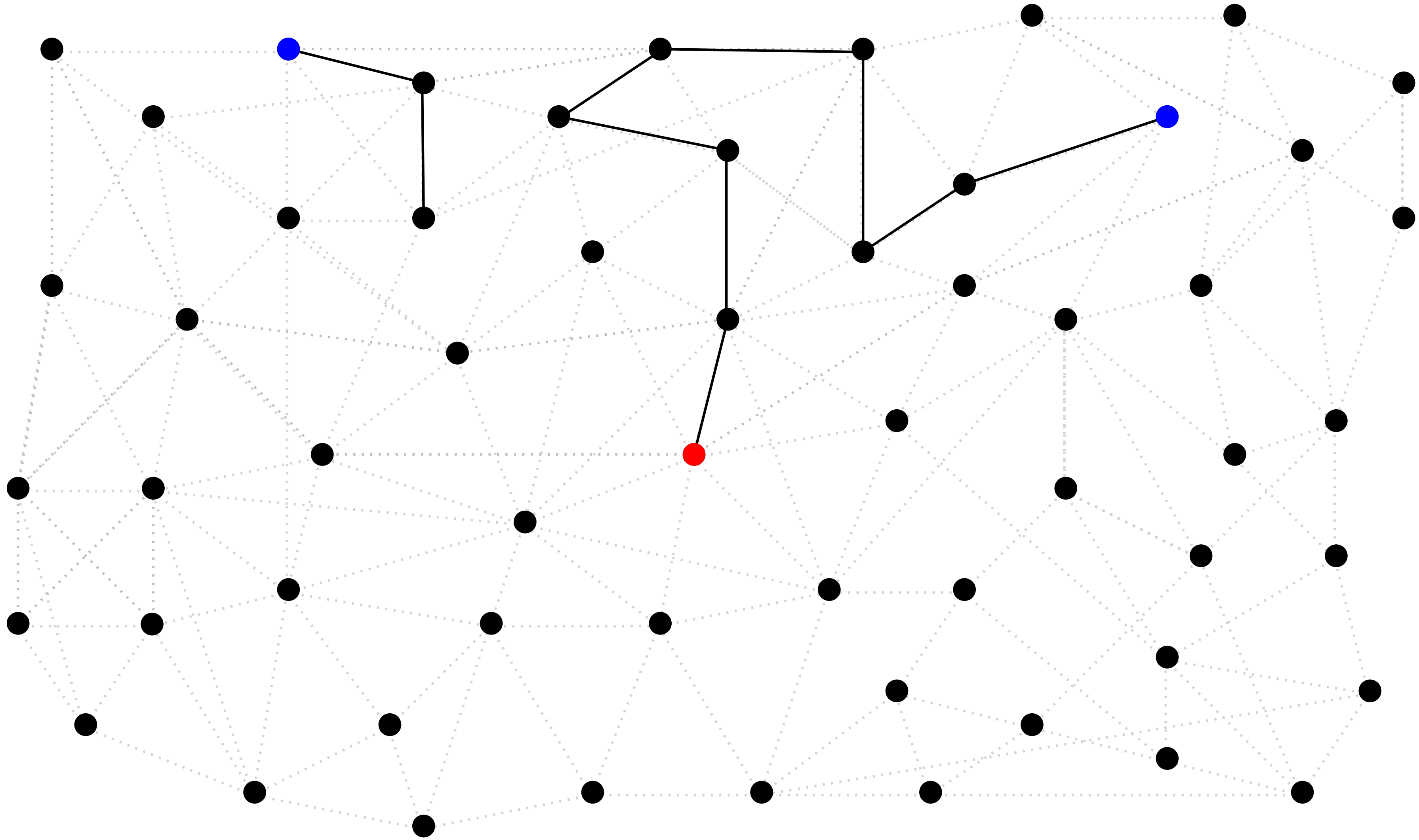

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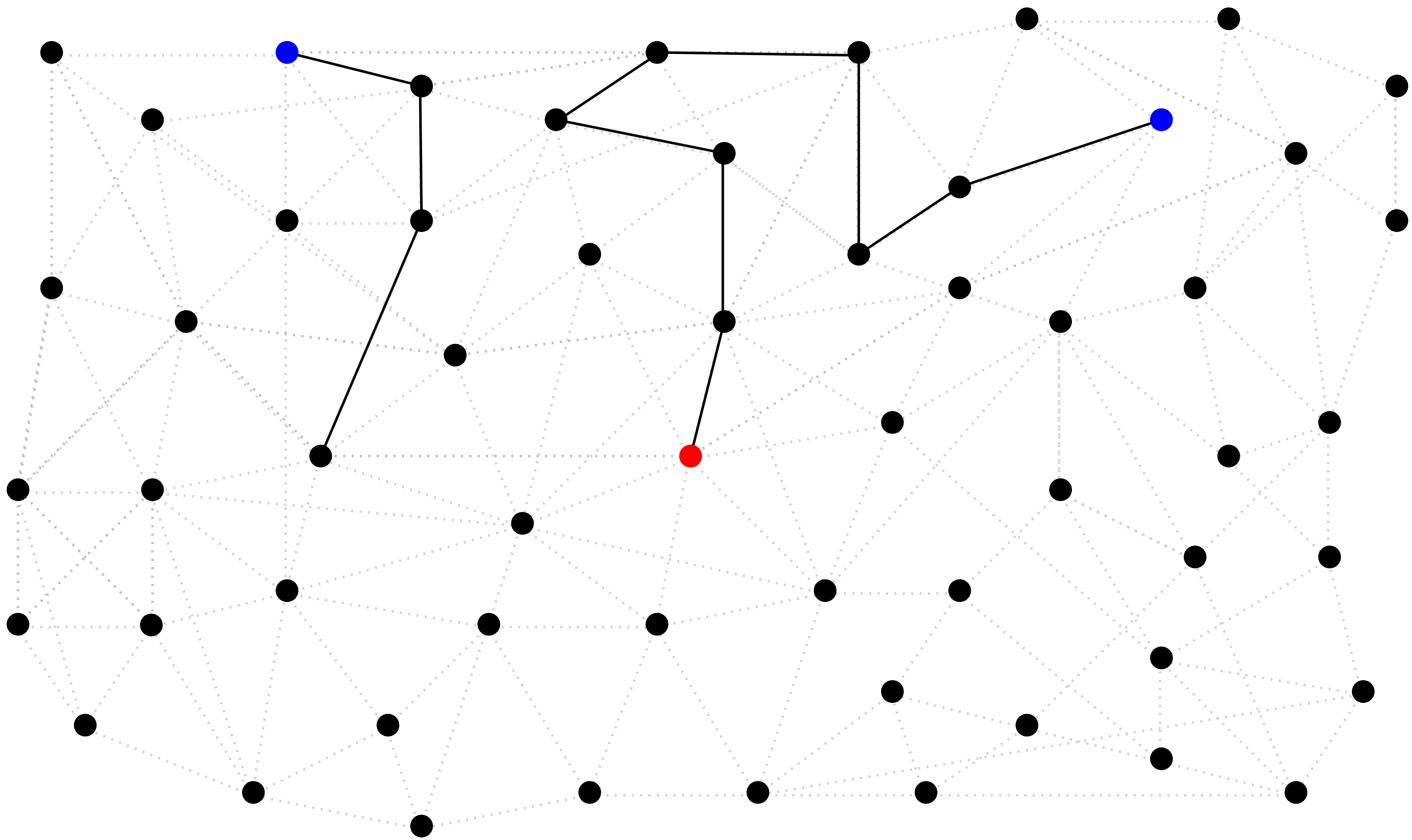

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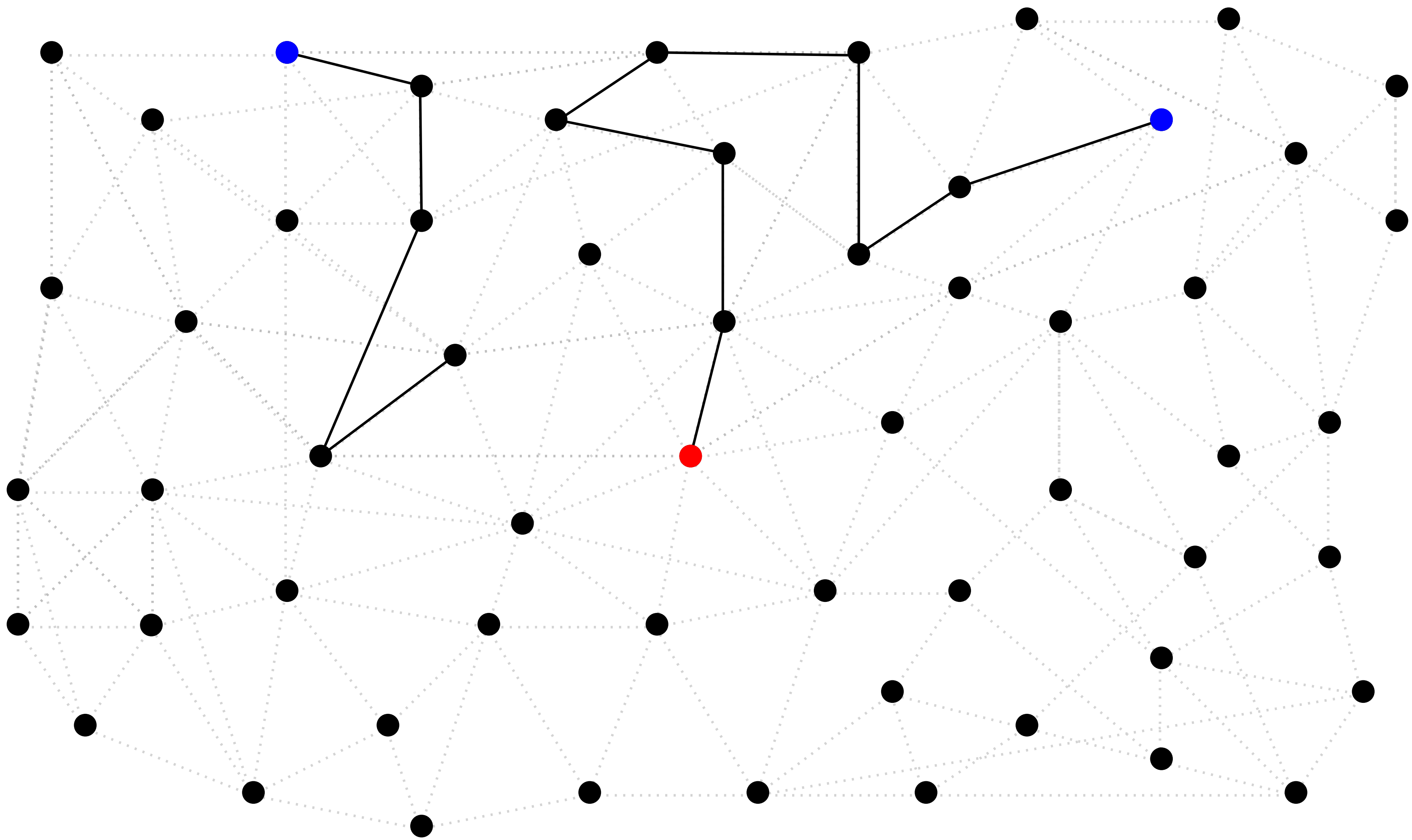

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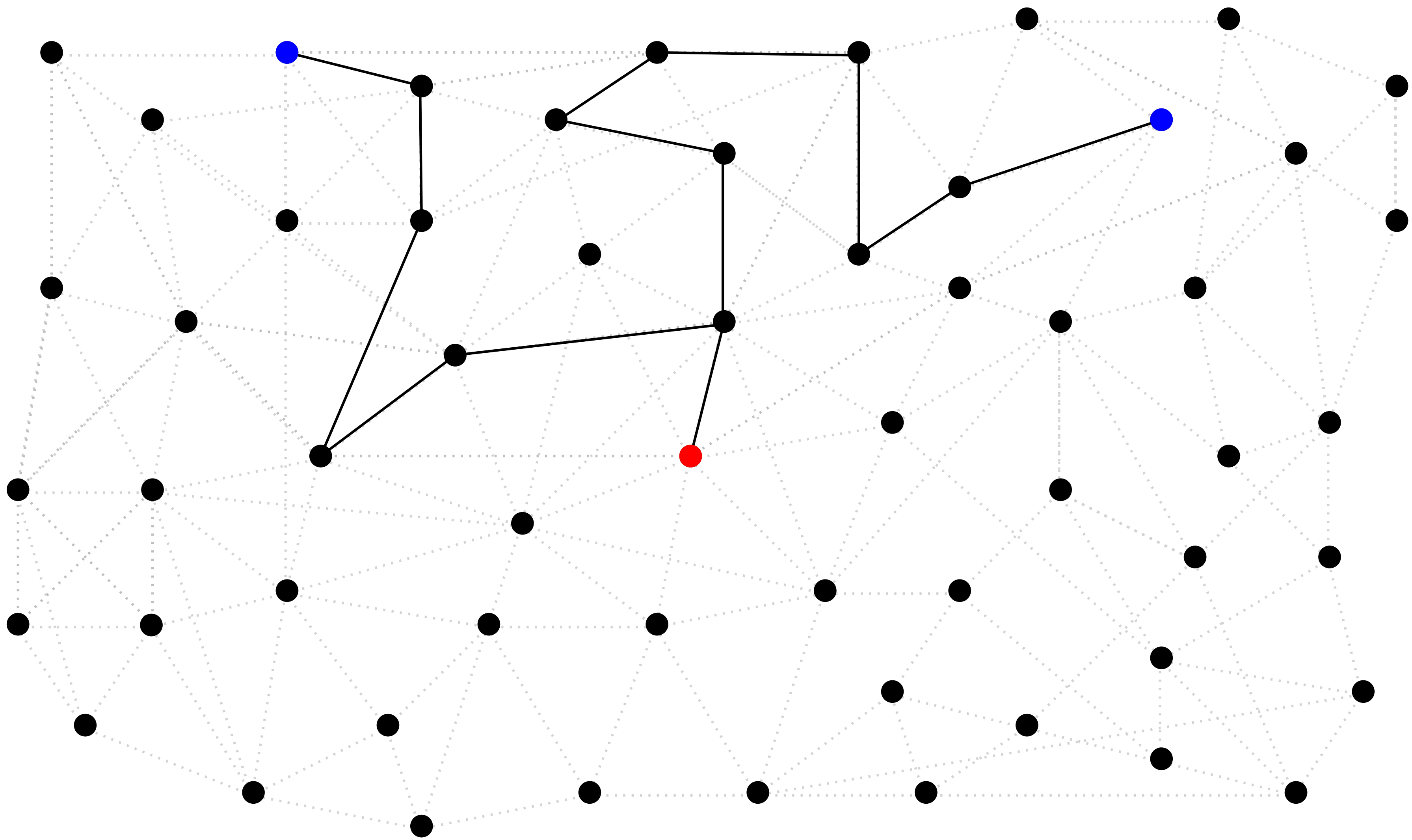

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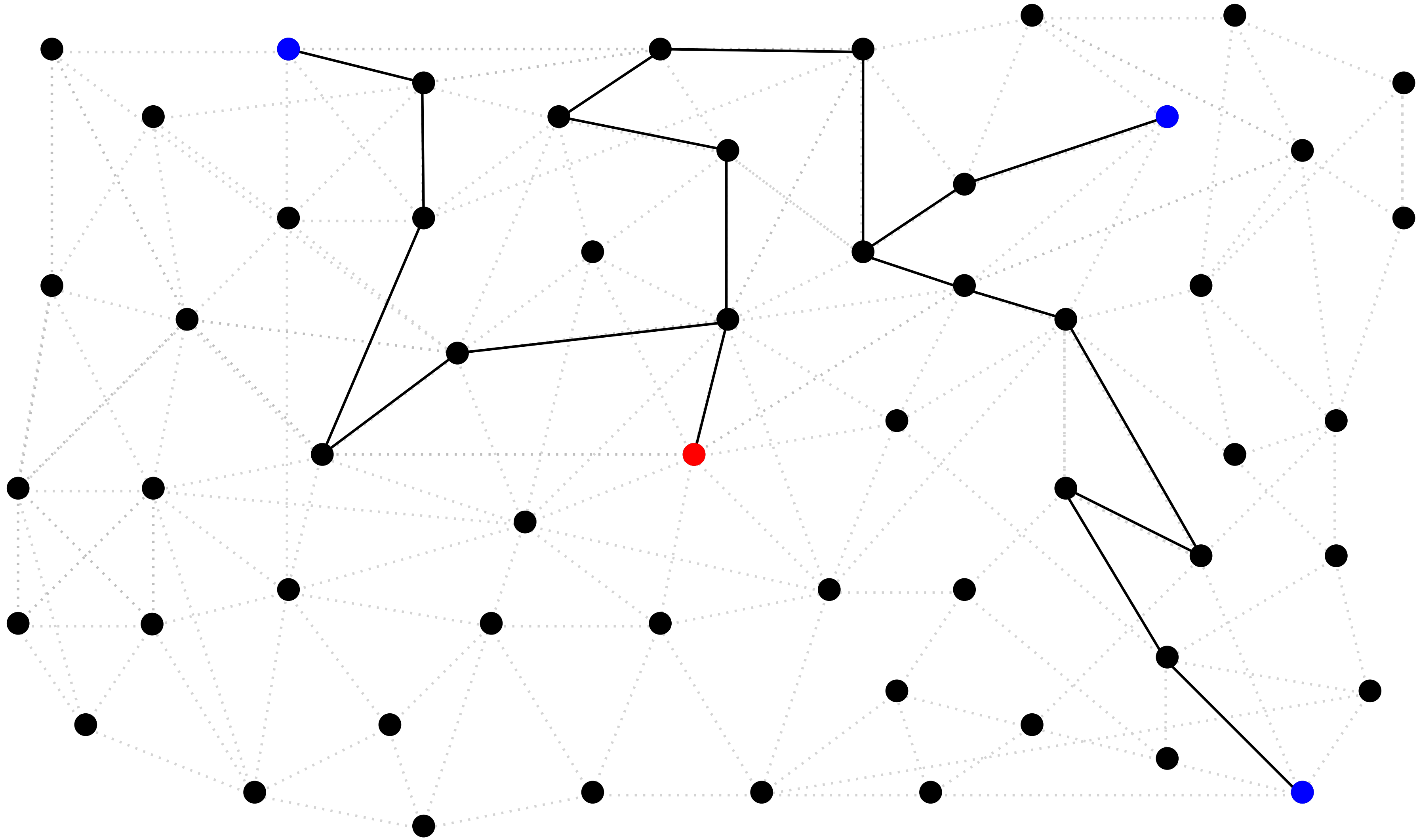

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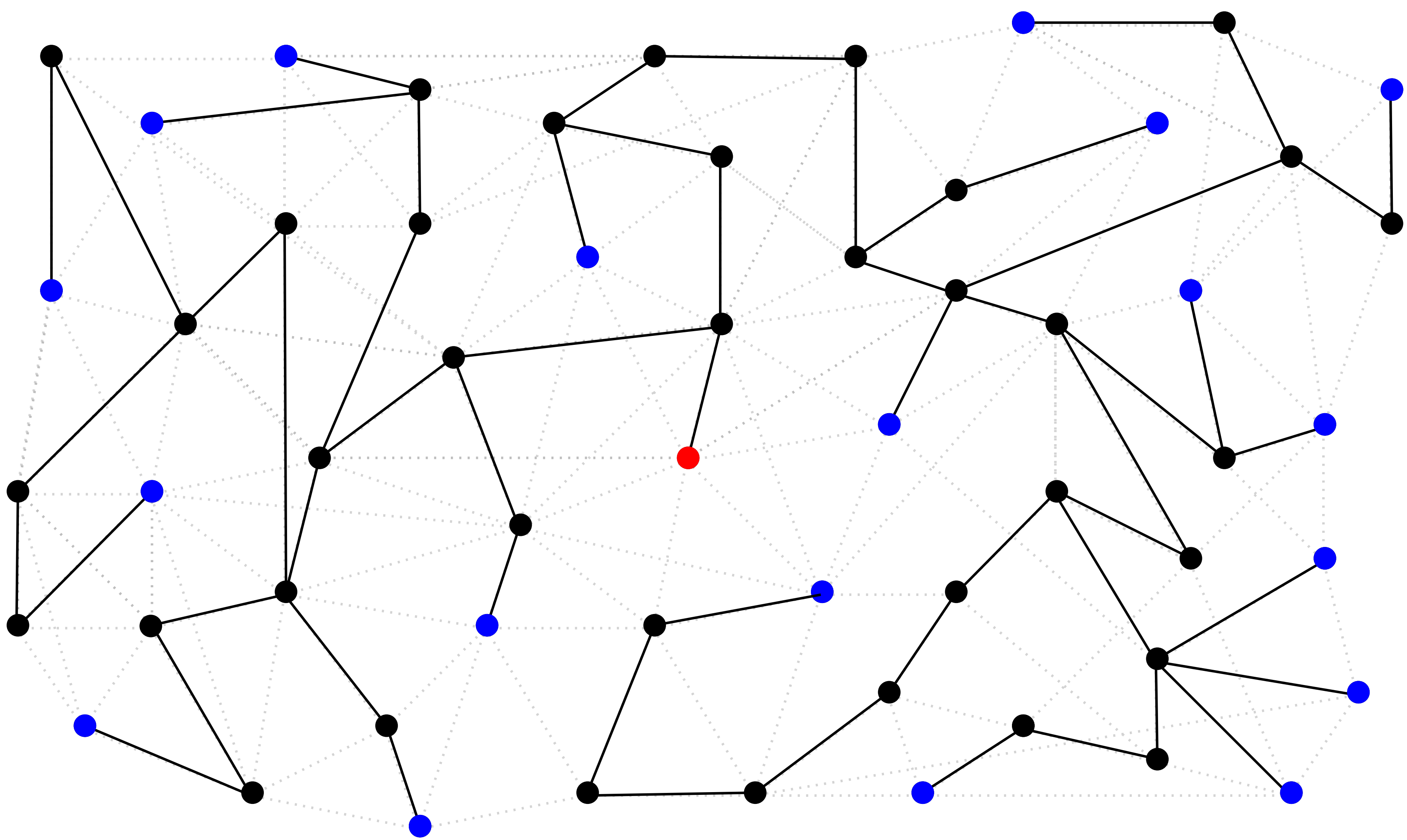

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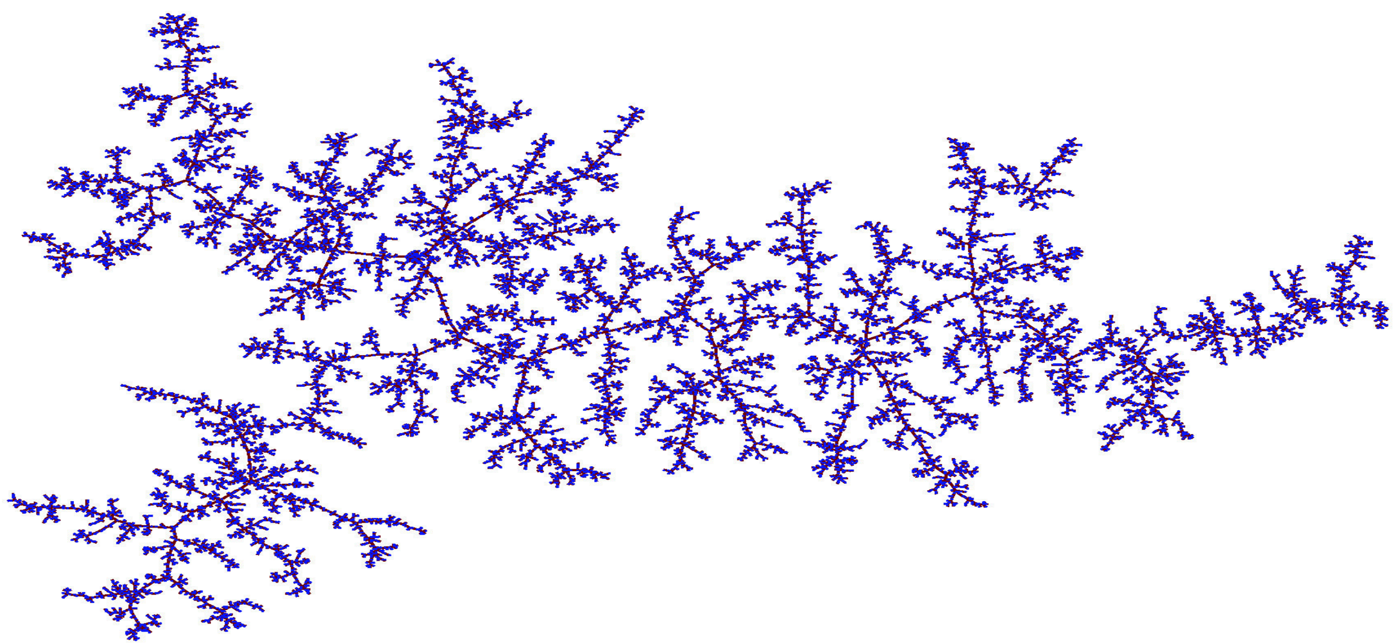

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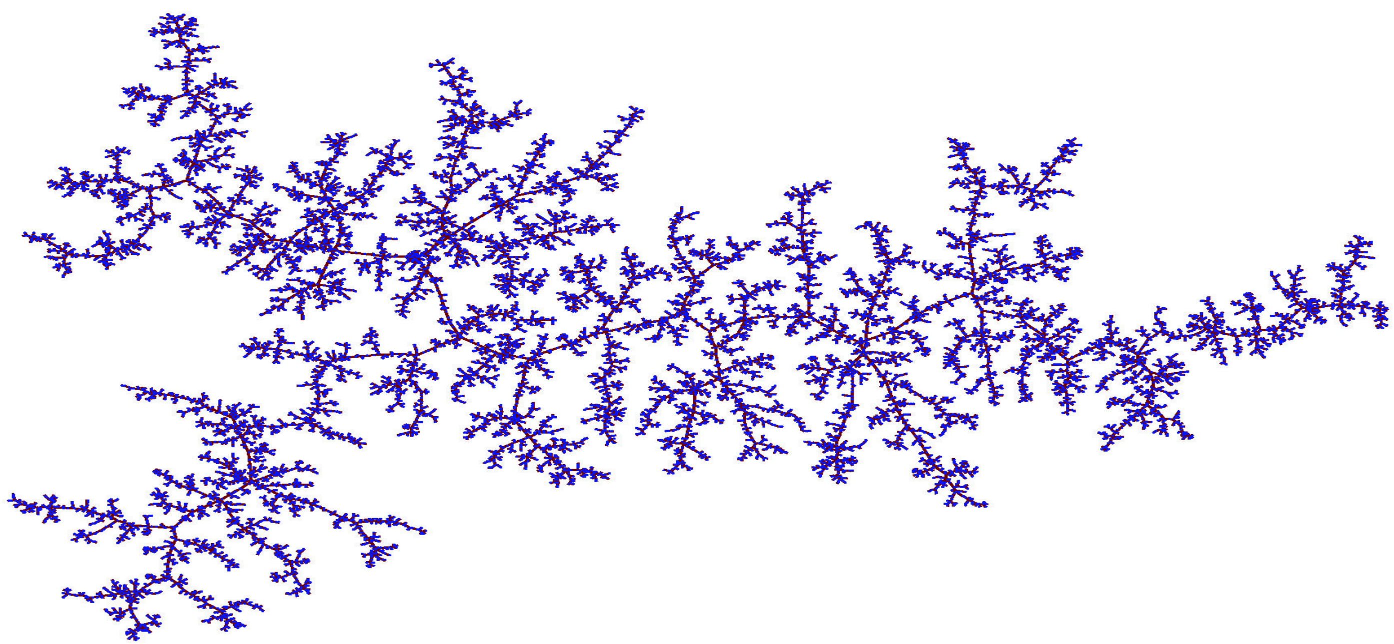

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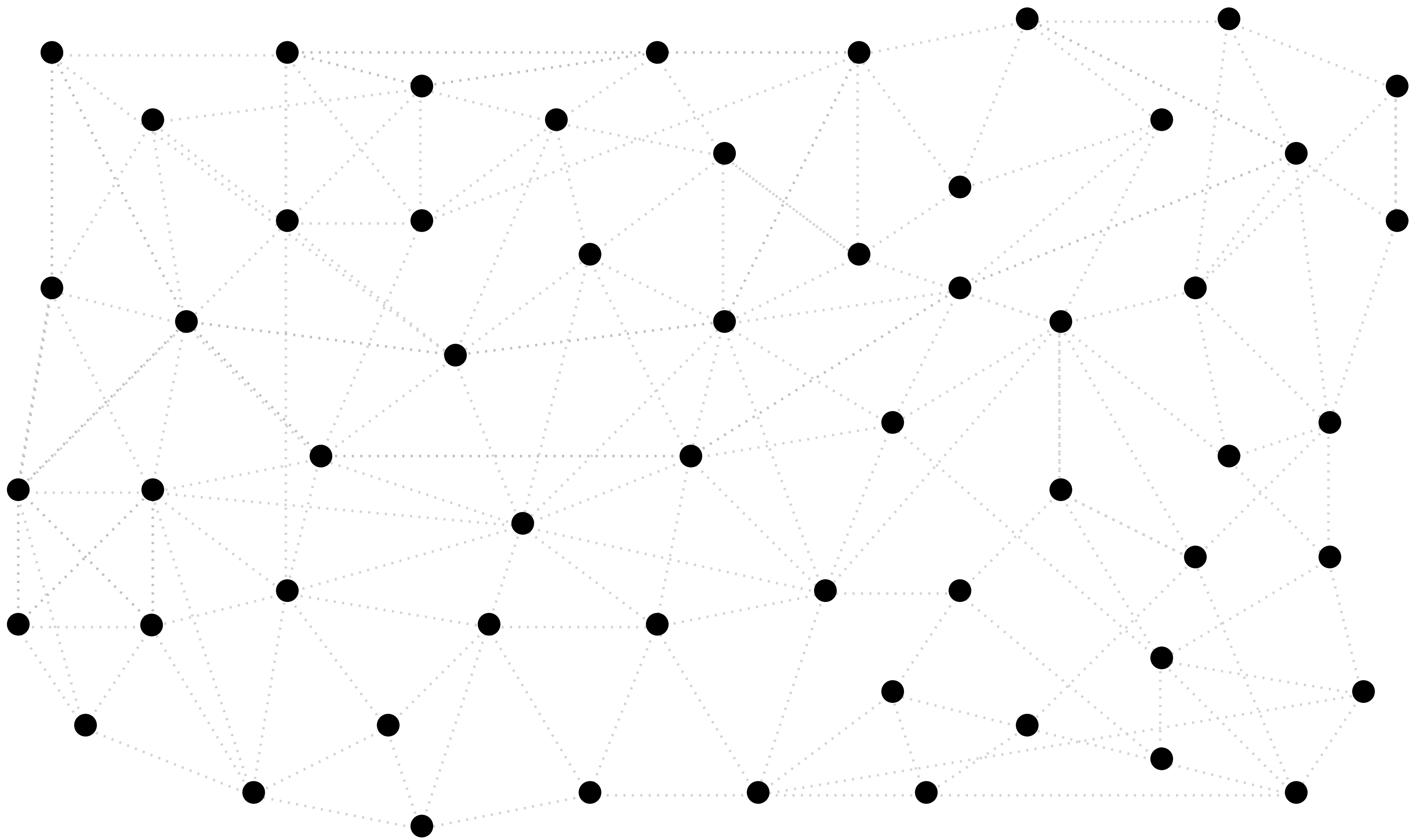

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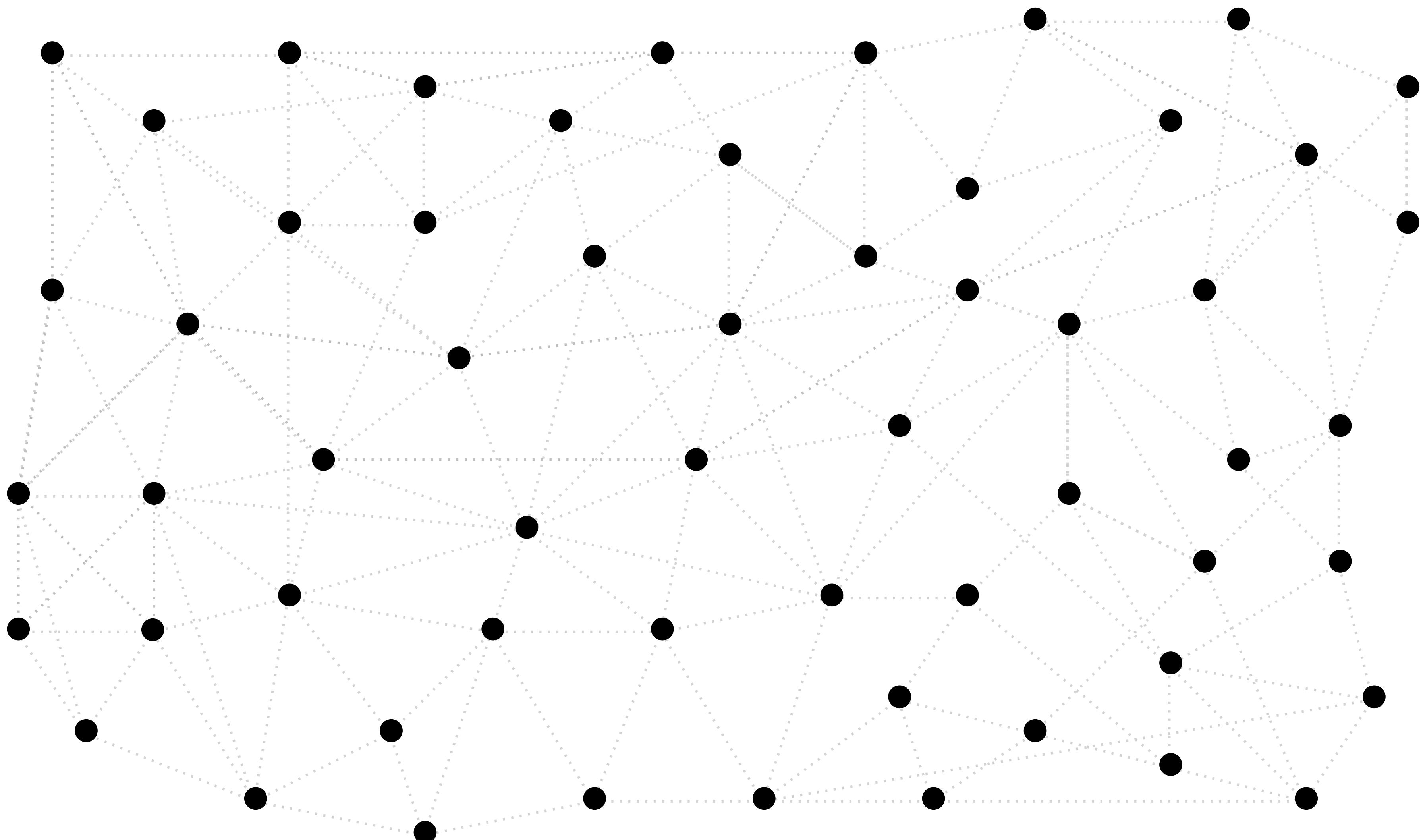

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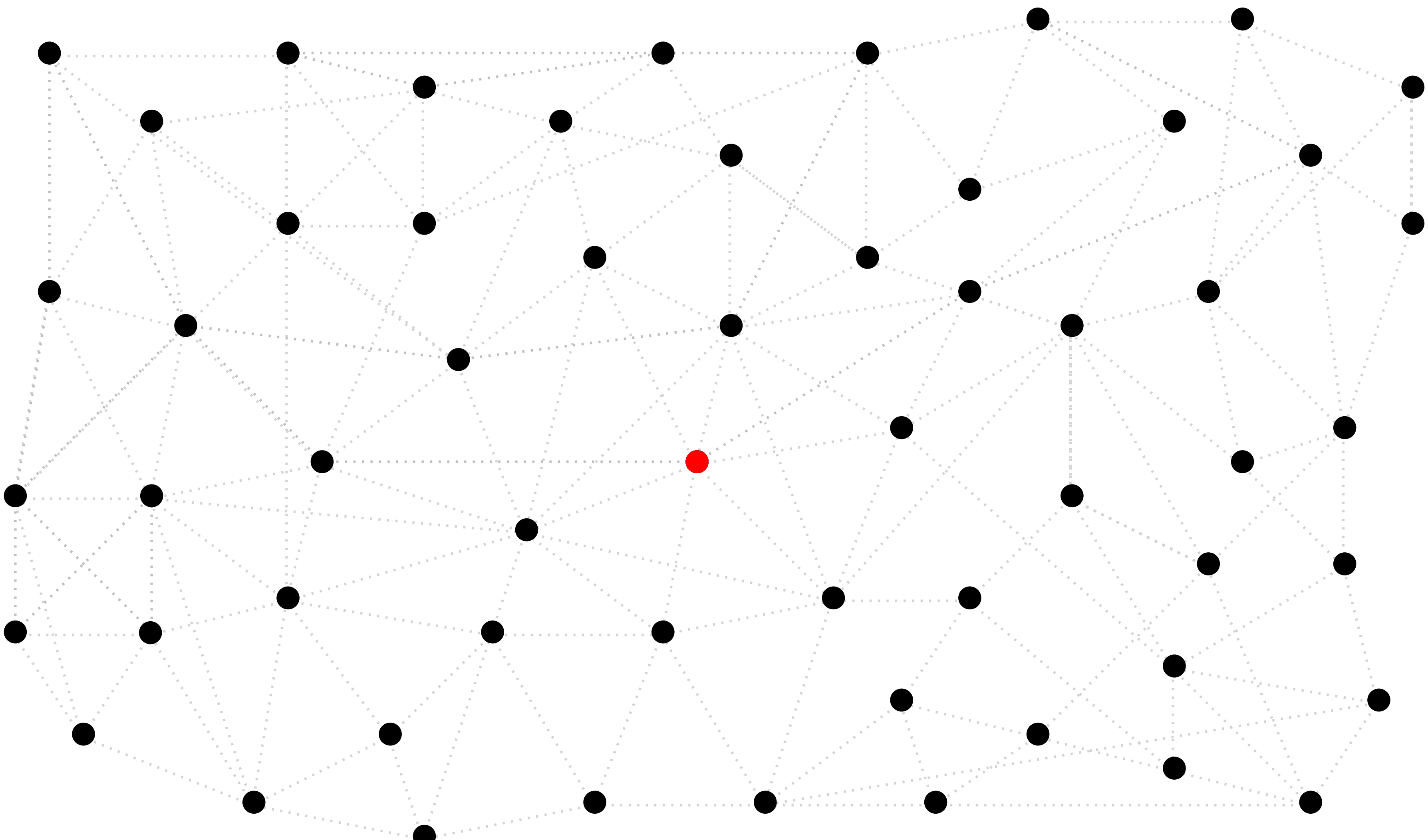

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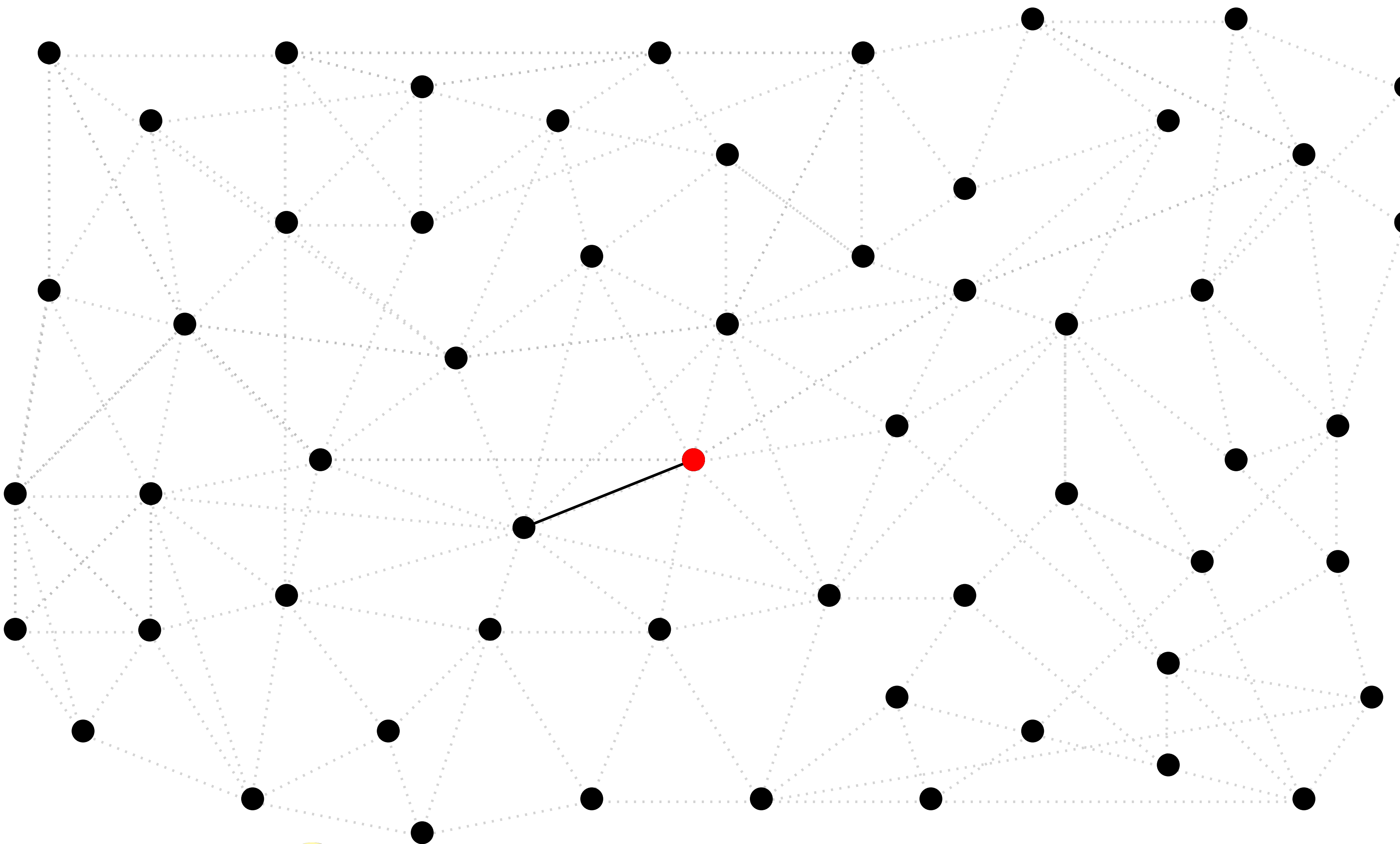

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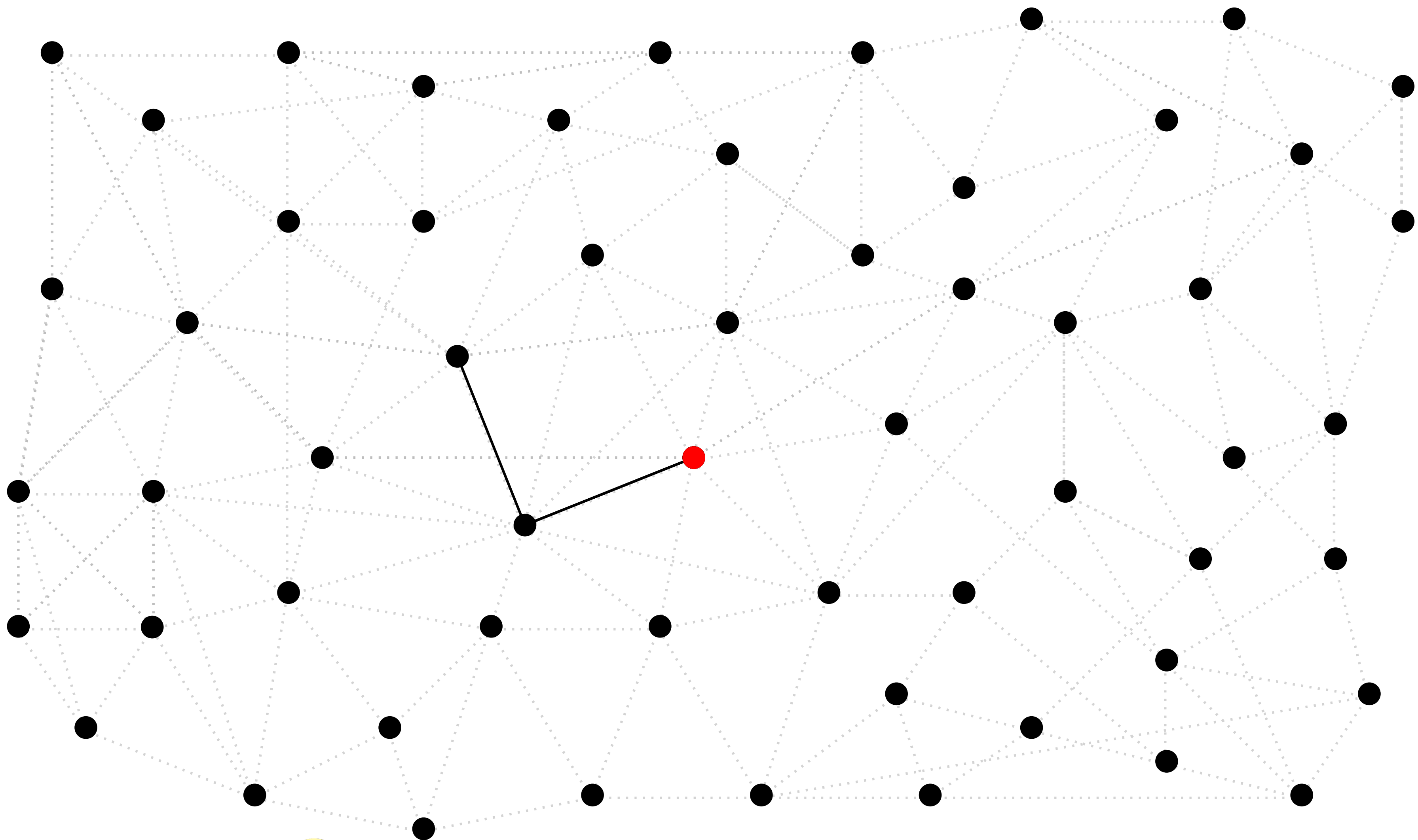

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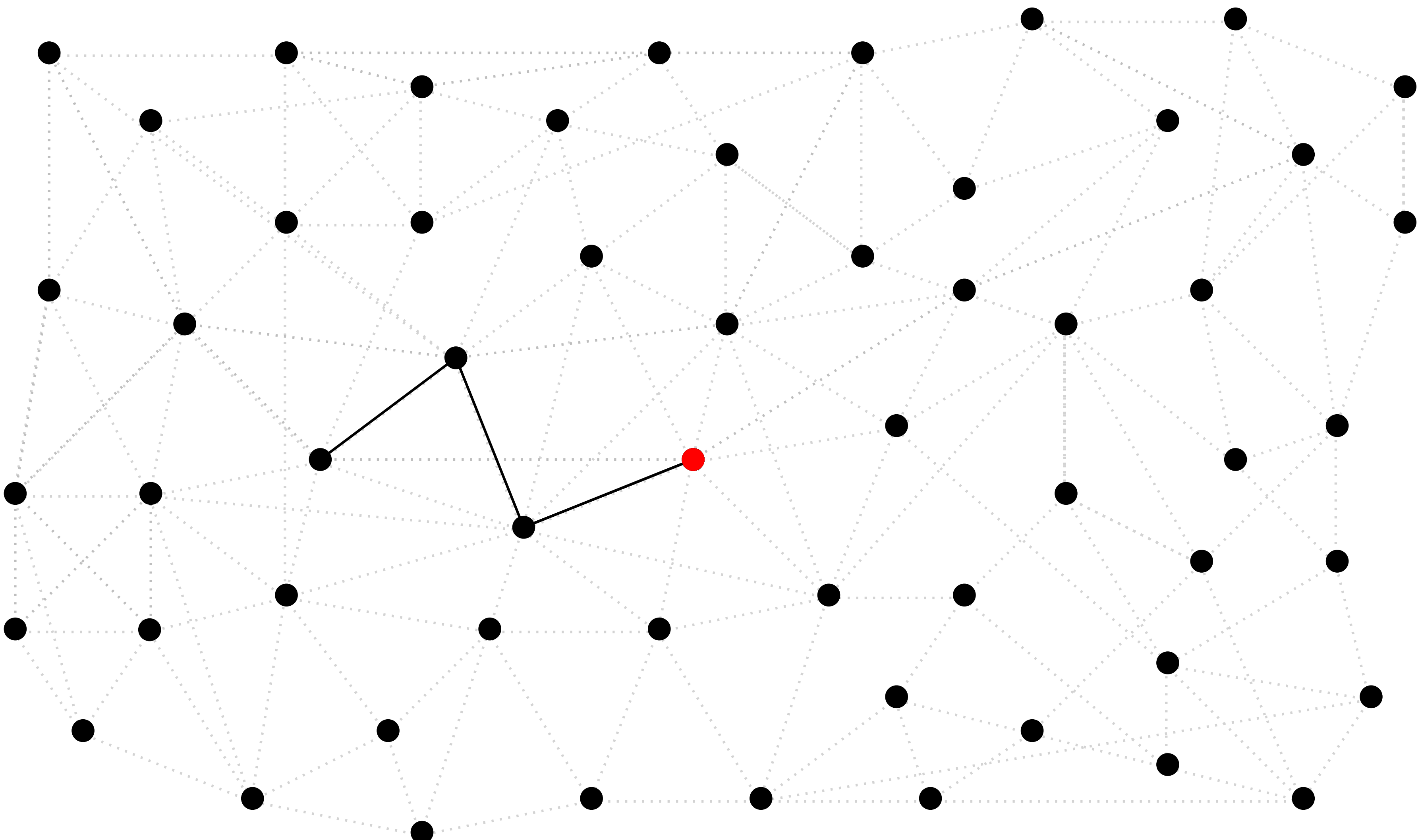

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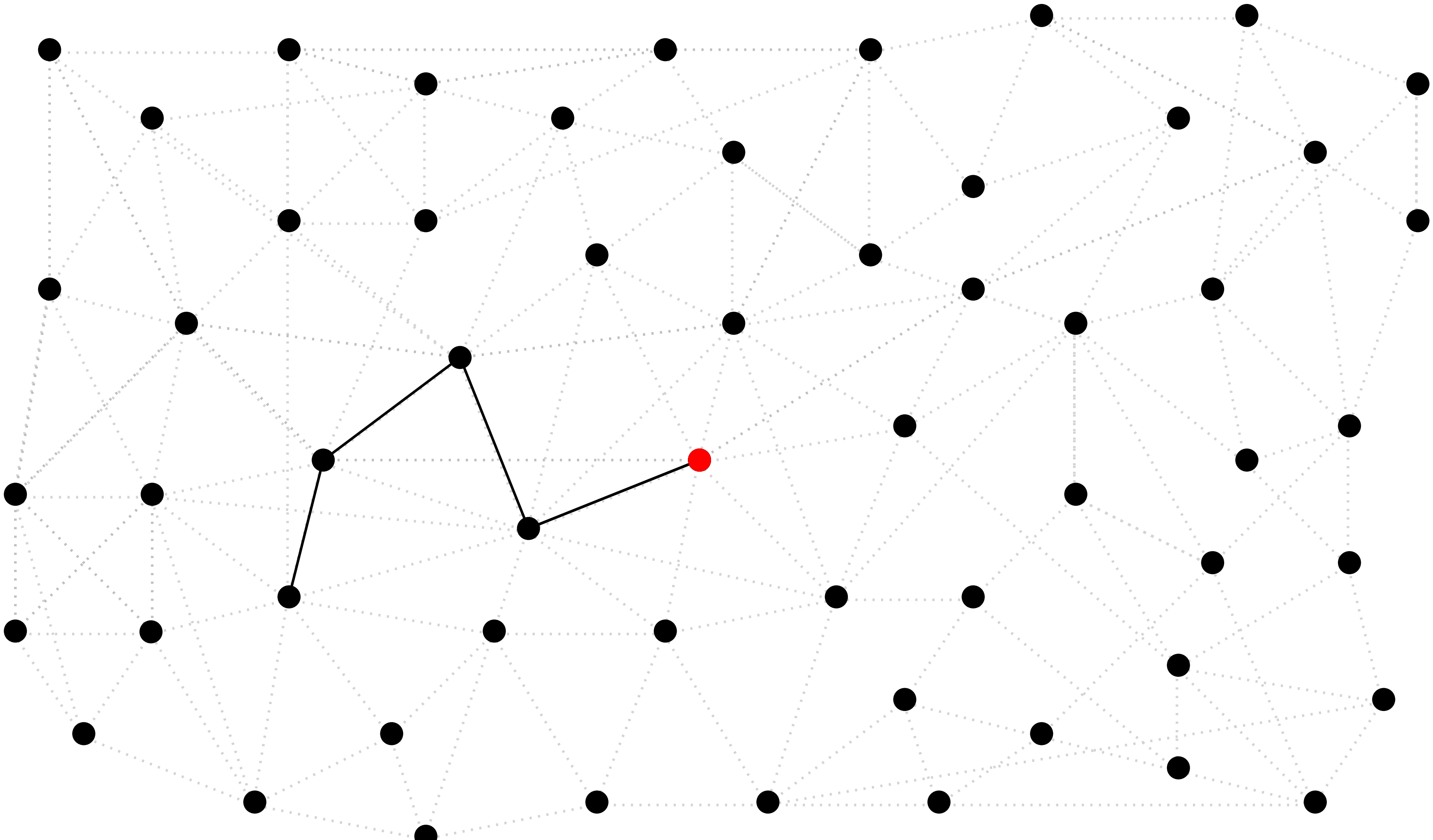

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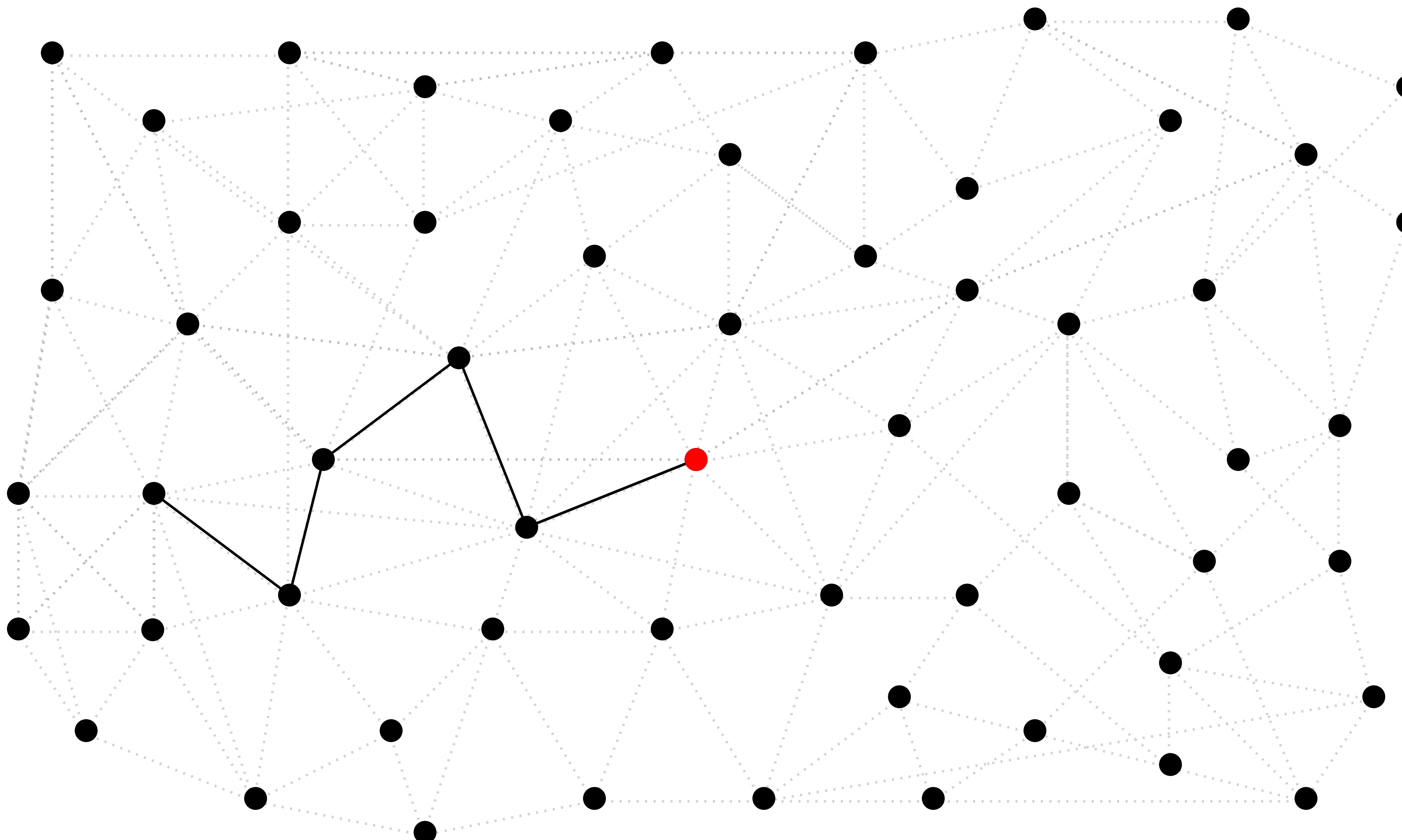

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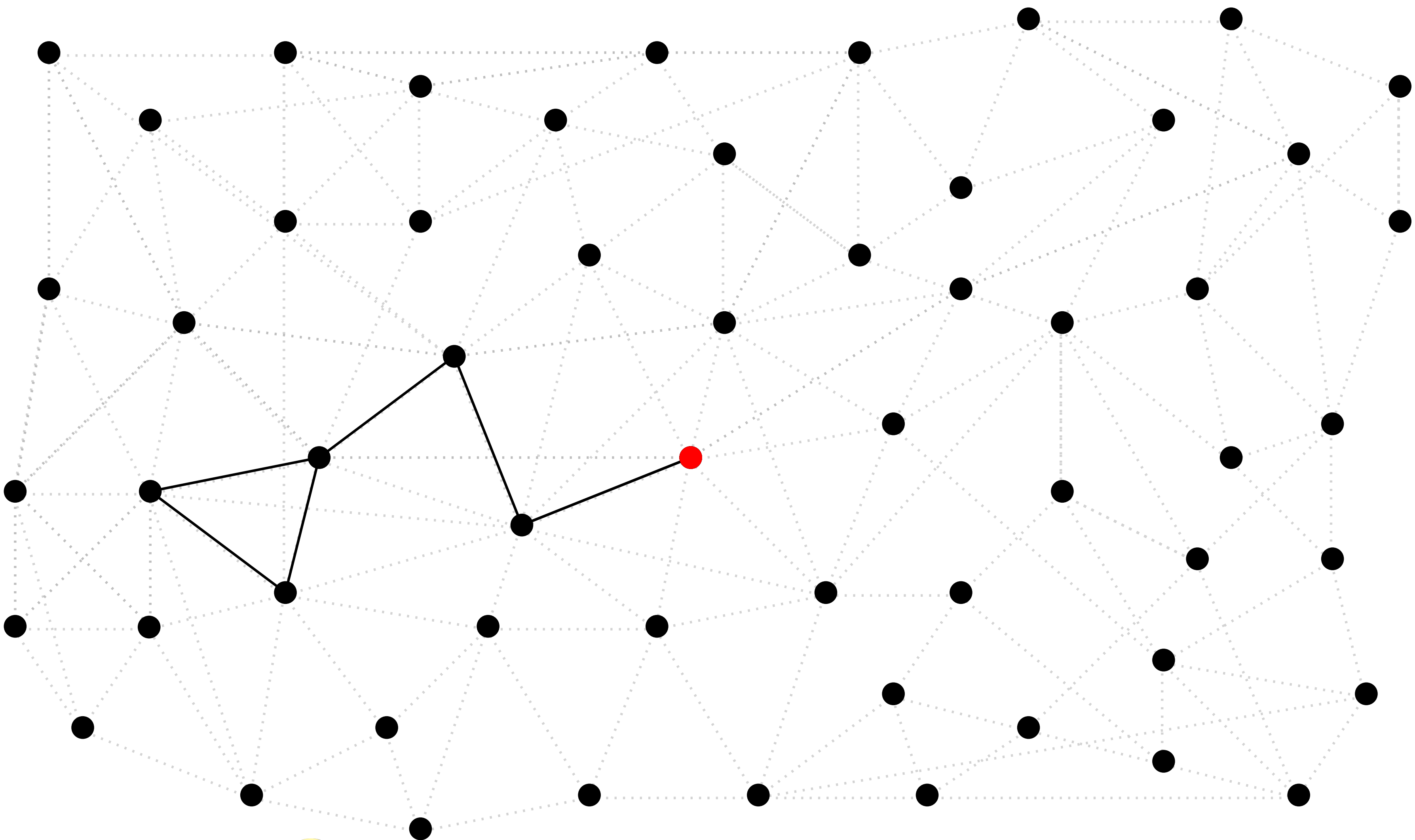

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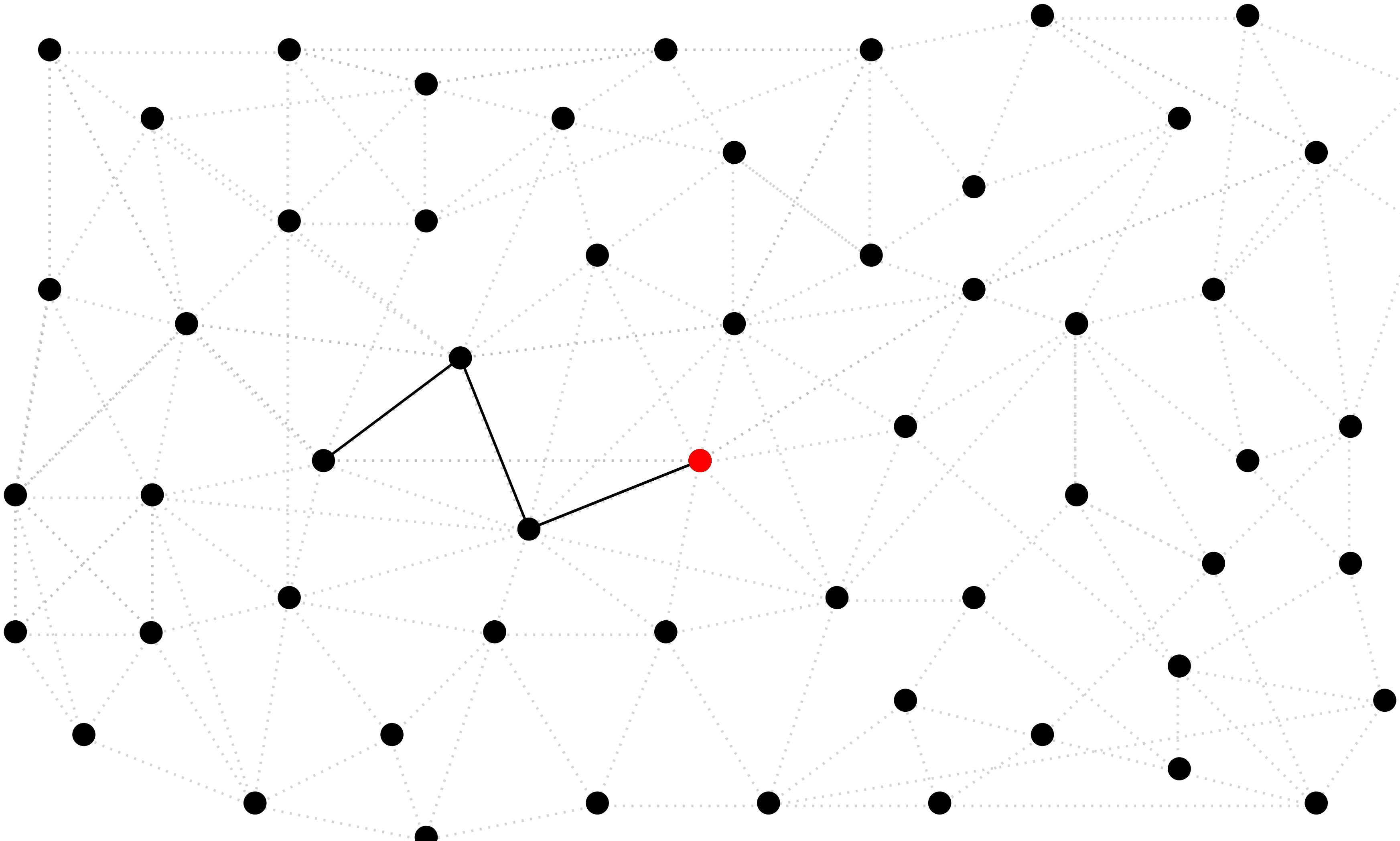

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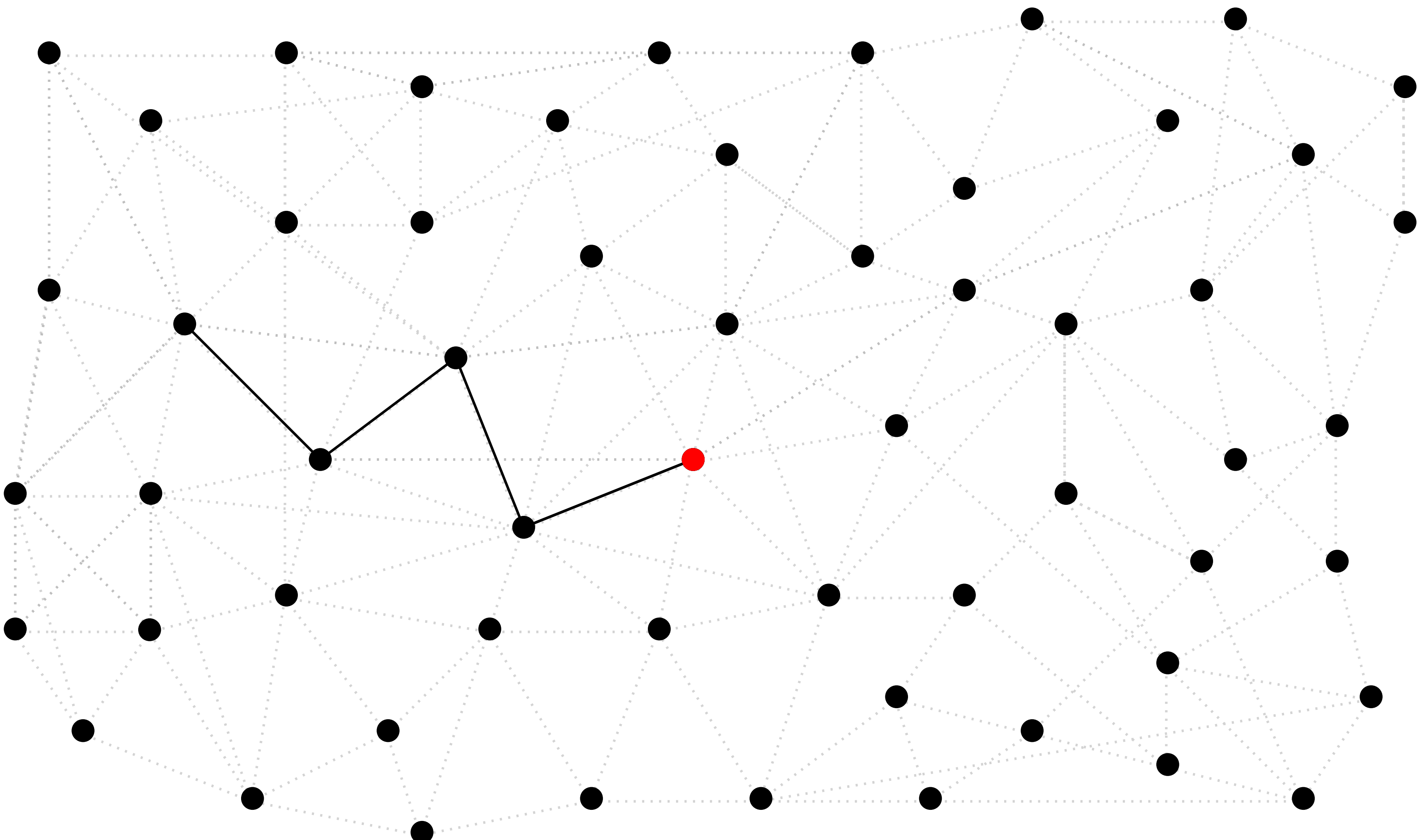

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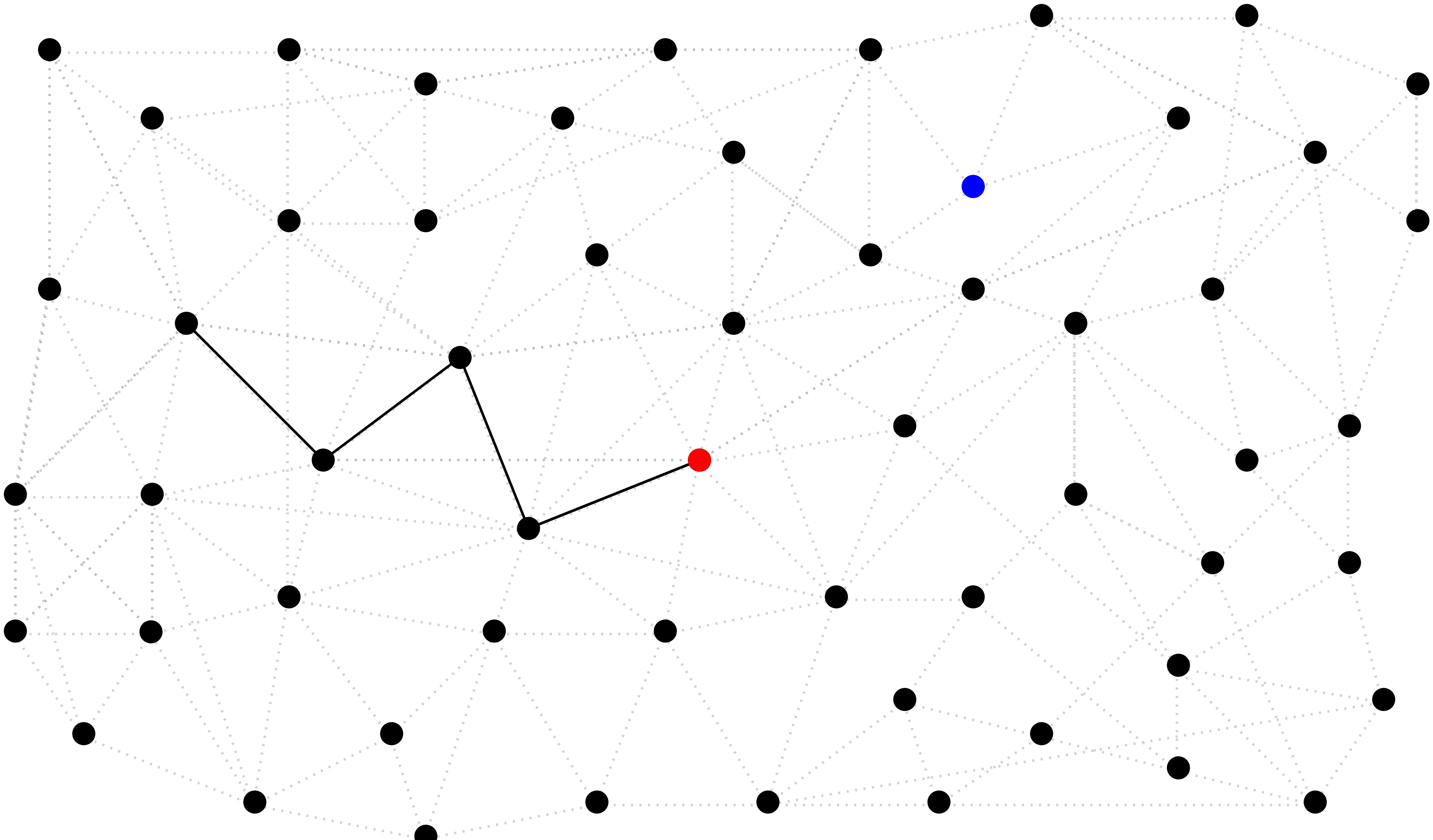

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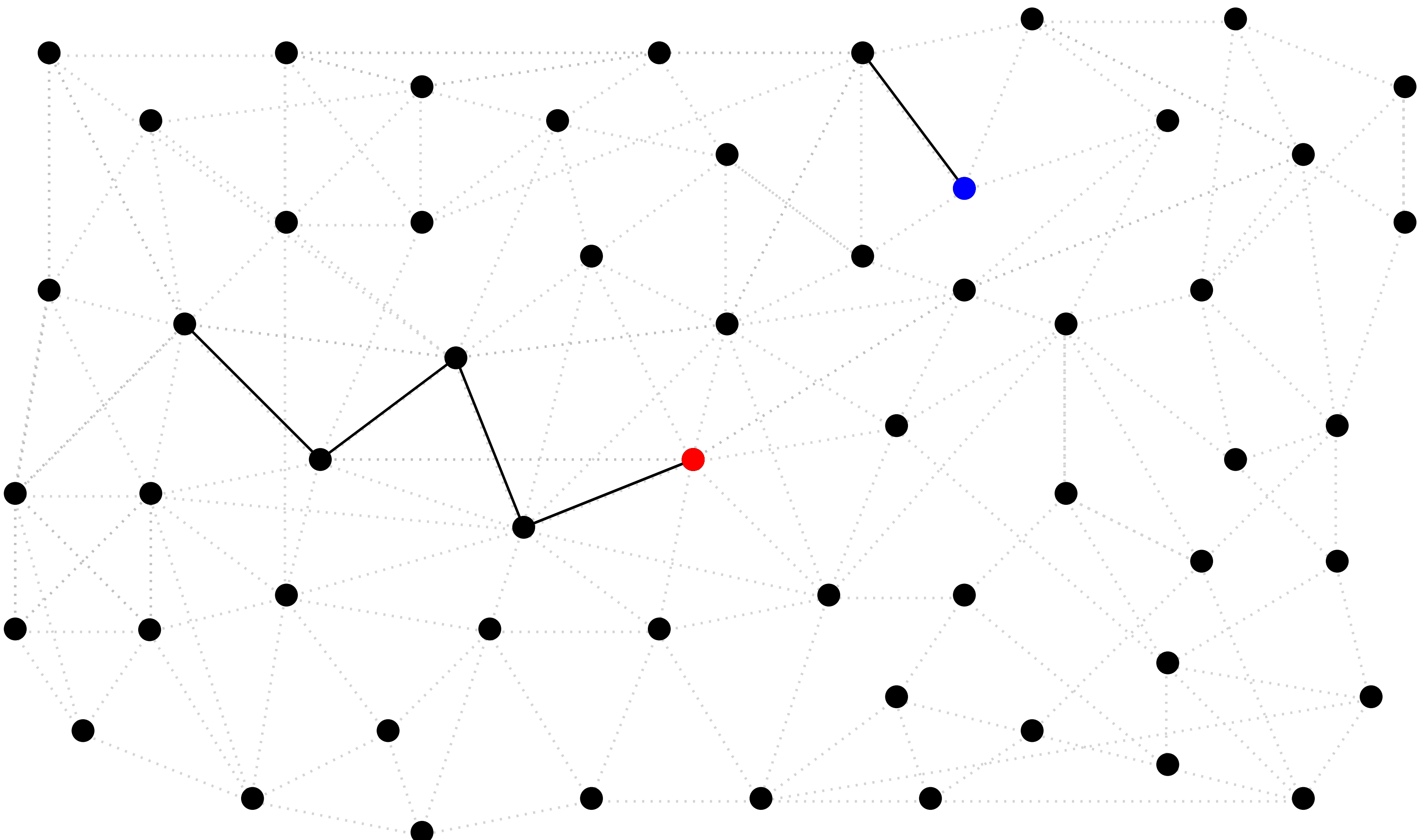

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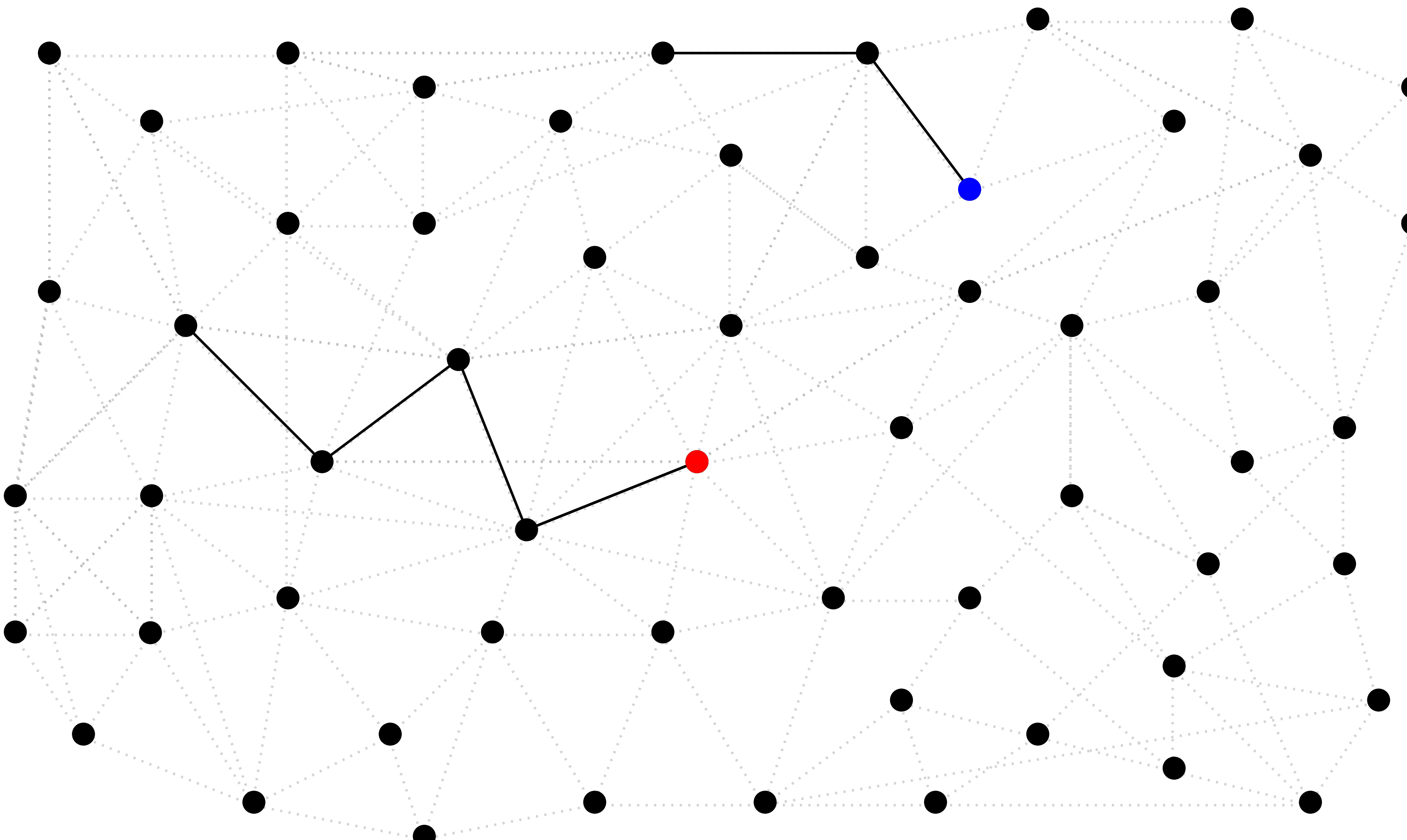

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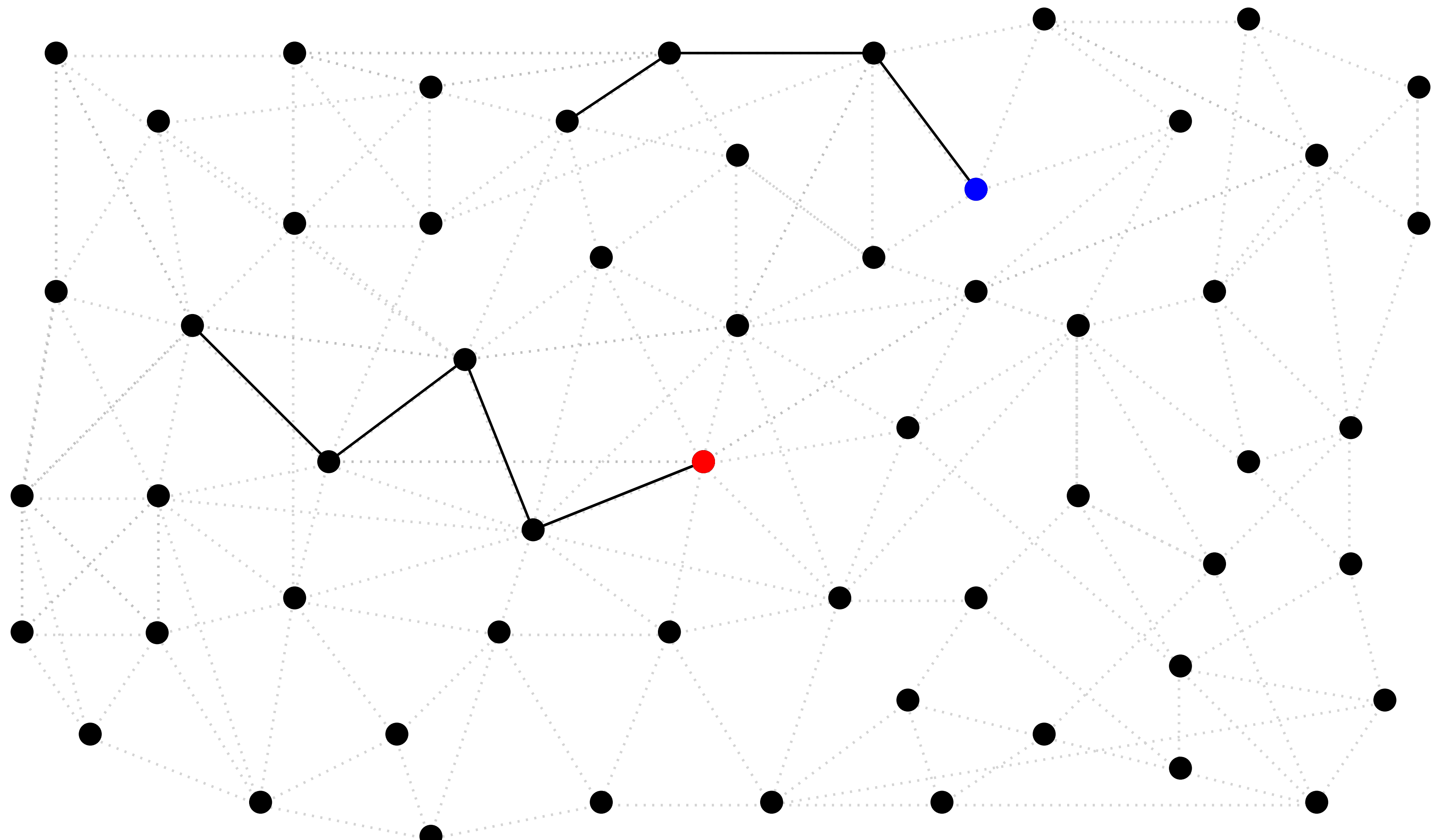

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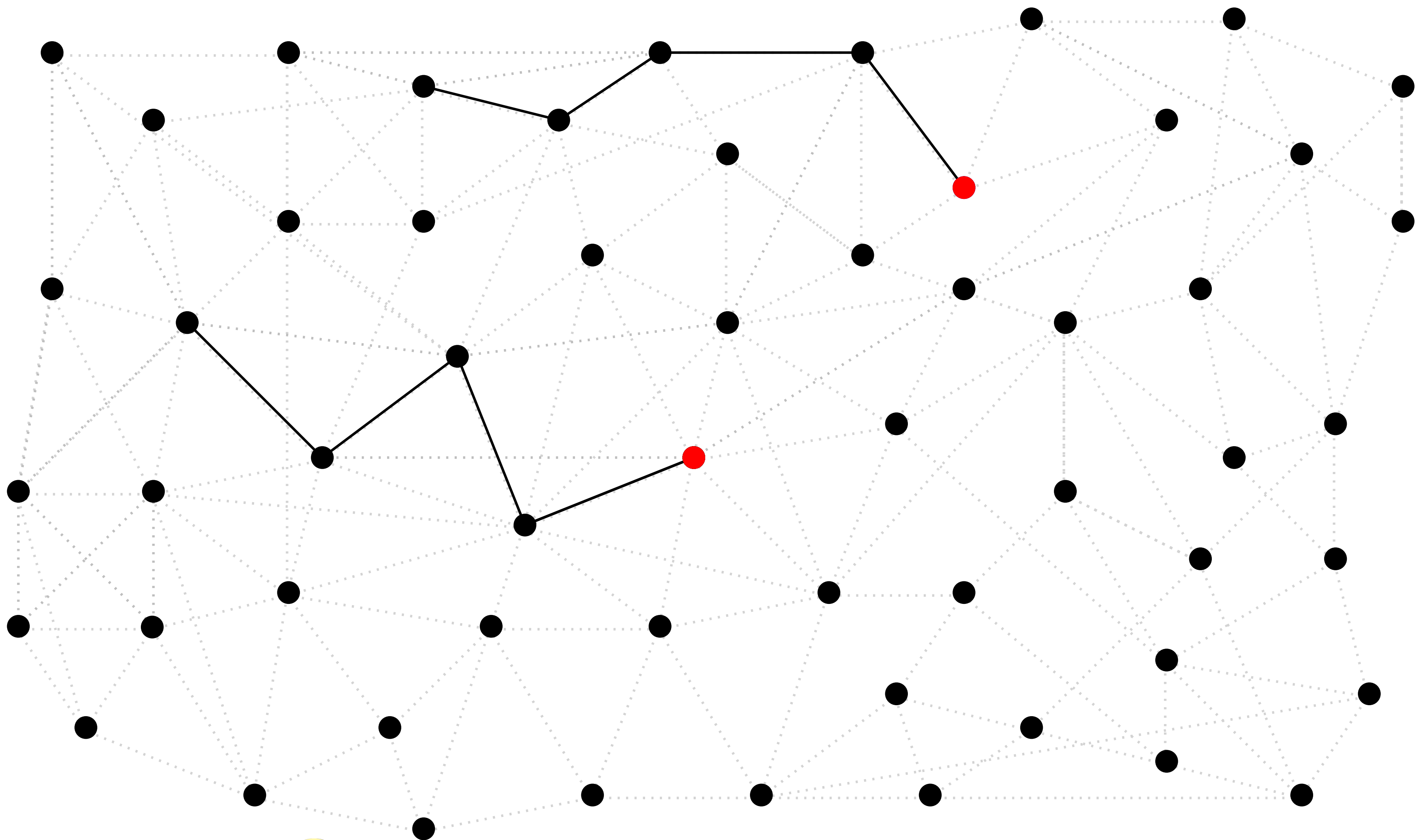

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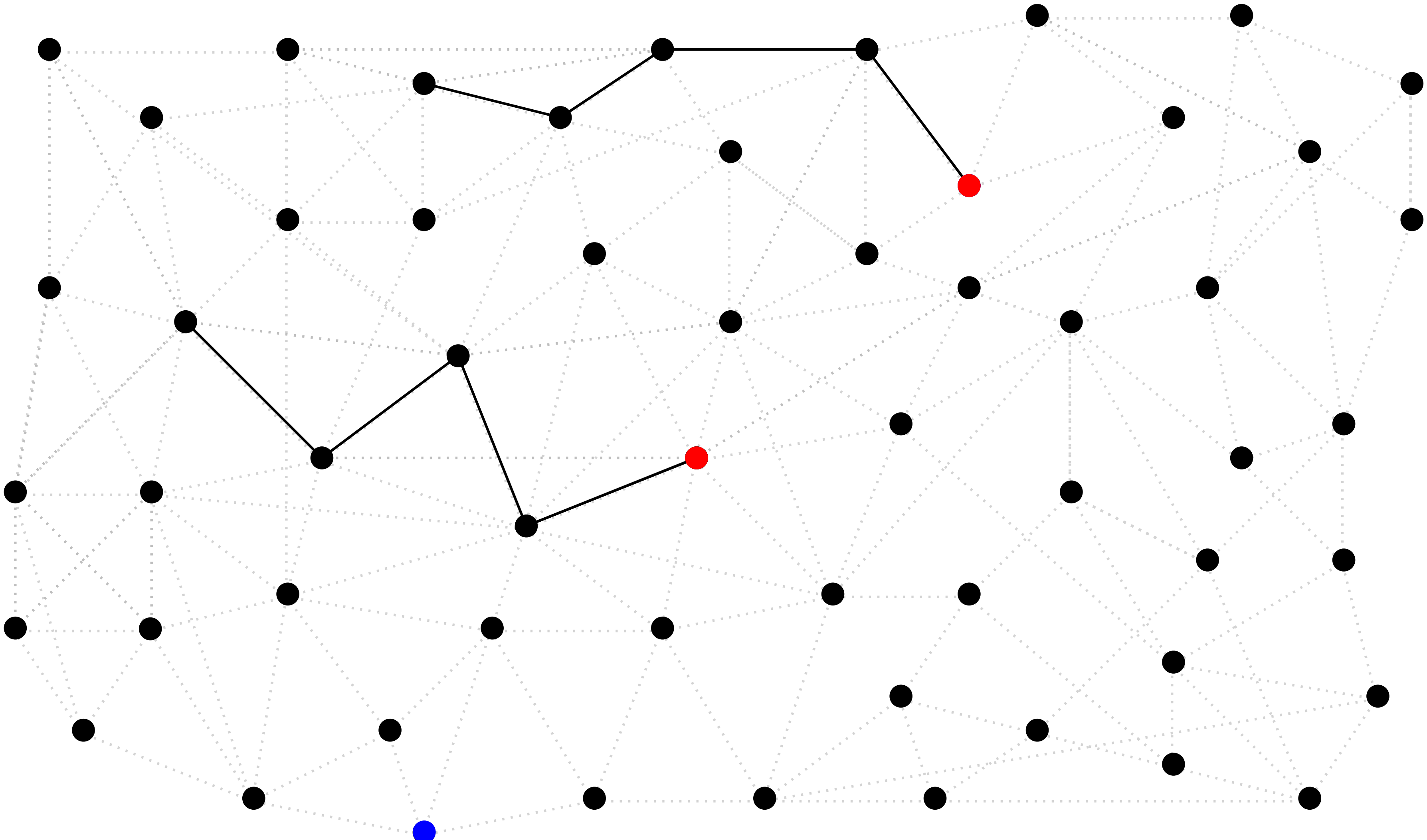

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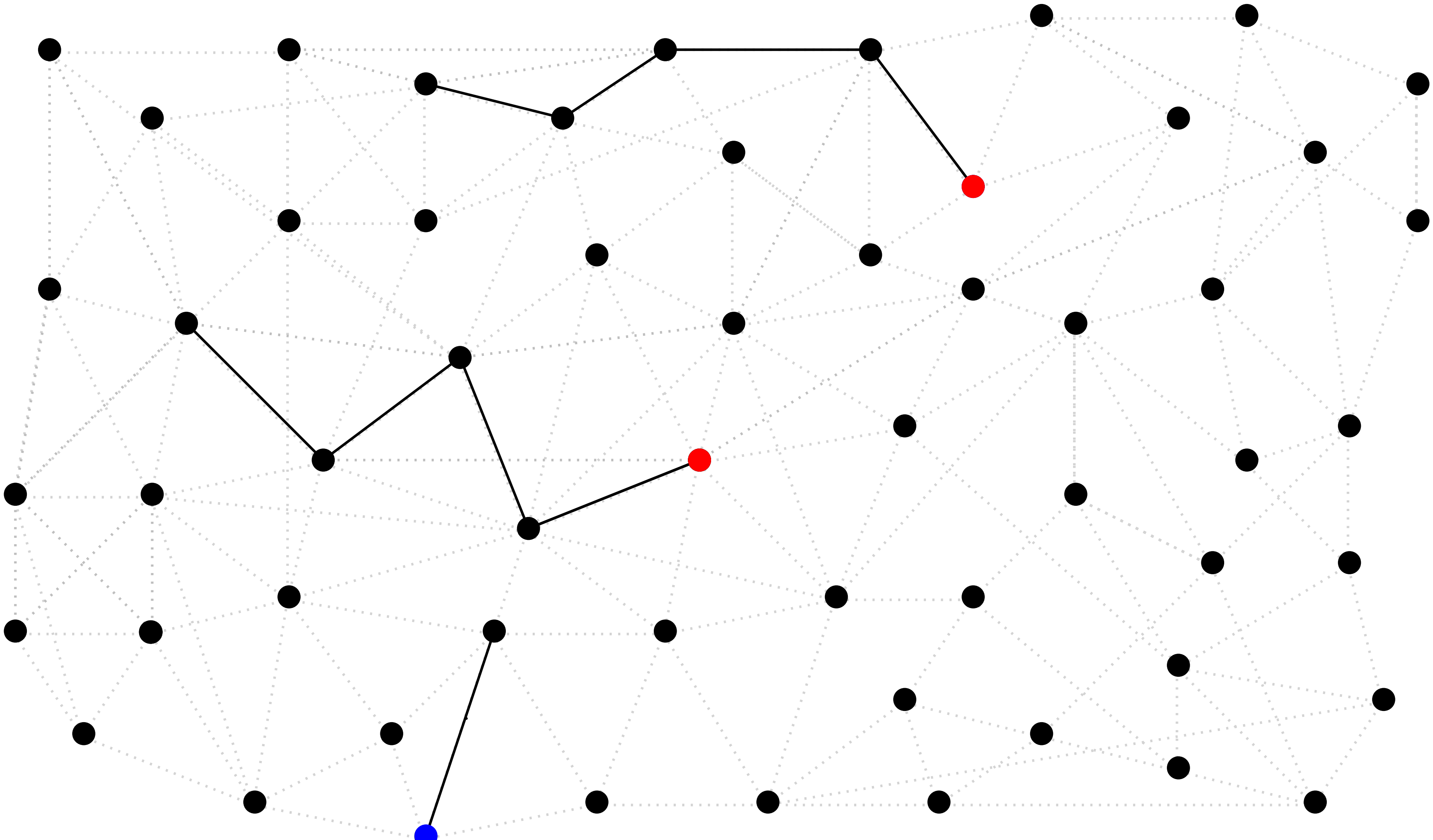

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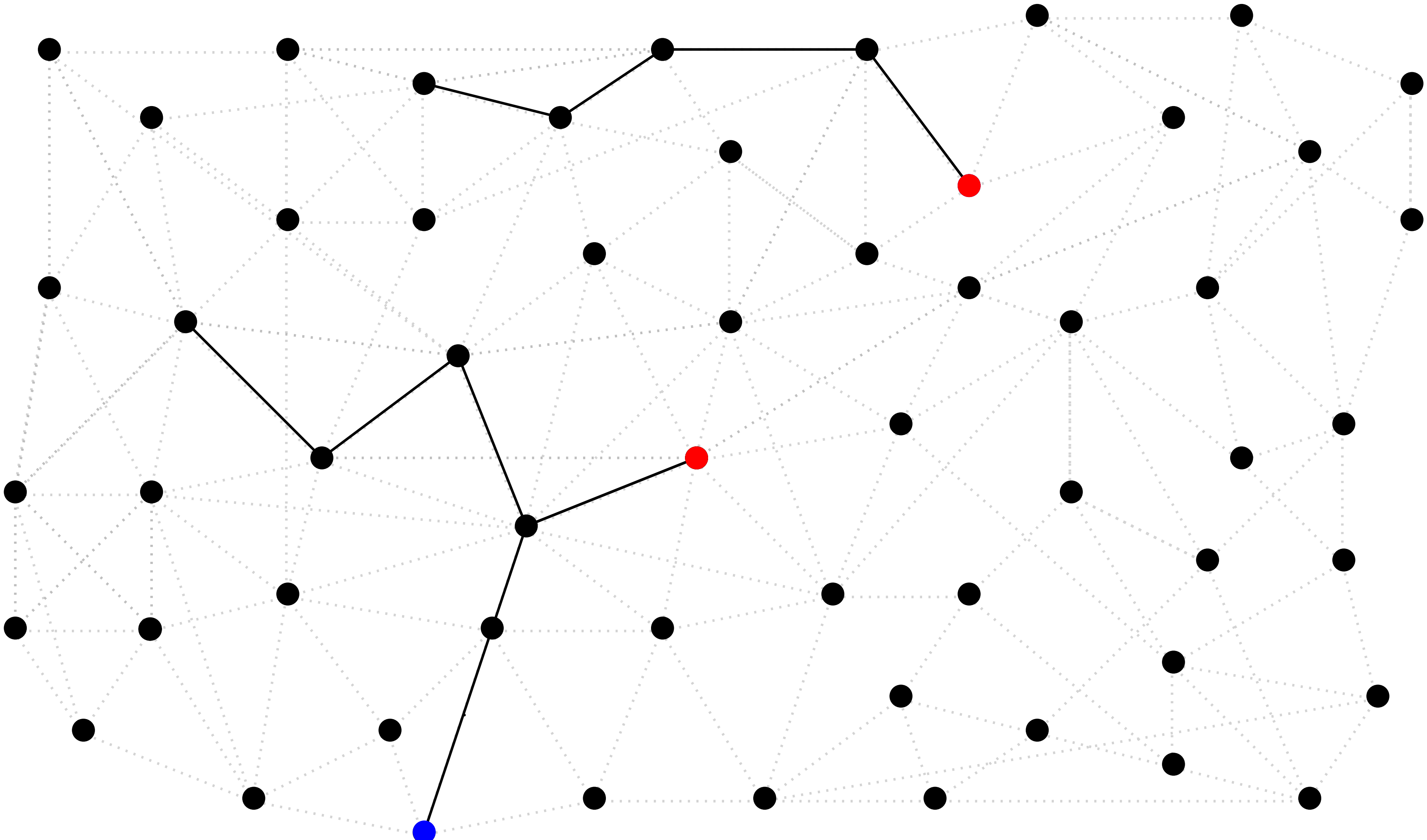

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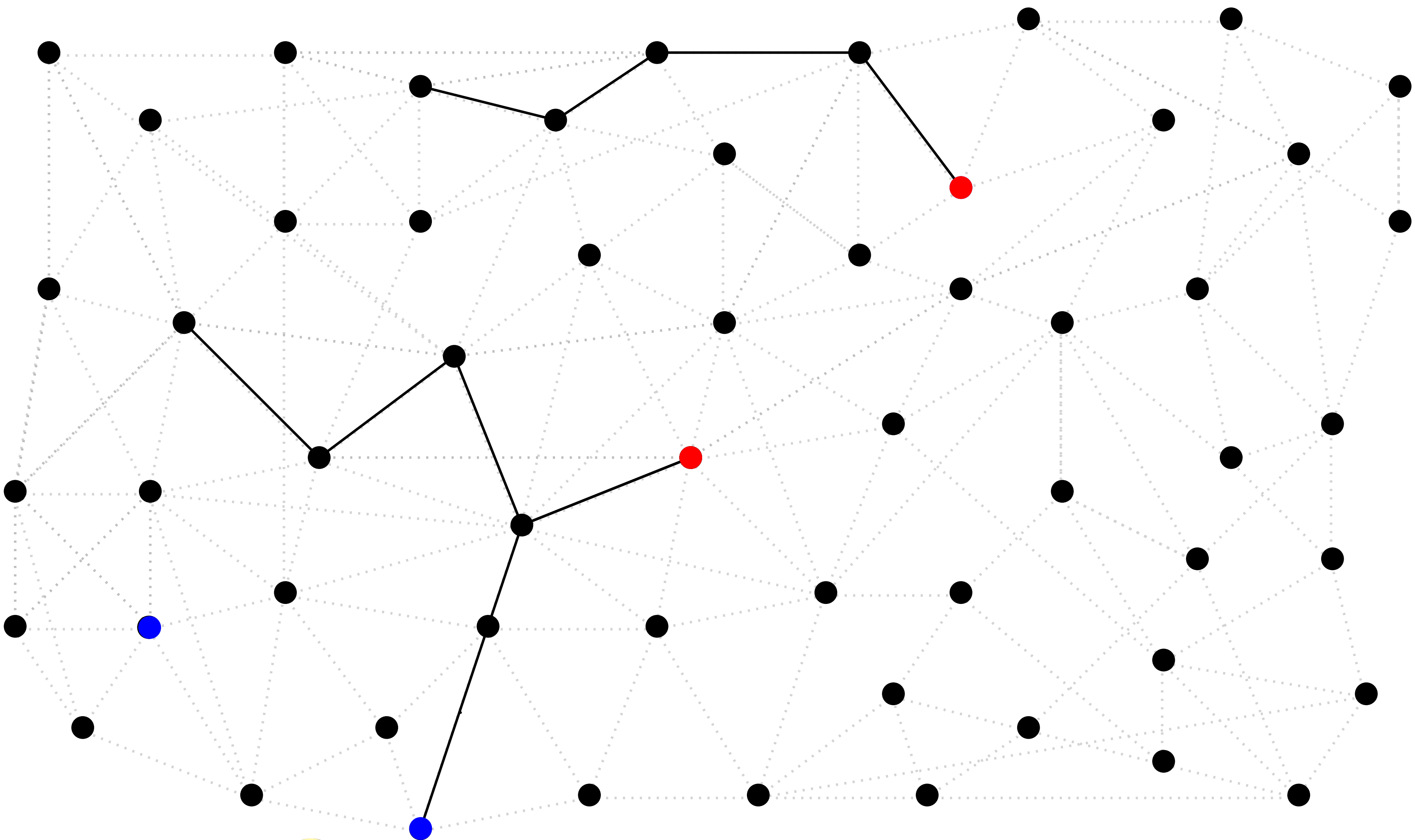

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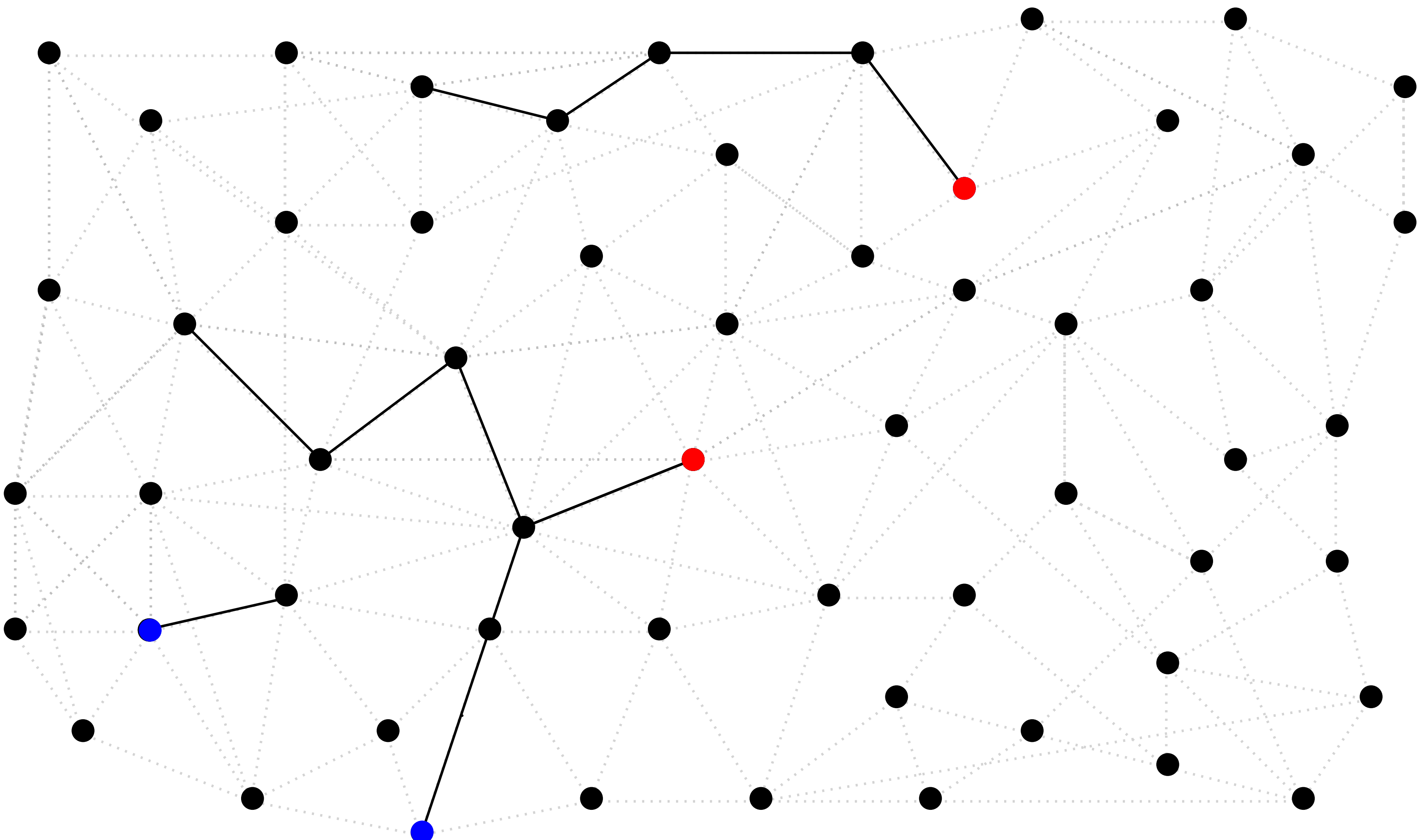

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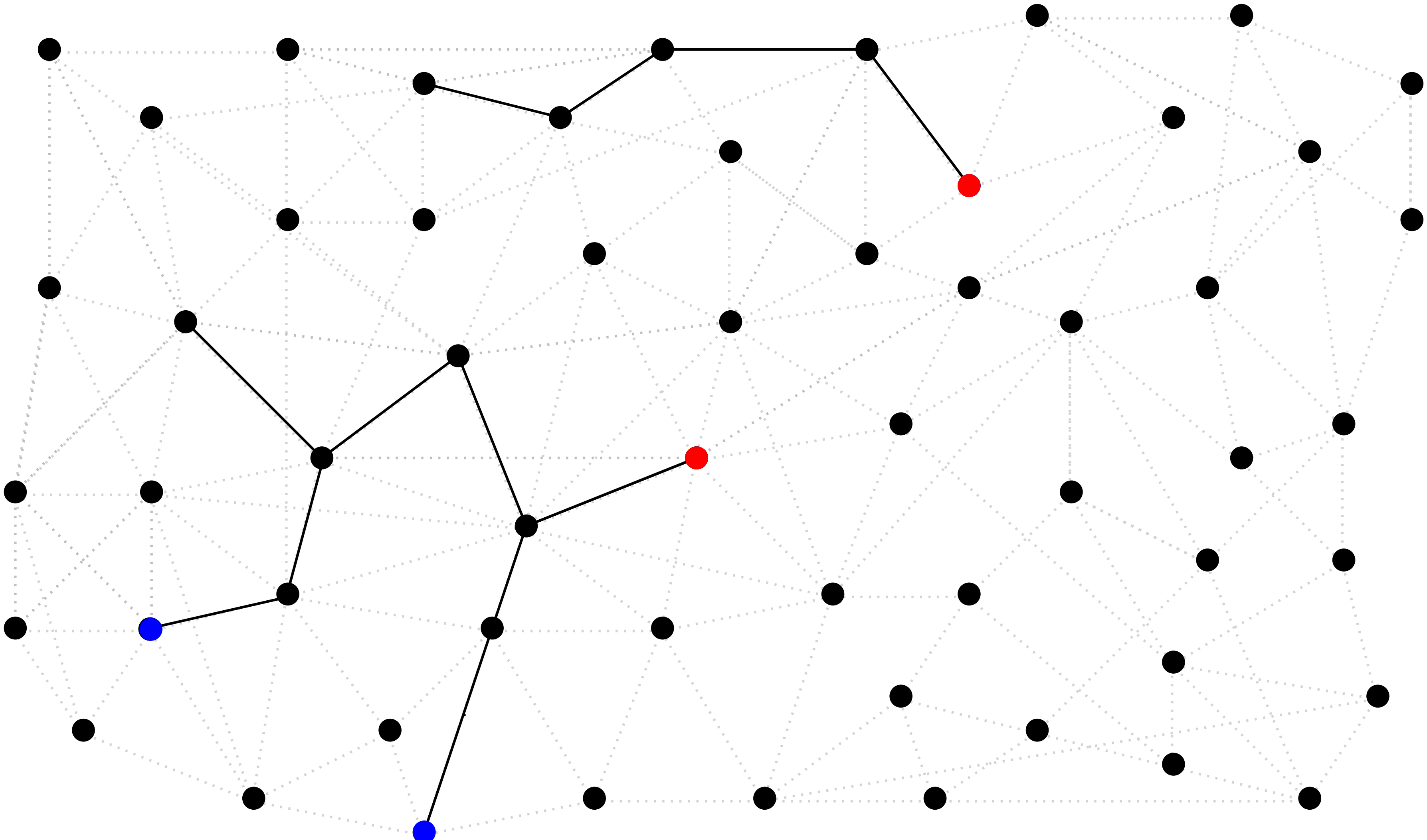

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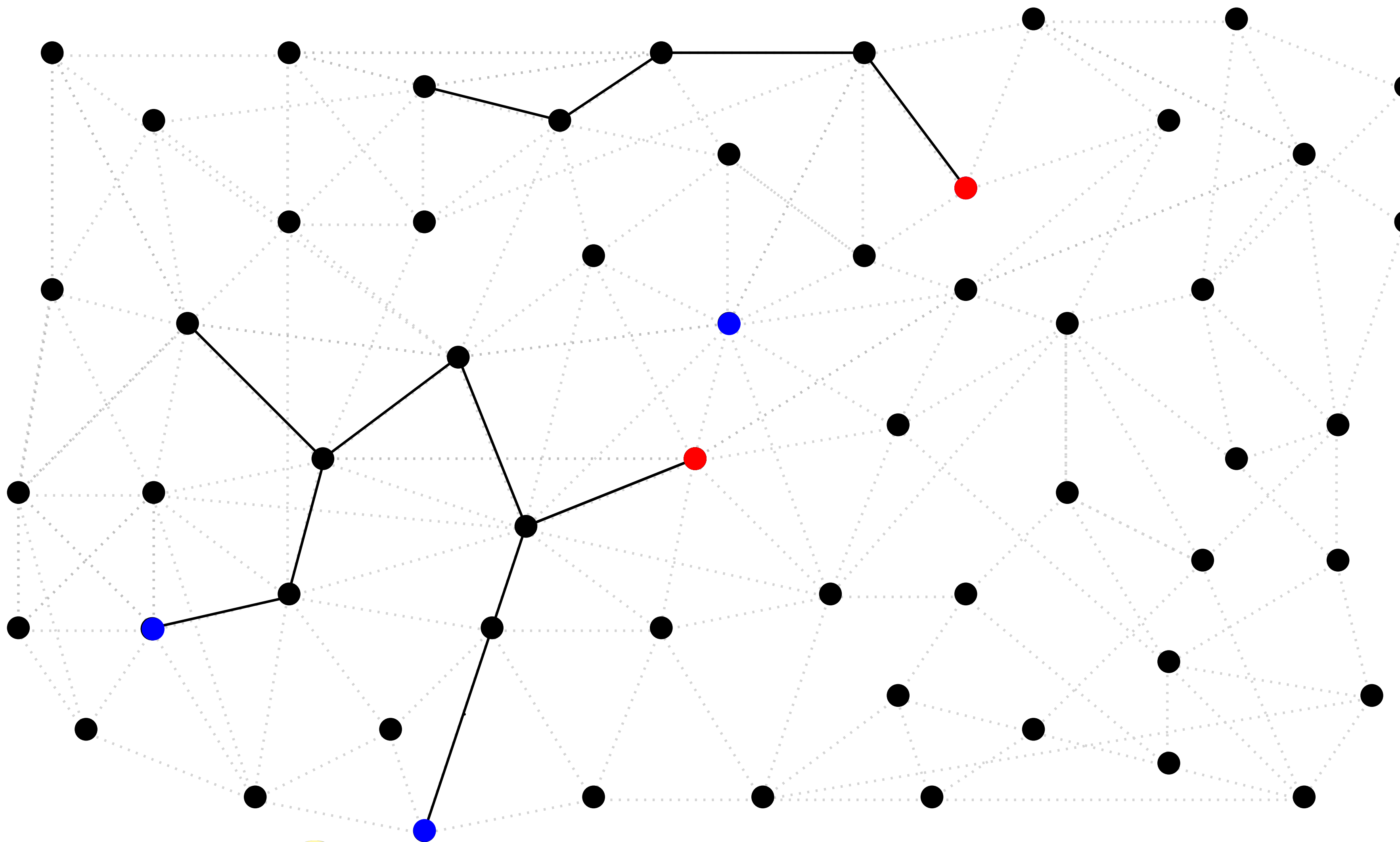

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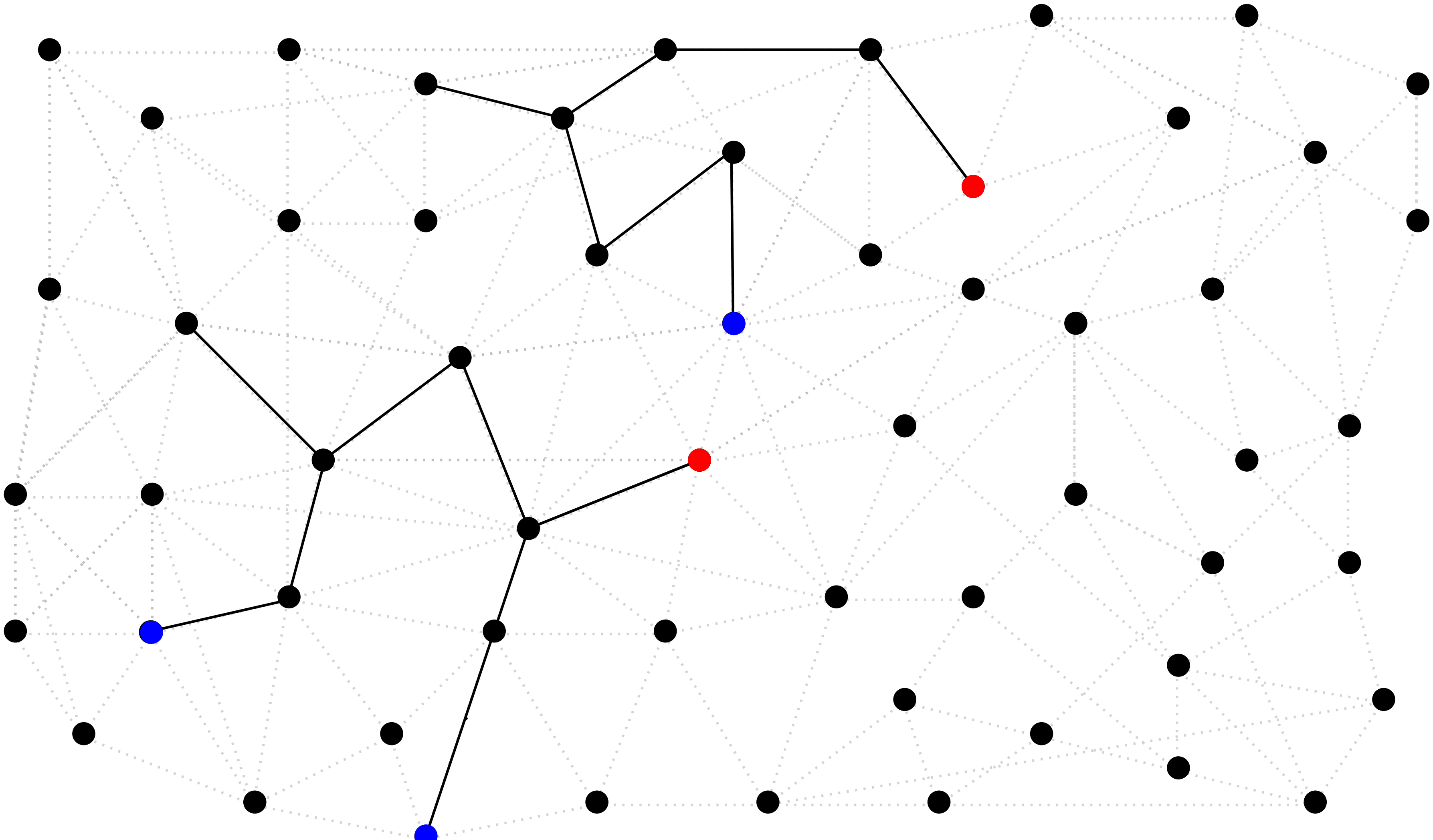

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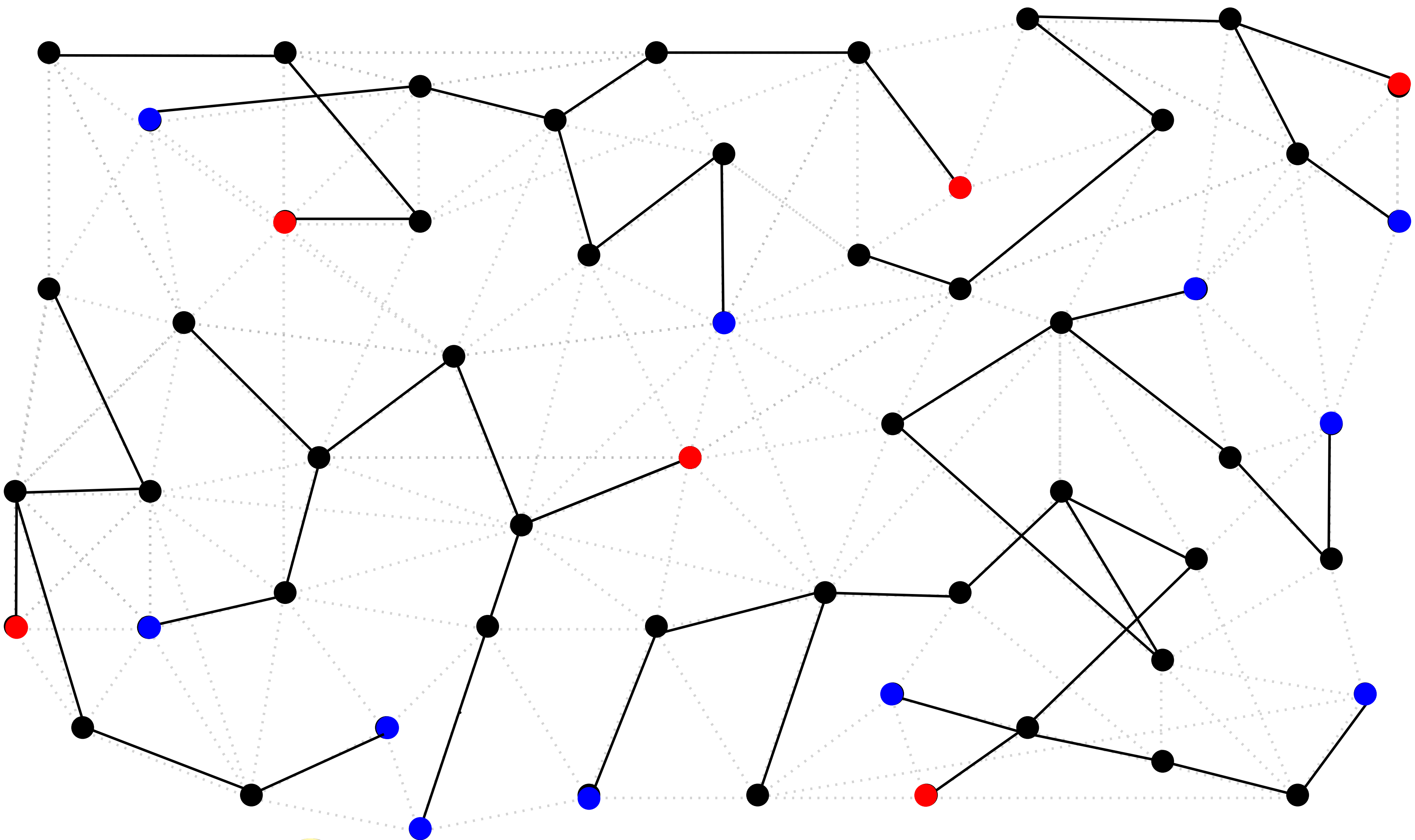

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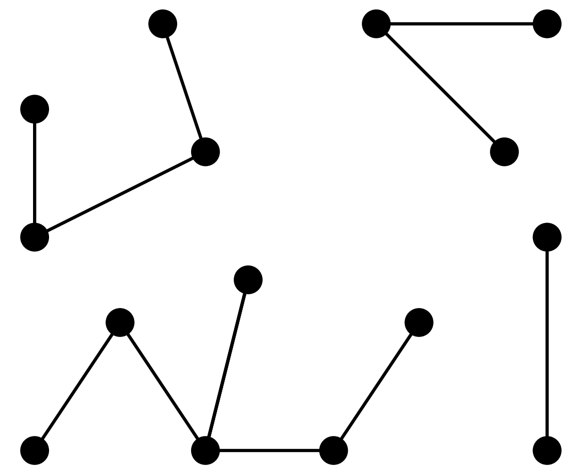
Motivation

We like to study evolution of random forests.

Why?
Understanding the state of matter, droplet formation in clouds, gravitational clustering, protein formation.



Sample of a random forest

We like to study scaling limits.

Why?
- Universality.
- Links the discrete to continuous.
- Links probability with geometry.

Scaling limit

We would like a metric to encapture how random graphs look when big.

We use the so called Gromov-Hausdorff-Prokhorov metric.

Graph like metrics

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$$d_H^X(A, B) = \inf\{\varepsilon > 0; A \subset B^\varepsilon \text{ and } B \subset A^\varepsilon\},$$
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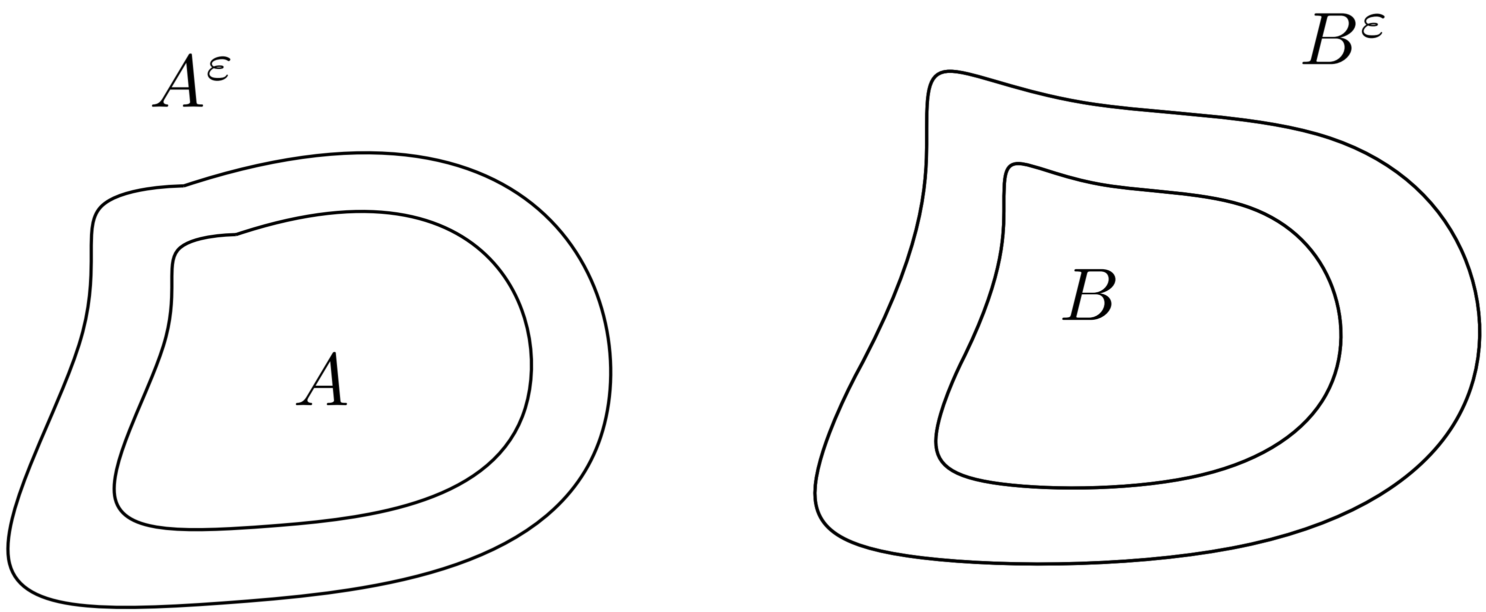

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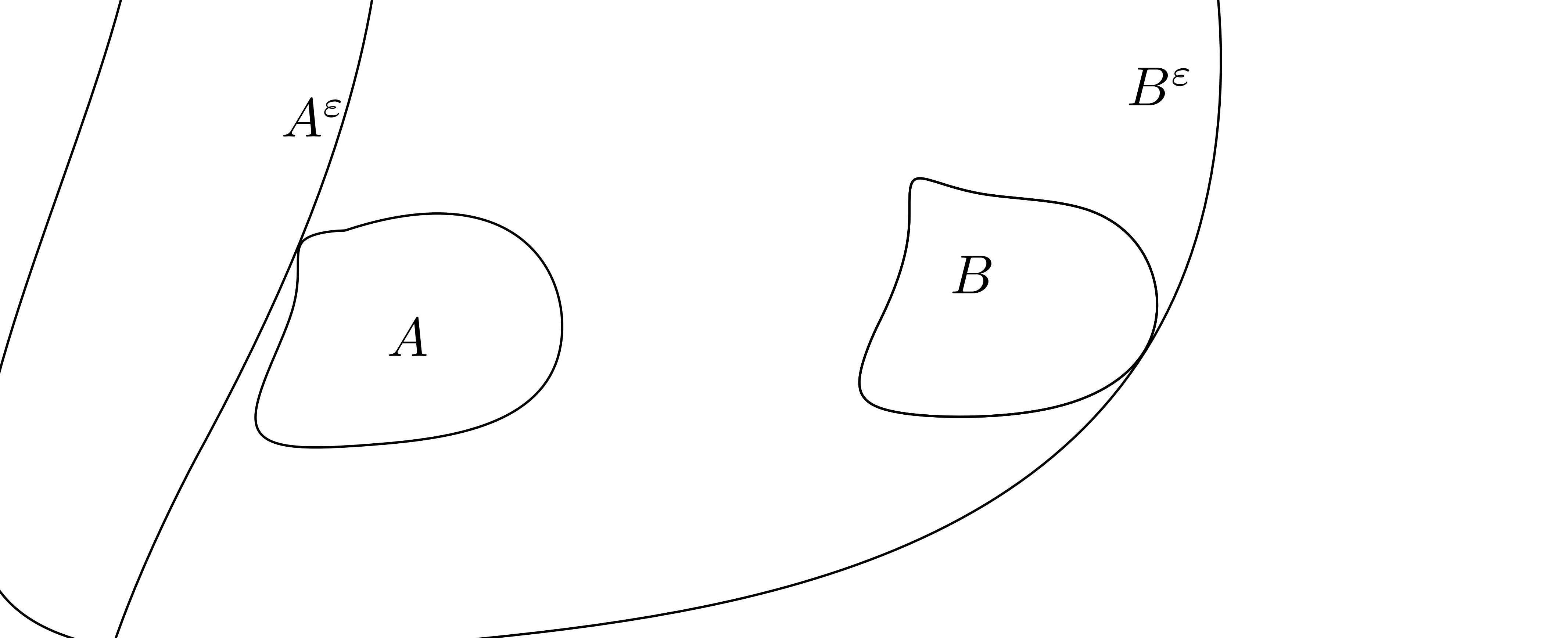

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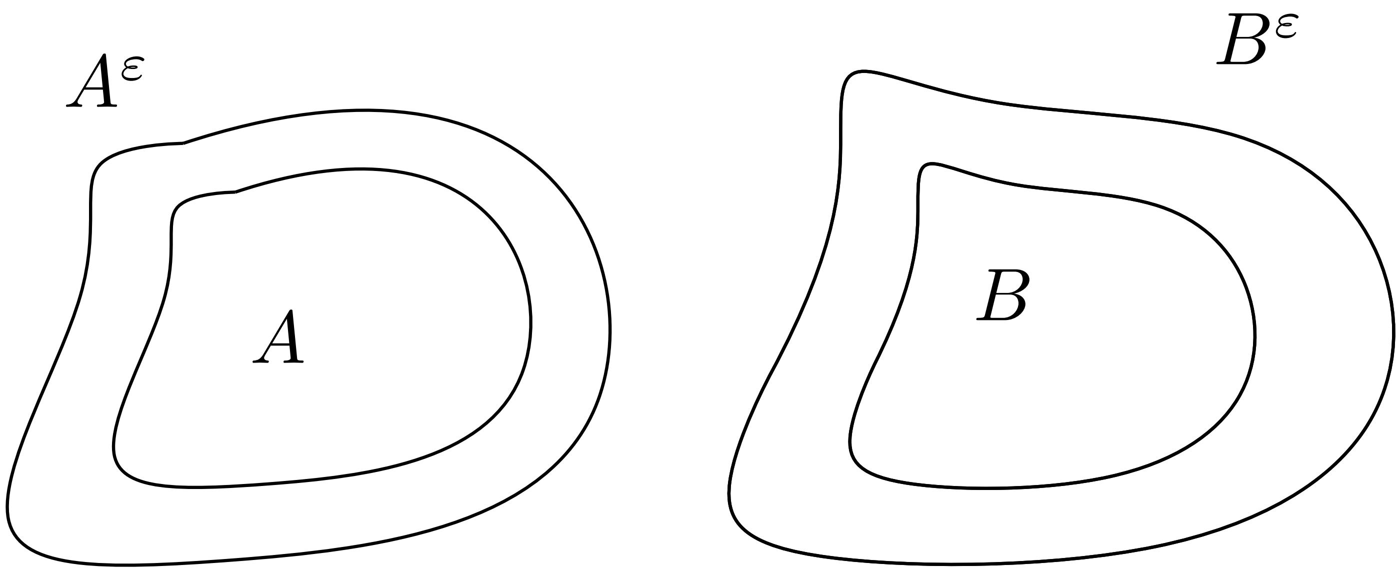

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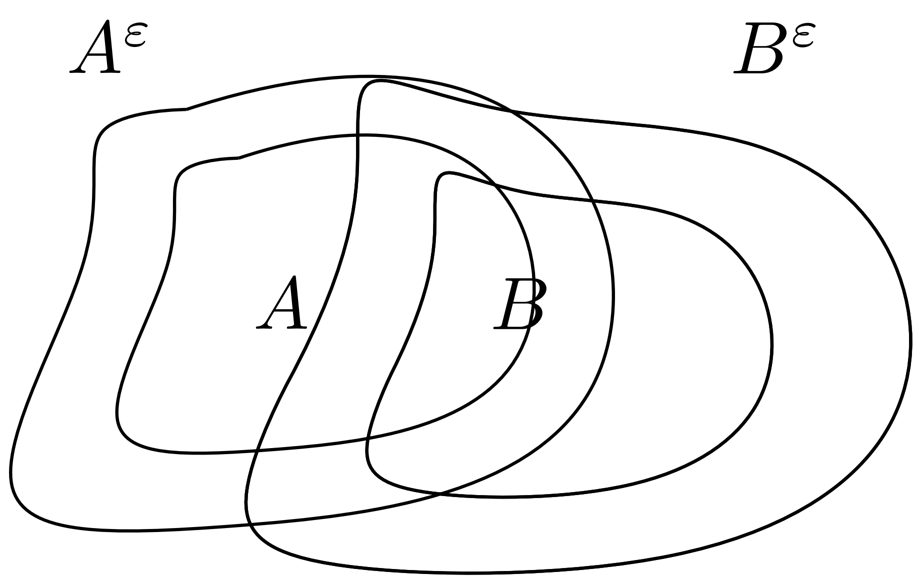

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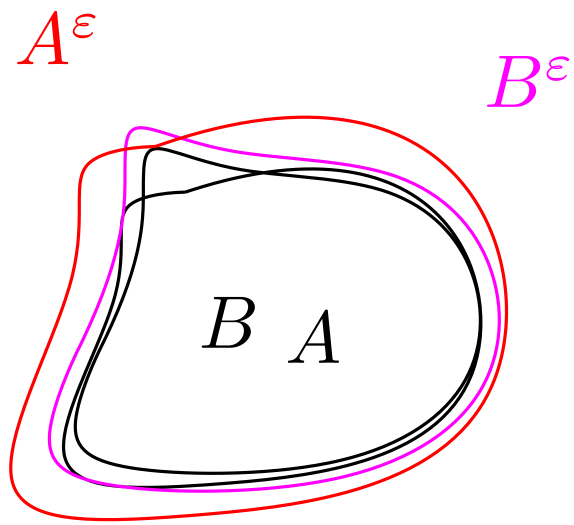

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We use the so called Gromov-Hausdorff-Prokhorov metric.

Prokhorov

Let (X, d) be a metric space and let μ, ν be two probability measures on the Borel σ-algebra of X. The *Prokhorov distance* $d_P(\mu, \nu)$ is defined by

$$d_P(\mu,\nu) := \inf\Big\{\varepsilon > 0 \,\Big|\, \mu(A) \leq \nu(A^\varepsilon) + \varepsilon \text{ and}$$

$$\nu(A) \leq \mu(A^\varepsilon) + \varepsilon, \text{ for all Borel sets } A \subset X\Big\},$$

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Gromov-Hausdorff-Prokhorov

$$d_{\mathrm{GHP}}\big((X, d_X, \mu_X), (Y, d_Y, \mu_Y)\big) :=$$

$$\inf_{Z,\phi,\psi} \Big\{ \max\big(d_H^Z(\psi(X), \phi(Y)),\ d_P^Z(\psi_* \mu_X, \psi_* \mu_Y)\big) \Big\},$$

This is a distance on the set of isometry-equivalent classes.

Graph like metrics

We would like a metric to encapture how random **forests** look when big.

Let $\mathcal{M}$ be the set of measured isometry-equivalence classes of compact measured metric spaces.

Define $\mathbb{L}_p$ to be the set of sequences (X_i, μ_i) in $\mathcal{M}^{\mathbb{N}}$ s.t.

$$\sum diam(X_i)^p + \sum \mu_i(X_i) < \infty.$$

$$d^p_{GHP}(F_1, F_2) = \left(\sum_{i \in \mathbb{N}} d_{GHP}(T_1^{(i)}, T_2^{(i)})^p\right)^p$$

$(\mathbb{L}_p, d^p_{GHP})$ is polish, i.e. separable and complete.

Recent advances

(Addario-Berry, Broutin, Goldschmidt, Miermont 17')

Consider the scaling limit of critical Erdős-Rényi random graphs on K_n

$$\mathbb{G}_\lambda^n = (\mathbb{G}_\lambda^{n,i})_{i\in\mathbb{N}}$$

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Consider the local limit of the forest, phase transition.

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Consider the local limit of the forest, phase transition.

(Archer, Shalev 24')

Show convergence of "dense" graphs to CRT in the GHP sense.

More known results

(Avena, Gaudillière 18')Distribution of forest of WA with stopping.

$$p = \frac{C}{C+|V(F)|} \qquad \mu^C(F) = \frac{C^{|F|}\prod_{T\in F}|V(T)|}{Z(C)} \qquad \mu^C_r(F) = \frac{C^{|F_r|}}{Z_r(C)}$$

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(Pittel 02')

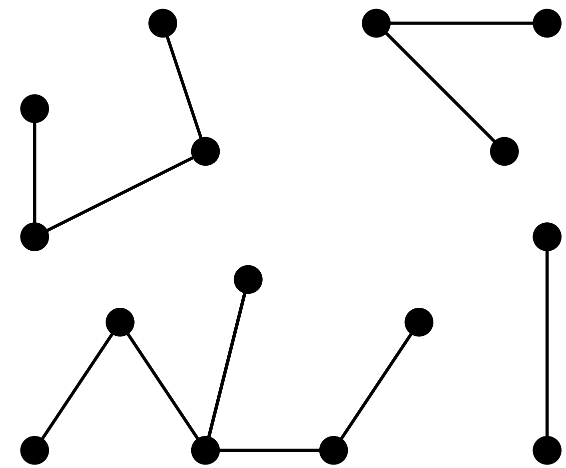
Branch distribution of WA, as stopping times of Bernouli variables.

Back to the model

Now pick a "nice family of graphs" with n vertices, for example K_n, or 'dense graphs'.

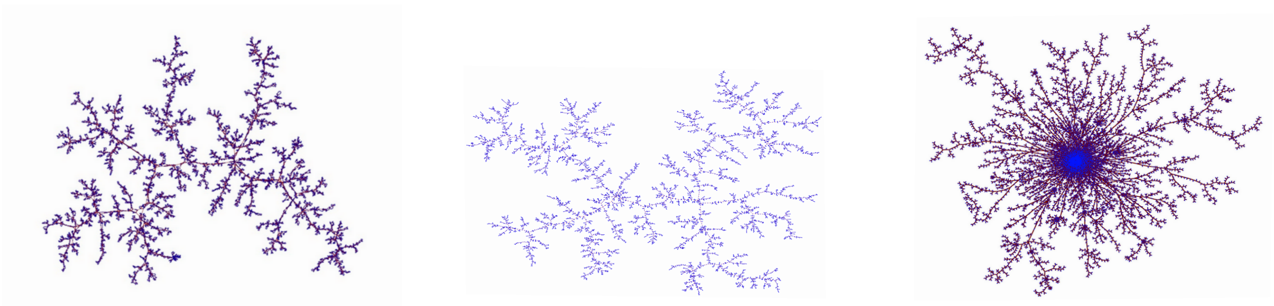
Then we can think of our model as a random forest of two variables (n, p)

We are interested in asymptotics of this model. (scaling limit)



Sample of a random forest

$\mathcal{F}(n,p) \rightarrow_{n\rightarrow\infty}$



Branch length distribution.

Let $\varepsilon = (\varepsilon_2 \cdots, \varepsilon_n)$ be a sequence of independent Bernoulli variables, $P(\varepsilon_j = \mathcal{B}) = j/(n + C(n))$, $P(\varepsilon_j = \mathcal{R}) = C(n)/(n + C(n))$; let $\varepsilon_1 = \mathcal{R}$. Let $t_1 = \min\{j \geq 1 : \varepsilon_{j+1} \in \{\mathcal{R}, \mathcal{B}\}\}$,
and recursively, if $\tau_s := 1 + \sum_{r=1}^{s} t_r < n$, then
$t_{s+1} = \min\{r > 1 : \varepsilon_{\tau_{s+r}} \in \{\mathcal{R}, \mathcal{B}\}\}$, and denote $t = \{t_1, t_2, \cdots\}$. Then

$$\mathbf{L} \equiv \mathbf{t};$$

in words, in distribution the sequence of branches lengths is the same as the sequence of time intervals between the success events $\{\varepsilon_j = 1\}$ for the Bernoulli sequence ε.

What is the right scale?

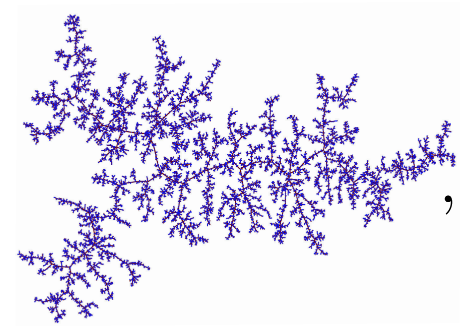
Let $n \in \mathbb{N}$ and $C(n) = O(n)$. Let $L = (L_i)_{i\in\mathbb{N}}$ be the lengths of the branches created by WGA with killing parameter $p(n) = C(n)/(n + C(n))$. Let $\mathbb{P}(l_i > L|M)$ be the probability that the length of the i-th branch created is greater than $L\sqrt{n}$ given that we have already created a forest of size $M\sqrt{n}$. Then asymptotically

$$\lim_{n\to\infty} \mathbb{P}\left(L_i > L\sqrt{n}|M\sqrt{n}\right) = \begin{cases} \exp\left(-LM - \frac{L^2}{2}\right) & \text{if } C(n) = o(\sqrt{n}) \\ \exp\left(-L(c+M) - \frac{L^2}{2}\right) & \text{if } C(n) \sim c\sqrt{n} \\ 0 & \text{if } C(n) = \omega(\sqrt{n}) \end{cases}$$

Phase transition

$$p(n) = \begin{cases} o(1/\sqrt{n}) \\ \sim c/\sqrt{n} \\ \omega(1/\sqrt{n}) \end{cases}$$

$\frac{\cdot}{\sqrt{n}}$



$, \cdot, \cdot, \cdot, \cdots$

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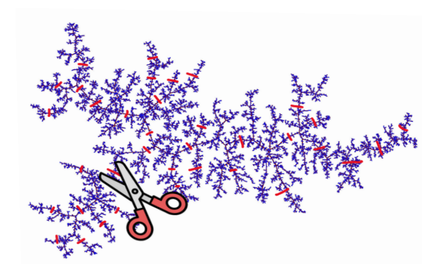
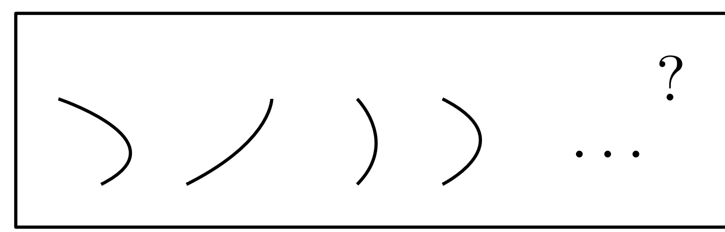


$\cdots$

$\cdot p(n)$



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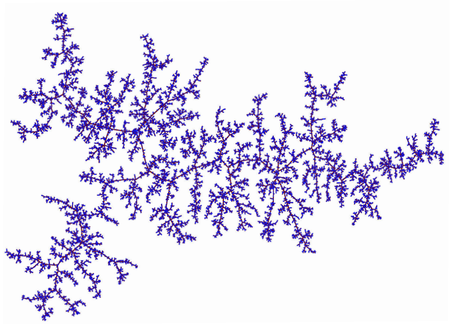

Aldous CRT or Brownian tree (91'-93')

Three equal constructions:

-Uniform spanning tree.

$UST(n)/\sqrt{n}$

-Brownian excursion.

$\mathbb{E}/\sim$

-Stick breaking.

$\overline{\cup_{k\in\mathbb{N}} T(k)}$

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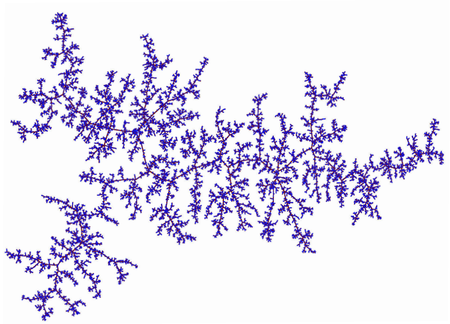

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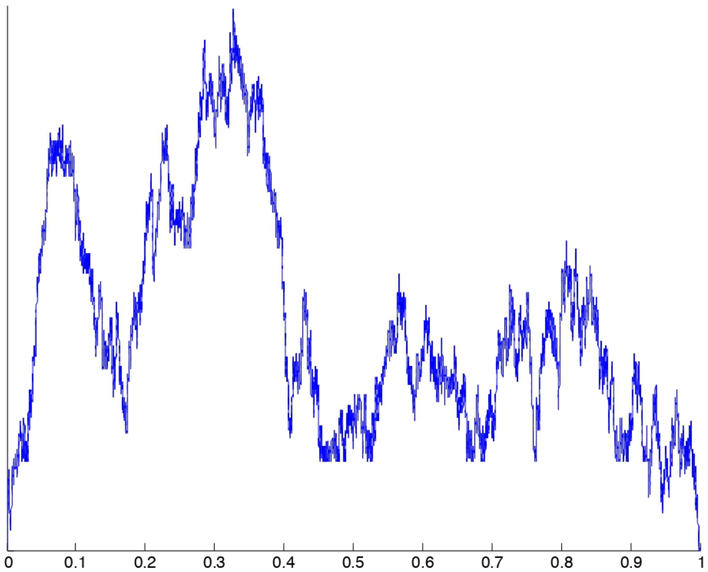

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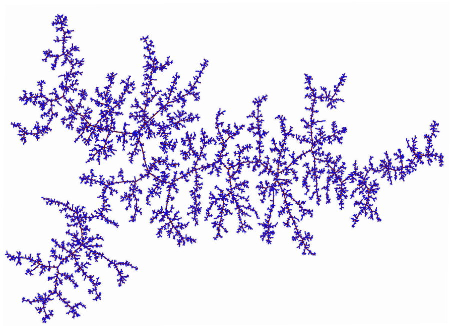

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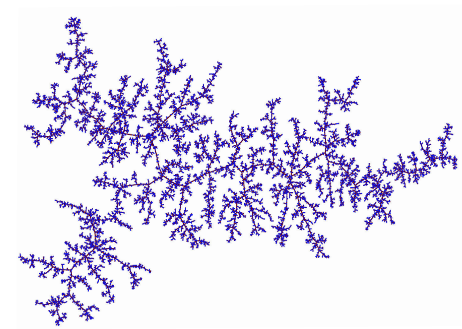

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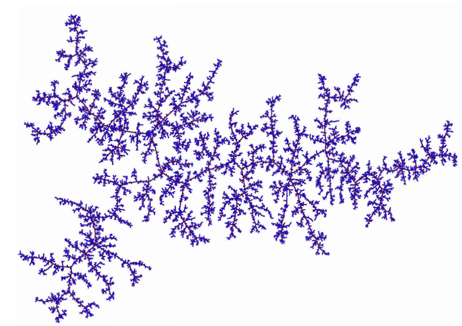

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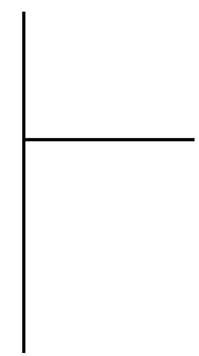

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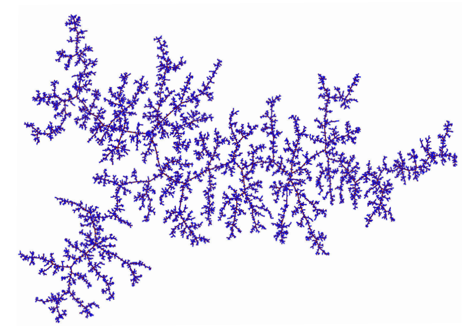

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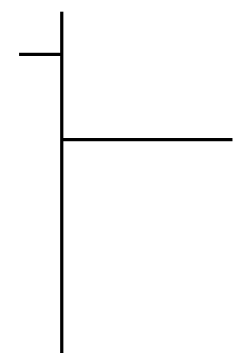

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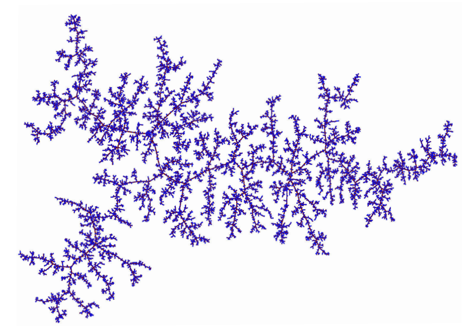

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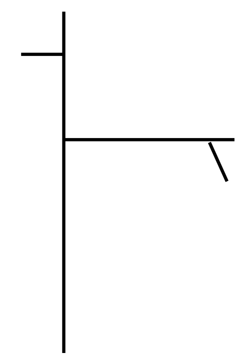

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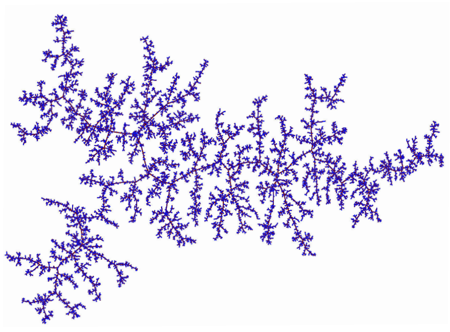

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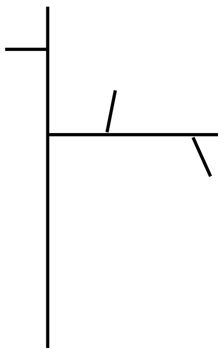

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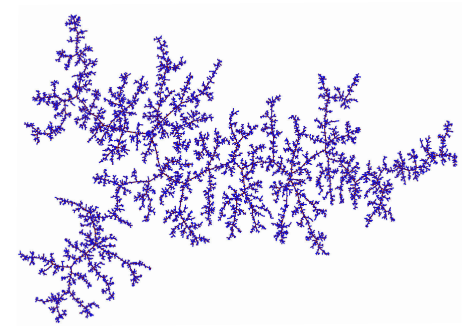

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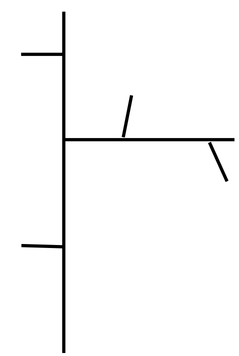

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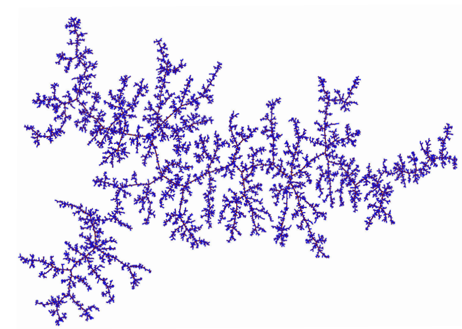

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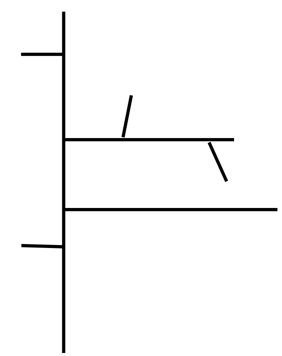

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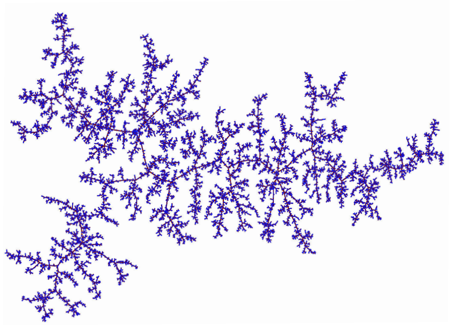

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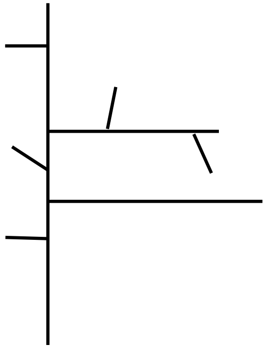

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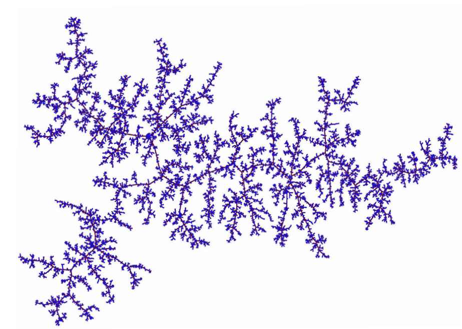

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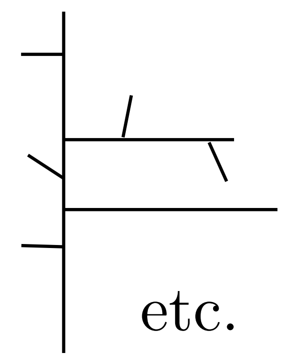

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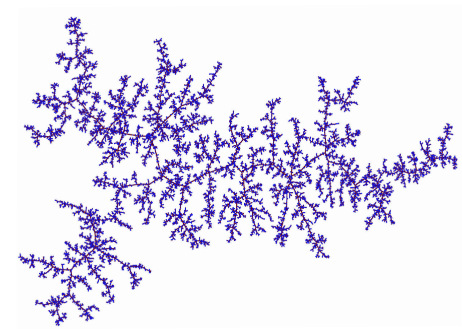

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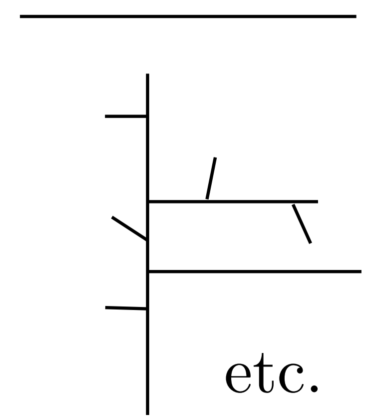

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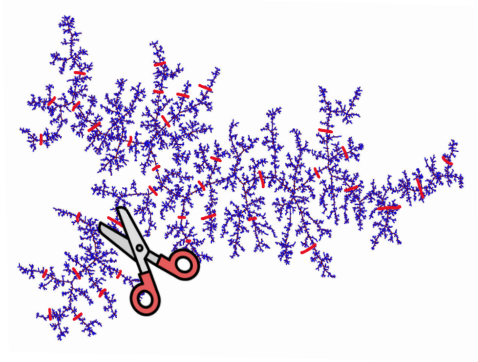

Just right

-Uniform spanning tree. (Edge Deletion)

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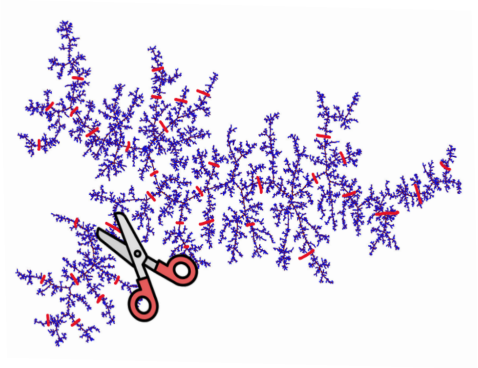

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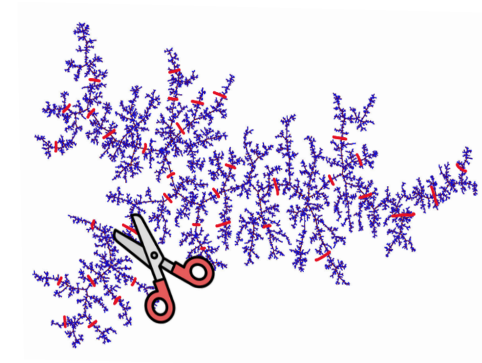

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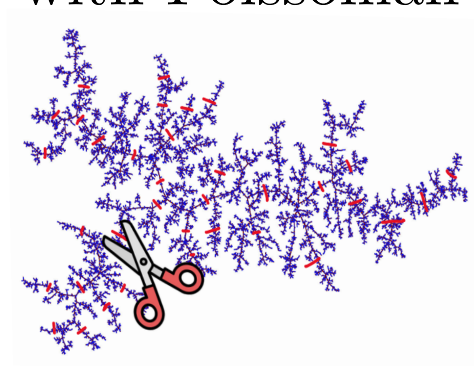

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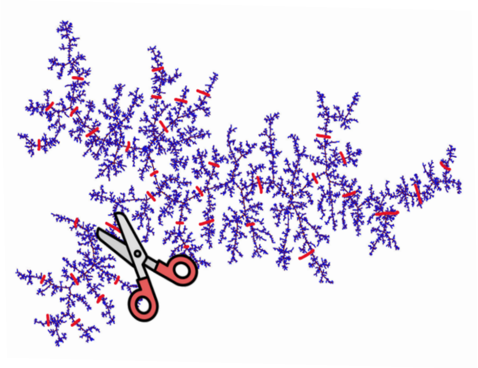

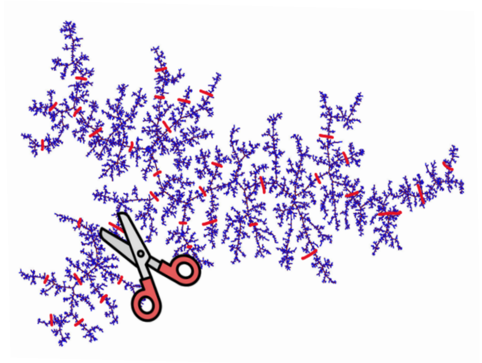

Conjecture
(Aldous-Pitman 98')

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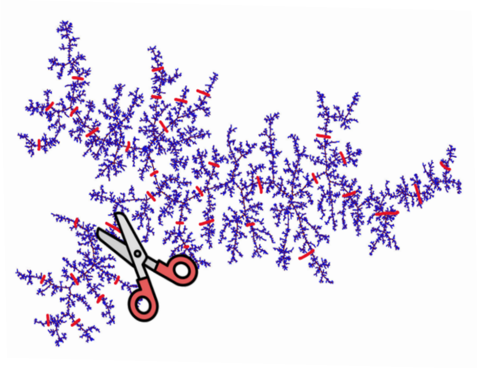

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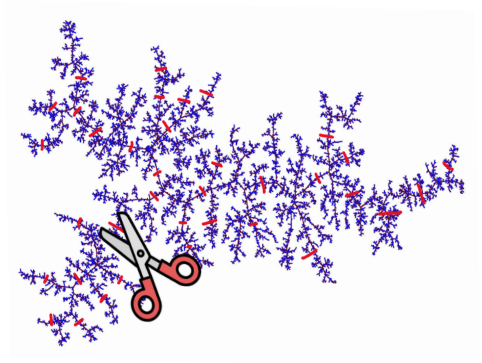

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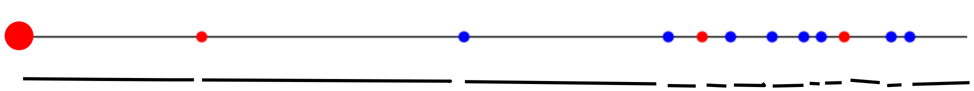

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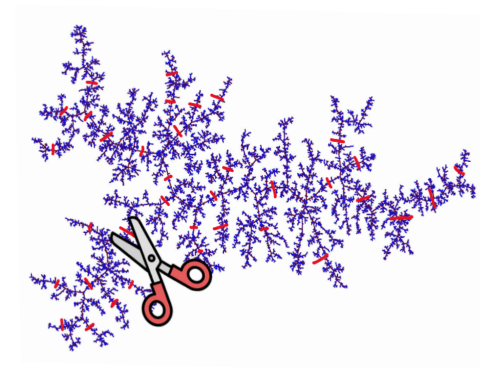

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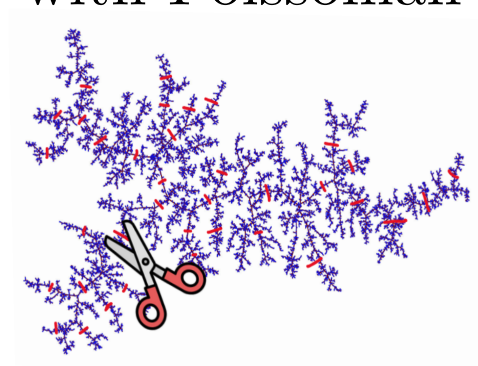

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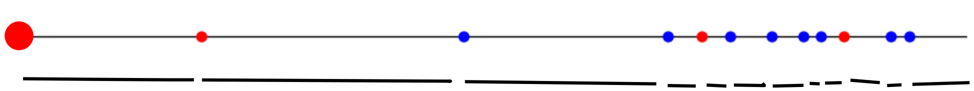

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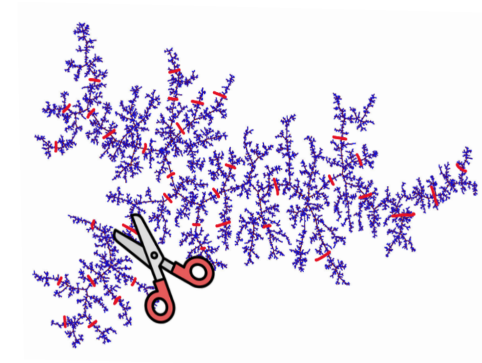

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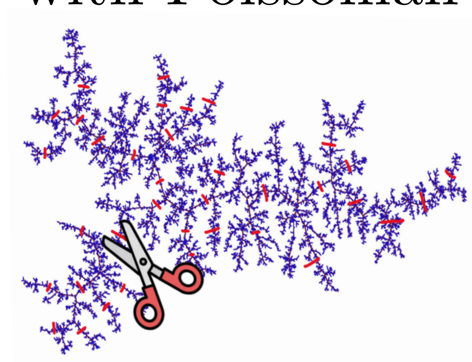

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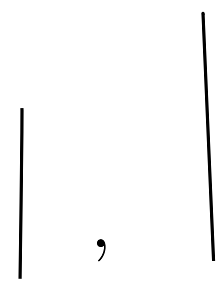

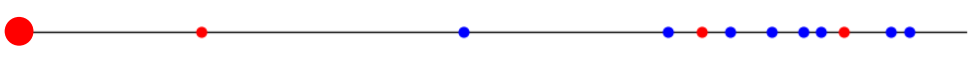

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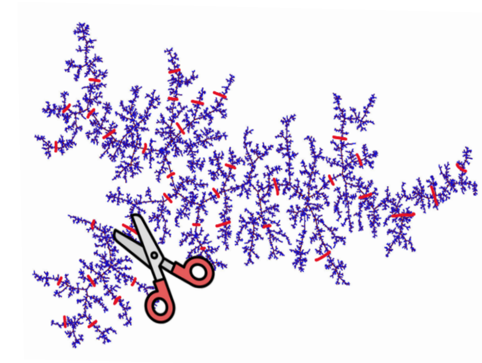

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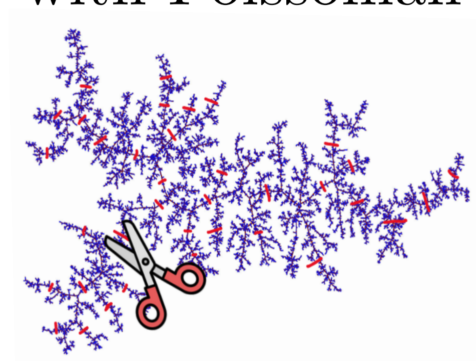

Conjecture
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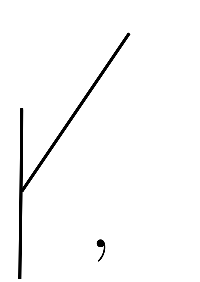

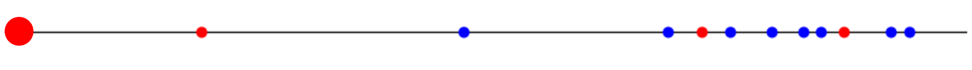

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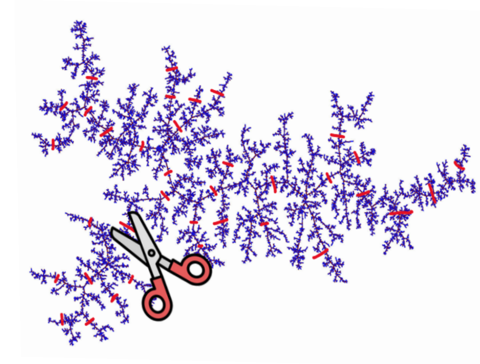

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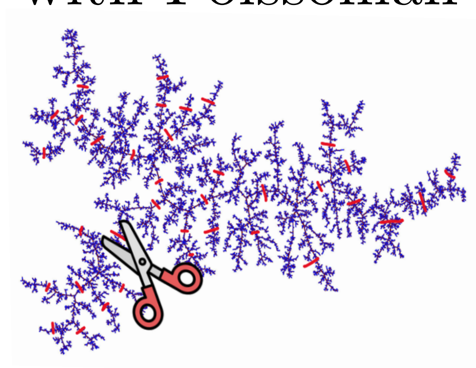

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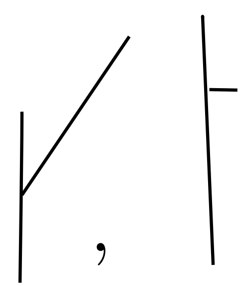

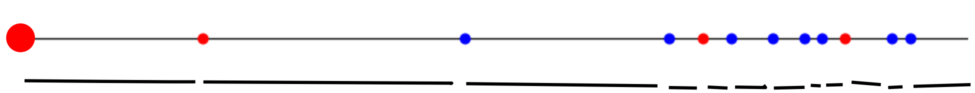

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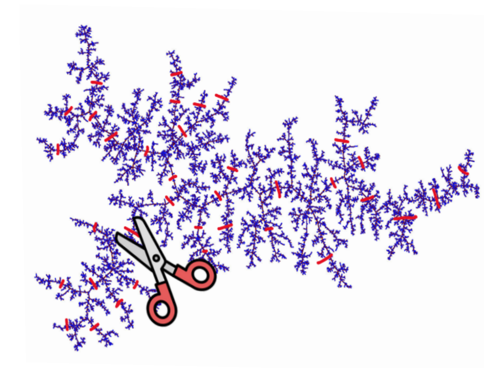

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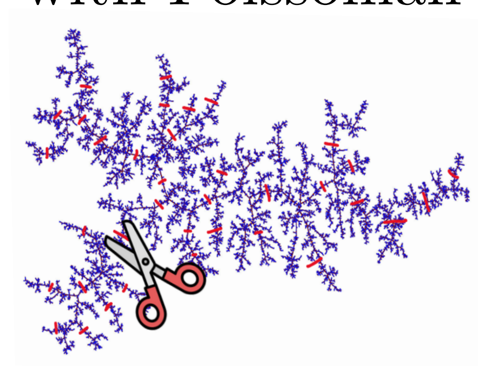

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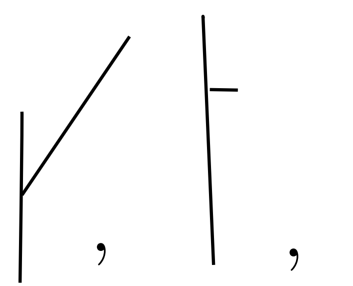

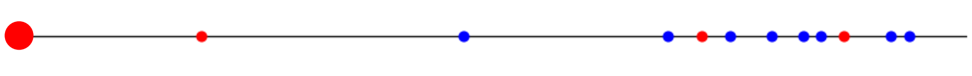

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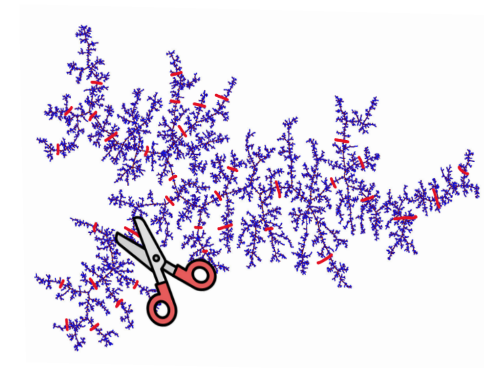

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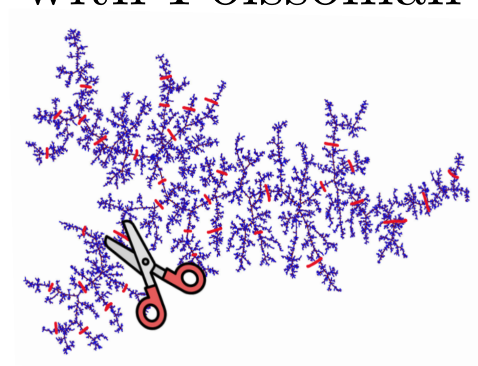

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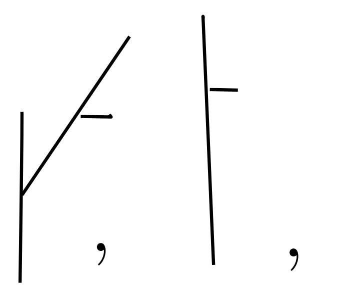

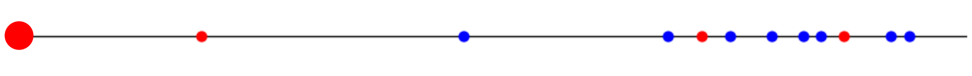

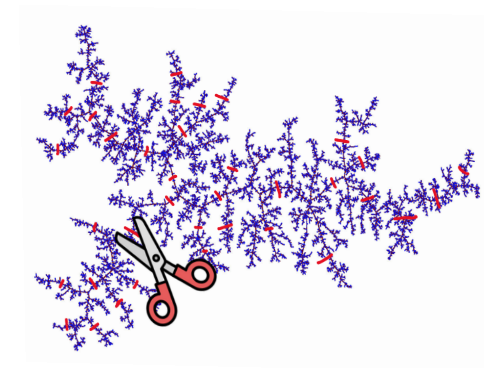

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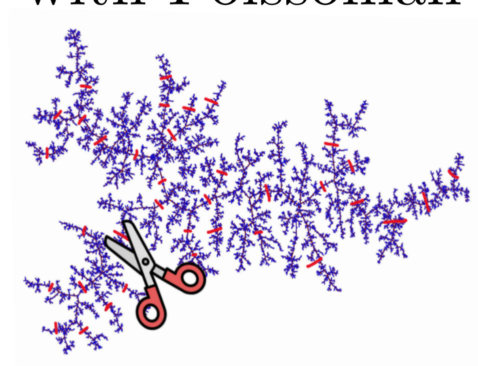

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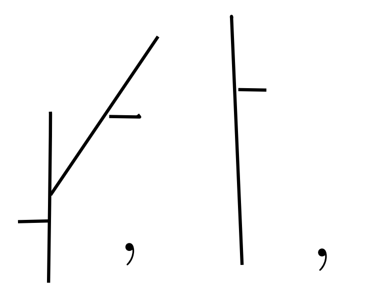

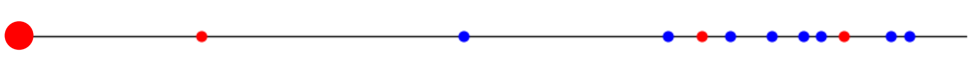

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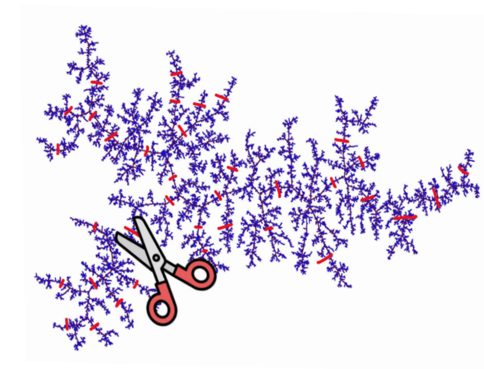

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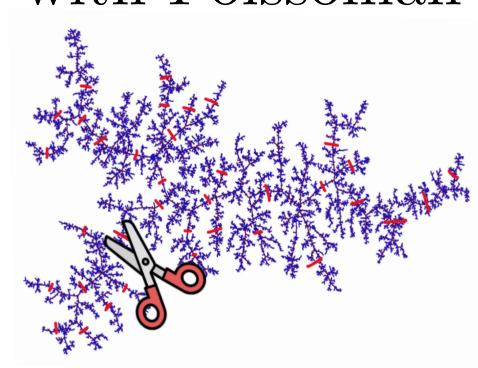

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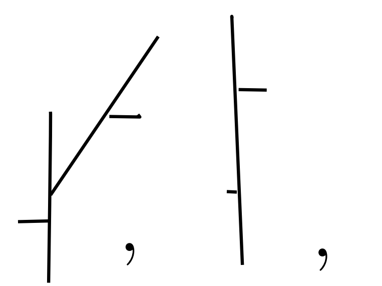

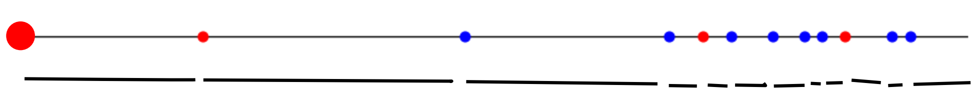

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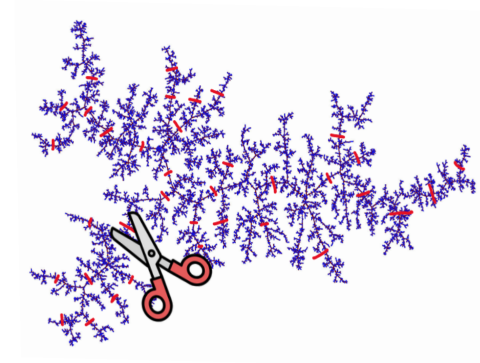

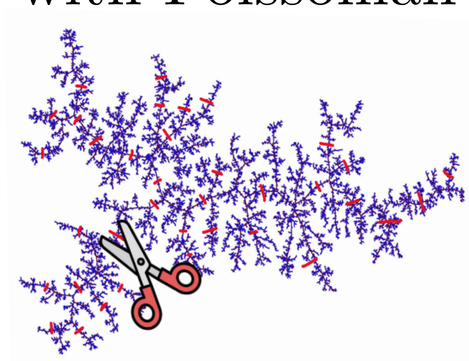

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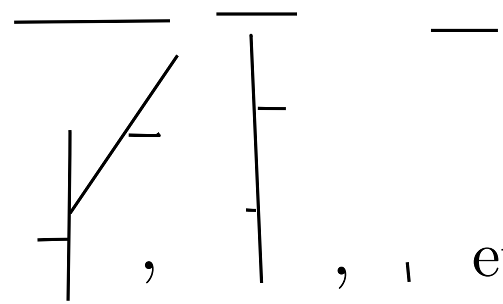
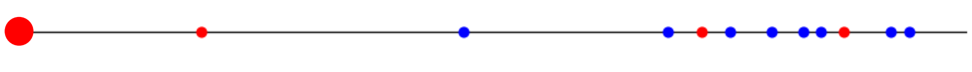


etc.

Conjecture
(Breki born in 98')



Aldous-Pitman fragmentation of the CRT

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(Aldous-Pitman 98') describe the evolution of the masses of the connected components of a Brownian CRT(T) cut according to a Poissonian rain P of intensity $d\lambda \otimes dt$ on $\mathrm{Sk}(T) \times \mathbb{R}_+$, where dt is the Lebesgue measure on $\mathbb{R}_+$ and λ is the length measure on the skeleton $\mathrm{Sk}(T)$ of T.

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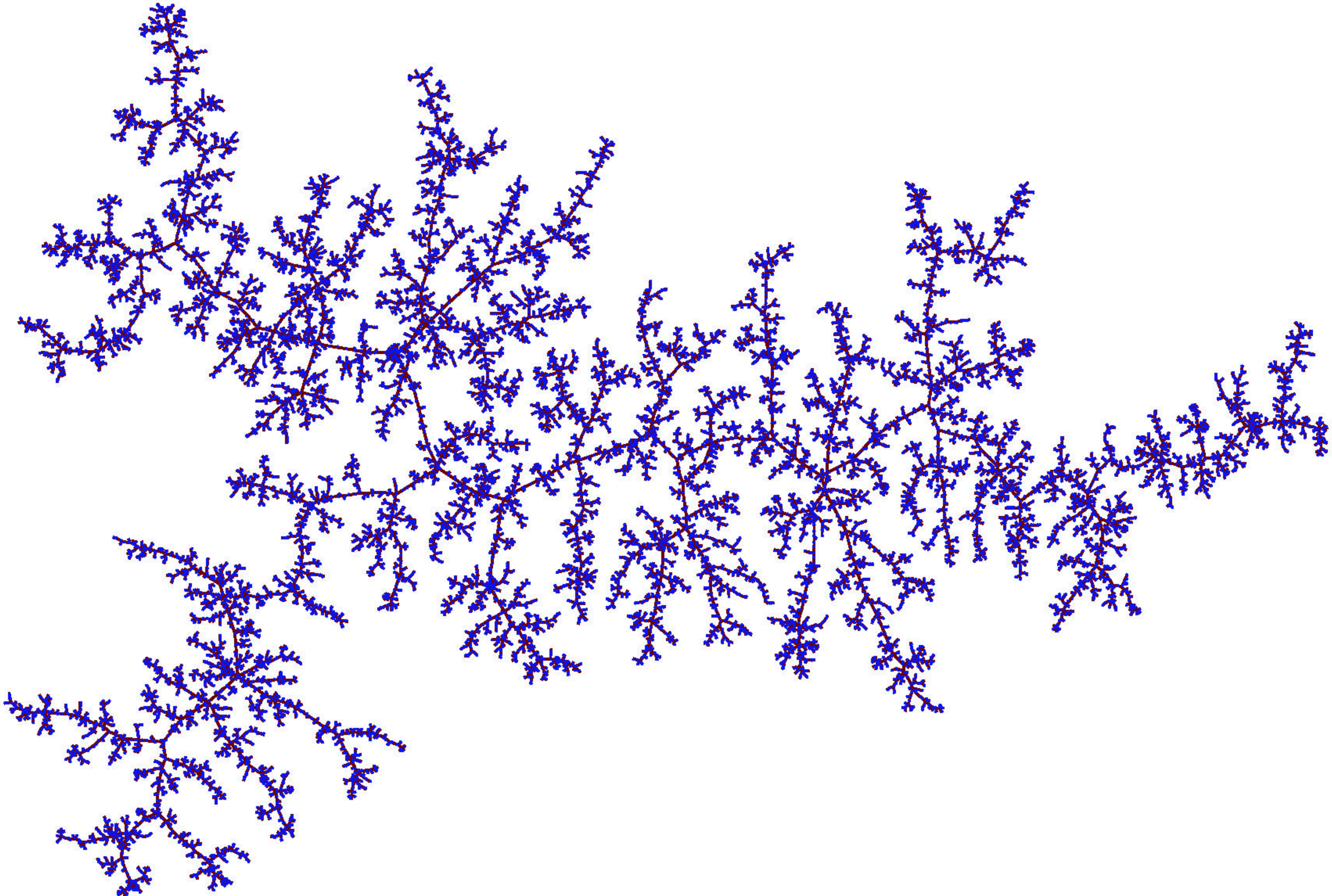
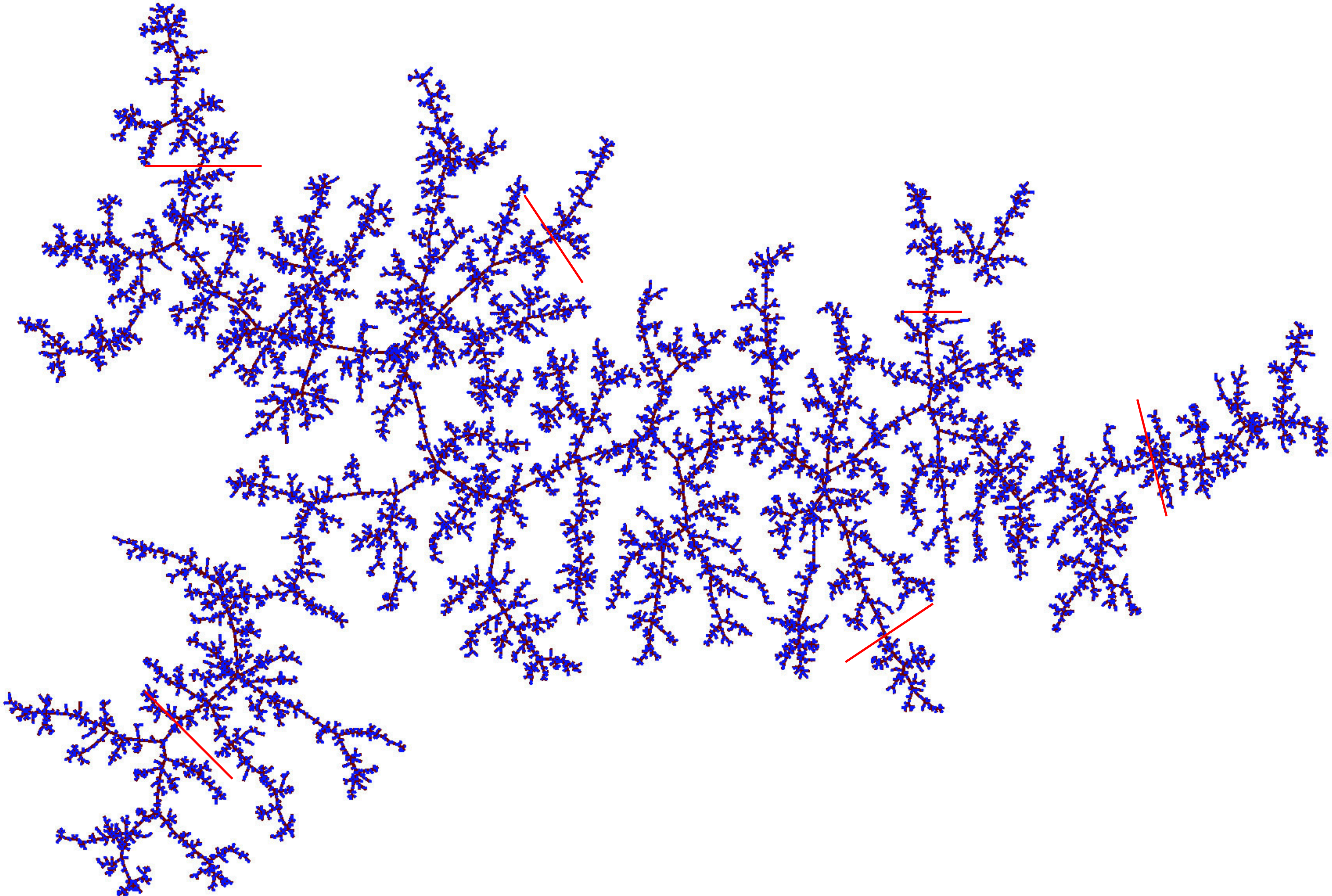
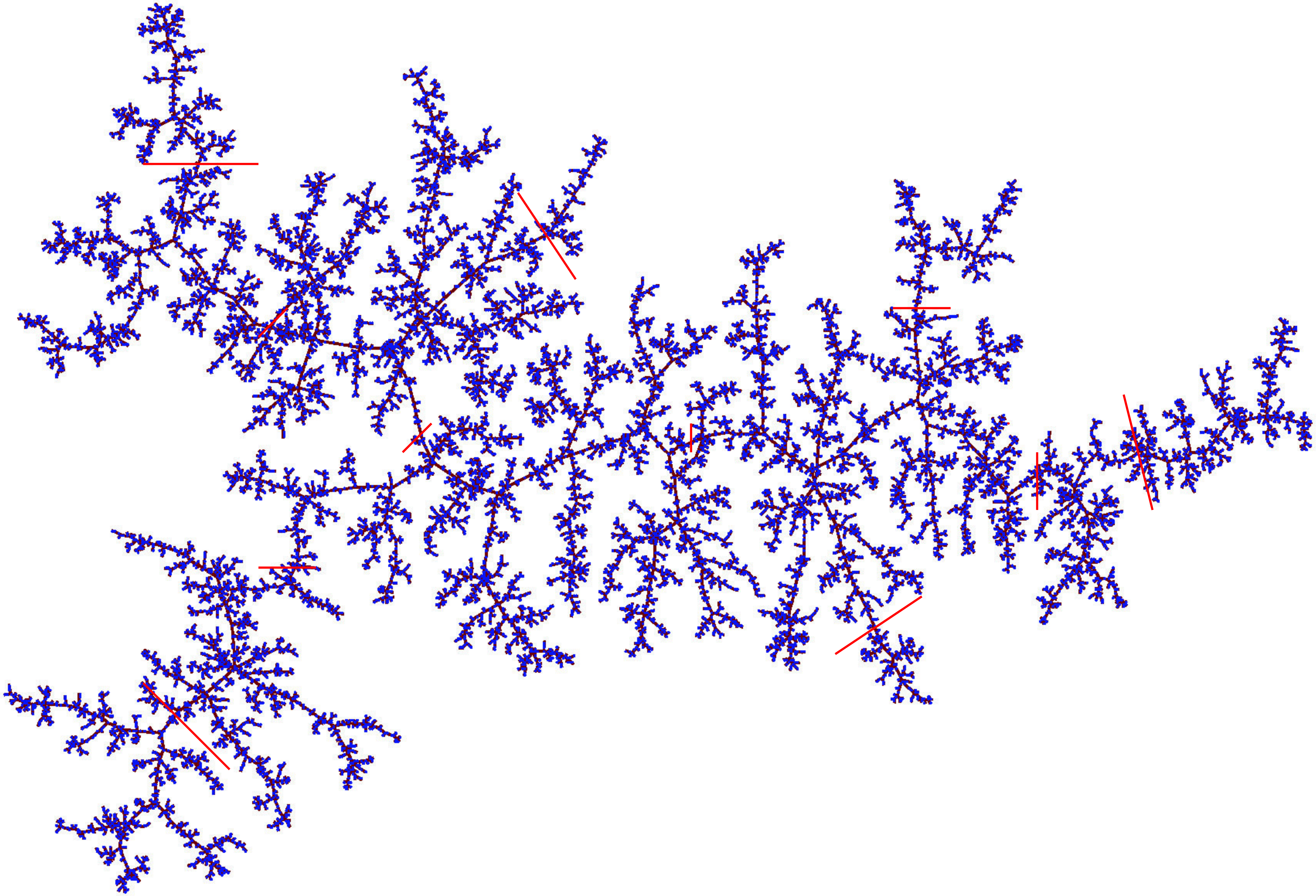
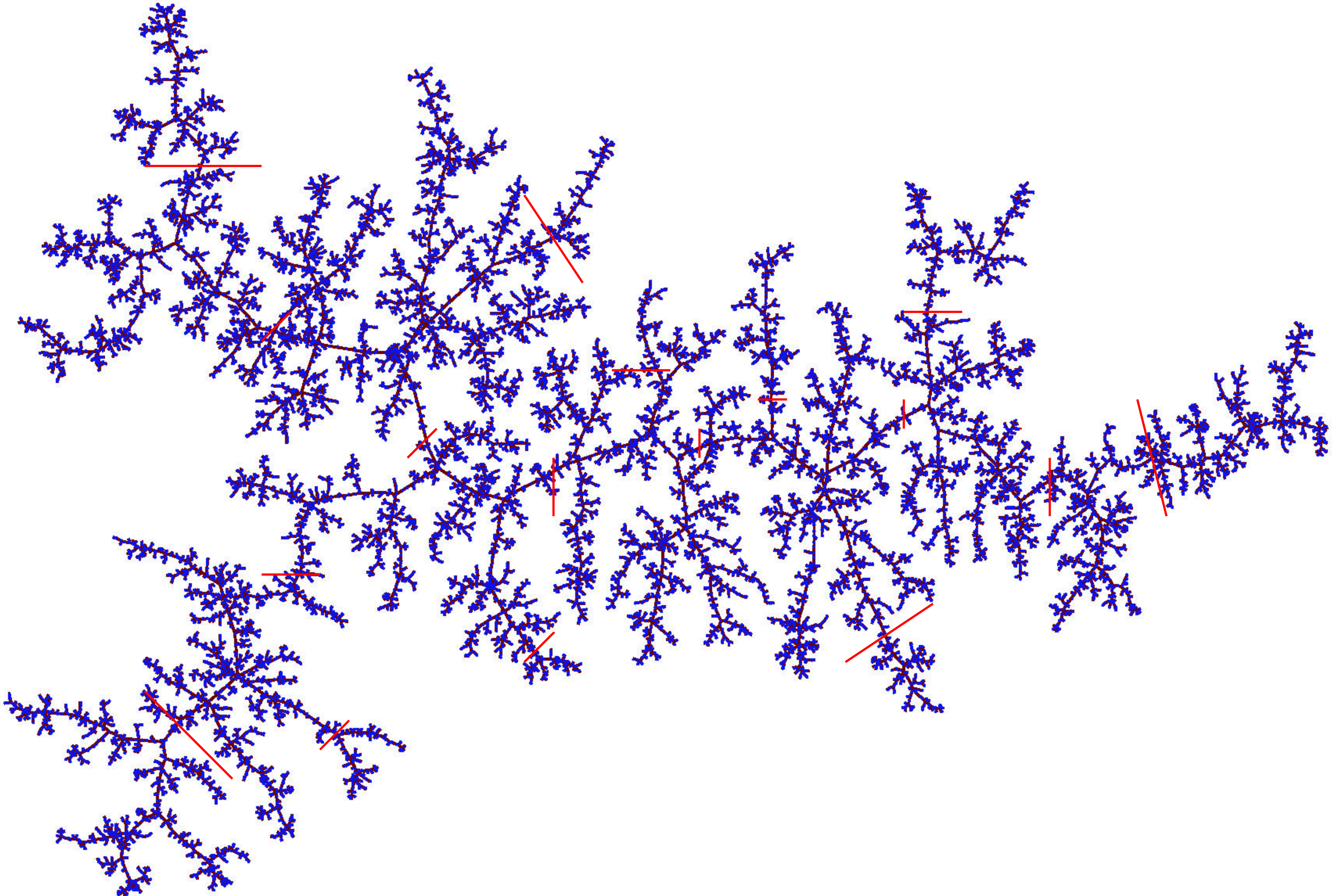
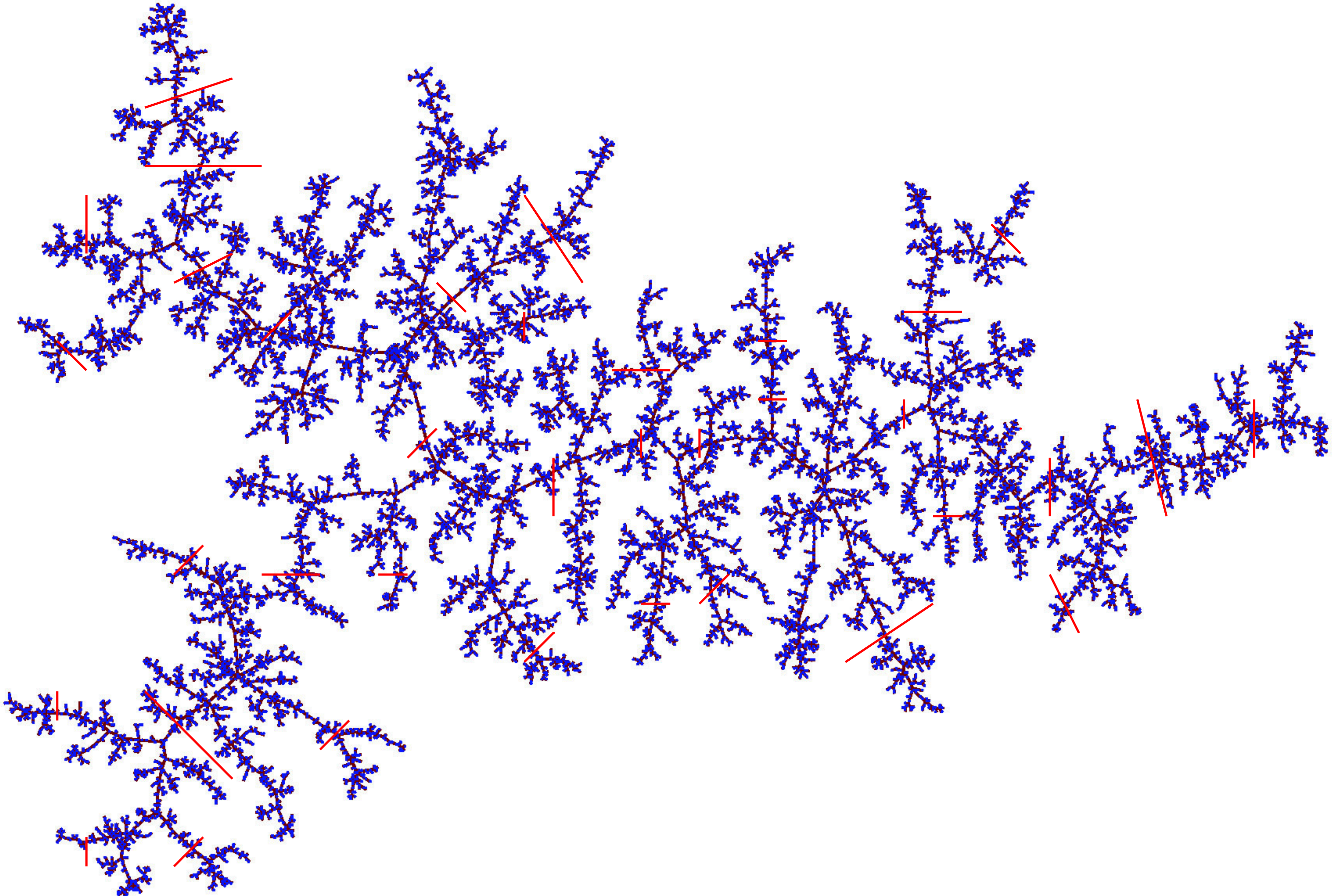
Although for a fixed t we expect the limit of the forest to converge to $\mathcal{F}_c$, we do not expect to obtain the distribution of the whole fragmentation process, because of the loop erasure.



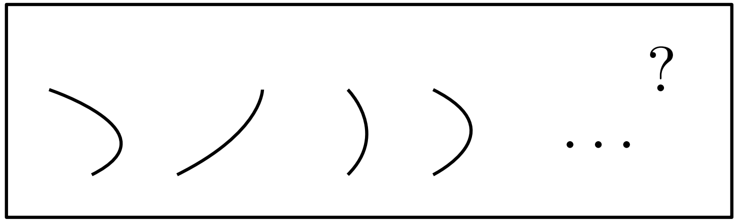








Fast stopping



The gluing happens at a large scale $\sim \sqrt{n}$,
but the cutting happens at a scale $\sim 1/p(n)$

Would depend on the topology of the forest?

No clear conjectures.

Further questions

(Work in progress) Wilson's algorithm on K_n with stopping parameter $\sim 1/c\sqrt{n}$ converges in distribution to the components of $\mathcal{F}_\lambda$ ordered in decreasing order in the space $(\mathbb{L}_4, d^4_{GHP})$.

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Does the same limit hold for more general graphs?

Can we show the same for Aldous-Broder algorithm?

Merci de votre attention,

ChercheuRs en probabiliTés.