Symmetry groups, semidefinite programs, and sums of squares

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1 Introduction

We will look at a fundamental problem in real algebraic geometry i.e. the existence and computation of a representation of a multivariate polynomial as a sum of squares (SOS). In other words, the question of finding $p_i \in \mathbb{R}[x], i = 1, \cdots, N$ such that

$$f(x) = \sum_{i=1}^{N} (p_i(x))^2.$$

This problem has applications in many fields of applied mathematics, such as continuous and combinatorial optimization as well as being theoretically interesting.

We will show a method that exploits symmetries in polynomials and semidefinite programming in order to get a reduction in the problem size in order to get faster and more accurate solutions may that be do to numerical conditioning or numerical errors.Word this better

The paper outlines the theoretical backgrond and gives the reader exaples explaining the definitions step by step in order to present an algorithm that is able to use the symetric properties of a polynomial that is invariant with respect to a certain representation and produces a solution to a semidefinite program given certain constraints.

2 Algorithm

Here we will present an algorithm that is the result of the paper and later explain certain concepts that we need to define in order to understand the algorithm

Algorithm I

Input: Linear representation ϑ of a finite group G on $\mathbb{R}^n$.

1. Determine all real irreducible representations of G.
2. Compute primary and secondary invariants θ_i, μ_j.
3. For each non-trivial irreducible representation compute the basis $b_1^i, \cdots, b_{r_i}^i$ of the module of equivariants.
4. For each irreducible representation i compute the corresponding matrix $\prod_i$.

Output: Primary and secondary invariants θ, μ and the matricies $\prod_i$.

Algorithm II

Input: Primary and secondary invariants θ, μ, matricies $\prod_i$ and $f \in \mathbb{R}[\theta]^G$.

1. Rewrite f in fundamental invariants giving $\tilde{f}(\theta, \mu)$.
2. For each irreducible representation determine $w_i(\theta)$ and thus the structure of the matrices $S_i \in \mathbb{R}[\theta]$.
3. Find a feasible solution of the semidefinite program corresponding to the constraints.

Output: SOS matrices S_i providing a generalized sum of squares decomposition of $\tilde{f}$.

3 Example

We will demonstrate the efficacy of this algorithm with an example.

4 Conclusion

Although it might seem cumbersome to find all the invariants of a representation....

[Gatermann_2004]

Hello World