Department of Mechanical Engineering
College of Engineering, University of the Philippines

ME11Lab: Term MP

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Instructions:

- This is a major activity covering the entire semester of ME11Lab. You are **strictly required** to use MATLAB® as your main computing tool and other computing tools which may be appropriate can be used in conjunction to, or as supplementary (for verification purposes), but not as replacement of MATLAB®.
- You are required to submit an MP Report entitled **"ME11Lab Term MP Report"** as an output requirement for this activity. You may include the MATLAB® script in the Report, which must be in `Courier New` font.
- You must submit the following:
  - The Term MP Report: hard copy and soft copy (PDF)
  - The M-files (compiled)

  Note: Both the PDF copy and M-files must be submitted through my email.
- **NOTE:** Always observe the *Honor Code: "You shall not take unfair advantage over your peers. You are strictly prohibited from DELIBERATELY copying the work of your classmates. HOWEVER, you may collaborate with your partner ONLY."*
- **Due Dates:** The due date for the Term MP Report (hard copy) will be on November 29, 2019 at the UP-DME Office, not later than 4:59PM. The soft copy (PDF) and M-files can be submitted on the same day but not later than 11:59PM. **Penalty** of 5% per working day until the **absolute deadline** on December 6, 2019 on or before 4:59PM (hard copy, DME Office) and 11:59PM (soft copy, through my email).

Computing Problems:

Problems are based on topics discussed in the Laboratory (ME11Lab) as well as the use of MATLAB®, in general.

1. Create a main script file that will allow the user to select on the following numerical methods of solving "$f(x)|_{x=c} = 0$" problems:

   (a) Bisection Method
   (b) Secant Method
   (c) Newton-Raphson Method
   (d) Regula-Falsi Method

   where c is the solution to the problem. Make sure to incorporate the IVT checker whenever applicable. The user must have a choice of which method to be used. The input will usually be the following: the function $f(x)$, the interval $[a, b]$, and tolerance ε. The output will be the solution c (or root) and the plot of $f(x)$ versus $x \in [a, b]$. Note that the user must be able to select which method to be used. [**Hint**: Use the `switch` built-in function in MATLAB®.]

2. Use the M-file developed in Prob.1 to solve the following problem: A quarterback throws a pass to his wide receiver running a route. The quarterback releases the ball at a height of h_Q. The wide receiver is supposed to catch the ball straight down the field 60 [ft] away at a height of h_R.

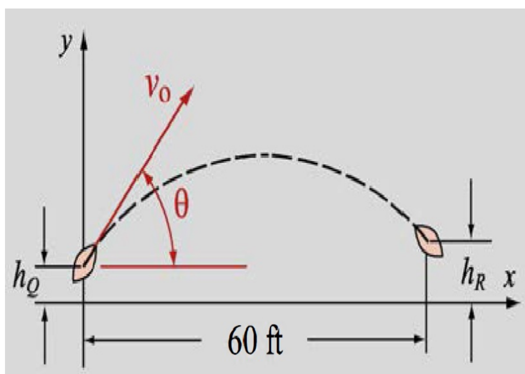

Figure 1: Figure for Problem 2: A quarterback throws the football to the receiver.

Recall the kinematic coordinates of projectile motion, (x, y), are defined as follows:

$$x = x_o + v_o \cos\theta\, t \tag{1a}$$

$$y = y_o + v_o \sin\theta\, t - \frac{1}{2}gt^2 \tag{1b}$$

(a) Show that, if $x_o = 0$ and $y_o = h_Q$, the vertical position of the football's motion is given by:

$$y = h_Q + x\tan\theta - \frac{x^2 g}{2v_o^2\cos^2\theta} \tag{2}$$

(b) At $x = 60$ [ft] and $y = h_R$, find the angle θ such that the receiver will catch the football. Note the following given data: $h_Q = 6.5$ [ft], $h_R = 6.85$ [ft], while the initial velocity of the throw is $v_o = 65$ [fps]. Use the *bisection method* to find the solution and use $\varepsilon = 0.01°$. Note that the acceleration due to gravity is $g = 32.2$ [ft/s^2]. [**Hint:** Set up first the "$f(\theta) = 0$" format of the problem, based on Eq. (2).]

3. Use the M-file developed in Prob.1 to solve the following problem: The force F acting between a particle with a charge q and a round disk of radius R and charge Q is given by the equation:

$$F = \frac{Qqz}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \tag{3}$$

where $\varepsilon_o = 0.885\times10^{-12}\,[\mathrm{C}^2/(\mathrm{Nm}^2)]$ is called the permittivity constant. Suppose that the disk has charge $Q = 9.5\times10^{-6}\,[\mathrm{C}]$ and radius $R = 0.1\,[\mathrm{m}]$ while the particle has charge $Q = 2.75\times10^{-5}\,[\mathrm{C}]$, find the distance z from the disk (see Fig. 2 for reference) such that the electrostatic force is $F = 0.5\,[\mathrm{N}]$. Use the *Newton-Raphson method* in finding the solution within accuracy of $\varepsilon = 1\times10^{-5}\,[\mathrm{m}]$.

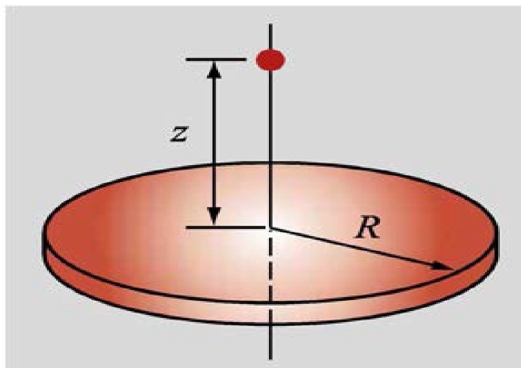

Figure 2: Figure for Problem 3: Electrostatic force between a disk and a particle.

4. A trough of length L has a cross section in the shape of a semicircle with radius R. Initially, it is fully-filled with water at volume $V_o\,[\mathrm{m}^3]$; after some time, the volume of water discharged through the hole at the bottom of the trough is $V_d\,[\mathrm{m}^3]$.

   (a) Derive the equation of volume of the remaining water as function of the height, h (from the top of the trough to the surface of remaining water passing along the radial direction), *i.e.*, derive the equation for $V = V(h)$. [**Hint:** You can do this manually by using some simple geometry principles.]

   (b) Suppose that $L = 10\,[\mathrm{m}]$, $R = 1.5\,[\mathrm{m}]$, and V_d is 40% of initial volume, find the depth of remaining water, within accuracy of $\varepsilon = 0.001\,[\mathrm{m}]$. Use the *regula falsi method* developed in Prob.1 to find the solution.

5. Create a script file that will require the user to input a function $f(x)$ and the vector $\{\mathbf{X}\}$ containing nodes X_j. These nodes serve as the interpolating points where $(n+1)$ is the total number of nodes. The user must have the option to choose between the following output:

   (a) The Vandermonde polynomial $\mathcal{V}_n(x)$ of order n as well as the plot of $f(x)$ and $\mathcal{V}_n(x)$ versus $[a, b]$, where $a = X_1$ and $b = X_{n+1}$.

   (b) The Lagrange polynomial $\mathcal{L}_n(x)$ of order n as well as the plot of $f(x)$ and $\mathcal{L}_n(x)$ versus $[a, b]$, where $a = X_1$ and $b = X_{n+1}$.

6. Use the program developed Prob. 5 to solve the following problem: The torsion stress factor for rectangular cross-section prismatic bars made of homogeneous and isotropic materials (*i.e.*, conventional materials such as metals) is given by Danao and Cabrera as follows:

$$K = \frac{B}{A} \tag{4}$$

where

$$A = \frac{1}{3}\left\{1 - \frac{192}{\pi^5 c}\left[\tanh\left(\frac{\pi c}{2}\right) + \sum_{m=1}^{\infty}\frac{\tanh\left[\frac{(2m+1)\pi c}{2}\right]}{(2m+1)^5}\right]\right\} \tag{5a}$$

$$B = 1 - \frac{8}{\pi^2}\left[\frac{1}{\cosh\left(\frac{\pi c}{2}\right)} + \sum_{m=1}^{\infty}\frac{1}{(2m+1)^2\cosh\left[\frac{(2m+1)\pi c}{2}\right]}\right] \tag{5b}$$

It should be noted that since the functions $A = A(c)$ and $B = B(c)$ are both functions of the *aspect ratio* c (*i.e.*, $c = b/a$, where a and b are the half-length dimensions of the narrow and wide sides of the rectangle, respectively), then the torsion stress factor K should also be a function of c, *i.e.*, $K = K(c)$. For given values of $\{c_j\} = \{1.0; 1.5; 2.0; 3.0; 5.0\}$, find the Langrange polynomial $\mathcal{L}(c)$ to replace the complicated-looking function of torsion stress factor $K(c)$. Also, plot $K(c)$ and $\mathcal{L}(c)$ versus $c \in [1.0, 5.0]$ and put a legend in the graph.

7. A fin is an extended surface used to transfer heat from a base material (at $x = 0$) to an ambient space. Heat flows from the base material through the base of the fin, through its outer surface, and through the tip (see Fig. 3 for reference). Measurement of the temperature distribution along a fin gives the following data (see attached text file):
`ME11Lab_TermMP_Problem7_Data_HeatTransferThruAFin`

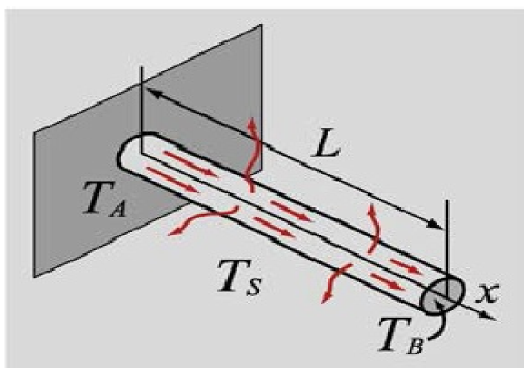

Figure 3: Figure for Problem 7: Heat transfer through a fin.

The fin as length of $L = 10\,[\mathrm{cm}]$. The heat flux per unit area (of the fin's cross-section), q_x, is given by:

$$q_x = -\kappa\frac{\mathrm{d}T}{\mathrm{d}x} \tag{6}$$

where κ is the fin's thermal conductivity. Compute the heat flux $q_x(x_j)$ at each node x_j (for $j = 1, 2, 3, ..., 11$) over the entire length of the fin. Also, plot the discrete heat flux $q_x(x_j)$ versus $x_j \in [0, L]$, for each $j = 1, 2, 3, ..., 11$. Note that $\kappa = 240.5\,[(\mathrm{W/m})/\mathrm{K}]$ is the fin's *thermal conductivity*. [**Hint:** Approximate the first-order derivative in Eq. (6) using the three-point forward difference at $x = 0$ and the three-point backward at $x = L$; as for the interior nodes, use the three-point midpoint difference scheme.]

8. A 30 [ft]-long uniform beam is simply supported at the left end ($x = 0$) and clamped at the right end ($x = L$). The beam is subjected to the triangular load shown (see Fig. 4 for reference). The deflection of the beam is given by the differential equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{M}{EI} \tag{7}$$

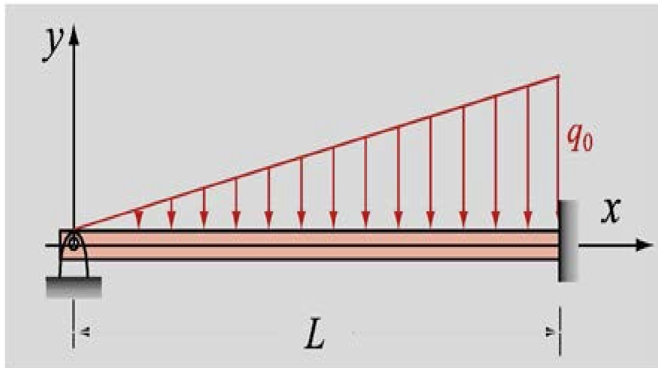

Figure 4: Figure for Problem 8: Simply-supported beam at left end and clamped at right end.

where $M = M(x)$ is the bending moment which is a function of x- coordinate (lengthwise direction of the beam), while $E = 29 \times 10^6$ [psi] is the elastic modulus and $I = 730\,[\mathrm{in}^4]$ is the beam's cross-sectional moment of inertia. It should be noted that the deflection $y = y(x)$.

(a) Develop a script file that calculates and plots the bending moment $M(x)$ versus $x \in [0, L]$. Use the data given in the attached text file:
`ME11Lab_TermMP_Problem8_Data_BeamSimplySupportedAndClamped`

(b) Plot $M(x)$ versus $x \in [0, L]$.

9. One problem for numerical solutions of first-order ODEs (to follow).....

10. One problem for numerical solutions of second-order ODEs (to follow).....