Lecture 1

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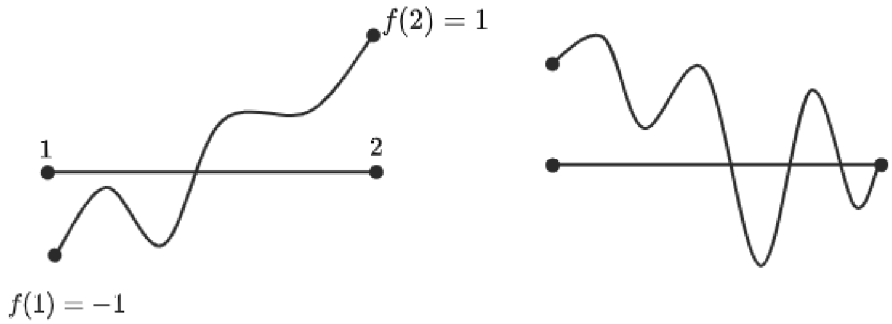

2 A Sneak Preview

Definition

$$\lim_{n \to a} f(x) = L$$

If x is close enough to a, $f(x)$ will be as close to L as you like. If $a, b \in \mathbb{R}$, the distance between a, b is expressed as $|a - b|$. If ϵ means how close we want $f(x)$ to be to L, what we are looking for is $|f(x) - L| < \epsilon$. For any $\epsilon > 0$, there exists a $\delta > 0$, such that for any x, if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x - 2} = \lim_{x \to 2} x + 2 = 4$$

Prove $\lim_{x \to 3} 7 - 2x = 1$. First, say what this means. For any $\epsilon > 0$, $\exists \delta > 0$, such that if for any x, if $0 < |x - 3| < \delta$, then $|\underbrace{(7 - 2x)}_{F(x)} - \underbrace{1}_{L}| < \epsilon$. Let ϵ be an arbitrarily chosen positive real number. Let $\delta =?$

<u>Scratch Work:</u>

We need

$$\begin{aligned}
|(7 - 2x) - 1| &< \epsilon \\
|6 - 2x| &< \epsilon \\
2|3 - x| &< \epsilon \\
2|x - 3| &< \epsilon \\
|x - 3| &< \frac{3}{2}
\end{aligned}$$

So we say $\delta = \frac{3}{2}$. Let x be arbitrary. Assume $0 < |x - 3| < \delta = \frac{3}{2}$. Our goal is to show: $|(7 - 2x) - 1| < \epsilon$.

$$|(7 - 2x) - 1| = |6 - 2x| = 2|3 - x| = 2|x - 3| < 2 \cdot \frac{3}{d} = 3. \checkmark$$

We know $\lim_{x \to 5} x^2 = 25$. What does this mean? For each $\epsilon > 0$, there is $\delta > 0$, such that if $0 < |x - \underbrace{5}_{a}| < \delta$, then $|\underbrace{x^2}_{f(x)} - \underbrace{25}_{L}| < \epsilon$. Choose $\epsilon > 0$ arbitrarily.

<u>Scratchwork:</u>

Aim: $|x^2 - 25| < \epsilon$.

$$\begin{array}{ll} |x^2 - 25| & < \epsilon \\ |x-5||+5| & < \epsilon \\ \left[x - 5 < \dfrac{\epsilon}{|x+5|} \right] & \text{NOT!} \end{array}$$

Notice that if any δ works, then any smaller δ works. It's harmless and useful just to <u>require</u> $0 < |x - 5| < 1$.

$$\begin{aligned} |x-5| &< 1 \\ -1 < x - 5 &< 1 \\ 4 < x &< 6 \\ 9 < |x+5| = x + 5 &< 11 \end{aligned}$$

so

$$\frac{\epsilon}{11} < \frac{\epsilon}{|x+5|}$$

We let $|x - 5| < \frac{\epsilon}{11}$. Let $\delta = \min\left(1, \frac{\epsilon}{11}\right)$.