Perception and Dissonance:
Exploring Combinations of Common Musical Instruments

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Abstract

When two pure tones are played simultaneously, the human ear perceives the resulting sound very differently depending upon the frequency difference between the two tones. If the frequency difference is within a certain critical bandwidth, the beat phenomenon will occur, and the physiological result is the perception of roughness or musical dissonance. A mathematical way to describe the dissonance of two pure tones was developed by Sethares in 1993. In his research, he proposed a function that could be used to calculate the amount of dissonance that will be perceived when two tones with differing frequencies are played together. This function can be used to calculate the dissonance between complex sound sources that contain multiple tones (a.k.a. harmonics).

All harmonic instruments produce sound that is comprised of many different frequencies which are often integer multiples of the fundamental. The relative amplitudes of these partial frequencies are what characterize the unique timbre of each instrument. When two different notes are played on the same instrument the partials will interfere with each other and create some amount of dissonance. The amount of dissonance generated in this way can then be calculated across a range of frequency differences to produce a dissonance curve for each instrument. The local minima on these curves represent frequency ratios which will result in the least dissonance, or greatest consonance. The purpose of my research is to examine the dissonance curves of many different dual combinations of harmonic instruments. We construct a parameter that describes the dissonance of these combinations.

Background

Each note played by a musical instrument is a combination of many frequencies vibrating simultaneously. For harmonic instruments, these frequencies form a harmonic series. This means each of the frequencies in the set (called partials) are integer multiples of the fundamental frequency. Each instrument's partials are produced at different amplitudes. The relative difference in amplitude between these partials is the main contributor to an instrument's timbre. When two complex notes are played together, their frequency difference as well as their individual timbres create a certain amount of consonance (pleasantness) and dissonance (roughness). Many theories have been proposed to explain this phenomenon.

Pythagoras noticed that the ratio of lengths between two vibrating strings was related to how dissonant they would sound when played together. He concluded that the simpler the ratio the more consonant the sound would be. The canonical consonant intervals: octave, perfect fifth, and perfect fourth have ratios: 2:1, 3:2, 4:3 respectively. While the most common dissonant intervals: the minor second, and the tritone have intervals of 16:15 and 25:18.

Plomp and Levelt (1965) attempted to explain this theory with the concept of a critical bandwidth. They theorized that there is a certain frequency bandwidth that two tones must be within to produce dissonance, and that the point of maximum dissonance occurs at around 25% of the critical bandwidth. They also proposed the idea that for complex tones, the total dissonance of the sound can be calculated by considering the dissonance between each pair of partials. Sethares (1993) further refined these ideas and presented a mathematical model to quantify the total dissonance that a complex sound will produce. He noticed that complex timbres often produce great consonance at frequency ratios corresponding to common scale intervals. Thus, instruments can be matched with scales that have many notes at consonant points for their timbre to achieve greater overall consonant sounds.

Purpose

Since each harmonic instrument has a unique set of partials, any two instruments played together will create a unique acoustical interaction. Some amount of dissonance will be present for each pair. One could then ask: Which two instruments, when played together, produce the least dissonance? The answer to this question could find application in the way that composers select instruments for their compositions. The purpose of this project is to define a mathematical parameter to describe how well two harmonic instruments pair together to minimize dissonance, then use this parameter to rank each pair of 22 different harmonic instruments from most dissonant to most consonant.

Methods

I used audio recordings of 22 different harmonic instruments, each consisting of a single note played at a frequency of 440 hertz. Using the Python libraries wavfile and scipy, I read the audio files and performed Discrete Fourier Transforms on each to obtain each instrument's frequency spectrum. From the spectra, I selected the frequencies and corresponding amplitudes that were the most prominent for each instrument. Partials with very small amplitudes were ignored. The spectrum and selected partials for the clarinet are shown in figure 3d.

The partials that I extracted from the spectra were then used with Sethares's dissonance function (Equation 1) to generate plots showing the dissonance between each pair of instruments over a range of frequency ratios. They are shown in figures: 4a, 4b, 4c, and 4d. Figures 4e and 4f show overlaid dissonance curves of one instrument compared to all 21 others.

$$D_F = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} d(f1_i, f2_j, v1_i, v2_j)$$

$$d(f_1, f_2, v_1, v_2) = v_1 v_2 (e^{\frac{-ad^*|f_2 - f_1|}{s_1 f_1 + s_2}} - e^{\frac{-bd^*|f_2 - f_1|}{s_1 f_1 + s_2}})$$

Equation 1: Sethares's dissonance function. f_1 and f_2 are the two partial frequencies. v_1 and v_2 are the two partial amplitudes. n_1 and n_2 are the number of partials in the two sets. d is the point of maximum dissonance: 0.24. a, b, s_1, and s_2 are fit parameters*

To determine how well the instruments paired together within a particular cardinality (the number of note divisions in a tuning). I used Krantz and Douthett's Integrated Scale Desirability Function (ISDF) function (Equation 2). This function returns a numeric value in the range (-0.5, 0.5) and involves an integration over a spectrum's dissonance function. The closer the value is to 0.5, the better the spectrum is suited to the cardinality c. For my analysis I used a c value of 12 as this is the dominant cardinality that the majority of western music uses. I calculated ISDF values for every possible pair of the 22 instrument samples (231 values) and ranked them.

$$K'_{F,b}(c) = \frac{5C'_{F,b}}{(C'_{F,b} - 1)} + \frac{20}{\ln(b)(1 - C'_{F,b})} \int_1^b \frac{D'_F(y)}{y} \left| \left\{ c\frac{ln(y)}{\ln(b)} + \frac{1}{2} \right\} - \frac{1}{2} \right| dy$$

Equation 2: Krantz and Douthett's Integrated Scale Desirability Function (ISDF). c is the cardinality. $D'_F(y)$ is the normalized dissonance function. b is the interval of closure. $C'_{F,b}$ is a scaling constant. y is the frequency ratio.

Results

After running each combined spectrum through the ISDF function with a cardinality of 12, the pairs were ranked according to their values. The top 10 and bottom 10 values are shown in figure 1. It is readily apparent that 9 of the top 10 combinations contain the tenor saxophone and that 6 of the bottom 10 combinations contain the vibes. This result makes little intuitive sense. The prevalence of these two instruments is likely just due to the number of partials that were extracted from their spectra. The tenor saxophone had the most partials and the vibes had the least. If ISDF is plotted against number of partials a clear linear trend can be seen (figure 2). Therefore, I conclude that the ISDF is not a useful measure of instrument coupling for combined spectra. Further research should consider possible ways to normalize instruments with respect to their number of partials for a better comparison.

Top 10 Instrument Pairs	ISDF
Harpsichord and Tenor Saxophone	0.254
Nylon Guitar and Tenor Saxophone	0.246
Mandolin and Tenor Saxophone	0.245
Steel String Guitar and Tenor Saxophone	0.245
Piano and Tenor Saxophone	0.243
Banjo and Tenor Saxophone	0.241
Harp and Tenor Saxophone	0.235
Oboe and Tenor Saxophone	0.235
Tenor Saxophone and Violin	0.230
Piano and Soprano Saxophone	0.227

Bottom 10 Instrument Pairs	ISDF
Harp and Vibes	0.028
Banjo and Vibes	0.028
Steel String Guitar and Vibes	0.039
Harpsichord and Vibes	0.043
Harp and Steel String Guitar	0.043
Banjo and Harp	0.044
Nylon Guitar and Vibes	0.047
Mandolin and Vibes	0.048
Banjo and Steel String Guitar	0.064
Banjo and Clarinet	0.069

Figure 1: Top 10 and Bottom 10 instrument pairs ranked by their ISDF values

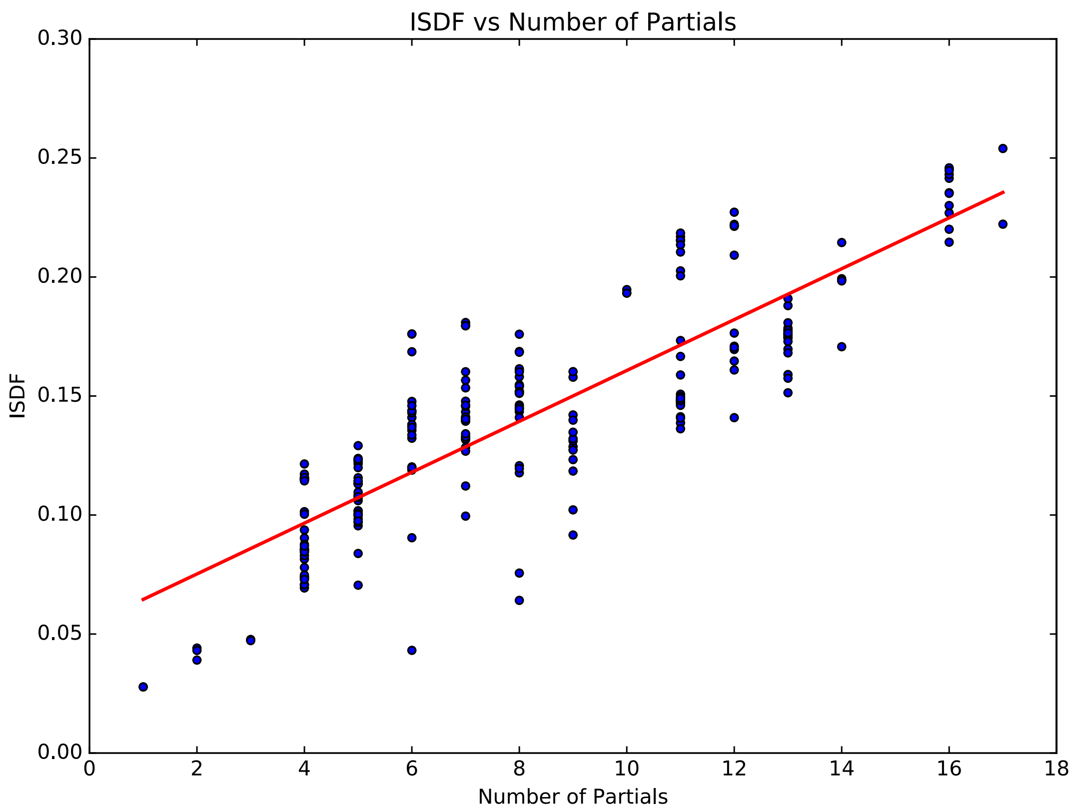


Figure 2: ISDF value as a function of Number of Partials

Instrument spectra

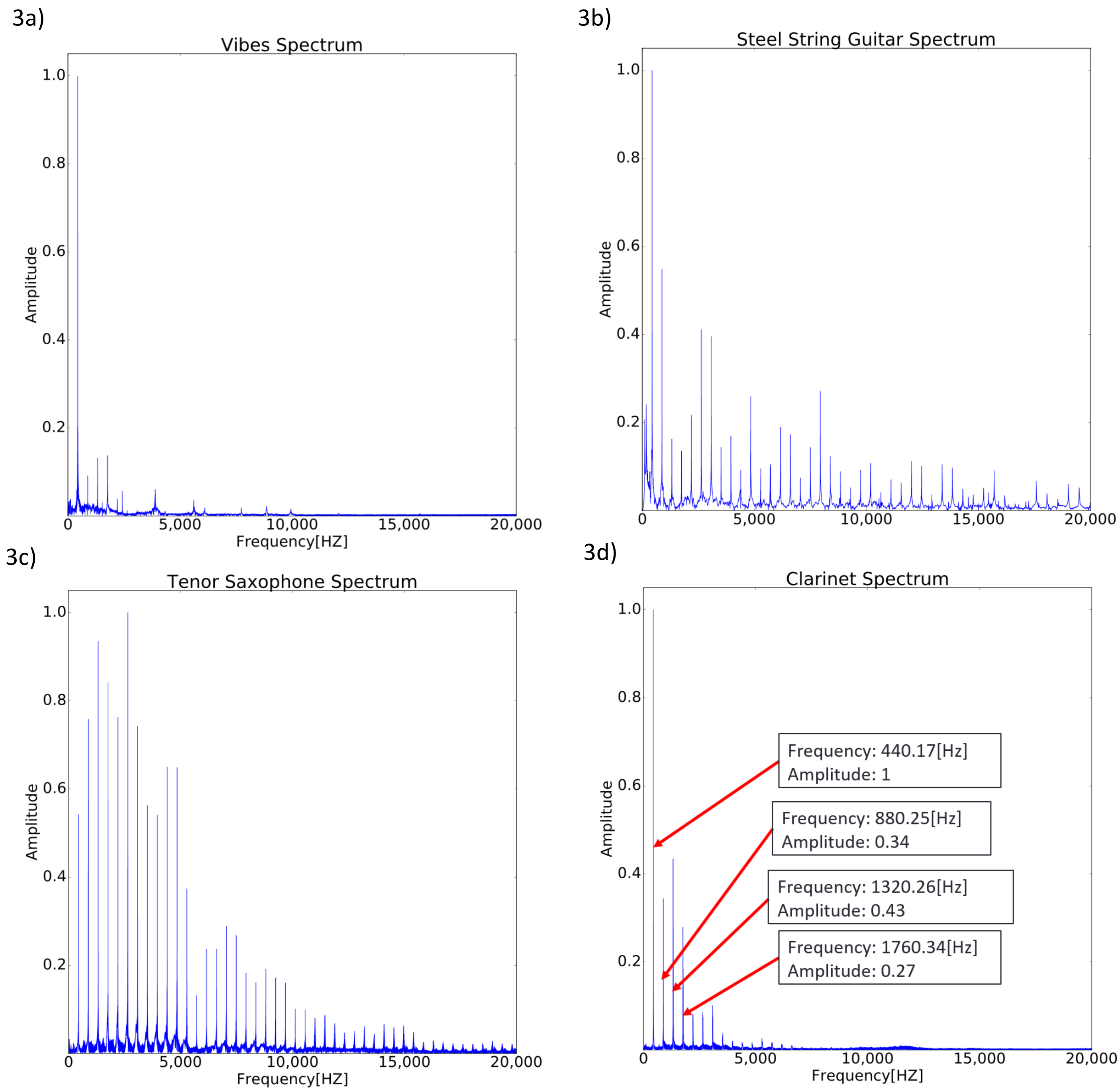
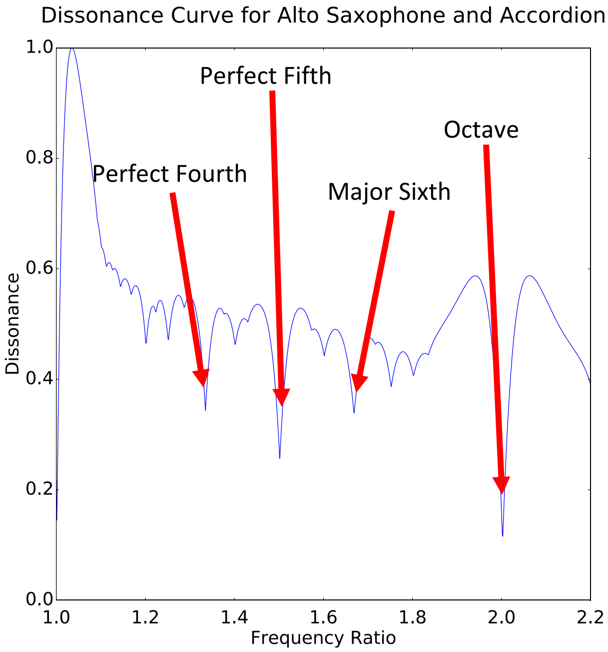


Figure 3: Frequency Spectra: a. Vibes, b. Steel String Guitar, c. Tenor Saxophone, d. Clarinet with chosen partials

Dissonance Curves

4a)



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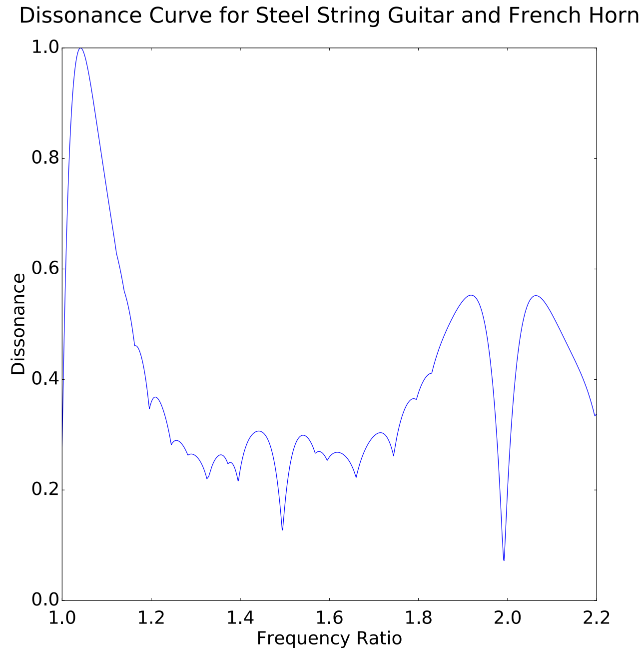

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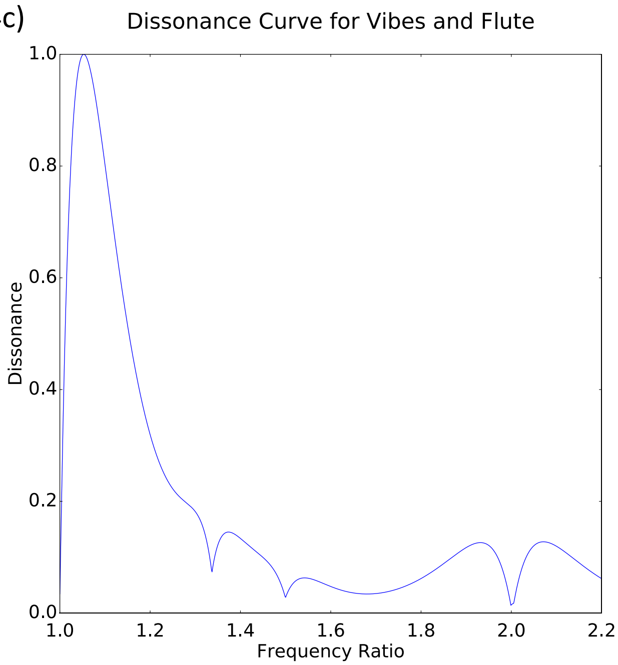

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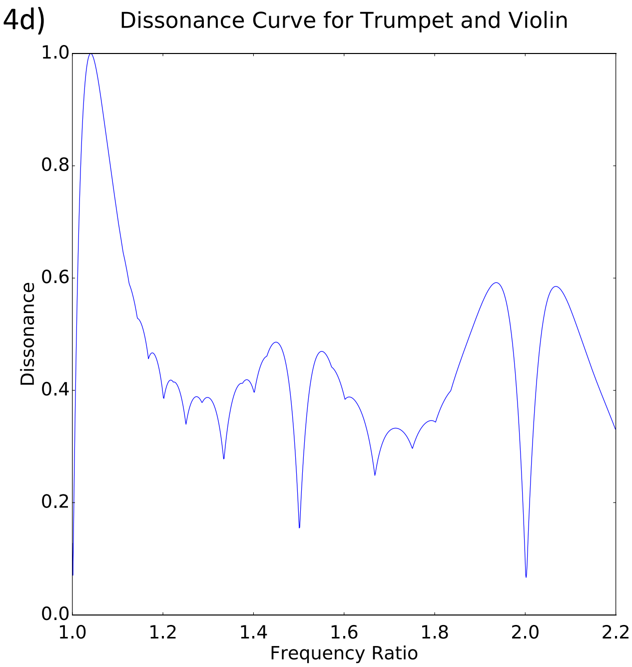

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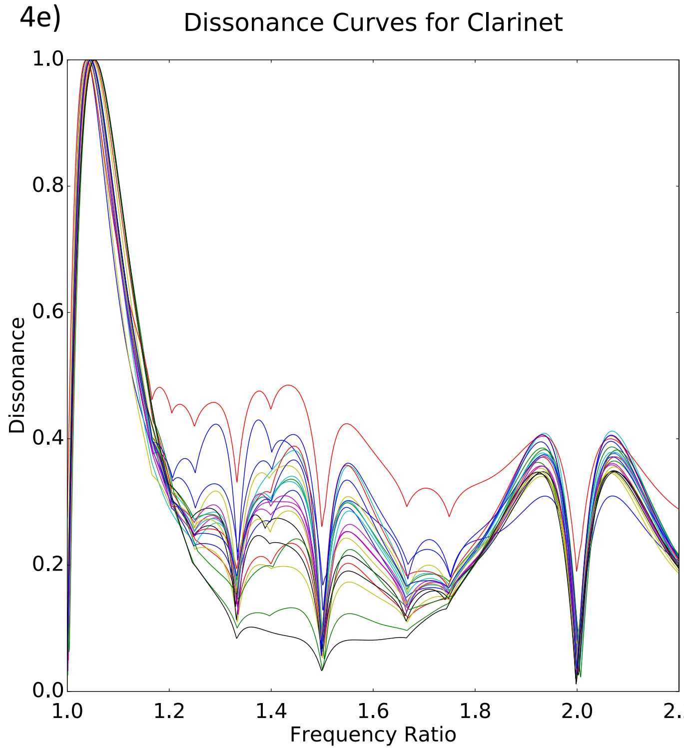

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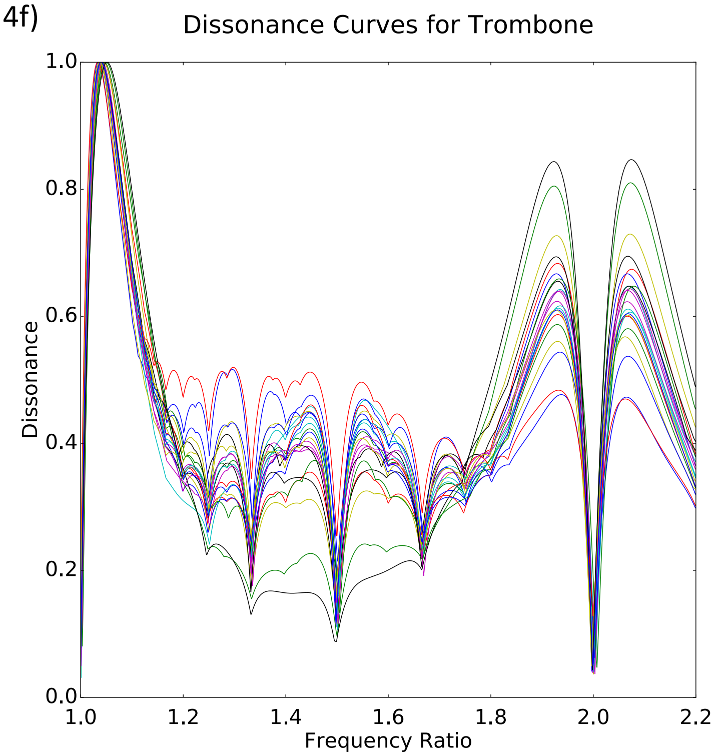

Figure 4: Dissonance curves: a. Alto Saxophone and Accordion, b. Steel String Guitar and French Horn, c. Vibes and Flute, d. Trumpet and Violin, e. Clarinet and all others, f. Trombone and all others

References

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3. Krantz, R. J., & Douthett, J. (2006). The Integrated Scale Desirability Function: A Musical Scale Consonance Measure Based on Perception Data. In Bridges London: Mathematics, Music, Art, Architecture, Culture (pp. 183-190). Tarquin Publications.