

Time Complexity (Medium)

What is the tightest bound for the worst case in the following function according to the input n ?

1.

```
int f(int n)
```

```
{
```

```
    int count = 0;
```

```
    for (int i = 0; i < n; i+=10)
```

```
        for (int j = i; j > 0; j--)
```

```
            count = count + 1;
```

```
    return count;
```

```
}
```

$$\frac{n}{10} \cdot \frac{n}{10}$$

(4 marks)

☐

$O(1)$

☐

$O(\log n)$

☐

$O(n)$

☐

$O(n \log n)$

☒

$O(n^2)$

2.

```
// Assume  $n \geq m$ 
```

```
int gcd(n,m)
```

```

{
    if (n%m ==0) return m;

    n = n%m;

    return gcd(m, n);
}

```

(4 marks)

☐ O(1)

☒ O(log n)

☐ O(n)

☐ O(n log n)

☐ O(n²)

3.

```


int f(int n) {
    int i, j, k = 0;

    for (i = n/2; i <= n; i++)
        for (j = 2; j <= n; j = j * 2)
            k = k + n/2;

    return k;
}

```

$\frac{n}{2}$
 $\log_2(n)$



(4 marks)

- ☐ $O(1)$
- ☐ $O(\log n)$
- ☐ $O(n)$
- ☒ $O(n \log n)$
- ☐ $O(n^2)$

4.

```
int f(int n)
```

```
{
```

```
    if (n <= 1)
```

```
        return n;
```

```
    return f(n-1) + f(n-1);
```

```
}
```

$$2T(n-1) = 2^k T(n-k) \\ k = n-1$$

(4 marks)

- ☐ $O(\log n)$
- ☐ $O(n)$
- ☐ $O(n \log n)$
- ☐ $O(n^2)$

☒ $O(2^n)$ Recursion tree

5.

```
int f(int n)
{
    if (n <= 10)
        return n;
    return f(n/4) + f(n/4) + f(n/4) + f(n/4);
}
```

$$4^{\log_4 n} = n$$

(4 marks)

☐ $O(1)$

☐ $O(\log n)$

☒ $O(n)$

☐ $O(n \log n)$

☐ $O(n^2)$

☐ $O(2^n)$



6.

```
// What is the time complexity for f(n)?
```

```
int f(int n)
```

```

{
    for (int i=2; i<n; i*=2) |  $O(\log_2 n)$ 
        g(n);

    return 0;
}

int g(int m)
{
    if (m>1)
        g(m-10);

    return 0;
}

```

(4 marks)

- ☐ $O(1)$
- ☐ $O(\log n)$
- ☐ $O(n)$
- ☒ $O(n \log n)$
- ☐ $O(n^2)$

// What is the time complexity for f(n)?

```
int f(int n)
{
    for (int i=0; i<n; i*=2)
        g(n);
    return 0;
}
```

```
int g(int m)
{
    if (m>1)
        g(m-10);
    return 0;
}
```

$O(n)$

(4 marks)

☐ $O(1)$

☐ $O(\log n)$

☐ $O(n)$

☐ $O(n \log n)$

☐ $O(n^2)$



Infinite loop

Time Complexity (Advanced)

What is the tightest bound for the worst case in the following function according to the input n ?

8.

```
int f(int n)
```

```
{
```

```
    int count = 0;
```

```
    for (int i = n; i > 10; i /= 2)
```

```
        for (int j = 0; j < i; j++)
```

```
            count += 1;
```

```
    return count;
```

```
}
```

$\nearrow \log n$

$\searrow 2^n$



(4 marks)



$O(1)$



$O(\log n)$



$O(n)$



$O(n \log n)$



$O(n^2)$

9.

```
// assuming the input arr is an array with size > n

void f(int n, int arr[])
{
    int i = 0, j = 0;

    for(; i < n; ++i)
        while(j < n && arr[i] < arr[j])
            j++;
}
```

(4 marks)

- ☐ $O(1)$
- ☐ $O(\log n)$
- ☒ $O(n)$
- ☐ $O(n \log n)$
- ☐ $O(n^2)$

10.

```
void f(int n)
{
    int i, j;

    for (i=1; i<=n; i++)
        for (j=1; j<=log(i); j++)
```



```
printf("hi");
```

```
}
```

(4 marks)

☐ $O(\log n)$

☐ $O(n)$

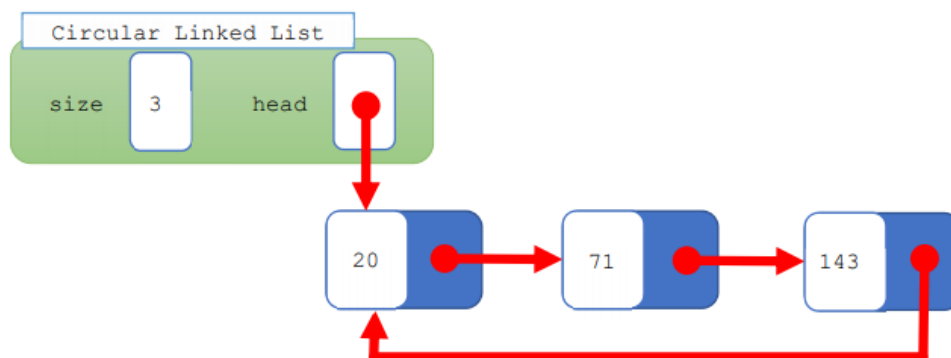
☒ $O(n \log n)$

☐ $O(n^2)$

☐ $O(n^2 \log n)$

T/F

11. Circular (Singly) Linked List is a special Linked List with one head pointer pointing to the first vertex/element, each vertex points to its immediate next vertex, and the last vertex pointing back to the first vertex (figure from CS2040C Practical Exam S2 2021). In a Circular Linked List, each vertex is thus only pointed by exactly one pointer and there is no vertex that points to NULL.



(1 mark)

☐ True

☒ False

12.

There are N non-negative fractions in $\frac{A}{B}$ format where the numerator is a non-negative 32-bit signed integer and the denominator is a positive 32-bit signed integer, e.g., $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$. We can sort N fractions (in ascending order) in $O(N \log N)$ time using Heap Sort because we can compare if fraction A/B is smaller than or equal to fraction C/D by testing if $A \cdot D \leq B \cdot C$.

(1 mark)

☒ True

☐ False

13.

A Binary Tree contains $|V|$ distinct integers. We run inorder traversal on it starting from the root. We will always see its $|V|$ integers listed in ascending order.

(1 mark)

☐ True

☒ False

14.

In a Binary Max Heap of $N = 13$ distinct integers, it is possible that the 3rd largest integer is stored as one of the leaves.

(1 mark)

☐ True

☒ False

15.

A hash table implementation uses Open Addressing (e.g., Linear Probing) as the collision resolution mechanism. We know in advance that there are at most N keys that will be inserted into the hash table and there is no deletion. If we set the initial table size $M = 2 \cdot N$, we will never need to resize/grow the table size for this specific set up.

(1 mark)

☐ True

☐ False

☒ False

16. On the same simple undirected unweighted graph $G = (V, E)$, it is possible that running Depth-First-Search from a source vertex s visits a different subset of vertices compared to running Breadth-First-Search from the same source vertex s .

(1 mark)



☐ True

☒ False

17. Bellman-Ford algorithm runs in $O(V^2)$ instead of $O(VE)$ on a simple directed weighted planar graph $G = (V, E)$.

(1 mark)



☒ True

☐ False

18. The $O((V+E) \log V)$ Dijkstra's algorithm is the fastest algorithm to correctly solve the Single-Source Shortest Paths (SSSP) problem on a simple weighted Directed Acyclic Graph $G(V, E)$.

(1 mark)

☐ True

☒ False

19. Given a set S that contains N points in a 2D Cartesian plane. It is possible that all the N points lie on the boundary of the convex hull of S .

(1 mark)

☒ True

☐ False

20.

We can sort N integers $\{a_1, a_2, \dots, a_N\}$ by mapping/projecting them into a 2D Cartesian plane $\{(a_1, a_1^2), (a_2, a_2^2), \dots, (a_N, a_N^2)\}$, computing the Convex Hull of these N points in $O(N \log N)$ time, and enumerating the points from the leftmost to the rightmost along the Convex Hull in $O(N)$.

(1 mark)

<input checked="" type="radio"/> True
<input type="radio"/> False

MCQ

21.

An AVL Tree has $|V| = 1023$ vertices. Which of the following is an impossible height h (number of edges from the root to the deepest leaf) for such an AVL Tree?

(1 mark)

<input checked="" type="radio"/> $h = 8$ edges
<input type="radio"/> $h = 9$ edges
<input type="radio"/> $h = 10$ edges
<input type="radio"/> $h = 11$ edges
<input type="radio"/> $h = 15$ edges

22.

In class, you have seen that a balanced BST with N vertices can be augmented with size of subtree (weight) attribute in each vertex so that we can compute the (1-based) rank of a given vertex v in its subtree in $O(1)$.

Namely, the weight $w = v.\text{left}.\text{weight} + 1$. We will treat $v.\text{left}.\text{weight} = 0$ if $v.\text{left} == \text{NUL}$. (Same rule goes for the case that $v.\text{right} == \text{NULL}$.)

Which of the following term is equivalent to $w = v.\text{left}.\text{weight} + 1$ for each node?

(1 mark)

<input type="text"/>

☐ `v.right.weight`

☐ `N - v.right.weight - 1`

☒ `N - v.right.weight`



☐ `N - v.right.weight + 1`

☐ `2 * v.right.weight`

- 23.** Which of the following is NOT a valid Binary Max Heap of $N = 7$ positive integers?
- The Binary Max Heap is given as an array with 0-based indexing with index 0 being the root/max element.
- (1 mark)

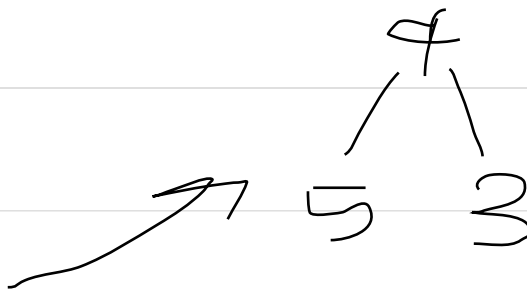
☐ [7, 6, 5, 4, 3, 2, 1]

☐ [7, 5, 6, 4, 3, 2, 1]

☐ [7, 4, 6, 1, 2, 3, 5]

☐ [7, 6, 4, 5, 3, 2, 1]

☒ [7, 4, 6, 5, 3, 2, 1]



- 24.** A Union-Find Disjoint Set (UFDS) data structure initially has $n = 1000$ disjoint sets. After a series of successful `unionSet(i, j)` operations (where two initially disjoint sets are merged), the UFDS currently has 500 disjoint sets.

What is the maximum possible size of any of these disjoint sets?

(1 mark)



<input type="radio"/>	100	X
<input type="radio"/>	499	
<input type="radio"/>	500	
<input checked="" type="radio"/>	501	
<input type="radio"/>	1000	X

25. We insert $N = 7$ integers $\{1, 5, 3, 4, 19, 7, 6\}$ one by one into an initially empty Hash Table with table size $M = 17$. The hash function used is $h(v) = v \% M$. We use Linear Probing as the collision resolution mechanism.

Which 8th integer among the given options below, if inserted afterwards, will collide with an existing integer and then trigger 7 more Linear Probing steps?

(1 mark)

<input type="radio"/>	12
<input type="radio"/>	14
<input type="radio"/>	15
<input checked="" type="radio"/>	18
<input type="radio"/>	45

26. A simple undirected unweighted graph $G = (V, E)$ has $|V| = 100$ vertices. It is also known that its diameter is 1 and all its 100 vertices have degree of 99.

How many edges $|E|$ that it has?

(1 mark)

☐ Can be any non-negative integer between $0 \leq |E| \leq 4950$ edges

☐ $|E|$ must be 99 edges



☐ $|E|$ must be 100 edges

☐ $|E|$ must be 294 edges

☒ $|E|$ must be 4950 edges

27. Running Depth-First-Search(root) on a (not necessarily balanced) Binary Search Tree containing at least 8 distinct integers will produce the same sequence of visited vertices as (note that DFS/BFS here will go to the vertex with lower number first/left child before exploring the vertex with higher number/right child):

(1 mark)

☐ Breadth-First-Search(root)



☐ Inorder-Traversal(root)

☒ Preorder-Traversal(root)

☐ Postorder-Traversal(root)



☐ None of the above

28. We ran Bellman-Ford algorithm on a simple directed weighted graph $G = (V, E)$. After Bellman-Ford algorithm stops, we do one last check: We see if we can still relax any edge (u, v) in E . This additional last check returns false, i.e., no more edge can be relaxed.

What can we conclude from this statement?

(1 mark)

- ☐ The Shortest-Path values found by Bellman-Ford may still contain wrong values
- ☐ $G(V, E)$ does not have cycle
- ☐ $G(V, E)$ is a planar graph
- ☐ $G(V, E)$ has 7 faces
- ☒ None of the above

29. The number of topological sorts of a Singly Linked List of $N = 5$ vertices (head) $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$ (tail) is:
(1 mark)

- ☐ 0, as Singly Linked List is not a Directed Acyclic Graph (DAG)
- ☒ 1
- ☐ 24
- ☐ 120
- ☐ Too many to count

30. A graph $G = (V, E)$ is a Planar Graph. It is also a simple connected undirected graph, has $|V| = 10$ vertices, and its planar embedding has $|F| = 3$ faces.

How many edges $|E|$ that it has?

(1 mark)

- ☐ Can be any non-negative integer between $0 \leq |E| \leq 45$ edges

☐

|E| must be 10 edges

☒

|E| must be 11 edges

☐

|E| must be 12 edges

☐

None of the above

Hashing

31.

Fill in the blanks

(10 marks)

We have a hash table T with 11 entries with indices from 0 to 10.

We will hash integers into the hash table T. And the hashing function h is the sum of digits, e.g. $h(1234) = 10$. And we will use open addressing with linear probing.

After the following operations, what will be the contents in all the slots in the final hash table? If an entry is occupied by a number, put the number in the blank. If the entry is empty, put a uppercase "E" and if the entry is deleted, put a uppercase "D".

insert(44)
insert(96)
insert(555)
insert(11)
insert(2988)
insert(59)
delete(555)
insert(68)
delete(44)



Please fill in the following hash table after the above operations. Please note there your answer is case sensitiv and you should NOT put any space into your answers. So an "E " will be a wrong answer even the answer is "E".

Index 0: 1
Index 1: 2
Index 2: 3
Index 3: 4

Index 4: 5
Index 5: 6
Index 6: 7
Index 7: 8
Index 8: 9
Index 9: 10
Index 10: 11

Enter the correct answer below.

1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
4	<input type="text"/>
5	<input type="text"/>
6	<input type="text"/>
7	<input type="text"/>
8	<input type="text"/>
9	<input type="text"/>
10	<input type="text"/>
11	<input type="text"/>

32.

Fill in the blanks

(10 marks)

We have a hash table T with 11 entries with indices from 0 to 10.

We will hash integers into the hash table T. And the hashing function h is the sum of digits, e.g. $h(1234) = 10$. And we will use open addressing with linear probing.

After the following operations, what will be the contents in all the slots in the final hash table? If an entry is occupied by a number, put the number in the blank. If the entry is empty, put a uppercase "E" and if the entry is deleted, put a uppercase "D".

insert(44)
insert(96)
insert(555)
insert(11)
insert(2988)

insert(59)
delete(96)
insert(101)
delete(555)



Please fill in the following hash table after the above operations. Please note there your answer is case sensitiv and you should NOT put any space into your answers. So an "E " will be a wrong answer even the answer is "E".

Index 0: 1
Index 1: 2
Index 2: 3
Index 3: 4
Index 4: 5
Index 5: 6
Index 6: 7
Index 7: 8
Index 8: 9
Index 9: 10
Index 10: 11

Enter the correct answer below.

1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
4	<input type="text"/>
5	<input type="text"/>
6	<input type="text"/>
7	<input type="text"/>
8	<input type="text"/>
9	<input type="text"/>
10	<input type="text"/>
11	<input type="text"/>

33.

Fill in the blanks

(10 marks)

We have a hash table T with 11 entries with indices from 0 to 10.

We will hash integers into the hash table T. And the hashing function h is the sum of digits, e.g. $h(1234) = 10$. And we will use open addressing with linear probing.

After the following operations, what will be the contents in all the slots in the final hash table? If an entry is occupied by a number, put the number in the blank. If the entry is empty, put a uppercase "E" and if the entry is deleted, put a uppercase "D".

insert(44)
insert(96)
insert(555)
insert(11)
insert(2988)
insert(59)
delete(96)
insert(29456)
delete(555)

Please fill in the following hash table after the above operations. Please note there your answer is case sensitive and you should NOT put any space into your answers. So an "E " will be a wrong answer even the answer is "E".

Index 0: 1
Index 1: 2
Index 2: 3
Index 3: 4
Index 4: 5
Index 5: 6
Index 6: 7
Index 7: 8
Index 8: 9
Index 9: 10
Index 10: 11

Enter the correct answer below.

1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
4	<input type="text"/>
5	<input type="text"/>
6	<input type="text"/>
7	<input type="text"/>
8	<input type="text"/>
9	<input type="text"/>
10	<input type="text"/>
11	<input type="text"/>

34.

Fill in the blanks

(10 marks)

We have a hash table T with 11 entries with indices from 0 to 10.

We will hash integers into the hash table T. And the hashing function h is the sum of digits, e.g. $h(1234) = 10$. And we will use open addressing with linear probing.

After the following operations, what will be the contents in all the slots in the final hash table? If an entry is occupied by a number, put the number in the blank. If the entry is empty, put a uppercase "E" and if the entry is deleted, put a uppercase "D".

insert(44)
insert(555)
insert(96)
insert(11)
insert(2988)
insert(59)
delete(96)
insert(6838)
delete(555)

Please fill in the following hash table after the above operations. Please note there your answer is case sensitive and you should NOT put any space into your answers. So an "E " will be a wrong answer even the answer is "E".

Index 0: 1
Index 1: 2
Index 2: 3
Index 3: 4
Index 4: 5
Index 5: 6
Index 6: 7
Index 7: 8
Index 8: 9
Index 9: 10
Index 10: 11

Enter the correct answer below.

1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
4	<input type="text"/>
5	<input type="text"/>
6	<input type="text"/>

7

8

9

10

11

Heap Extraction

35.

Fill in the blanks

(8 marks)

Here is a min-heap with array implementation, the first item in the left has index 0 in the array:

1 2 3 23 4 33 11 55 46 22 9

Please compute the min-heap array after **THREE** extractMin operations.

(Please note that Blank "/" below is the first entry of the array. If the index starts from 0, Blank "1" is at index 0.)

1 , 2 , 3 , 4 , 5 , 6 , 7 , 8

Enter the correct answer below.

1 Please enter a number for this text box.

2 Please enter a number for this text box.

3 Please enter a number for this text box.

4 Please enter a number for this text box.

5 Please enter a number for this text box.

6 Please enter a number for this text box.

7 Please enter a number for this text box.

8 Please enter a number for this text box.



36. Fill in the blanks

(8 marks)

Here is a min-heap with array implementation, the first item in the left has index 0 in the array:

11 23 21 33 27 22 67 44 46 45 76

Please compute the min-heap array after **THREE** extractMin operations.

(Please note that Blank "1" below is the first entry of the array. If the index starts from 0, Blank "1" is at index 0.)

____ 1 _____, ____ 2 _____, ____ 3 _____, ____ 4 _____, ____ 5 _____, ____ 6 _____, ____ 7 _____, ____ 8 _____

Enter the correct answer below.

- 1 Please enter a number for this text box.
- 2 Please enter a number for this text box.
- 3 Please enter a number for this text box.
- 4 Please enter a number for this text box.
- 5 Please enter a number for this text box.
- 6 Please enter a number for this text box.
- 7 Please enter a number for this text box.
- 8 Please enter a number for this text box.

37. Fill in the blanks

(8 marks)

Here is a min-heap with array implementation, the first item in the left has index 0 in the array:

11 23 45 33 27 69 67 44 97 46 76

Please compute the min-heap array after **THREE** extractMin operations.

(Please note that Blank "1" below is the first entry of the array. If the index starts from 0, Blank "1" is at index 0.)

1 , 2 , 3 , 4 , 5 , 6 , 7 , 8

Enter the correct answer below.

- 1 Please enter a number for this text box.
- 2 Please enter a number for this text box.
- 3 Please enter a number for this text box.
- 4 Please enter a number for this text box.
- 5 Please enter a number for this text box.
- 6 Please enter a number for this text box.
- 7 Please enter a number for this text box.
- 8 Please enter a number for this text box.

38.

Fill in the blanks

(8 marks)

Here is a min-heap with array implementation, the first item in the left has index 0 in the array:

33 35 45 44 46 69 67 97 76 61 81

Please compute the min-heap array after **THREE** extractMin operations.

(Please note that Blank "1" below is the first entry of the array. If the index starts from 0, Blank "1" is at index 0.)

1 , 2 , 3 , 4 , 5 , 6 , 7 , 8

Enter the correct answer below.

- 1 Please enter a number for this text box.
- 2 Please enter a number for this text box.
- 3 Please enter a number for this text box.
- 4 Please enter a number for this text box.
- 5 Please enter a number for this text box.
- 6 Please enter a number for this text box.
- 7 Please enter a number for this text box.

Single Source Shortest Path

39.

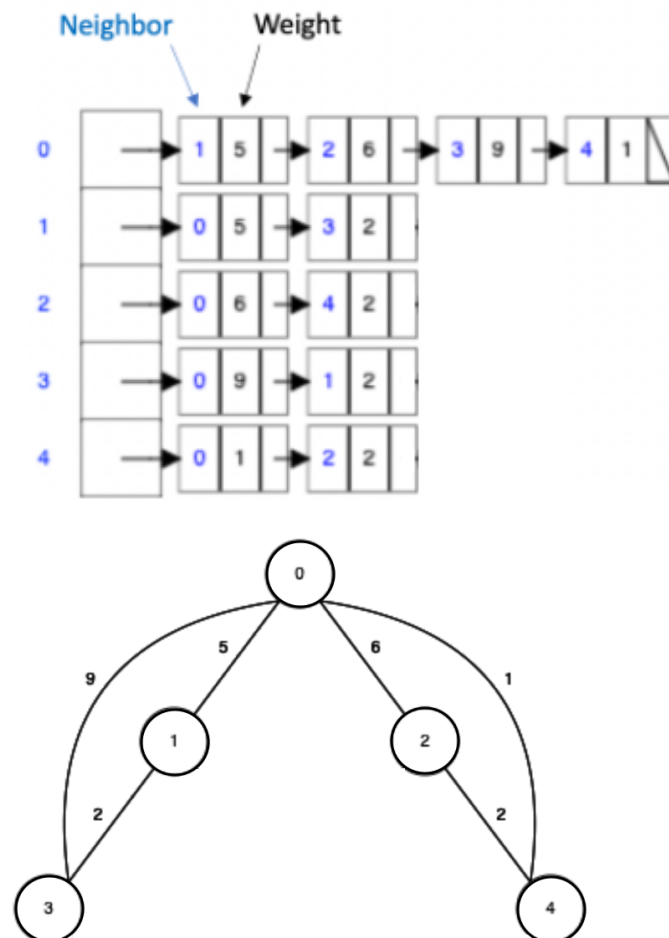
Fill in the blanks

(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the real question you have to answer.

Example

Given an undirected graph in an adjacency list:



The shortest distances to all the nodes from the source **Node 0** are:

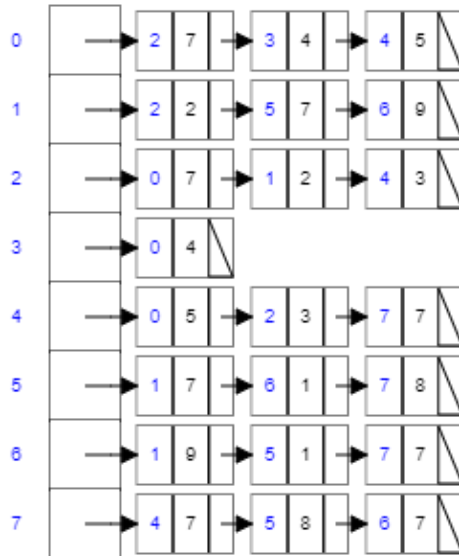
0: 0 1: 5 2: 3 3: 7 4: 1

And the shortest path from Node 0 to Node 2 is:

0 → 4 → 2

Real question

For this following graph represented in an adjacency list



What is the shortest distance for each node starting from the source Node 0?

0: 0 1: 1 2: 2 3: 3 4: 4 5: 5 6: 6 7: 7

Enter the correct answer below.

1 Please enter a number for this text box.

Character:
Limit: 3

2 Please enter a number for this text box.

Character:
Limit: 3

3 Please enter a number for this text box.

Character:
Limit: 3

4 Please enter a number for this text box.

Character:
Limit: 3

5 Please enter a number for this text box.

Character:
Limit: 3

6 Please enter a number for this text box.

Character:
Limit: 3

7 Please enter a number for this text box.

Character:
Limit: 3

40.

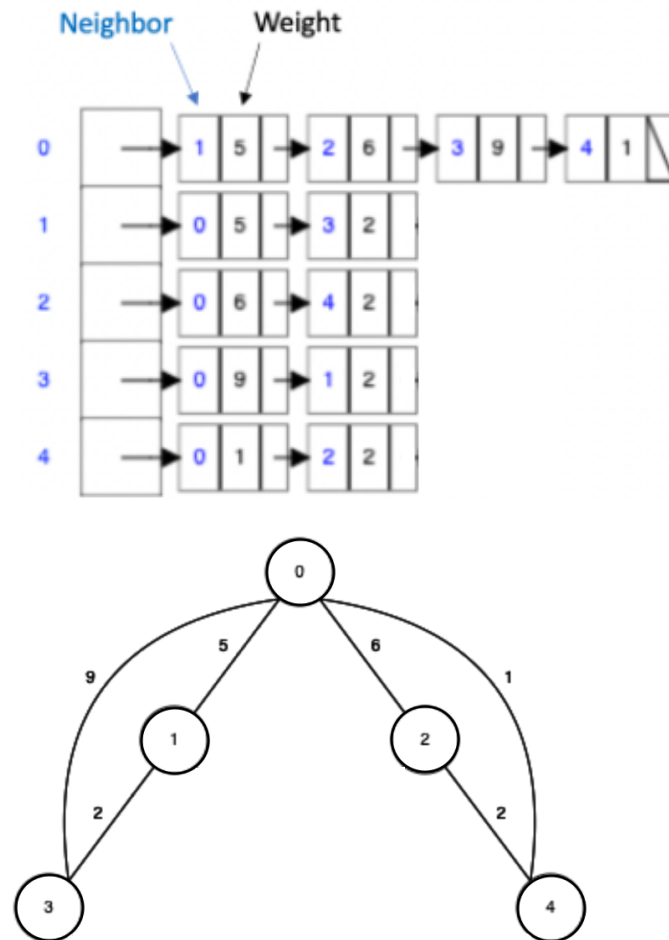
Fill in the blanks

(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the real question you have to answer.

Example

Given an undirected graph in an adjacency list:



The shortest distances to all the nodes from the source **Node 0** are:

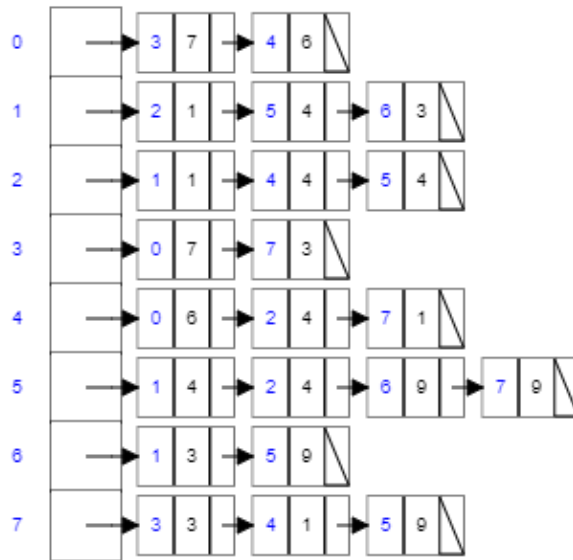
0: 0 1: 5 2: 3 3: 7 4: 1

And the shortest path from Node 0 to Node 2 is:

0 → 4 → 2

Real question

For this following graph represented in an adjacency list



What is the shortest distance for each node starting from the source Node 0?

0: 0 1: 1 2: 2 3: 3 4: 4 5: 5 6: 6 7: 7

Enter the correct answer below.

- 1 Please enter a number for this text box.
- 2 Please enter a number for this text box.
- 3 Please enter a number for this text box.
- 4 Please enter a number for this text box.
- 5 Please enter a number for this text box.
- 6 Please enter a number for this text box.
- 7 Please enter a number for this text box.

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

41.

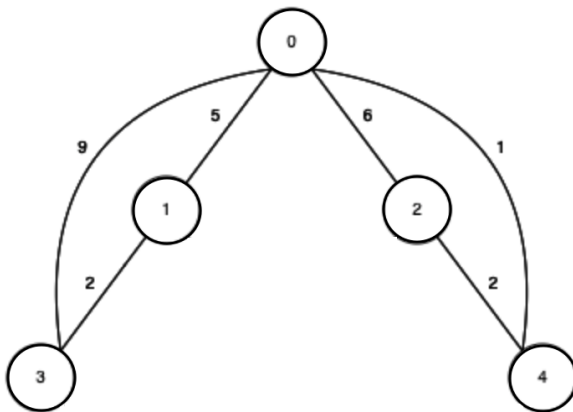
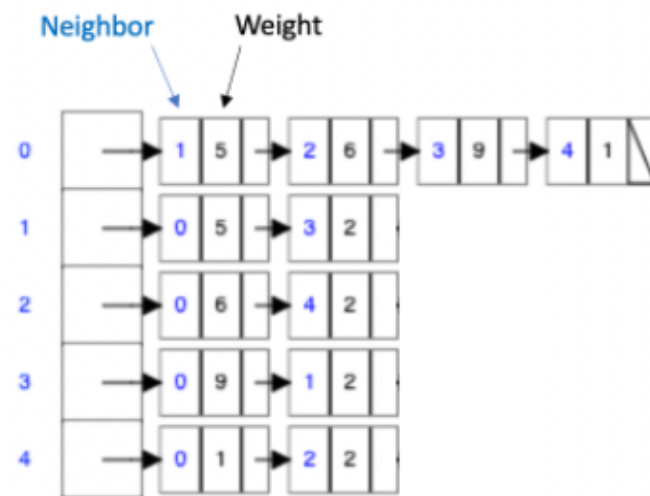
Fill in the blanks

(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the real question you have to answer.

Example

Given an undirected graph in an adjacency list:



The shortest distances to all the nodes from the source **Node 0** are:

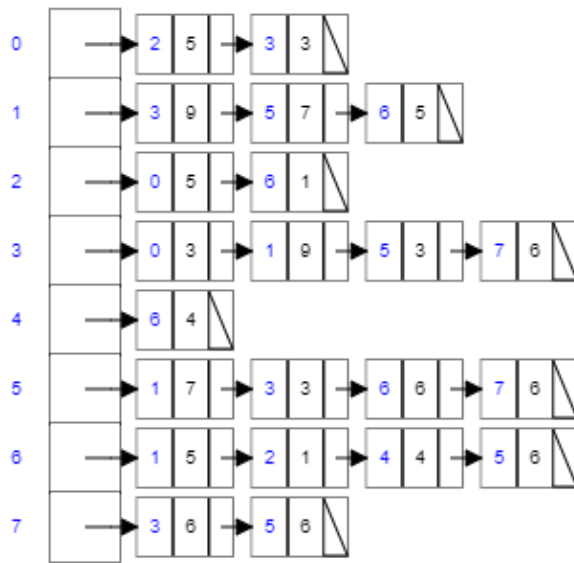
0: 0 1: 5 2: 3 3: 7 4: 1

And the shortest path from Node 0 to Node 2 is:

0 → 4 → 2

Real question

For this following graph represented in an adjacency list



What is the shortest distance for each node starting from the source Node 0?

0: 0 1: 1 2: 2 3: 3 4: 4 5: 5 6: 6 7: 7

Enter the correct answer below.

- 1 Please enter a number for this text box.
- 2 Please enter a number for this text box.
- 3 Please enter a number for this text box.
- 4 Please enter a number for this text box.
- 5 Please enter a number for this text box.
- 6 Please enter a number for this text box.
- 7 Please enter a number for this text box.

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

Character:
Limit: 3

42.

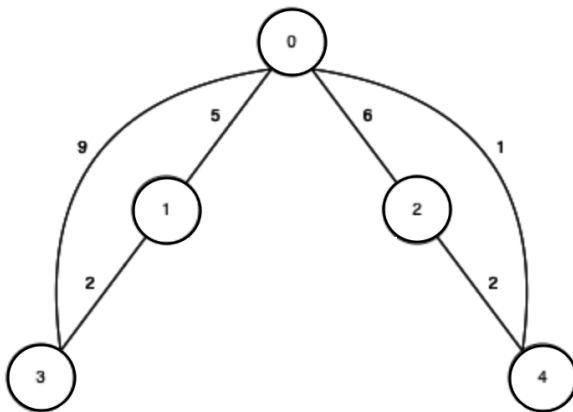
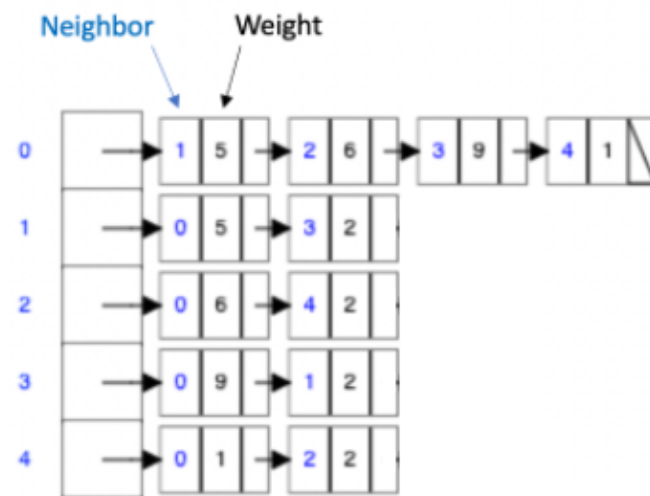
Fill in the blanks

(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the real question you have to answer.

Example

Given an undirected graph in an adjacency list:



The shortest distances to all the nodes from the source **Node 0** are:

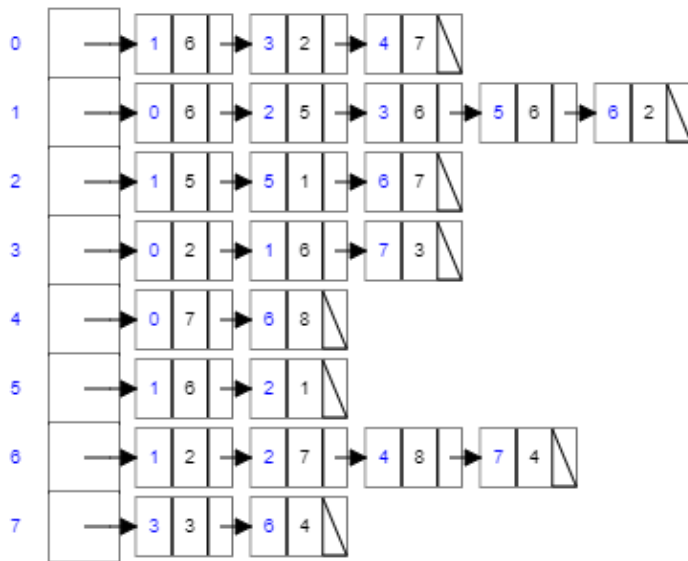
0: 0 1: 5 2: 3 3: 7 4: 1

And the shortest path from Node 0 to Node 2 is:

0 → 4 → 2

Real question

For this following graph represented in an adjacency list



What is the shortest distance for each node starting from the source Node 0?

0: 0 1: 1 2: 2 3: 3 4: 4 5: 5 6: 6 7: 7

Enter the correct answer below.

1 Please enter a number for this text box.

Character:
Limit: 3

2 Please enter a number for this text box.

Character:
Limit: 3

3 Please enter a number for this text box.

Character:
Limit: 3

4 Please enter a number for this text box.

Character:
Limit: 3

5 Please enter a number for this text box.

Character:
Limit: 3

6 Please enter a number for this text box.

Character:
Limit: 3

7 Please enter a number for this text box.

Character:
Limit: 3

MST

43.

Fill in the blanks

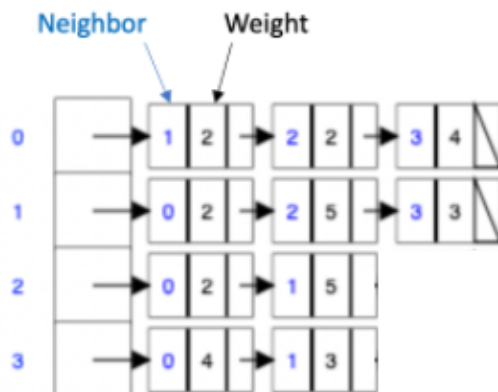
(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the *real* question you have to answer.

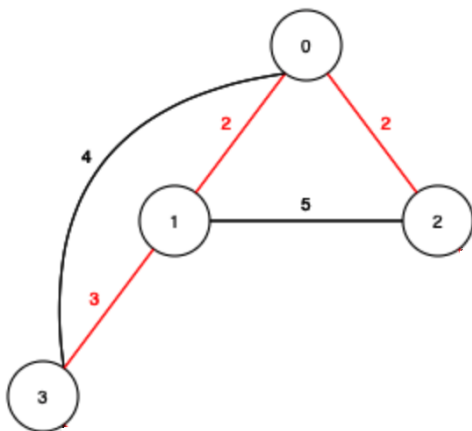
Note: Please be careful about the number "8" and "6". You have to read them carefully.

Example

Given an undirected graph in an adjacency list:



And the Minimal Spanning Tree (MST) should be the red edges below:

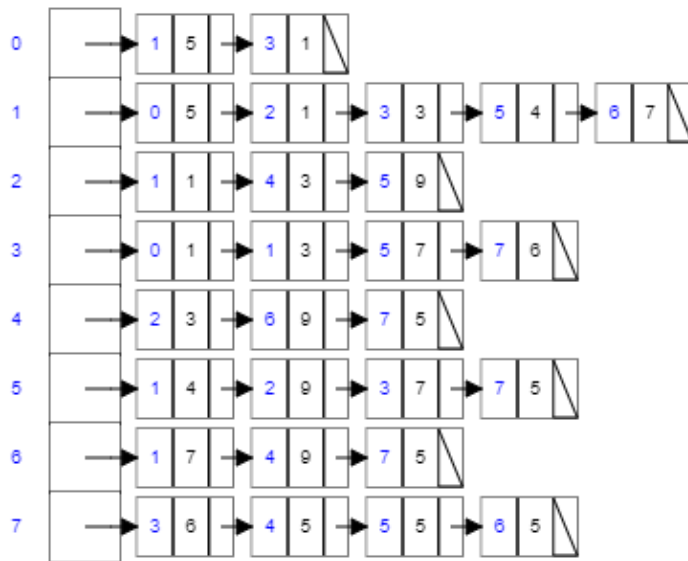


The total weight of the MST is $2 + 2 + 3 = 7$.

(And the edges are "01", "02" and "13".)

Real question

For this following graph represented in an adjacency list



The total weight of the MST of this graph is 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character:
Limit: 2

44.

Fill in the blanks

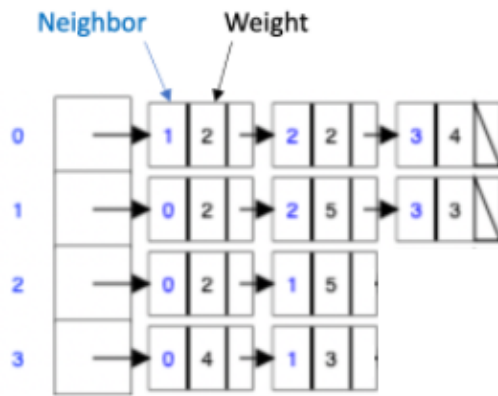
(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the real question you have to answer.

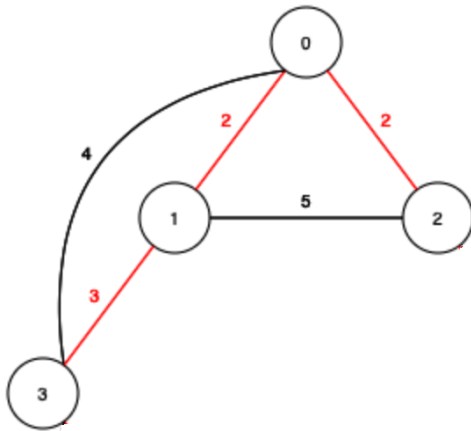
Note: Please be careful about the number "8" and "6". You have to read them carefully.

Example

Given an undirected graph in an adjacency list:



And the Minimal Spanning Tree (MST) should be the red edges below:

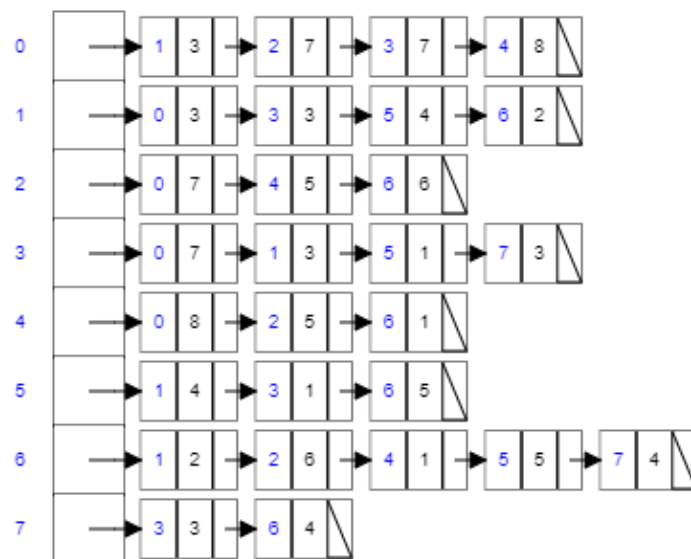


The total weight of the MST is $2 + 2 + 3 = 7$.

(And the edges are "01", "02" and "13".)

Real question

For this following graph represented in an adjacency list



The total weight of the MST of this graph is 1.

Enter the correct answer below.

1

Please enter a number for this text box.

Character:
Limit: 2

45.

Fill in the blanks

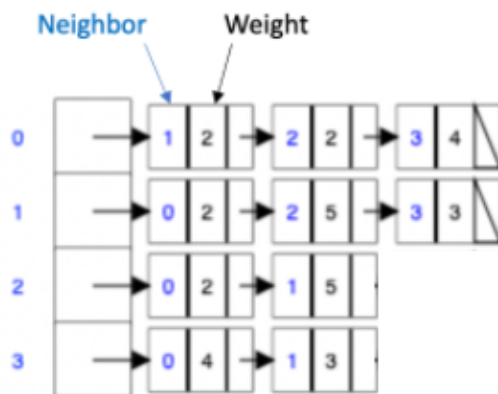
(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the *real* question you have to answer.

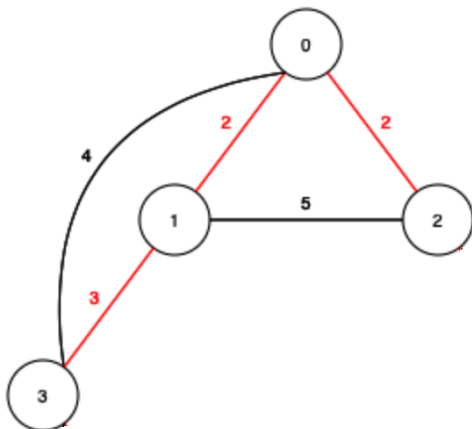
Note: Please be careful about the number "8" and "6". You have to read them carefully.

Example

Given an undirected graph in an adjacency list:



And the Minimal Spanning Tree (MST) should be the red edges below:

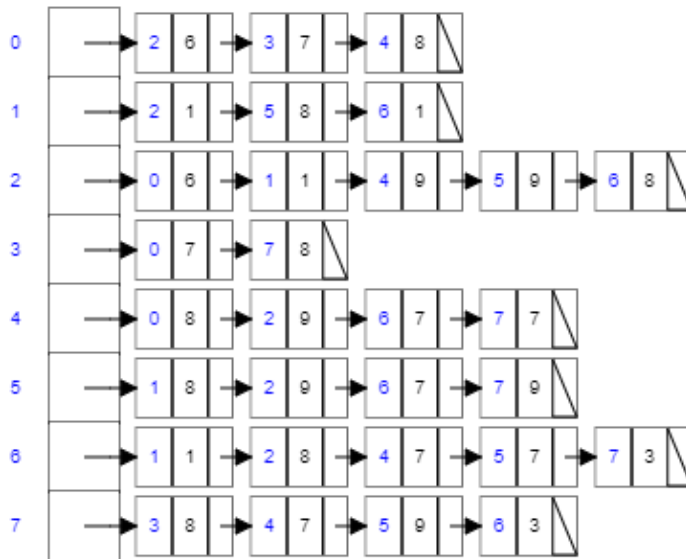


The total weight of the MST is $2 + 2 + 3 = 7$.

(And the edges are "01", "02" and "13".)

Real question

For this following graph represented in an adjacency list



The total weight of the MST of this graph is 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character:
Limit: 2

46.

Fill in the blanks

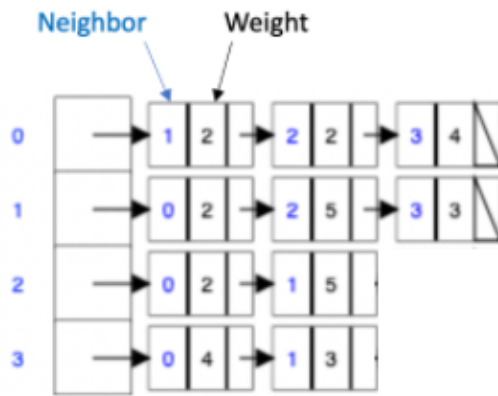
(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the ***real*** question you have to answer.

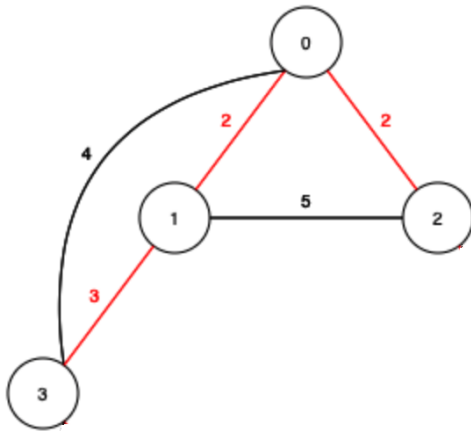
Note: Please be careful about the number "8" and "6". You have to read them carefully.

Example

Given an undirected graph in an adjacency list:



And the Minimal Spanning Tree (MST) should be the red edges below:

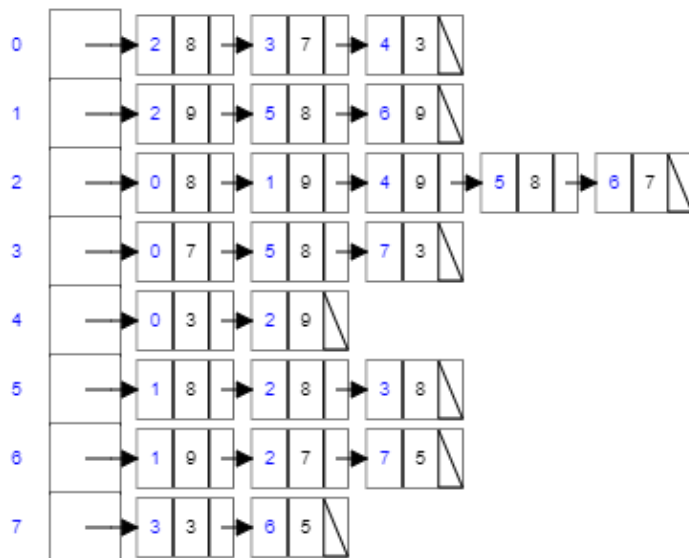


The total weight of the MST is $2 + 2 + 3 = 7$.

(And the edges are "01", "02" and "13".)

Real question

For this following graph represented in an adjacency list



The total weight of the MST of this graph is 1.

Enter the correct answer below.

1

Please enter a number for this text box.

Character
Limit: 2

47.

Fill in the blanks

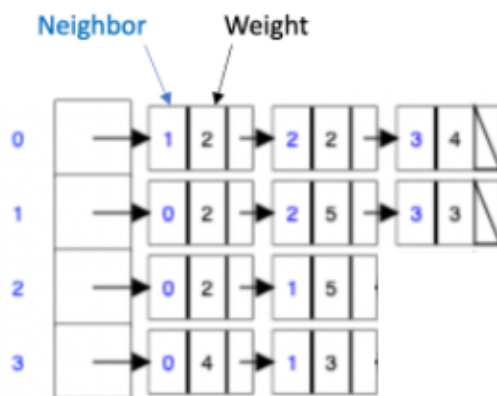
(8 marks)

The first part (green fonts) is an example to illustrate the settings and notations of this question. The second part (black fonts) is the *real* question you have to answer.

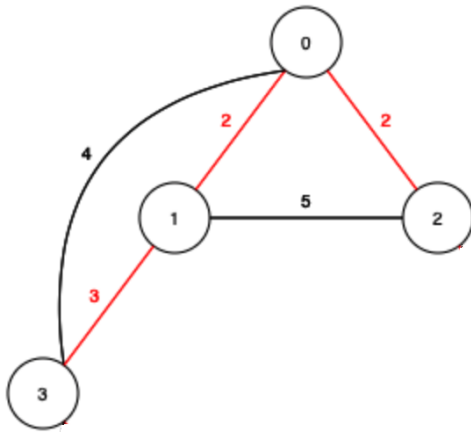
Note: Please be careful about the number "8" and "6". You have to read them carefully.

Example

Given an undirected graph in an adjacency list:



And the Minimal Spanning Tree (MST) should be the red edges below:

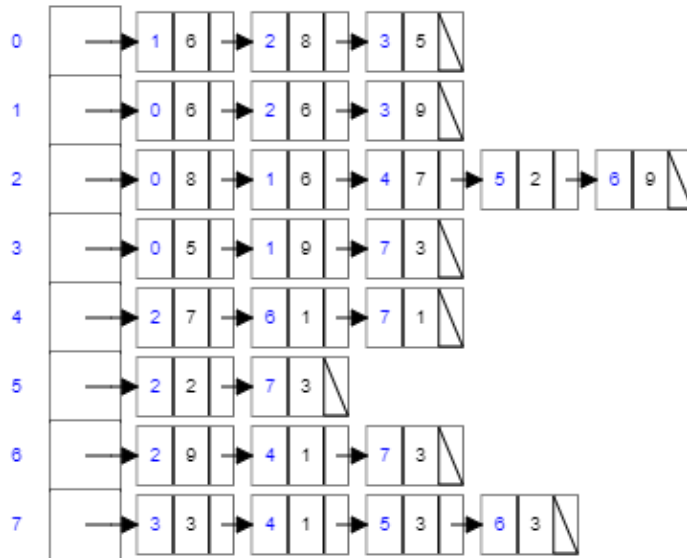


The total weight of the MST is $2 + 2 + 3 = 7$.

(And the edges are "01", "02" and "13".)

Real question

For this following graph represented in an adjacency list



The total weight of the MST of this graph is 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character Limit: 2

Problem Solving (Easy)

In a big city, there are n citizen. The government wants to find who are the 100 richest persons in the country. Assuming that the government can check the bank for each person and know how much he/she saves in the bank and uses that as the criteria for how rich a person is. You can assume n is a very large number, e.g. more than a million.

Note that if there are multiple possible solutions, give the one with the best time complexity.

48. Describe what is the algorithm and data structure you want to compute the answer.

And what is the time complexity for your algorithm according to n ?

(8 marks)



Word Limit:
Character Limit: 1000

49. In the next year, there are new immigrants migrating to this country. Assuming that you've already computed the top 100 richest persons in the questions above and whatever data structure you used for the first n people are computed. What will be the additional time complexity to update the top 100 richest persons for each new addition of immigrant?

(4 marks)



Word Limit:
Character Limit: 1000

Problem Solving (Medium)

Note that if there are multiple possible solutions, give the one with the best time complexity.

50. The Art of Delegation

It is another day in school, and you are a mastermind of "delegating" your works. Instead, you will go to your friends for "help", but secretly have them do all your works.

You have concluded that the more one of your M ($1 \leq M \leq 200\,000$) friends helps you, the more annoyed they become. You have also been able to determine how much more annoyed your friend gets everytime you ask them for help.

At the beginning of the day, a friend is initially X annoyed at you. That is their annoyance level. Everytime you ask them for help though, they become Y more annoyed at you - their annoyance level X increases by a constant amount Y so that $X = X + Y$.

You want to complete a project of N ($1 \leq N \leq 200\,000$) independent tasks solely with the "help" from your friends, but you need to be careful not to annoy any of them too much.

What is the best you can do?

Example 1: You have 4 friends and you need to ask for help 4 times. Initially, their annoyance levels are $X_1 = 1$, $X_2 = 2$, $X_3 = 3$, $X_4 = 4$, and their increases are $Y_1 = 2$, $Y_2 = 3$, $Y_3 = 4$, $Y_4 = 5$.

One optimal solution is to ask for help twice from friend 1, once from friend 2, and once from friend 3, in which case the final annoyance levels are: $X_1 = 1 + 2 \cdot 2 = 5$, $X_2 = 2 + 3 = 5$, $X_3 = 3 + 4 = 7$, $X_4 = 4$. The friend that is most annoyed at you is friend 3, whose annoyance level at you is 7.

Alternatively, you could ask friend 1 for help 3 times and friend 2 once, leaving you with $X_1 = 1 + 3 \cdot 2 = 7$, $X_2 = 2 + 3 = 5$, $X_3 = 3$, $X_4 = 4$.

Both strategies yield the same minimal answer (7), and this is your output.

Write your solution as an open ended essay (you can write in pseudocode) obeying the following grading rubric (total 10 marks):

- 2 marks to the correct problem name is identified.
- 3 marks if the correct data structure needed to solve this problem is used.

- 3 marks if the correct algorithm needed to solve this problem is used.
- 2 marks for the correct time complexity analysis.

(10 marks)



Word Limit:
Character Limit: 1000

51.

Fill in the blanks

(1 mark)

The Art of Delegation problem - test your understanding (1).

There are $N = 100\,000$ tasks that you have to do, but you only have $M = 1$ friend.

This only friend of yours has $X = 10$ and $Y = 7000$.

What is your only friend final annoyance level after all your N tasks are "delegated" to this poor friend? Your answer = 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character:
Limit: 10

52.

Fill in the blanks

(1 mark)

The Art of Delegation problem - test your understanding (2).

There is just $N = 1$ task that you have to do and you have $M = 2$ friends.

The first friend has $X = 1$ and $Y = 1000000$.

The second friend has $X = 999999$ and $Y = 1$.

What is the optimal maximum annoyance level among the 2 friends after "delegating" your only $N = 1$ task to that friend? Your answer = 1.

Enter the correct answer below.

1

1000000

Please enter a number for this text box.

Character Limit: 8

Problem Solving (Hard)

Note that if there are multiple possible solutions, give the one with the best time complexity.

PS: Do NOT attempt this section until you have cleared and possibly double checked your answers for the other sections.

53. New Board Game

Steven is designing a new 1D board game involving a fair 6-sided dice.

The rules of the game are as follows:

- The 1D board has $N+1$ cells.
The first cell is a special source cell.
The remaining N squares are labeled with a digit $[1, 2, 3, 4, 5, 6]$.
- The goal of the game is to start from the source cell and reach the last (rightmost) cell N .
- Players take turns rolling a fair 6-sided dice.
A player moves to the nearest cell after their current cell that has digit that matches the top of the recently rolled fair 6-sided dice.
- If there are no cells of the rolled digit after a player's current cell, that player's piece does not move and their turn ends.

Steven is interested in the length of play required for different board layouts. Steven would like to know the smallest number of dice rolls any one player would have to draw to complete this new game.

Example 1: If given the following board with $N = 9$ and values $[\text{source}, 1, 2, 3, 4, 5, 6, 6, 6, 6]$.

index = 0	1	2	3	4	5	6	7	8	$N = 9$
source (X)	1	2	3	4	5	6	6	6	6

Then the best first dice roll is a 6 which will advance a player from the source cell to the 6th cell immediately. From the 6th cell to the end (9th cell), only a dice roll that gives a 6 will advance the player. Therefore the smallest number of dice rolls is 4, as animated above.

Example 2: If given the following board with $N = 7$ and values $[\text{source}, 1, 2, 3, 4, 5, 6, 1]$, then the first dice roll can be any of digit $[1..6]$ but the best second dice roll must give a 1 so that we can reach the 7th cell in just 2 dice rolls.

You are given N ($1 \leq N \leq 200\,000$) and an array of $N+1$ integers called `board` that only contains digit $[1..6]$ (you can assume that the value of the 0th index - the source cell - can be ignored, i.e., you can assume `board[0] = 0`).

Your task is to compute an integer using the fastest possible algorithm (using appropriate data structure(s)): the minimum number of dice rolls required to move from the source cell to square N.

Write your solution as an open ended essay (you can write in pseudocode) obeying the following grading rubric (total 10 marks):

- 2 marks to the correct problem name is identified.
- 3 marks if the correct data structure needed to solve this problem is used.
- 3 marks if the correct algorithm needed to solve this problem is used.
- 2 marks for the correct time complexity analysis.

(10 marks)



Word Limit:
Character Limit: 1000

54.

Fill in the blanks

(1 mark)

New Board Game problem - test your understanding (1).

If given the following board with $N = 200\,000$ and values [source, 1, 1, 1, ..., 1], i.e., source and then 200 000 copies of 1s, what is the smallest number of dice rolls needed to solve the game?

Your answer = 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character Limit: 7

55.

Fill in the blanks

(1 mark)

New Board Game problem - test your understanding (2).

If given the following board with $N = 7$ and values [source, 1, 2, 1, 3, 2, 1, 2], what is the smallest number of dice rolls needed to solve the game?

Your answer = 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character Limit: 2

56.

Fill in the blanks

(1 mark)

New Board Game problem - test your understanding (3).

If given the following board with $N = 4$ and values [source, 6, 6, 1, 6], what is the smallest number of dice rolls needed to solve the game?

Your answer = 1.

Enter the correct answer below.

1 Please enter a number for this text box.

Character Limit: 2

Finally

57.

Please tell us something you appreciate or found it useful in CS2040C?

(1 mark)

Leetcode for Algo interviews

Word Limit:
Character Limit: 2000

Finish Quiz

Save For Later