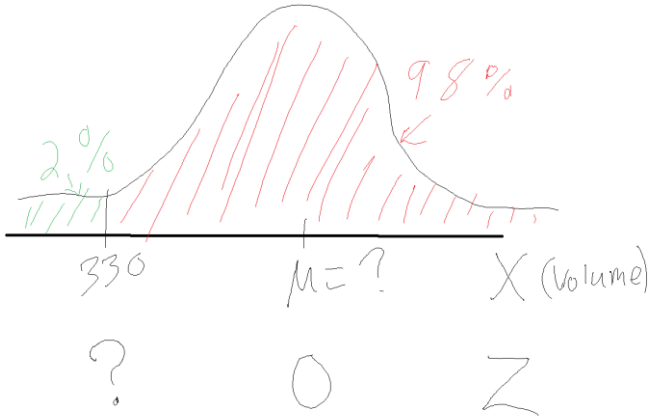
Take Home Assignment 4

Continuous Probability Distributions

(a) If the company wants 98% of all its cans of tonic water to have *at least* 330 ml as printed on the container, what **mean amount** should the filling machine be set to put in each can?

If 98% must be at least 330, then 2% can be below. The Z score for a left tail of 2% is -2.0537.

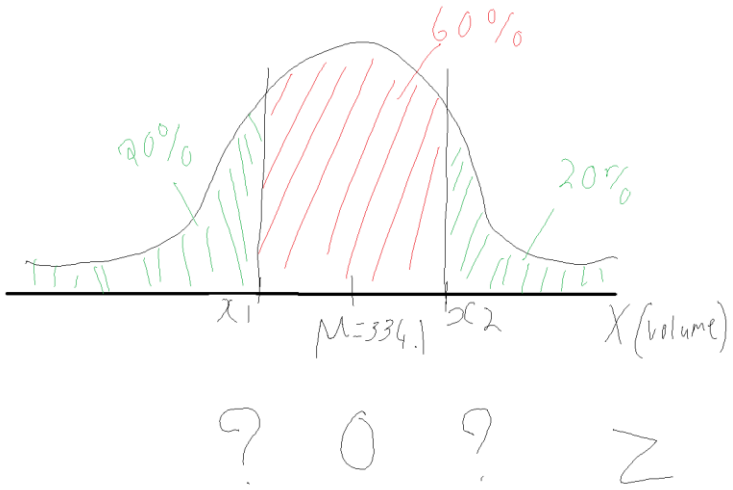


$$Z = \frac{x - \mu}{\sigma}$$
$$Z\sigma = x - \mu$$
$$Z\sigma - x = -\mu$$
$$x - Z\sigma = \mu$$
$$330 - -2.0537 * 2 = \mu$$
$$\mu = 334.1074$$

Therefore, the mean amount the machine should be set to is 334.1074ml to have 98% cans filled to at least 330ml.

(b) Using the mean obtained in part (a), find out

(i) the interval of the middle 60% of cans.



With a middle of 60%, that means there is a tail of 20% either side. A 20% tail on the right side has a Z score of 0.8416, which means the left side would have a score of -0.8416.

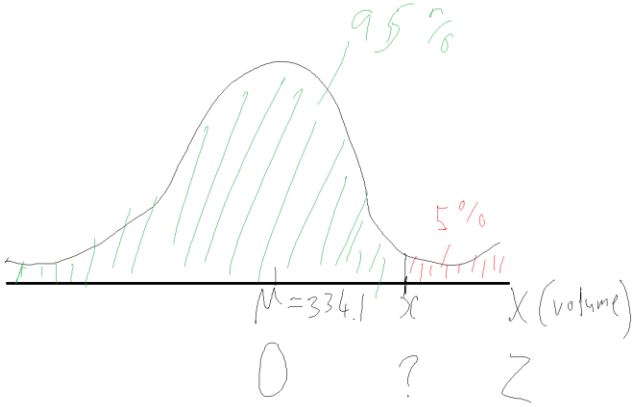
$$z_1 = -0.8416, z_2 = 0.8416$$

$$z_1 = \frac{x_1 - \mu}{\sigma}$$
$$z_1\sigma = x_1 - \mu$$
$$z_1\sigma + \mu = x_1$$
$$-0.8416 * 2 + 334.1074 = x_1$$
$$x_1 = 332.4242$$

$$z_2 = \frac{x_2 - \mu}{\sigma}$$
$$z_2\sigma = x_2 - \mu$$
$$z_2\sigma + \mu = x_2$$
$$0.8416 * 2 + 334.1074 = x_2$$
$$x_2 = 335.7906$$

Therefore, the interval of the middle 60% is 332.4242ml to 335.7906ml.

(ii) the minimum volume of the top 5% of cans.



A tail of 5% has a Z score of 1.6449.

$$z = \frac{x - \mu}{\sigma}$$
$$z\sigma = x - \mu$$
$$z\sigma + \mu = x_1$$
$$1.6449 * 2 + 334.1074 = x$$
$$x = 337.3972$$

Therefore, the minimum volume of the top 5% of cans is 337.3972ml