

## Take Home Assignment 5

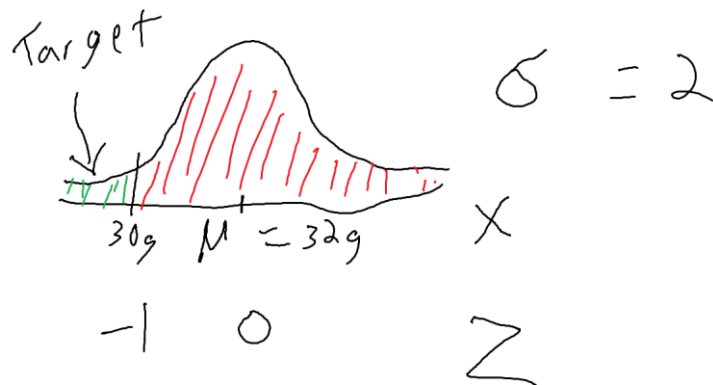
### Sampling Distributions

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- (a) You purchase one of these Party Buckets, empty the content and measure its weight (denoted  $X$ ). What is the chance that the weight ( $X$ ) is less than 30 oz?

$$\mu = 32, \sigma = 2$$



$$\begin{aligned} Z &= \frac{x - \mu}{\sigma} \\ Z &= \frac{30 - 32}{2} \\ Z &= \frac{-2}{2} \\ Z &= -1 \end{aligned}$$

$$Prob = P(X < 30g)$$

$$Prob = P(Z < -1)$$

$$Prob = 0.5 - P(-1 < Z < 0)$$

$$Prob = 0.5 - P(0 < Z < 1)$$

$$Prob = 0.5 - 0.3413$$

$$Prob = 0.1587$$

Therefore, there is a 15.87% chance that  $X$  is less than 30 oz.

(b) Suppose you purchase 4 buckets and measure their mean weight ( $\bar{X}$ ).

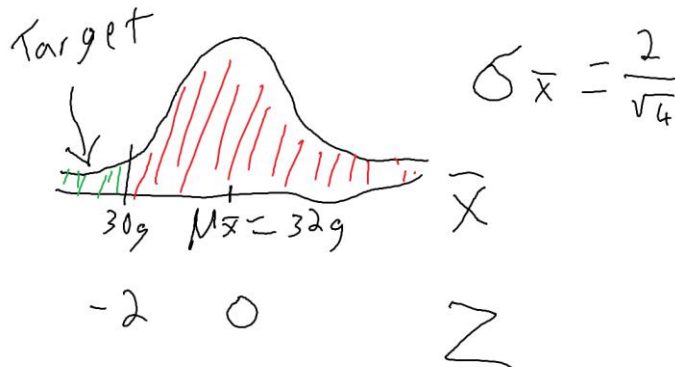
(i) Describe the probability distribution of the mean weight ( $\bar{X}$ ).

$$X: \mu = 32, \sigma = 2$$

$$\bar{X}: \mu_{\bar{X}} = \mu, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$\bar{X}: \mu_{\bar{X}} = 32, \sigma_{\bar{X}} = \frac{2}{\sqrt{4}} = 1$$

(ii) What is the probability the mean weight ( $\bar{X}$ ) is less than 30 oz?



$$Z = \frac{\bar{x} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

$$Z = \frac{30 - 32}{1}$$

$$Z = \frac{-2}{1}$$

$$Z = -2$$

$$Prob = P(X < 30g)$$

$$Prob = P(Z < -2)$$

$$Prob = 0.5 - P(-2 < Z < 0)$$

$$Prob = 0.5 - P(0 < Z < 2)$$

$$Prob = 0.5 - 0.4772$$

$$Prob = 0.0228$$

Therefore, there is a 2.28% chance that  $X$  is less than 30 oz.

(c) Now suppose you purchase 9 buckets for a huge party and measure the mean weight ( $\bar{X}$ ).

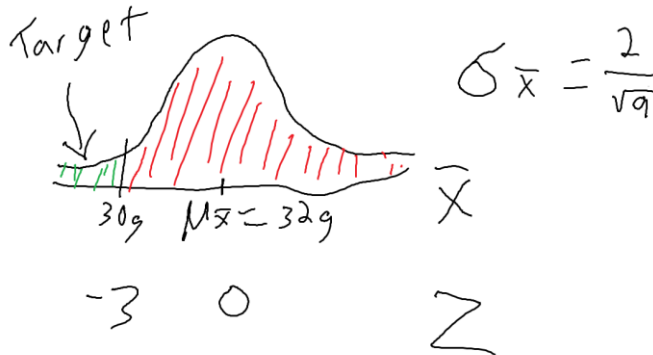
(i) Describe the probability distribution of the mean weight ( $\bar{X}$ ).

$$X: \mu = 32, \sigma = 2$$

$$\bar{X}: \mu_{\bar{X}} = \mu, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$\bar{X}: \mu_{\bar{X}} = 32, \sigma_{\bar{X}} = \frac{2}{\sqrt{9}} = 0.666$$

(ii) What is the probability the mean weight ( $\bar{X}$ ) is less than 30 oz?



$$Z = \frac{\bar{x} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

$$Z = \frac{30 - 32}{0.666}$$

$$Z = \frac{-2}{0.666}$$

$$Z = -3$$

$$Prob = P(X < 30g)$$

$$Prob = P(Z < -3)$$

$$Prob = 0.5 - P(-3 < Z < 0)$$

$$Prob = 0.5 - P(0 < Z < 3)$$

$$Prob = 0.5 - 0.4987$$

$$Prob = 0.0013$$

Therefore, there is a 0.13% chance that  $\bar{X}$  is less than 30 oz.

(d) Explain why your answer to (c)(ii) is different from (b)(ii).

As a sample size increases, the standard error from the mean decreases. This means that the probability of being close to the mean value would increase while, as seen in c(ii) and b(ii), the probability of being farther away from the mean value would decrease.